Dipole and quadrupole amplitudes and semi-inclusive observables at the LHC

Stéphane Munier

CPHT, École Polytechnique, CNRS

QCD at high density

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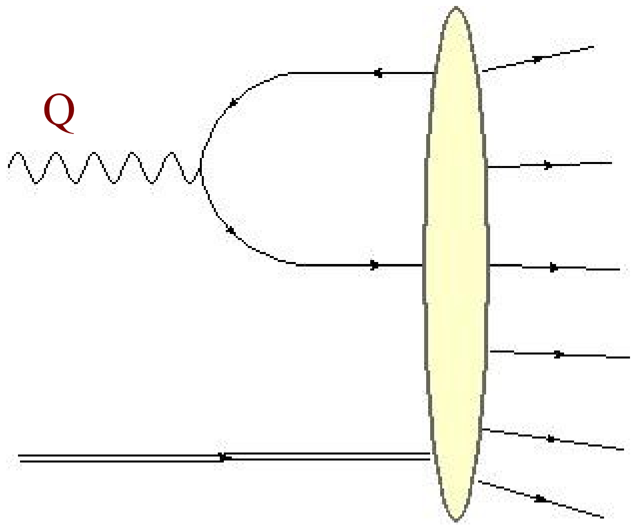
This regime is very interesting theoretically. Parton saturation may also have important phenomenological consequences at the LHC.

QCD at high density: How to test it?

At an ***electron-hadron collider:***



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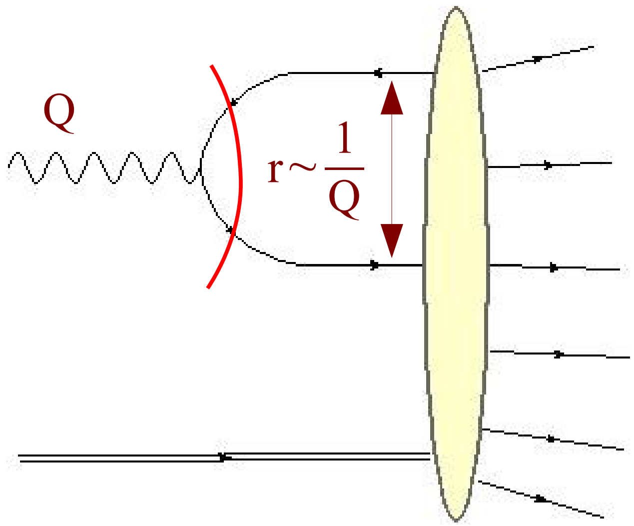
DIS can be understood as a dipole of "tunable" size r interacting with the target (p or A), that gets denser at higher energies.



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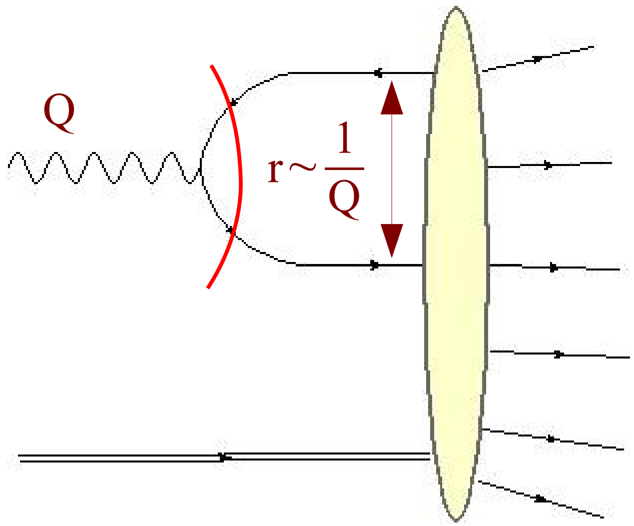

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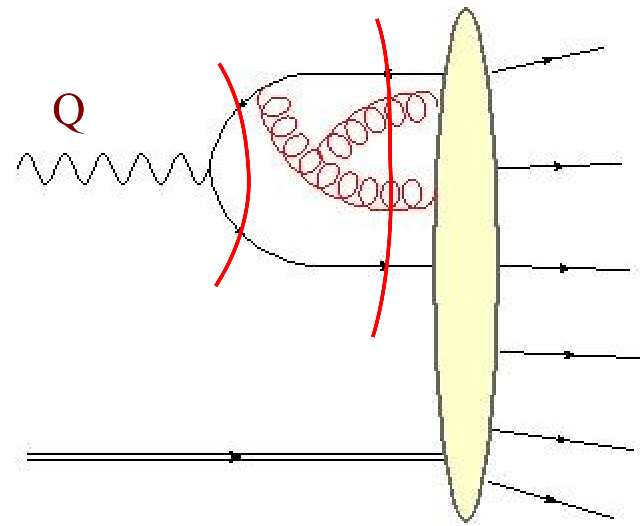


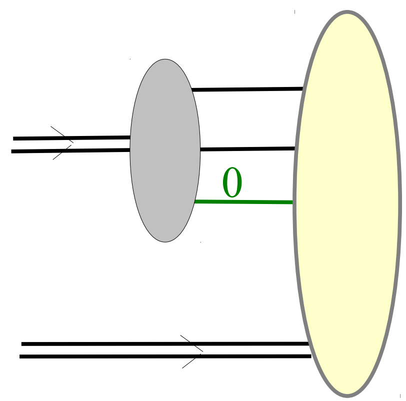
A lot of understanding of the dipole scattering amplitude was gained at HERA, at the border of the dense/saturation regime of QCD!

On the theoretical size, it is "easy" to formulate the QCD evolution of the dipole amplitude with the energy as ***radiative corrections to the dipole wave function***.

BFKL (at low density), BK, JIMWLK equations (accounting for high-density effects)

QCD at high density: How to test it?

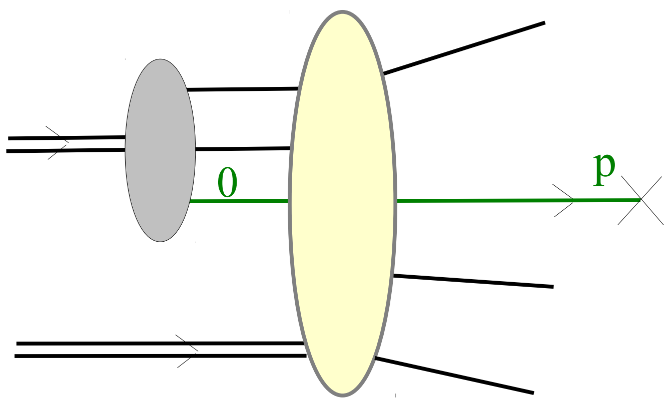
At a **hadron collider**, we need to find appropriate production processes:



QCD at high density: How to test it?

*At a **hadron collider**, we need to find appropriate production processes:*

- p_T *-broadening:*



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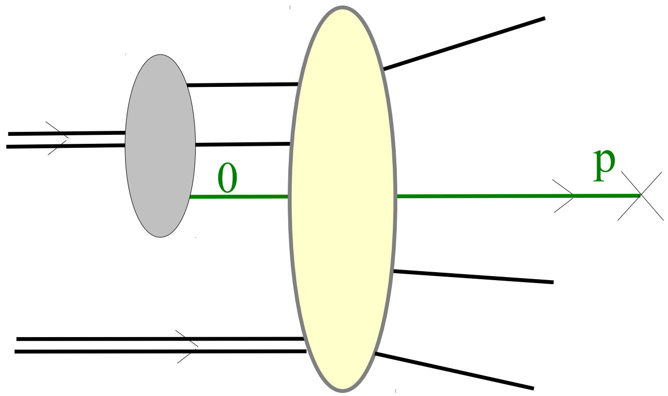

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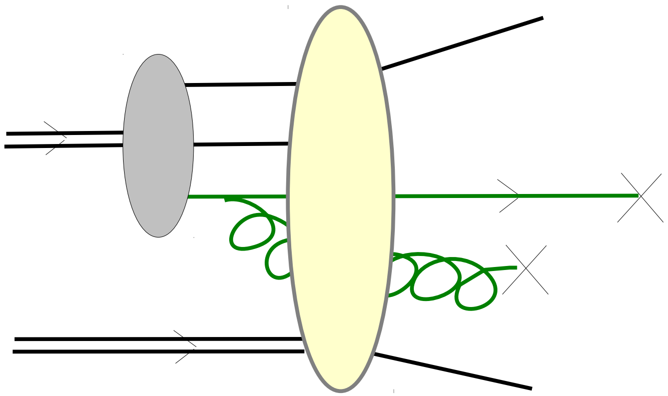
- Forward dijet azimuthal correlations:



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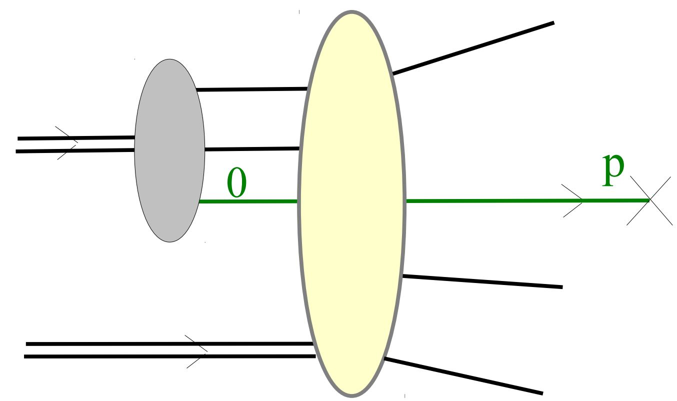

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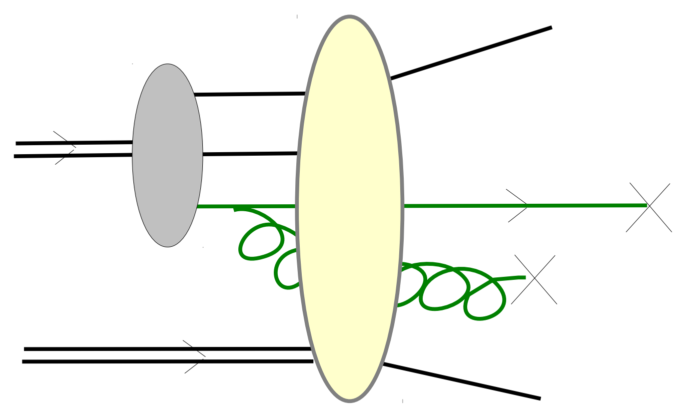

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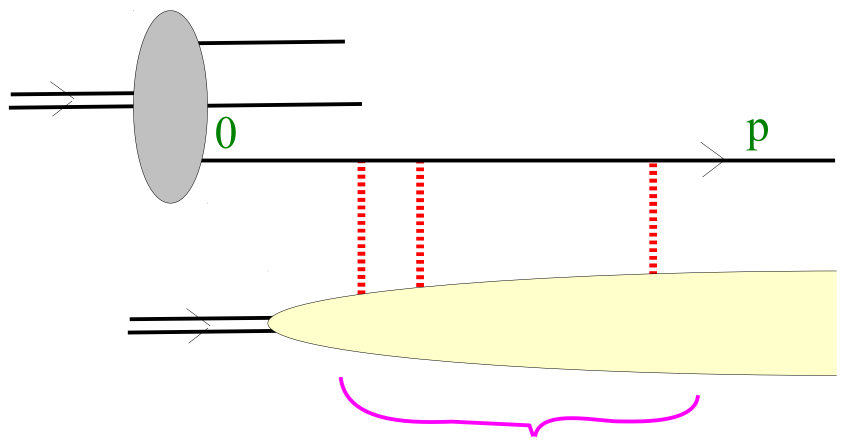
These observables are more tricky to formulate in QCD!

Outline

- Production processes in pA in terms of dipole/quadrupole amplitudes
- Robustness under quantum evolution
- Properties of the evolution

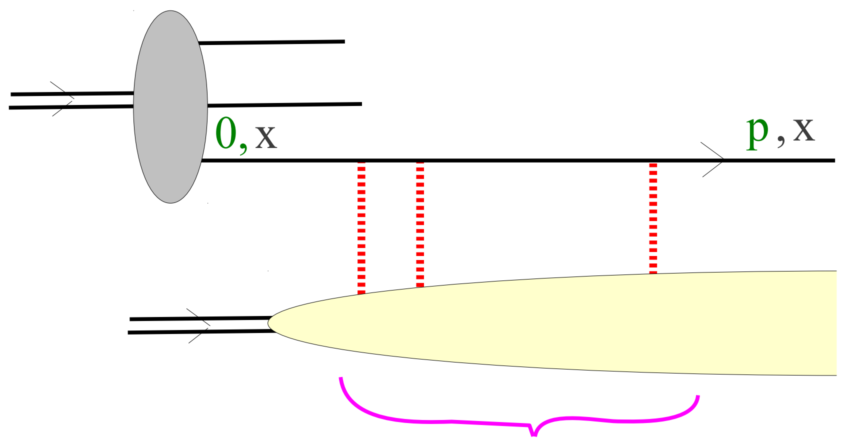
Formulation of p_T*-broadening*



time $t=-\infty$ $t=0$ $t=+\infty$

Initial state *Final state*

Formulation of p_T*-broadening*



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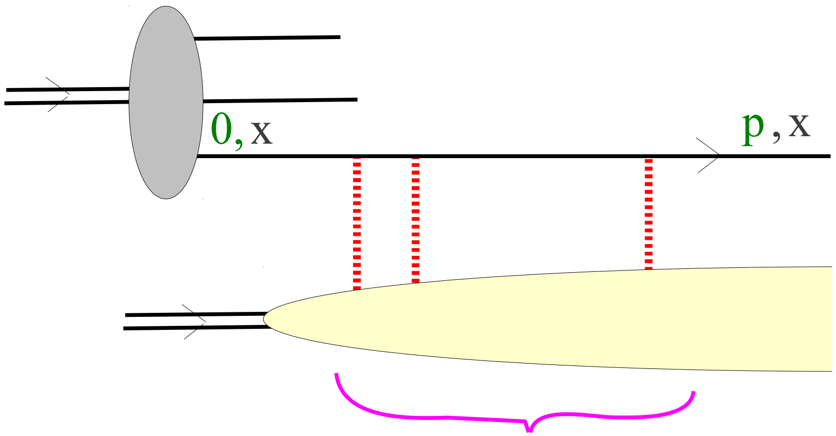

