

# Higgs production in gluon fusion at approximate N<sup>3</sup>LO

**Marco Bonvini**

DESY Hamburg

QCD@LHC, DESY Hamburg, September 5, 2013



*Work in collaboration with:*

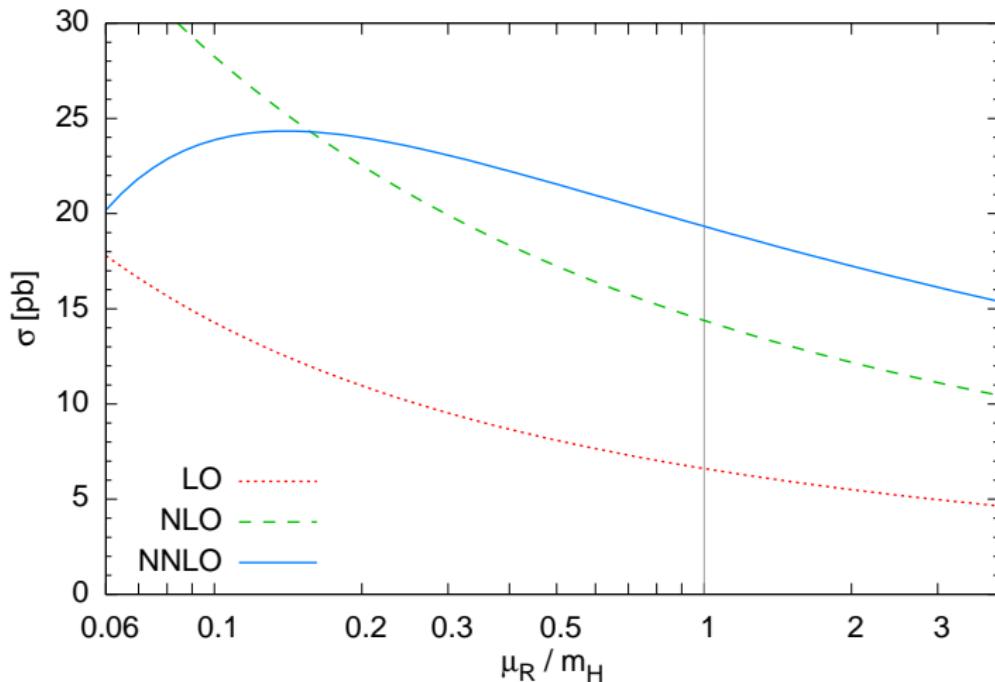
**Richard Ball, Stefano Forte, Simone Marzani, Giovanni Ridolfi**

arXiv:1303.3590

(see also arXiv:1306.6633)