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Wilson line

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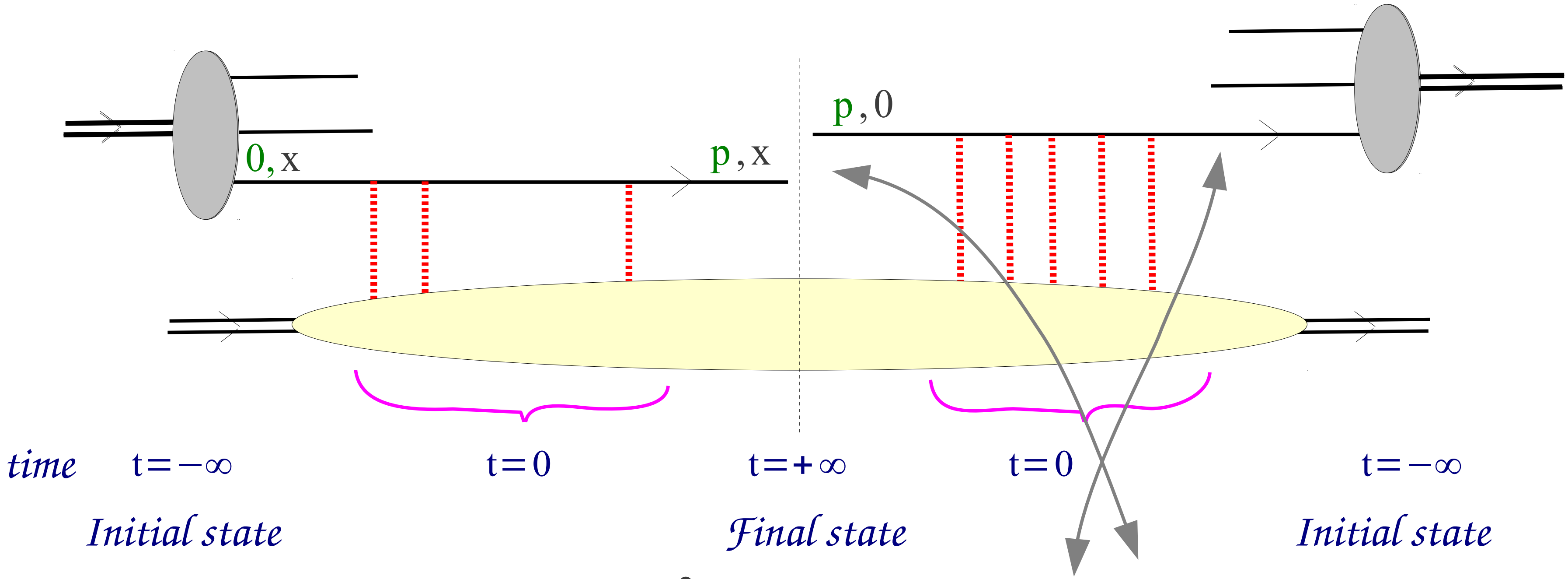
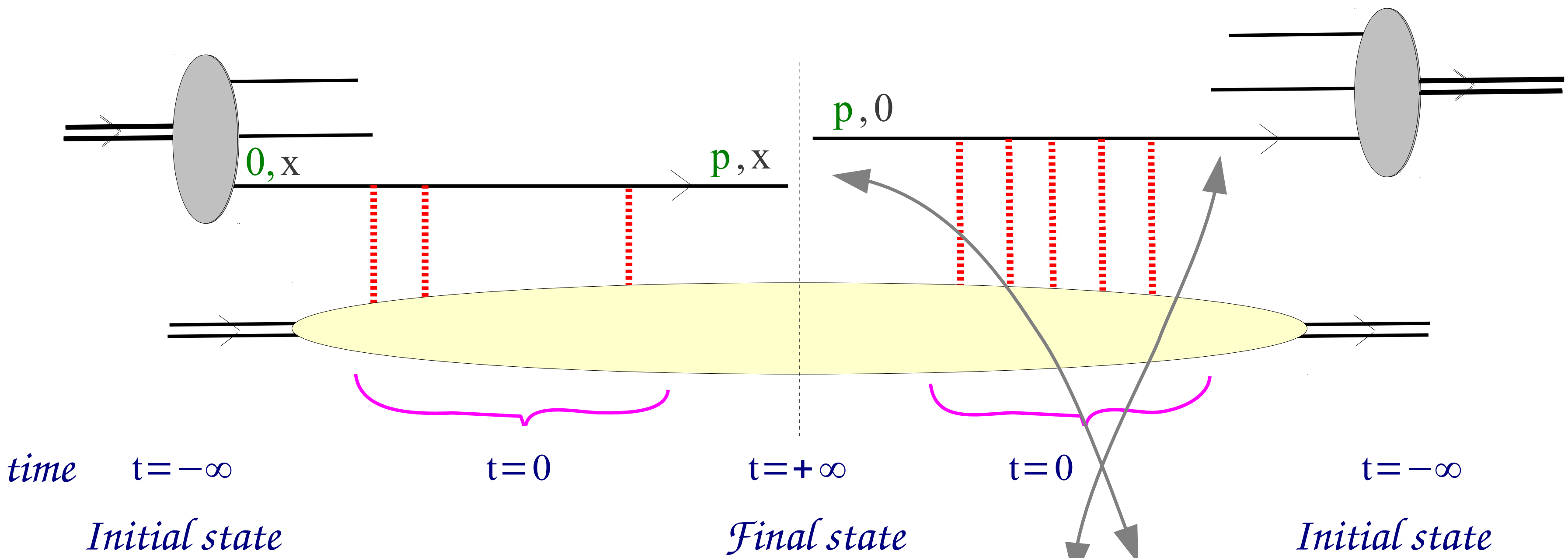


$$\frac{dN}{d^2p} \sim \int \frac{d^2x}{(2\pi)^2} e^{-ipx} \quad V_0^* V_x$$

Wilson lines

Formulation of p_T-broadening



$$\frac{dN}{d^2 p} = \int \frac{d^2 x}{(2\pi)^2} e^{-ipx} \left\langle \frac{1}{N_c} \mathrm{Tr}\, V_0^* V_x \right\rangle$$

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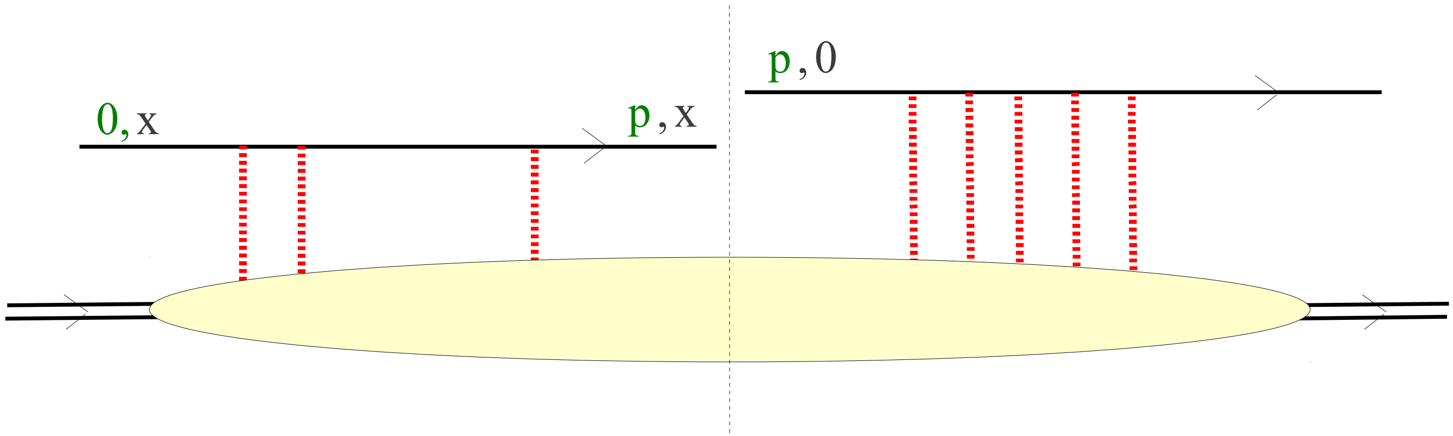

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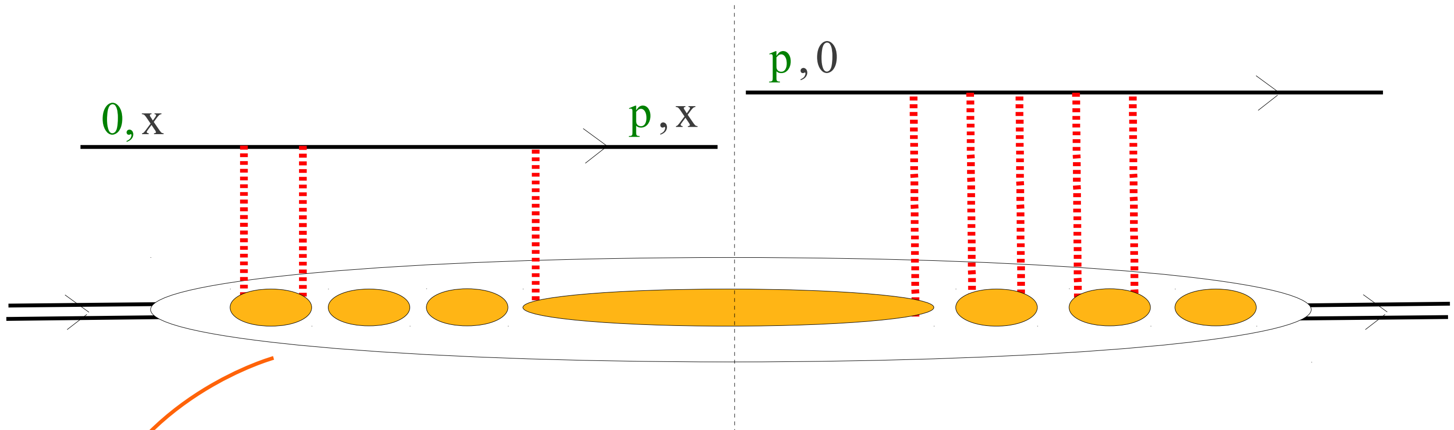

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McLerran-Venugopalan model
(assumes 2-gluon exchanges at most)

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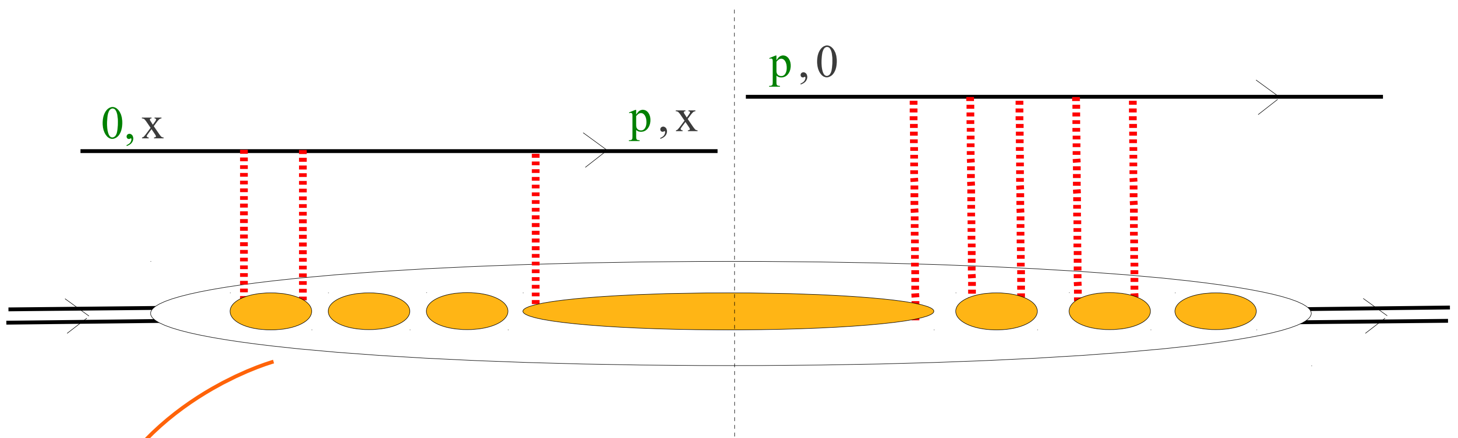

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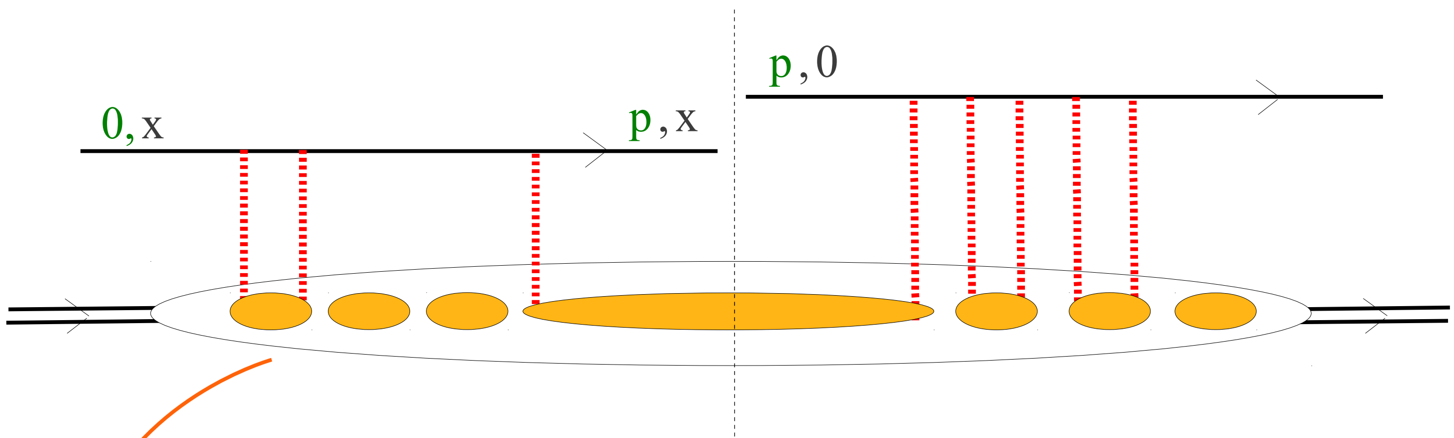
S-matrix element for the elastic scattering of a color dipole

Formulation of p_T-broadening



$$\frac{dN}{d^2 p} = \int \frac{d^2 x}{(2\pi)^2} e^{-i p x} \underbrace{S_{dipole}(x)}_{\text{S-matrix element for the elastic scattering of a color dipole}}$$

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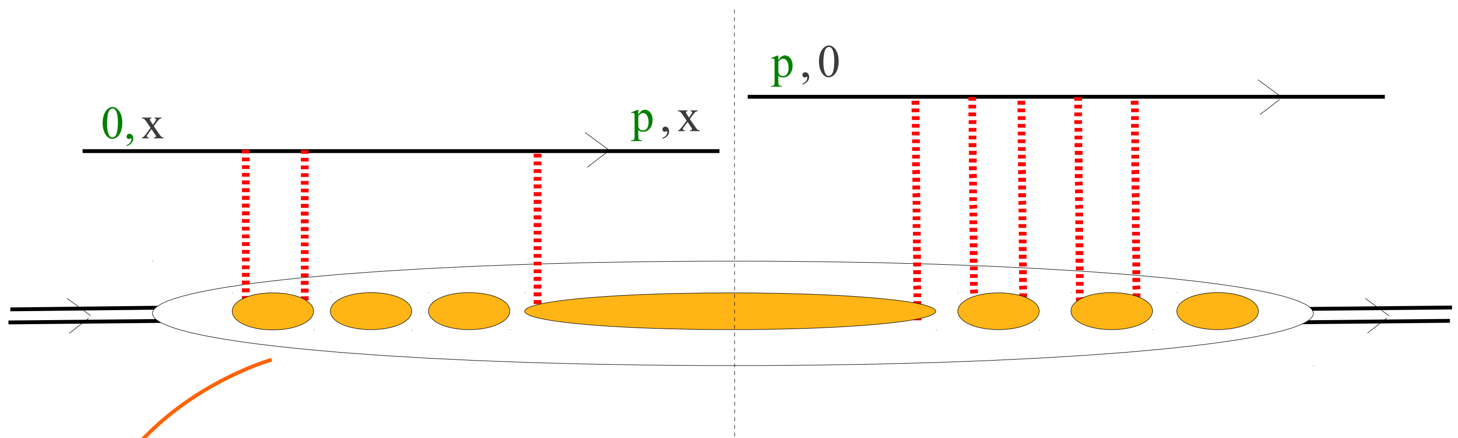


$$\frac{dN}{d^2 p} = \int \frac{d^2 x}{(2\pi)^2} e^{-ipx} \underbrace{S_{dipole}(x)}_{\text{S-matrix element for the elastic scattering of a color dipole}} \sim e^{-p^2/Q_s^2}$$

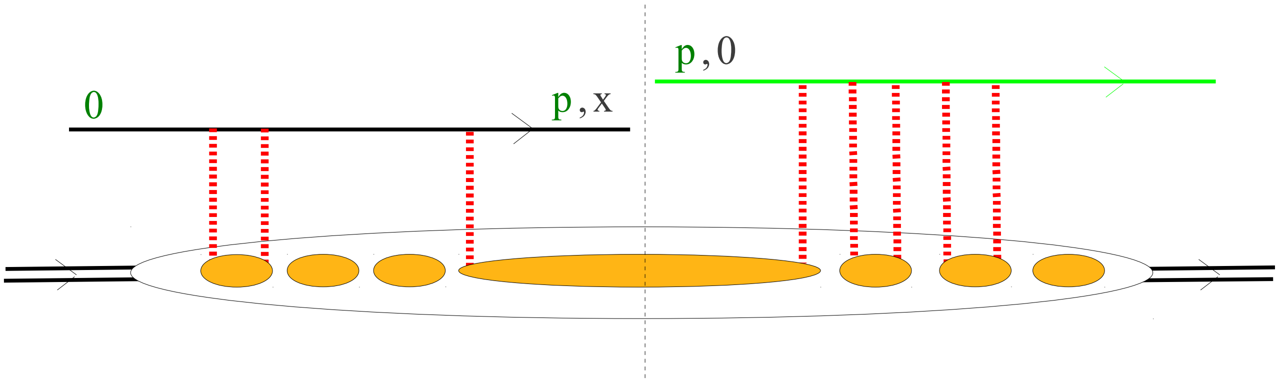
McLerran-Venugopalan model
(assumes 2-gluon exchanges at most)

S-matrix element for the elastic
scattering of a color dipole

$$S_{dipole}(x) = \exp\left(-\frac{x^2 Q_s^2}{4}\right)$$

Formulation of p_T*-broadening*

$$\frac{dN}{d^2 p} =$$



$$\frac{dN}{d^2 p} = \int \frac{d^2 x}{(2\pi)^2} e^{-i p x} S_{dipole}(x)$$

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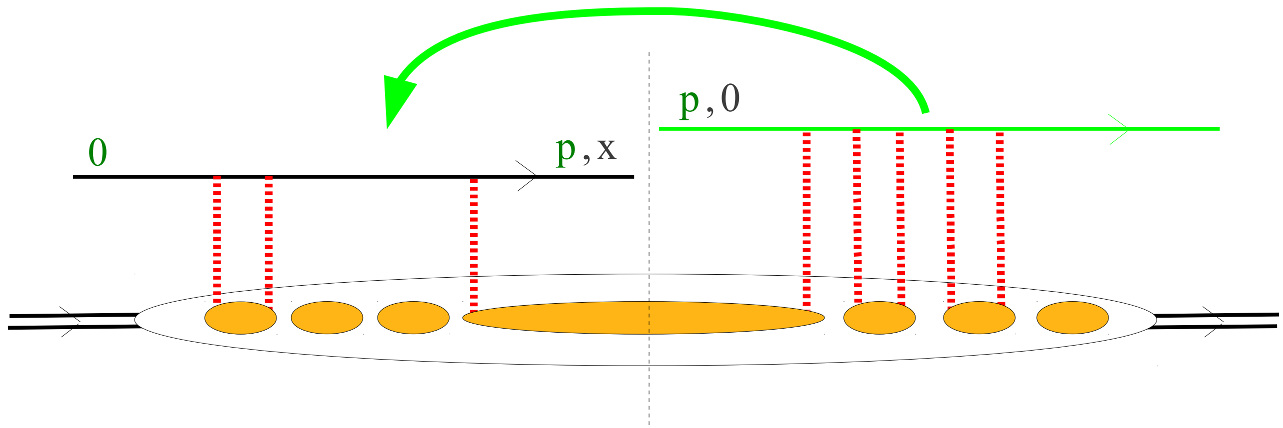

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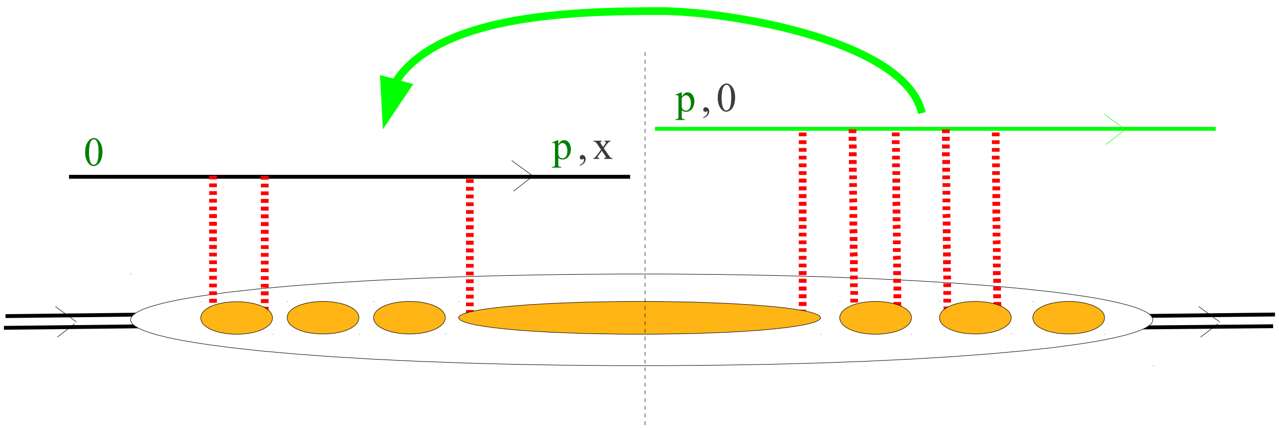

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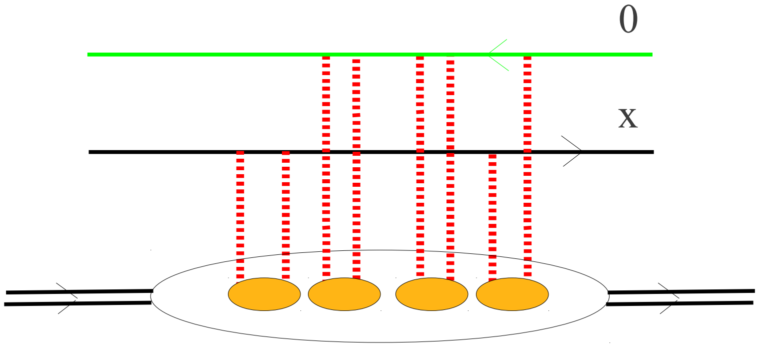

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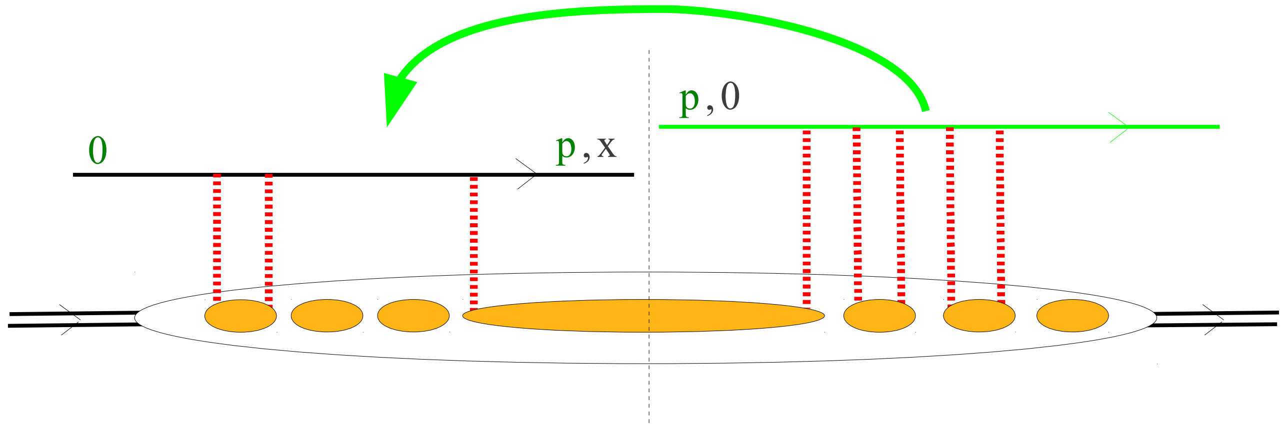

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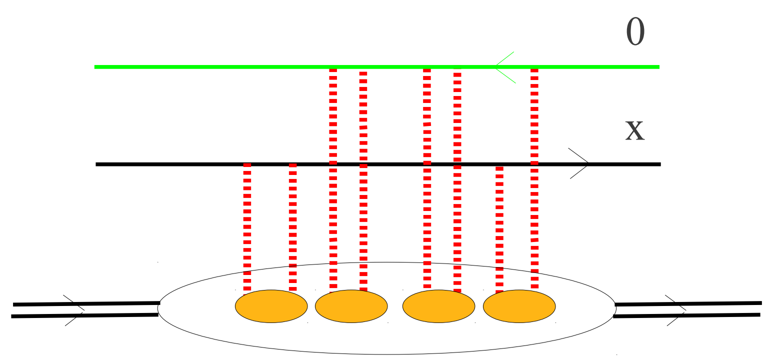
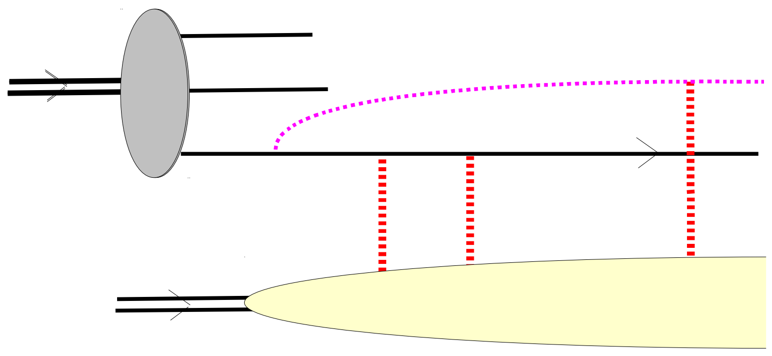


$$\frac{dN}{d^2p} = \int \frac{d^2x}{(2\pi)^2} e^{-ipx} S_{dipole}(x)$$

Intuitively: just bend the quark line in the complex conjugate amplitude to an antiquark line to transform it to a dipole amplitude!