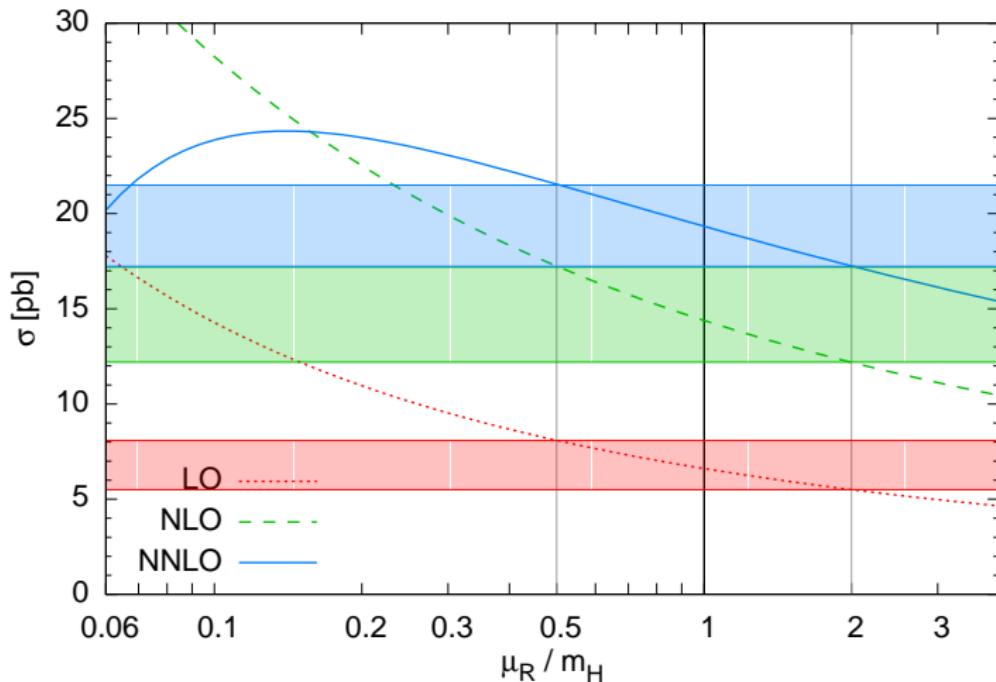
# Perturbative (in)stability to NNLO

Higgs cross section       $m_H = 125 \text{ GeV} @ \text{ LHC 8 TeV}$



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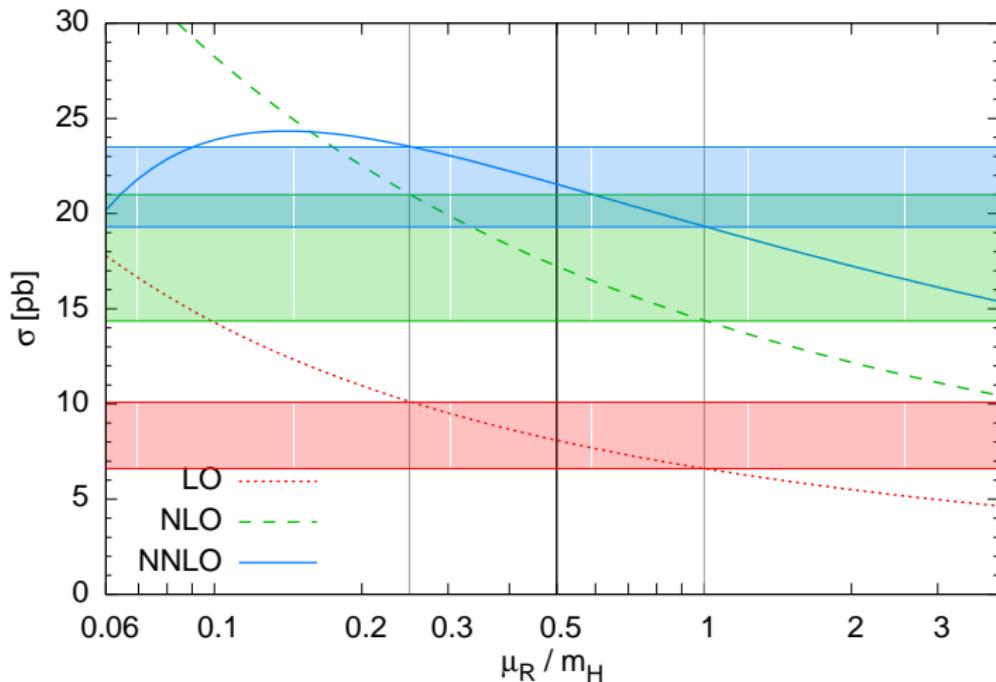
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$$1/2 < \mu_R/m_H < 2$$