Zakharov (1996...)

Formulation of dijet correlations



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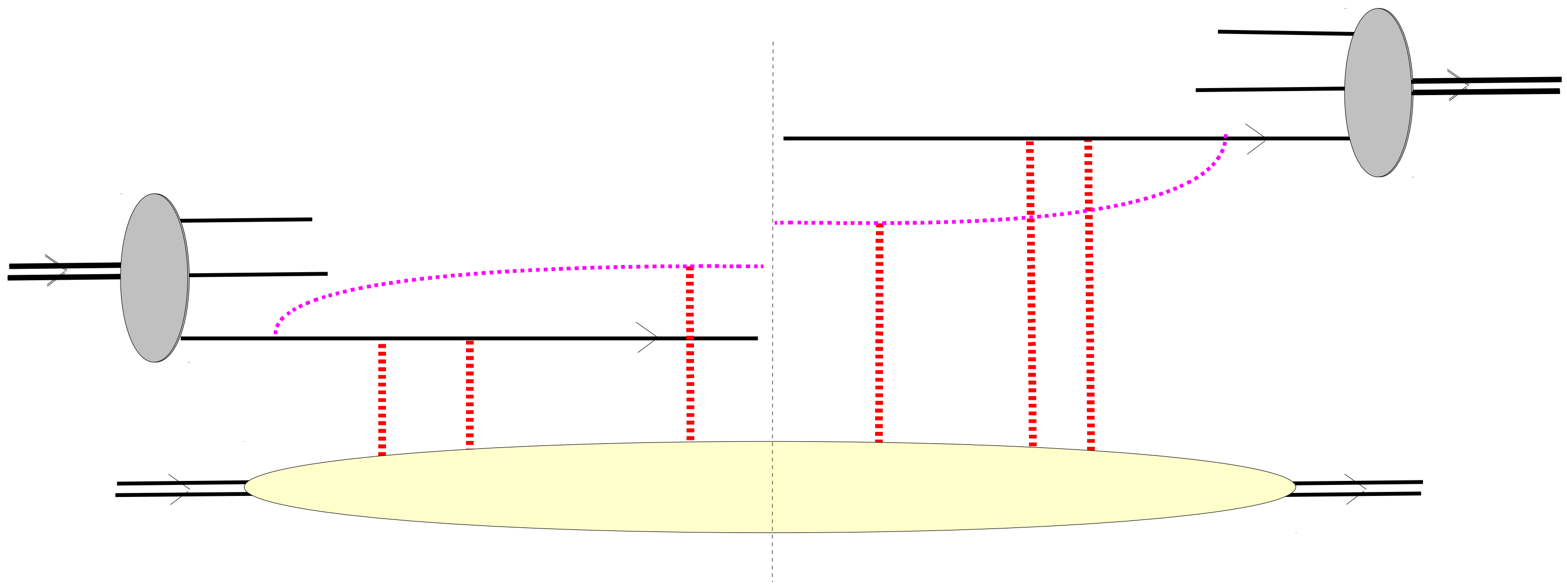

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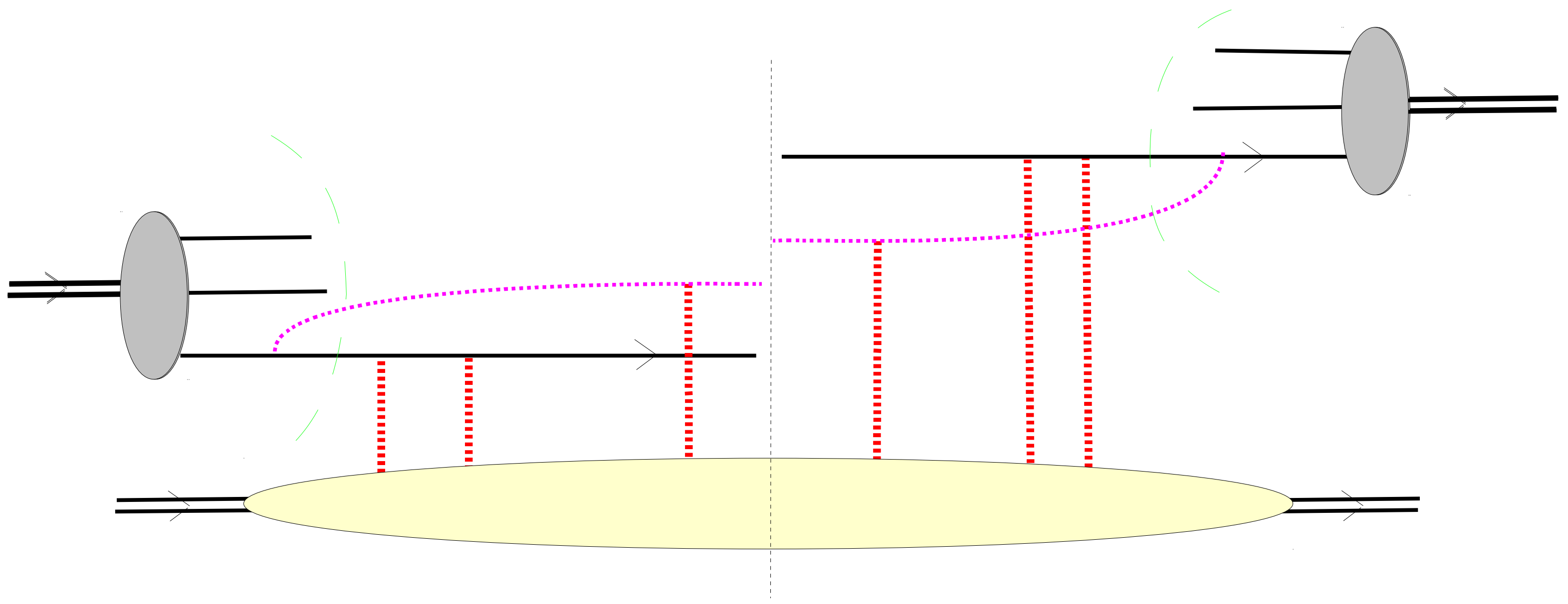

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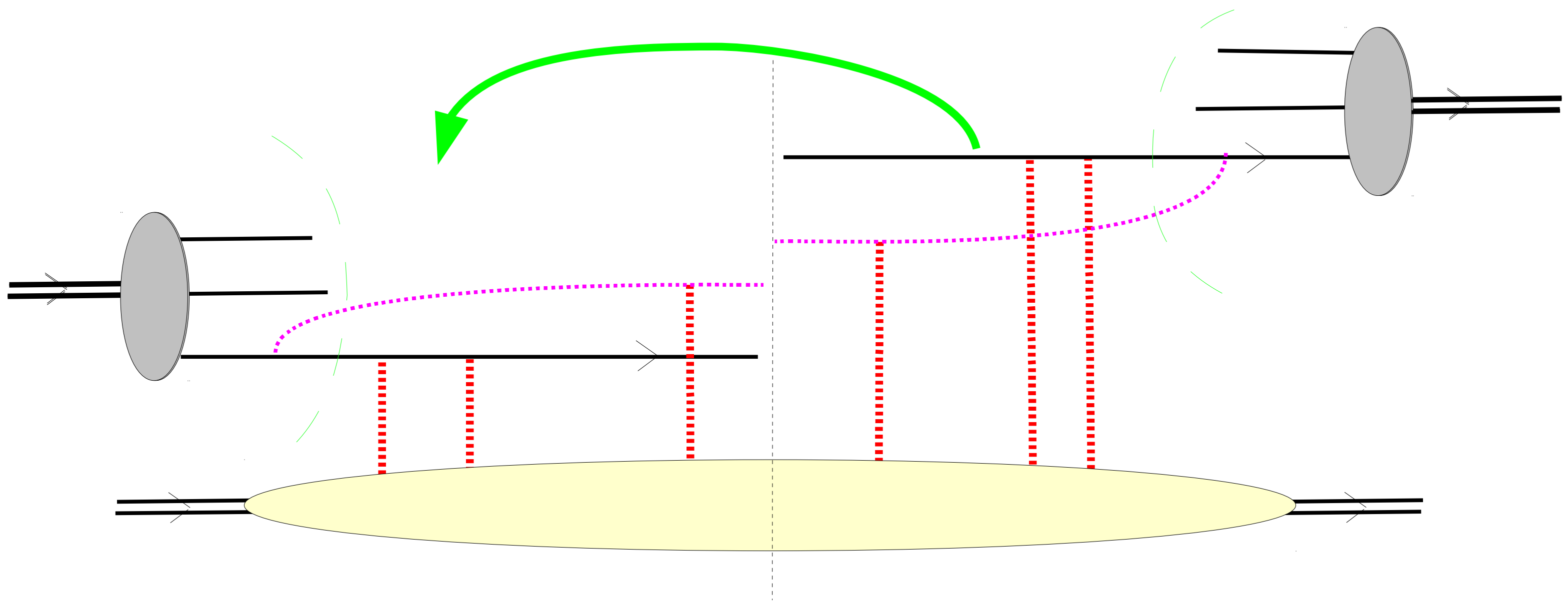

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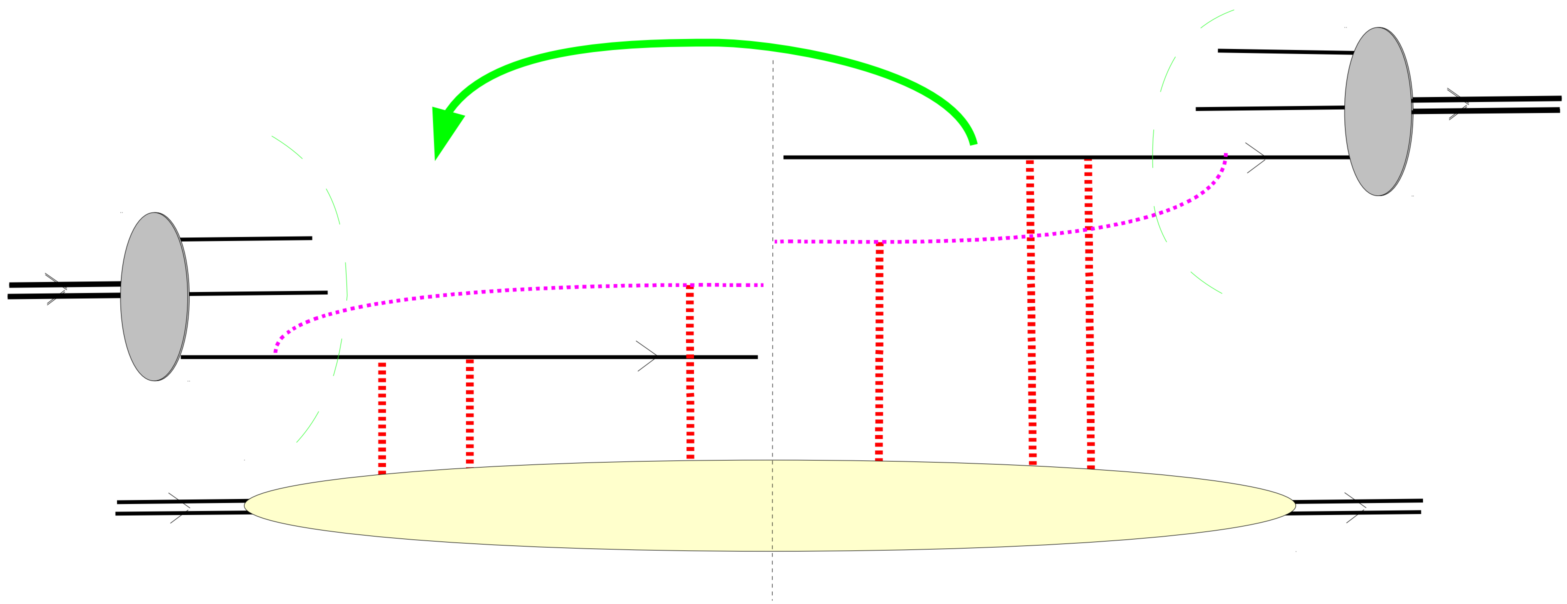

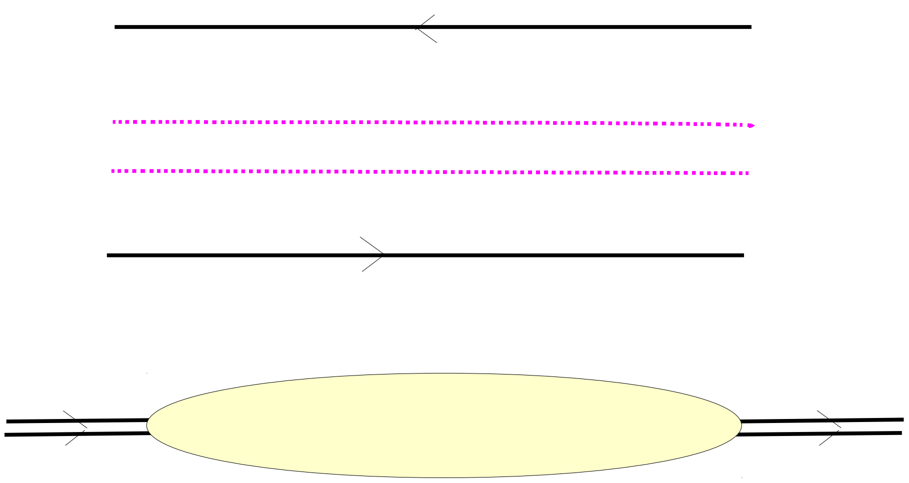

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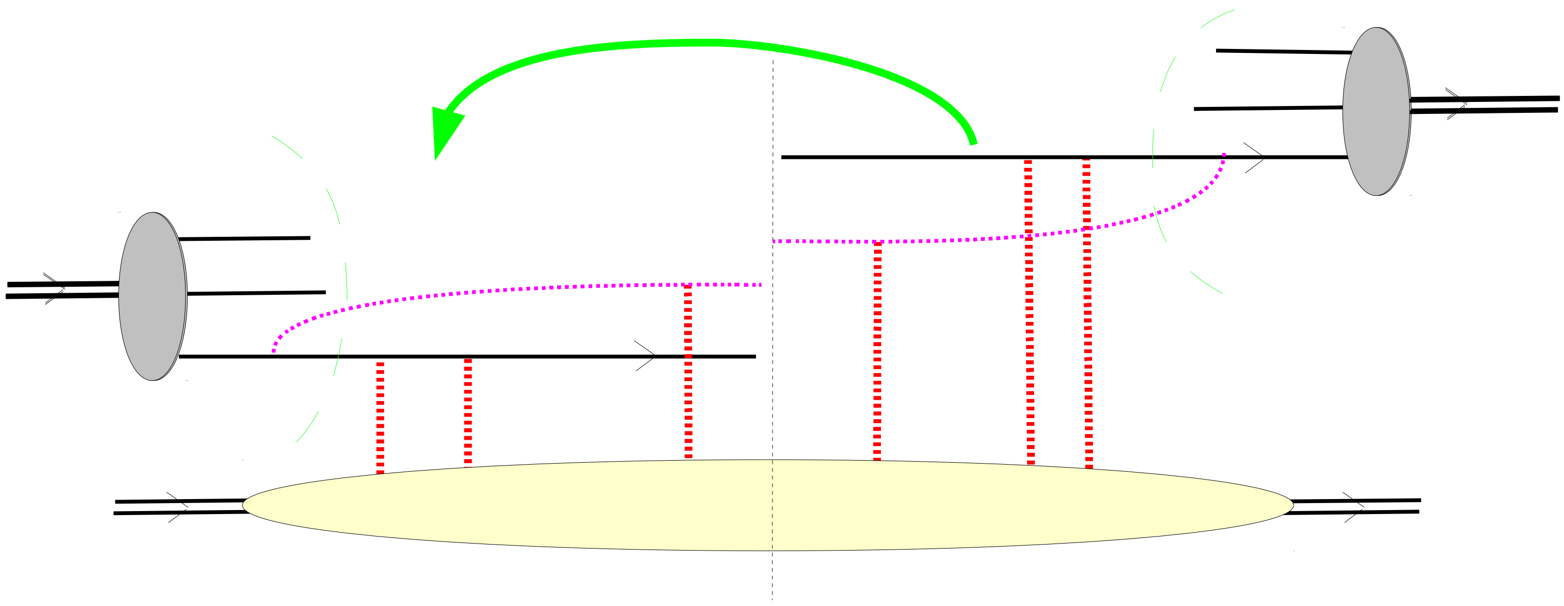
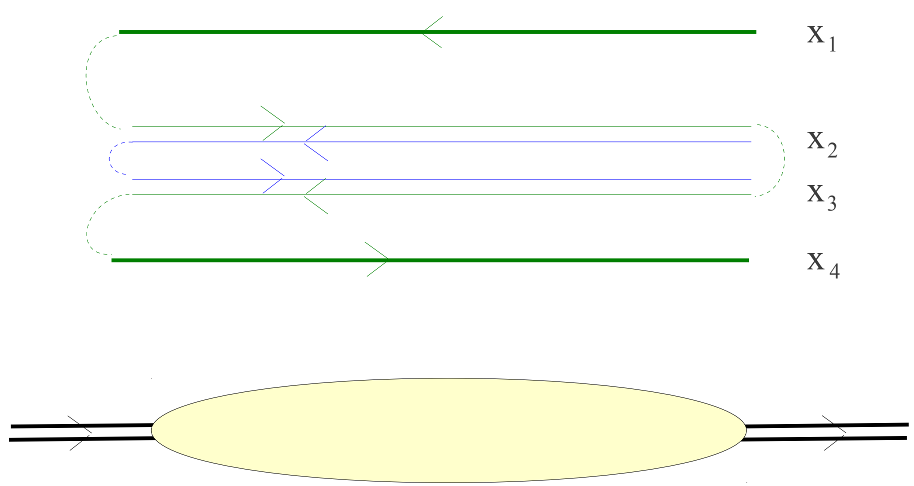


Large number-of-color limit



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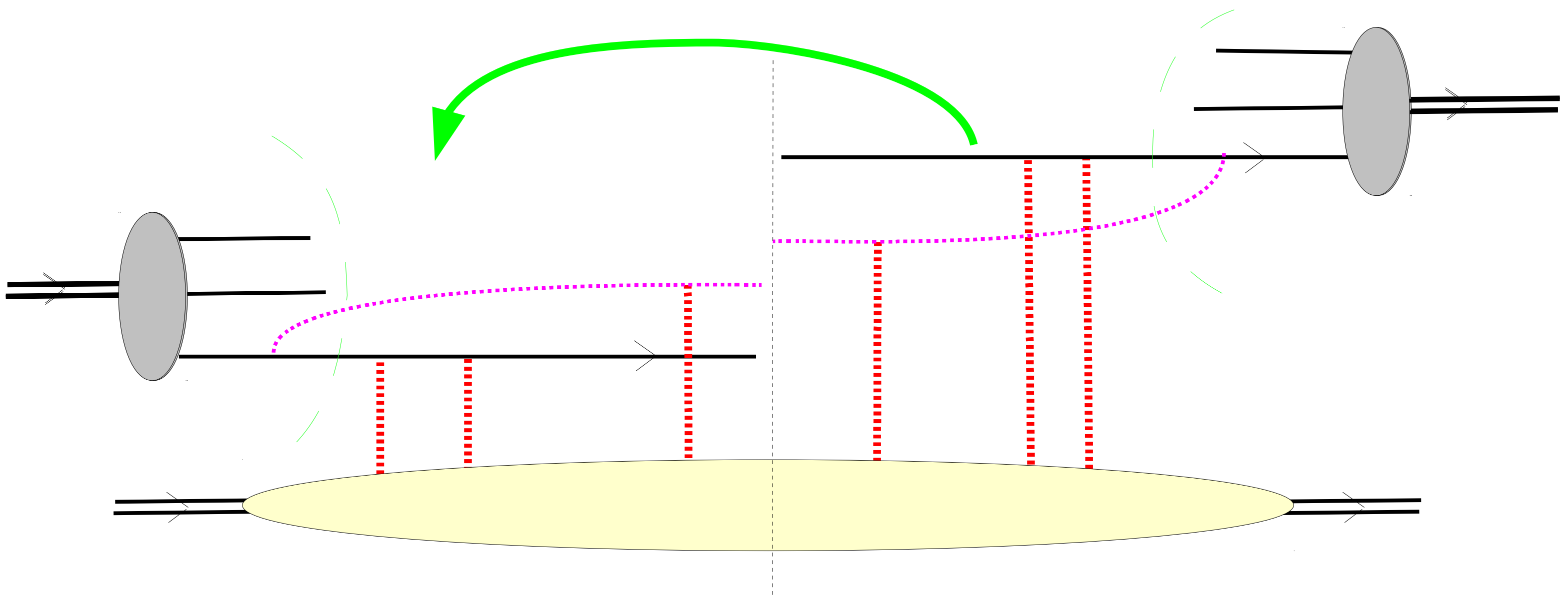

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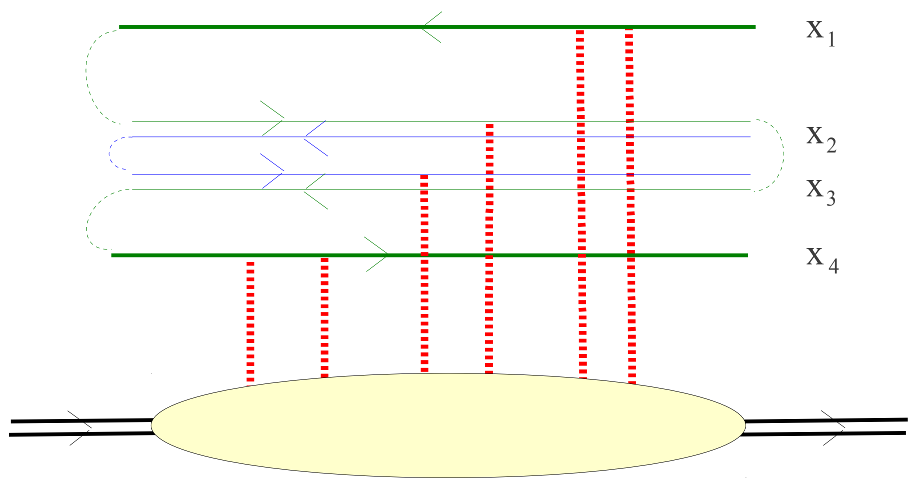

$$S \propto \left\langle \mathrm{Tr}\left(V_{x_2}^{*} V_{x_3} \right) \right\rangle$$

$$Q \propto \left\langle \mathrm{Tr}\left(V_{x_1}^{*} V_{x_2} V_{x_3}^{*} V_{x_4} \right) \right\rangle$$

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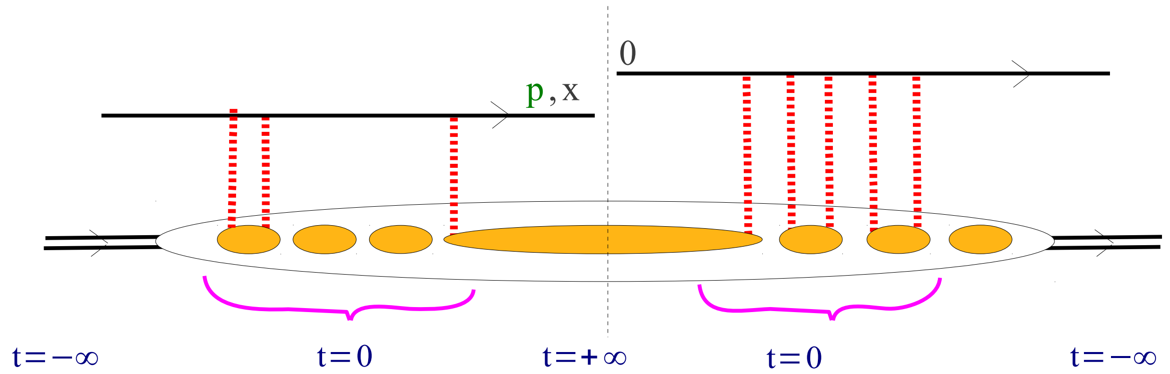
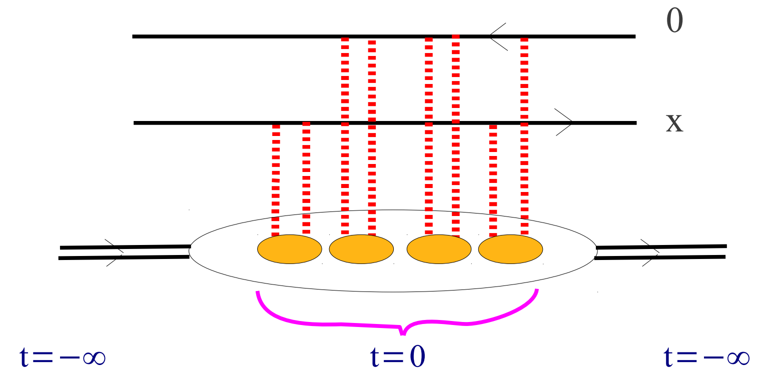
Quantum corrections



$$\frac{dN}{d^2p}=\int\frac{d^2x}{(2\pi)^2}e^{-ipx}\left\langle\frac{1}{N_c}\mathrm{Tr}\left(\quad V_0^*\qquad\qquad\right)\left(\quad V_x\qquad\qquad\right)\right\rangle$$



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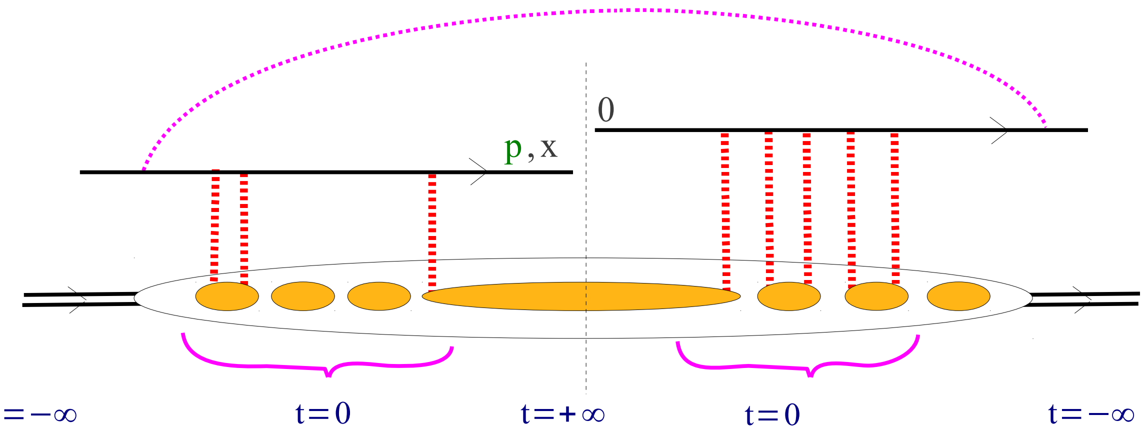