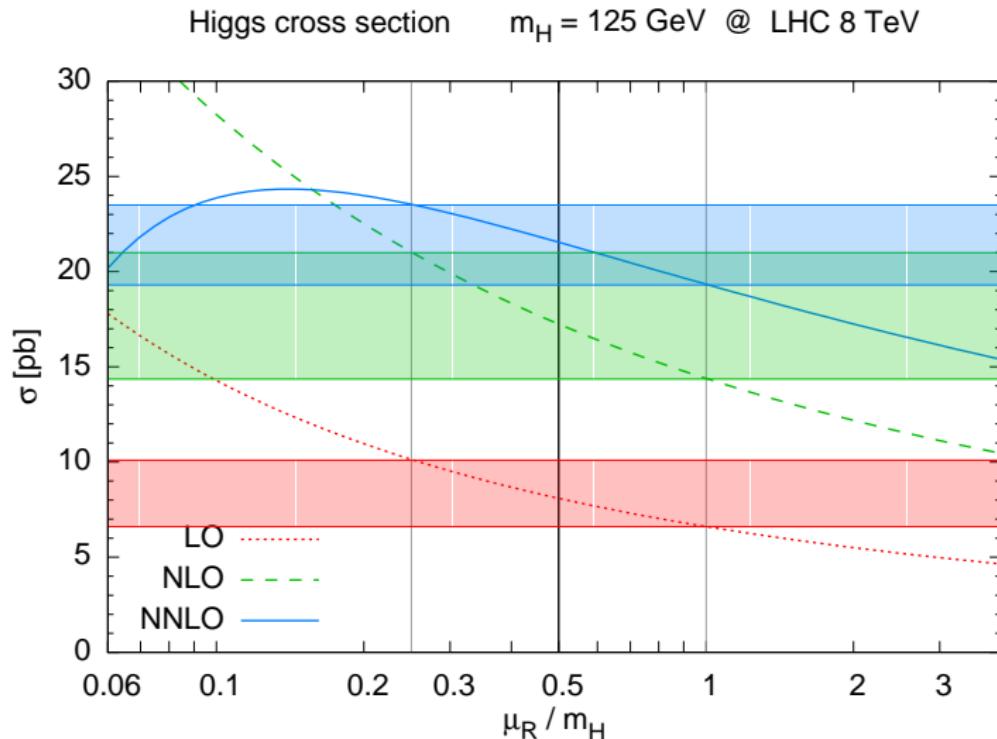
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Higgs cross section       $m_H = 125 \text{ GeV} @ \text{ LHC 8 TeV}$



$$1/4 < \mu_R/m_H < 1$$

# Perturbative (in)stability to NNLO



Convergence is slow! NNLO not definitive.

# Higgs production: *inclusive* cross section

$$\sigma(\tau) = \tau \sigma_0 \sum_{ij} \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ij}\left(\frac{\tau}{z}\right) C_{ij}(z, \alpha_s), \quad \tau = \frac{m_H^2}{s}, \quad z = \frac{m_H^2}{\hat{s}}$$

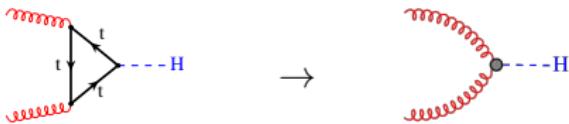
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• LO:

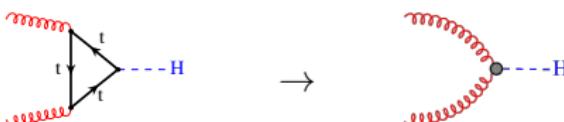


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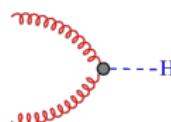
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- LO:



→



- NLO  $C_{ij}^{(1)}(z)$ :

- large  $m_t$  ( $\gg m_H$ ) approximation
- full  $m_t$  dependence

[Dawson 1991; Djouadi, Spira, Zerwas 1991]

[Spira, Djouadi, Graudenz, Zerwas 1995]

- NNLO  $C_{ij}^{(2)}(z)$ :

- large  $m_t$  approximation [Harlander, Kilgore 2002; Anastasiou, Melnikov 2002]
- expansion in  $m_H/m_t$  and in  $(1-z)$  [Harlander, Ozeren 2009]
- expansion in  $m_H/m_t$  [Pak, Rogal, Steinhauser 2010]
- finite  $m_t$  small- $z$  behavior [Marzani, Ball, Del Duca, Forte, Vicini 2008]

- NNLO + NNLL threshold resummation

[de Florian, Grazzini 2012]

# Towards N<sup>3</sup>LO

- N<sup>3</sup>LO  $C_{ij}^{(3)}(z)$ :
  - large  $m_t$  limit:
    - soft approximation [Moch, Vogt, 2005]
    - three loops [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser 2009]  
[Lee, Smirnov, Smirnov 2010]
  - $H + \text{jet}$  at NNLO [Gehrmann, Glover, Huber, Ikizlerli, Studerus 2010]  
[Boughezal, Caola, Melnikov, Petriello, Schulze 2013]
  - three emissions as an expansion in  $(1 - z)$  [Anastasiou, Duhr, Dulat, Mistlberger 2013]  
[Höschele, Hoff, Pak, Steinhauser, Ueda 2012]  
[Buehler, Lazopoulos 2013]
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  - with finite  $m_t$  dependence:
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**Remark:** the exact  $m_t$  dependence is needed not only for large  $m_H$ , but also for large  $\sqrt{s}$