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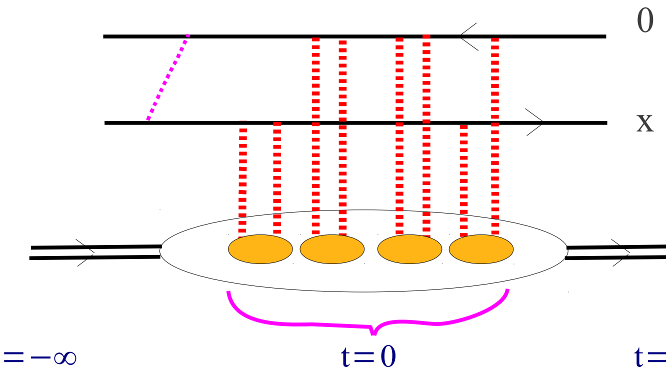

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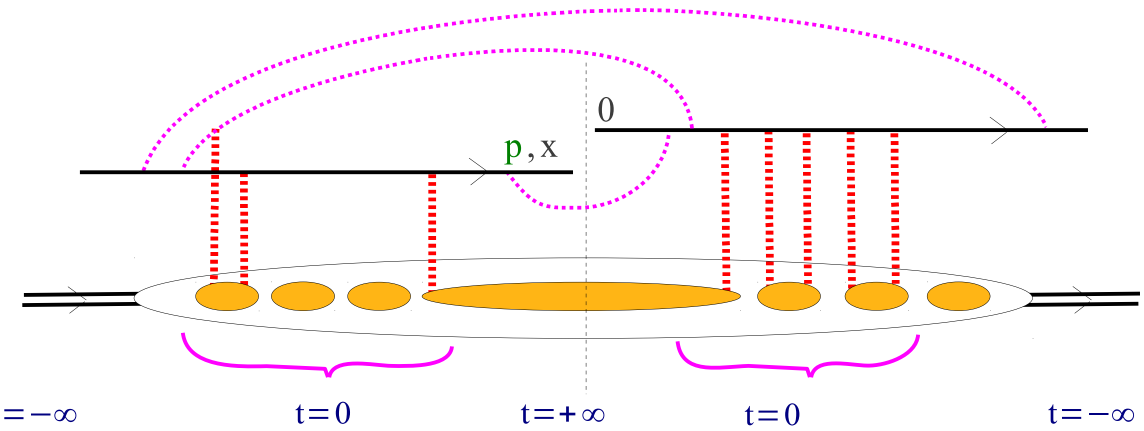

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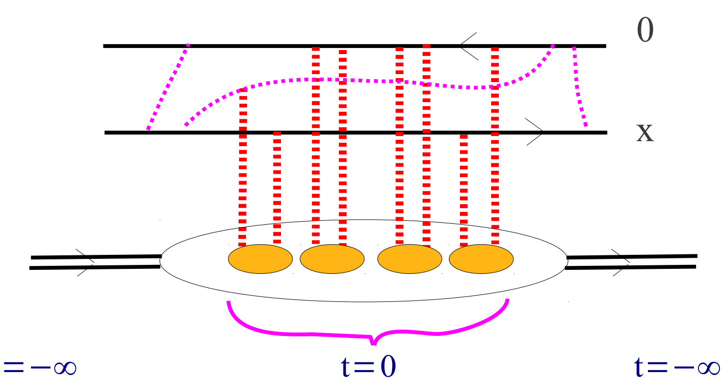

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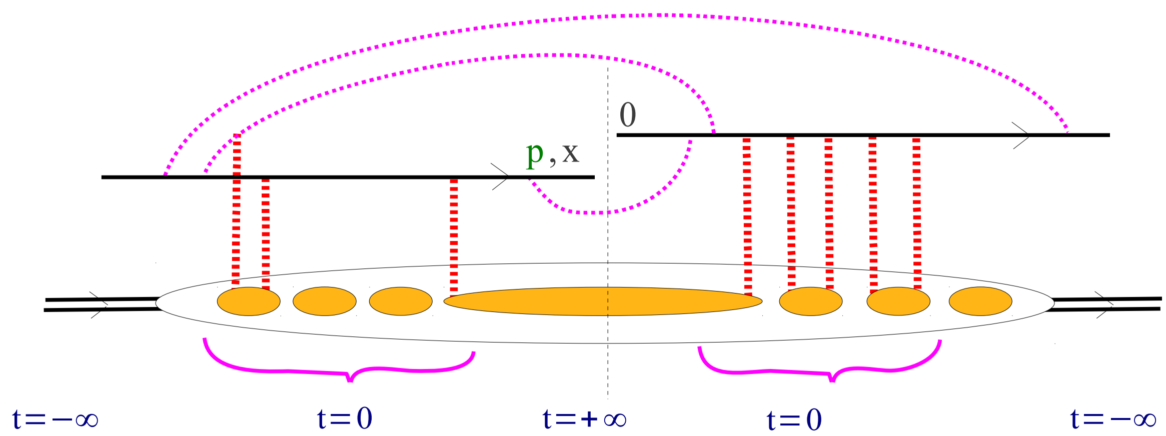
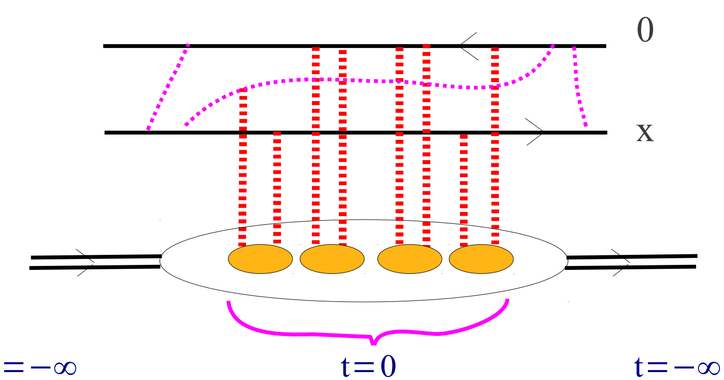


$$\frac{dN}{d^2 p} = \int \frac{d^2 x}{(2\pi)^2} e^{-ipx} \left\langle \frac{1}{N_c} \mathrm{Tr}\left(T\, V_0^* e^{-i\int d^4y\, L_I} \right) \left(T\, V_x\, e^{i\int d^4y\, L_I} \right) \right\rangle$$



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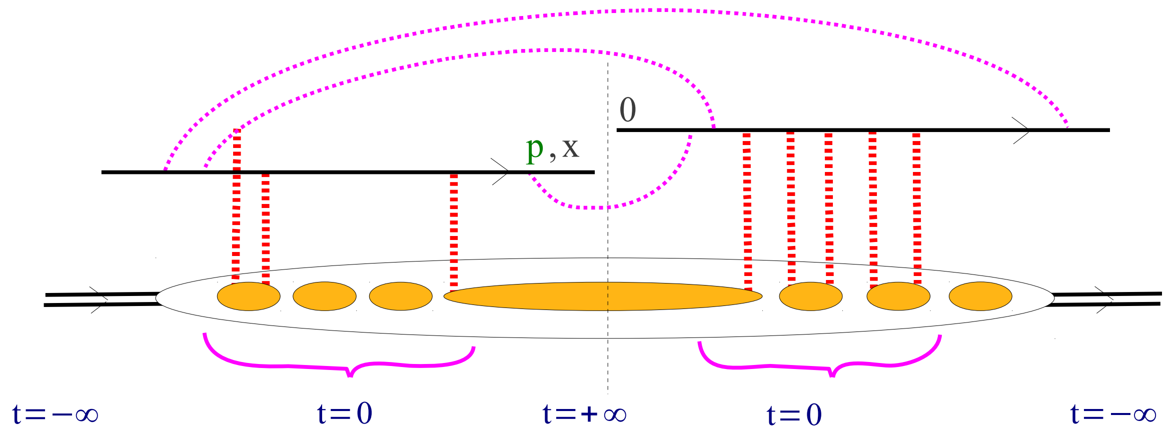

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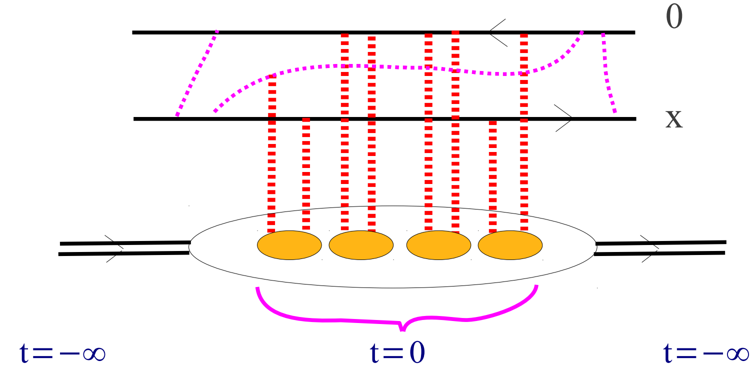

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?

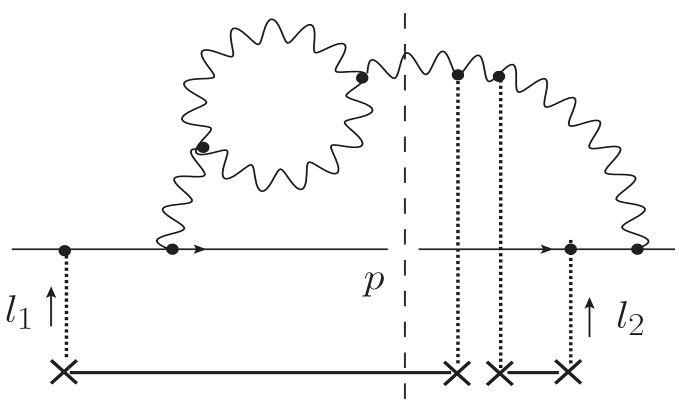
Quantum corrections: check at next-to-leading order

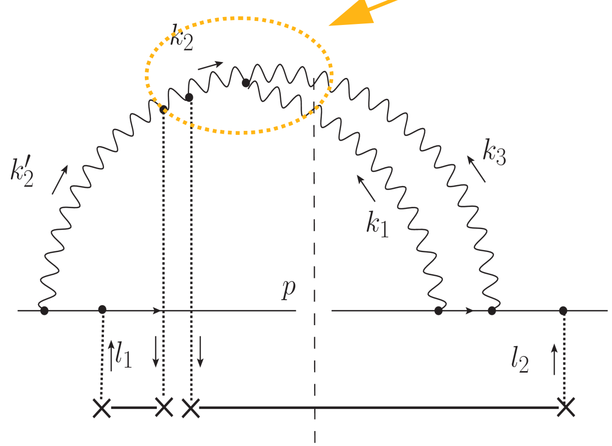
Hundreds of graphs on both *(broadening and dipole)* **sides!**

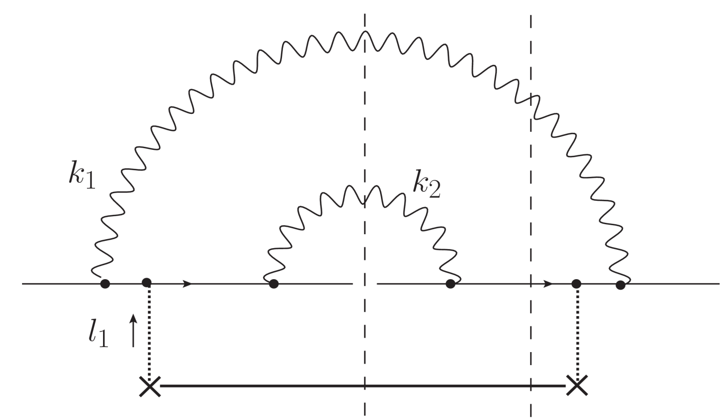
Mueller, Munier (2012)

May be grouped in 3 classes:

3-gluon vertex to compute exactly!







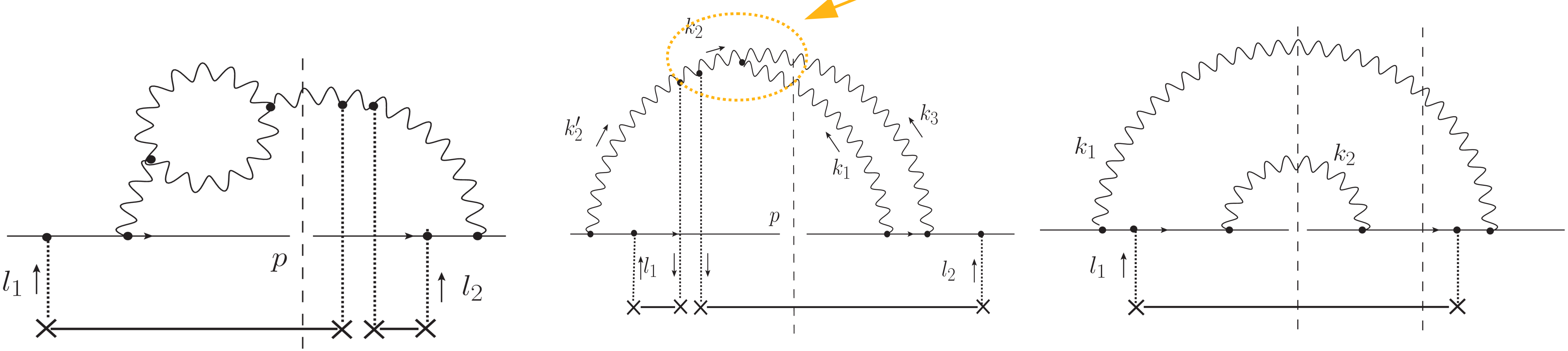
Quantum corrections: check at next-to-leading order

Hundreds of graphs on both** (broadening and dipole) **sides!