# Ingredients of our N<sup>3</sup>LO prediction

$gg$  channel only ( $\sim 97\%$  of the full NNLO):

$$C_{gg}(z, \alpha_s) \simeq C_{\text{soft}}(z, \alpha_s) + C_{\text{high-energy}}(z, \alpha_s)$$

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$$N \rightarrow \infty \quad N \rightarrow 1$$

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Mellin space:  $C_{gg}(\textcolor{violet}{N}, \alpha_s) = \int_0^1 dz z^{\textcolor{violet}{N}-1} C_{gg}(z, \alpha_s)$

(ordinary funct) (distribution)

Known analytic structure: poles in  $N = 1, 0, -1, -2, \dots$

## Side remark: saddle point

$$\sigma_{gg}(\tau) = \tau \sigma_0 \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} \tau^{-N} \mathcal{L}_{gg}(N) C_{gg}(N, \alpha_s), \quad \tau = \frac{m_H^2}{s}$$

The inversion integral is *largely* dominated by the region close to  
the saddle point  $N = N_0(\tau)$

[MB, Forte, Ridolfi 2012]

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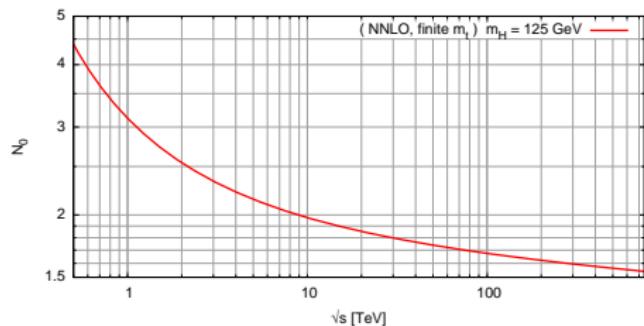
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The saddle point is *unique*, *real* and perturbatively stable

LHC (7 to 14 TeV):

$$N_0 \sim 2.1 - 1.9$$

## Soft part: $C_{\text{soft}}$

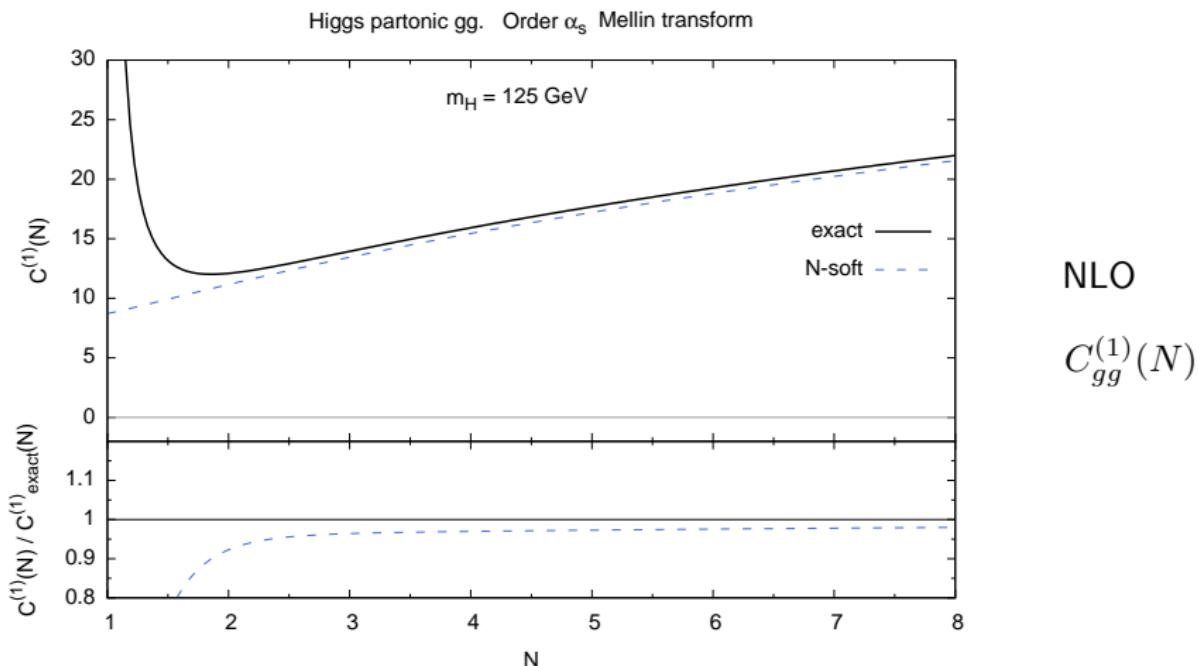
Expanding in powers of  $\alpha_s$  the soft-gluon (threshold) resummation expression