Mueller, Munier (2012)

May be grouped in 3 classes:

3-gluon vertex to compute exactly!

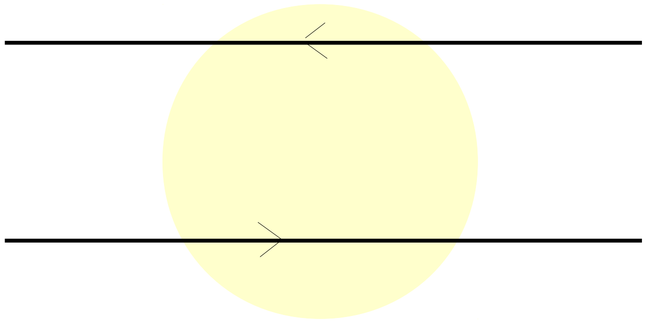


The correspondence between broadening and dipole scattering is preserved at NLO!
This statement is also true for the dijet/quadrupole correspondence.

Outline

- Production processes in pA in terms of dipole/quadrupole amplitudes
- Robustness under quantum evolution
- Properties of the evolution

Properties of the evolution – dipole amplitude



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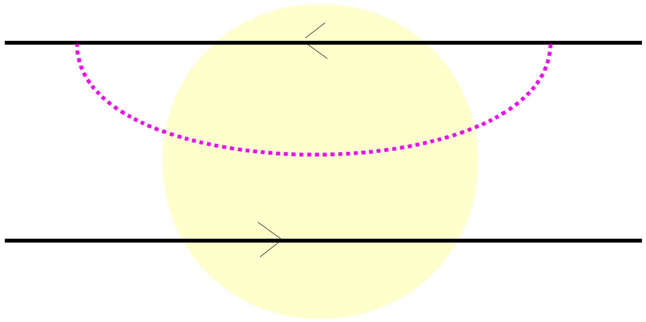
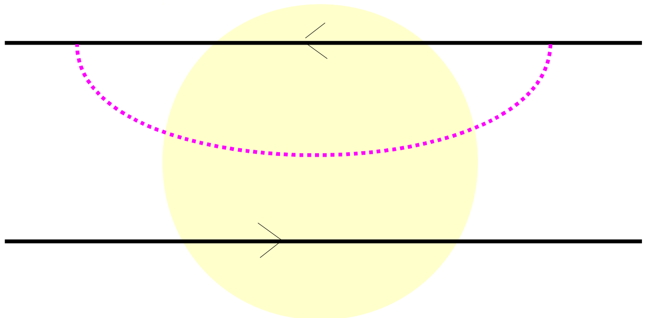


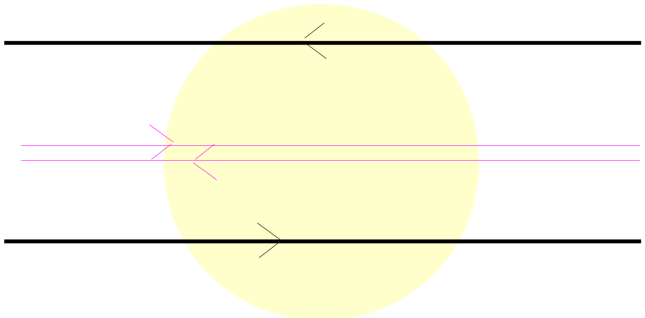
Properties of the evolution – dipole amplitude

Mueller (1993)

$$P_{d \to dd}$$

Properties of the evolution – dipole amplitude





Mueller (1993)

$$\frac{dS}{dy} = \int P_{d \rightarrow dd} \left(SS - S \right)$$

Balitsky-Kovchegov equation (1996-1999)

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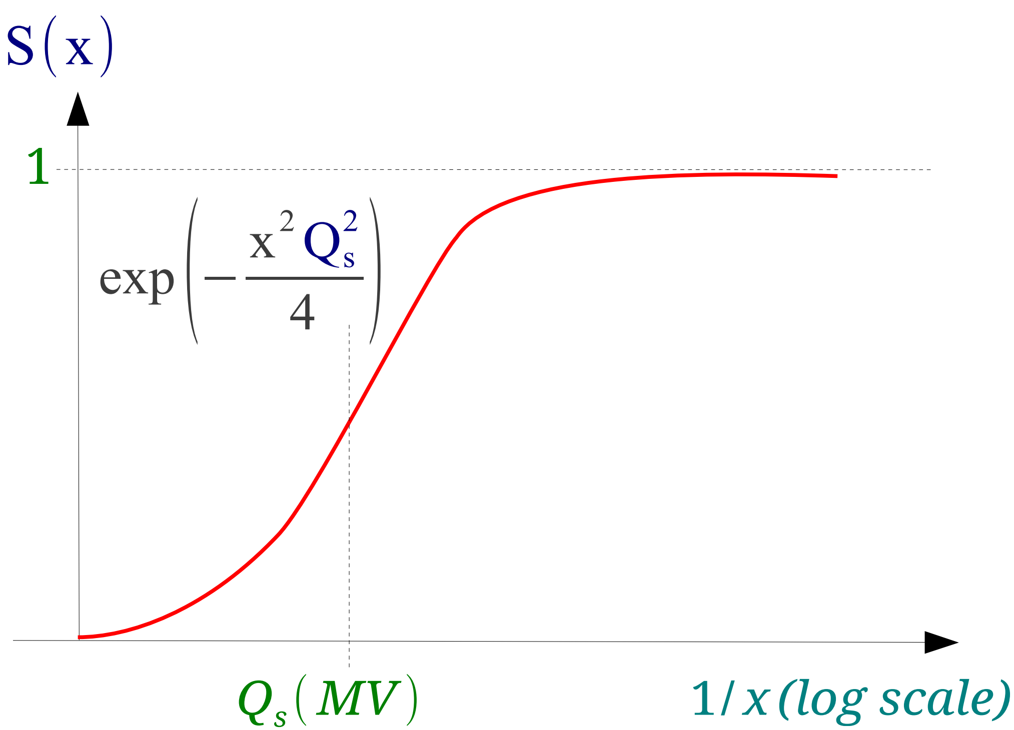

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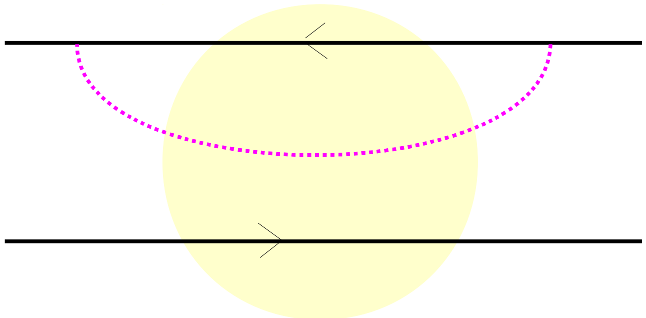

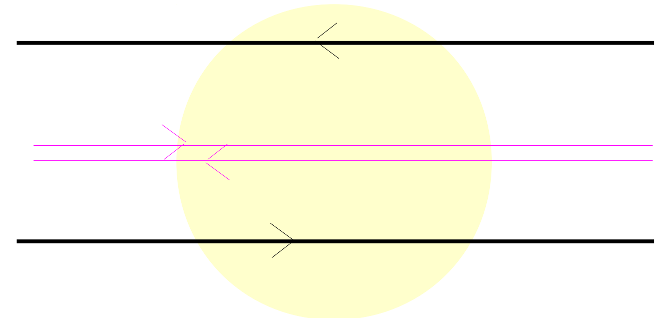

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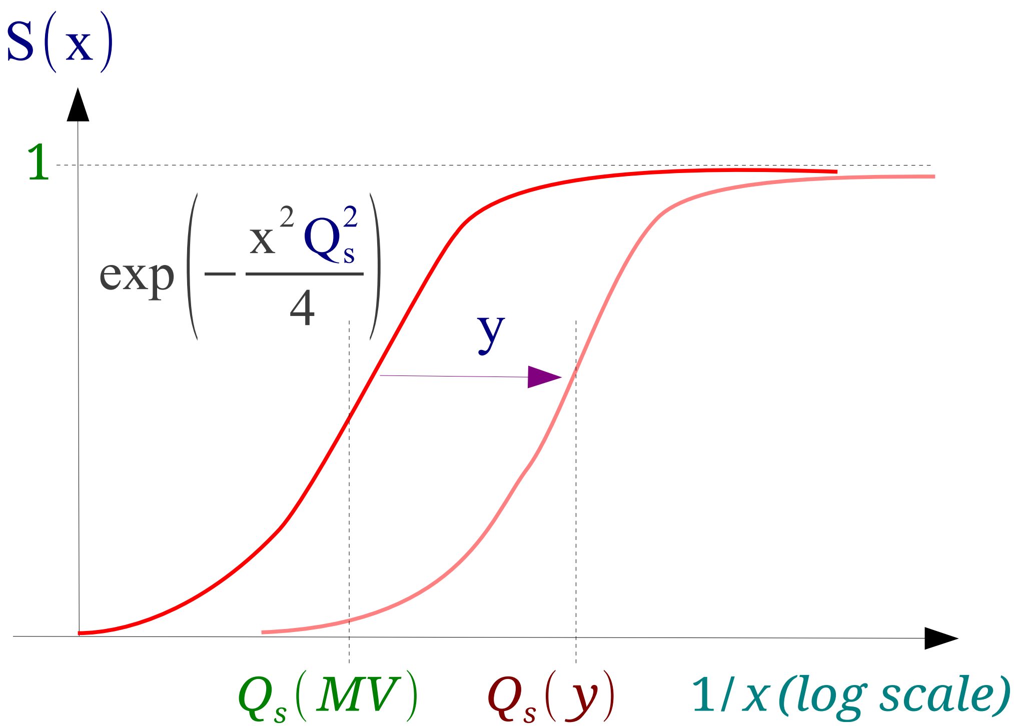
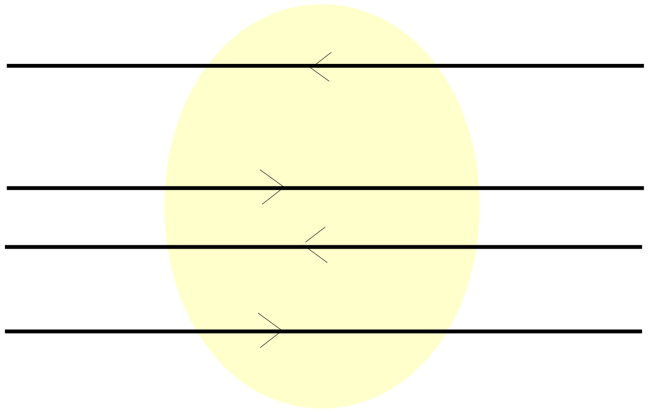


Solutions tend to traveling waves

at large rapidities

= geometric scaling

Properties of the evolution – quadrupole amplitude



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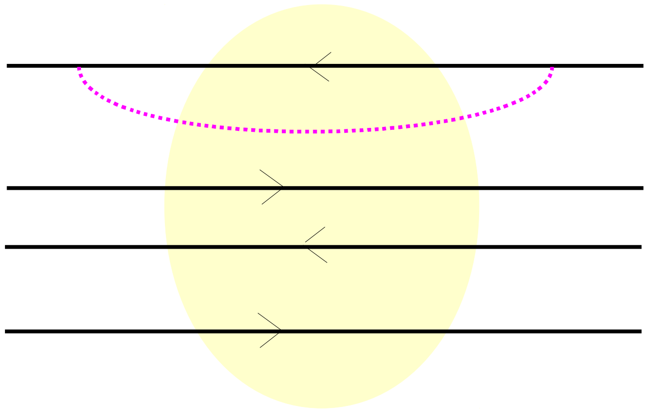

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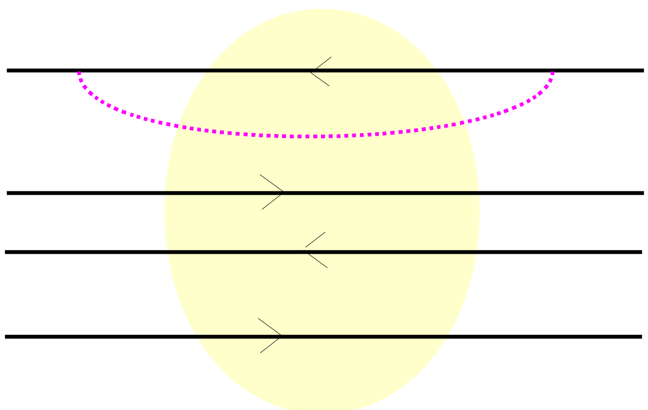

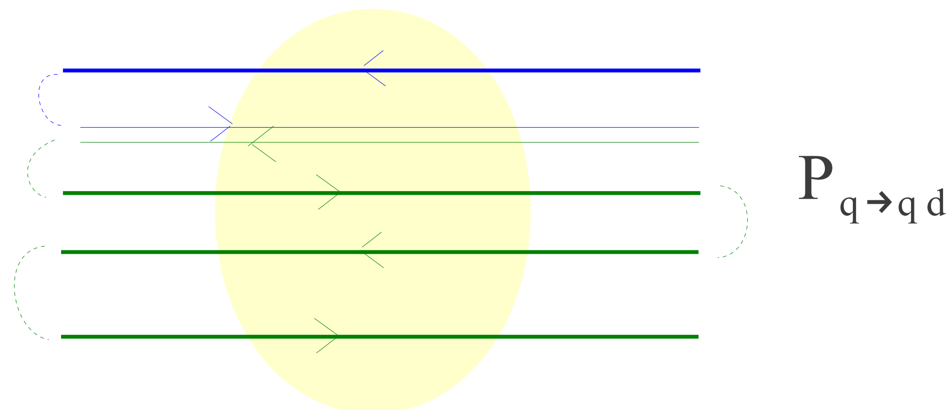

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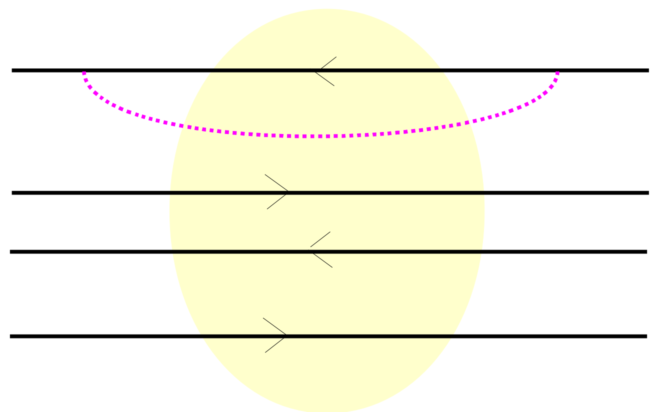
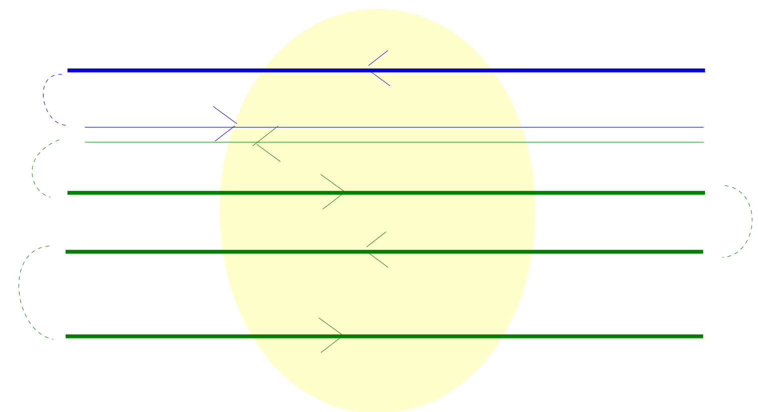
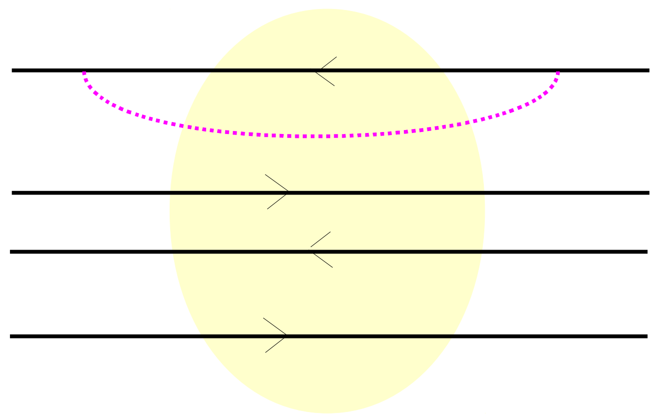


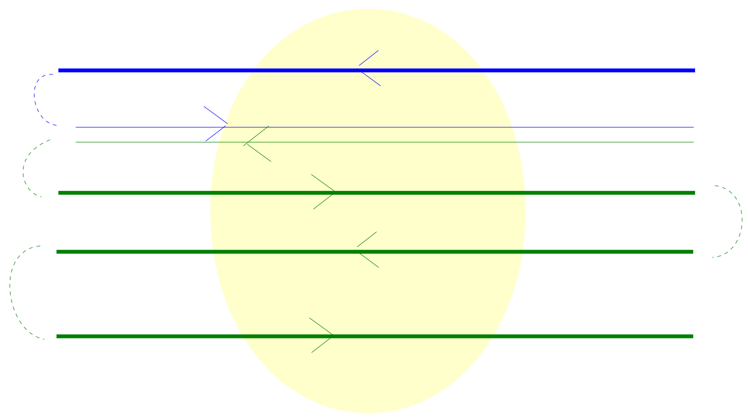


$$P_{q\to qd}$$

$$\frac{dQ}{dy} = \int P_{q\to q\,d}\left(QS - Q\right)$$

Properties of the evolution – quadrupole amplitude





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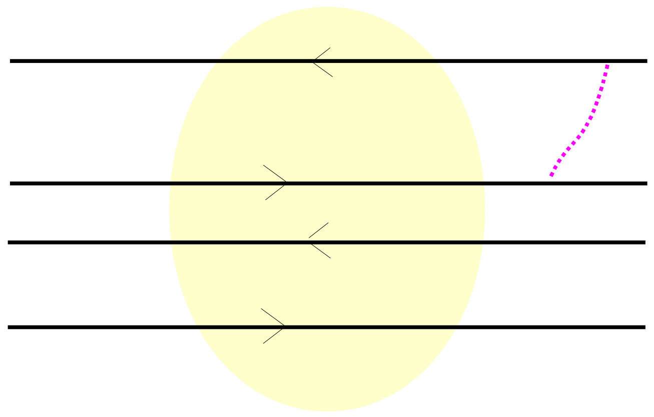

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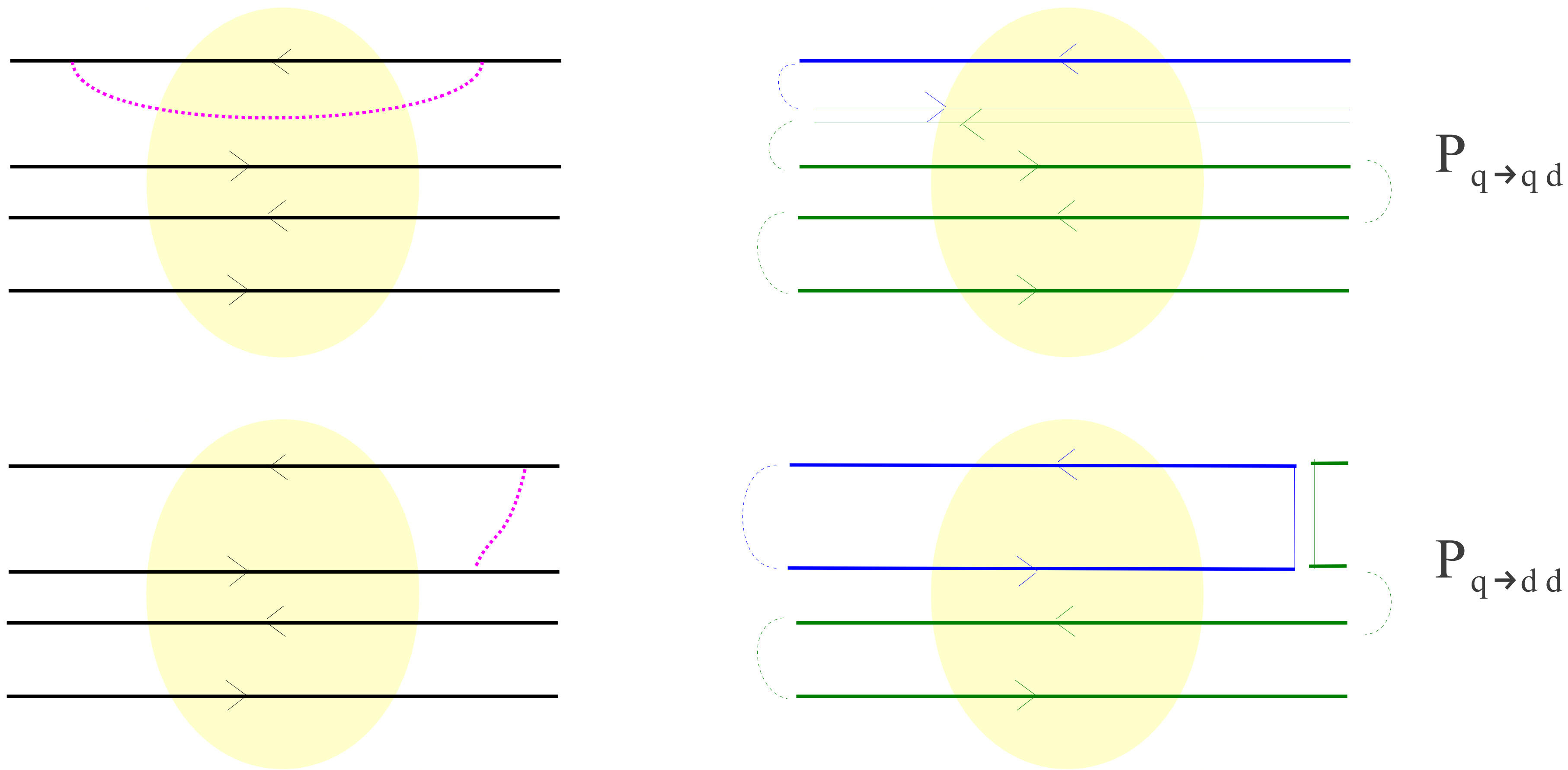
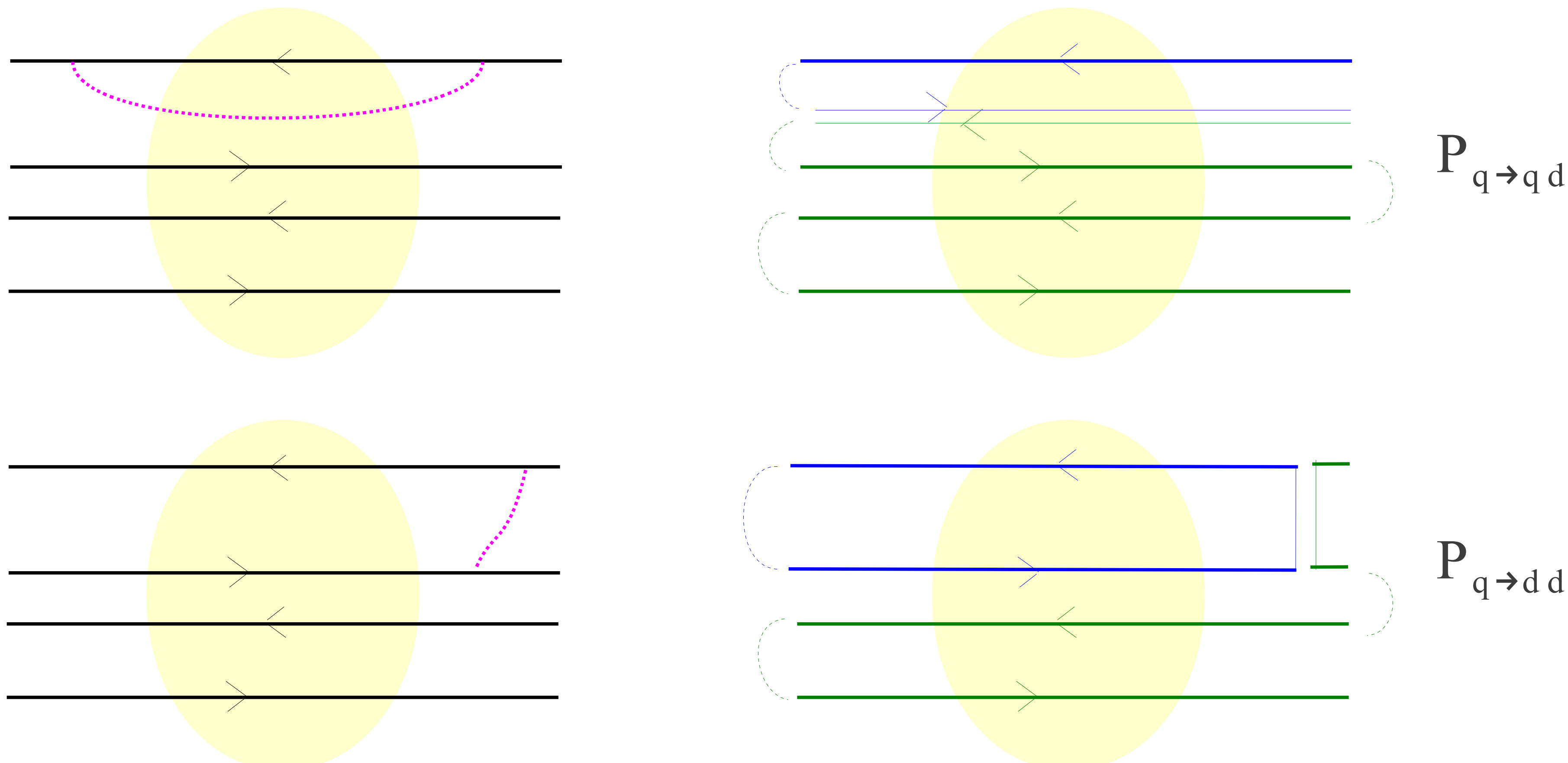


$$\frac{dQ}{dy} = \int P_{q \to q\,d}\left(QS - Q\right) + \int P_{q \to d\,d}\left(SS - Q\right)$$

Properties of the evolution – quadrupole amplitude



$$\frac{dQ}{dy} = \int P_{q\rightarrow q\,d}(QS - Q) + \int P_{q\rightarrow d\,d}(SS - Q)$$

Solutions also exhibit geometric scaling…

Similar properties as dipole amplitude, just more complicated!

Dominguez, Mueller, Munier, Xiao (2011)

Summary and outlook

- *One and two-particle inclusive observables at the LHC are related to* ***dipole*** *and* ***quadrupole*** *amplitudes respectively, at semi-classical level, but also when quantum corrections are included at least* ***to next-to-leading order.***

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Dominguez, Marquet, Stasto, Xiao (2013)

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Dipole and quadrupole amplitudes = the fundamental universal objects to describe the LHC small-x data?

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Dipole and quadrupole amplitudes = the fundamental universal objects to describe the LHC small-x data?

- *Can one understand more features of the quadrupole amplitude?*

Gaussian approximation:
Iancu, Triantafyllopoulos (2012)

- *Can one constrain this object experimentally?*