$$C_{gg}(N, \alpha_s) \stackrel{N \rightarrow \infty}{=} g_0\left(\alpha_s, \frac{m_H}{m_t}\right) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \dots\right]$$

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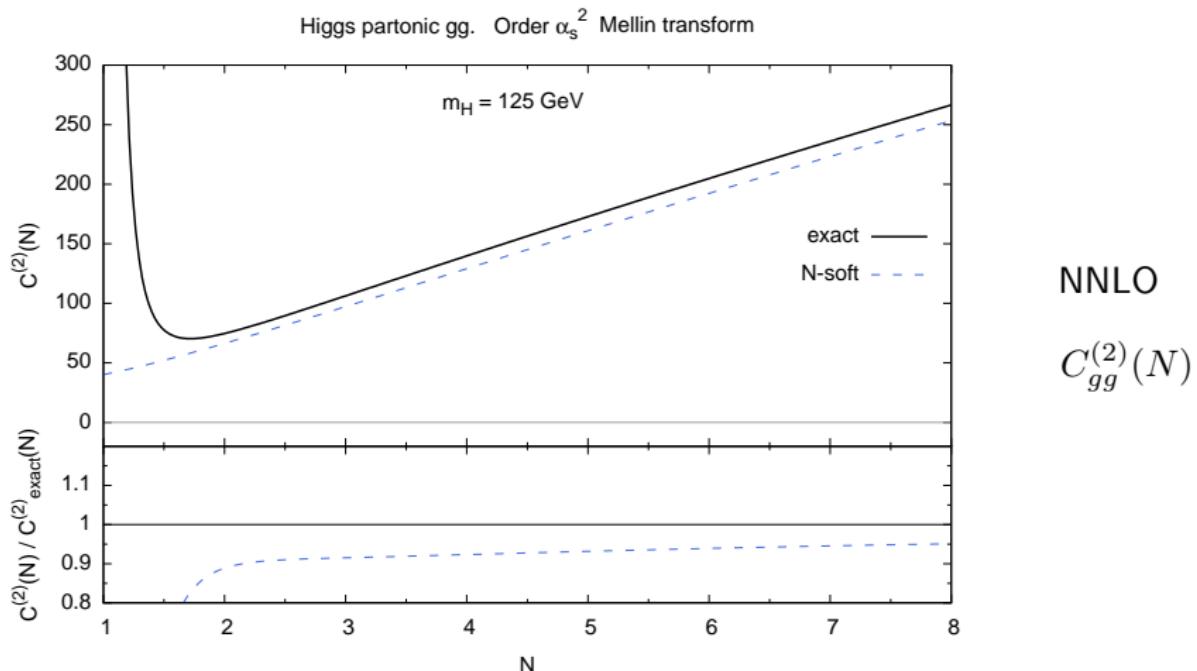
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cut in  $N \leq 0$ , in  $z$ -space linear combination of  $\left(\frac{\log^k \log \frac{1}{z}}{\log \frac{1}{z}}\right)_+$  **Not physical!**

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$$P_{gg} = \frac{C_A}{\pi} \frac{1 - 2z + 3z^2 - 2z^3 + z^4}{z(1-z)} \quad (z < 1)$$

[Krämer, Laenen, Spira 1997]

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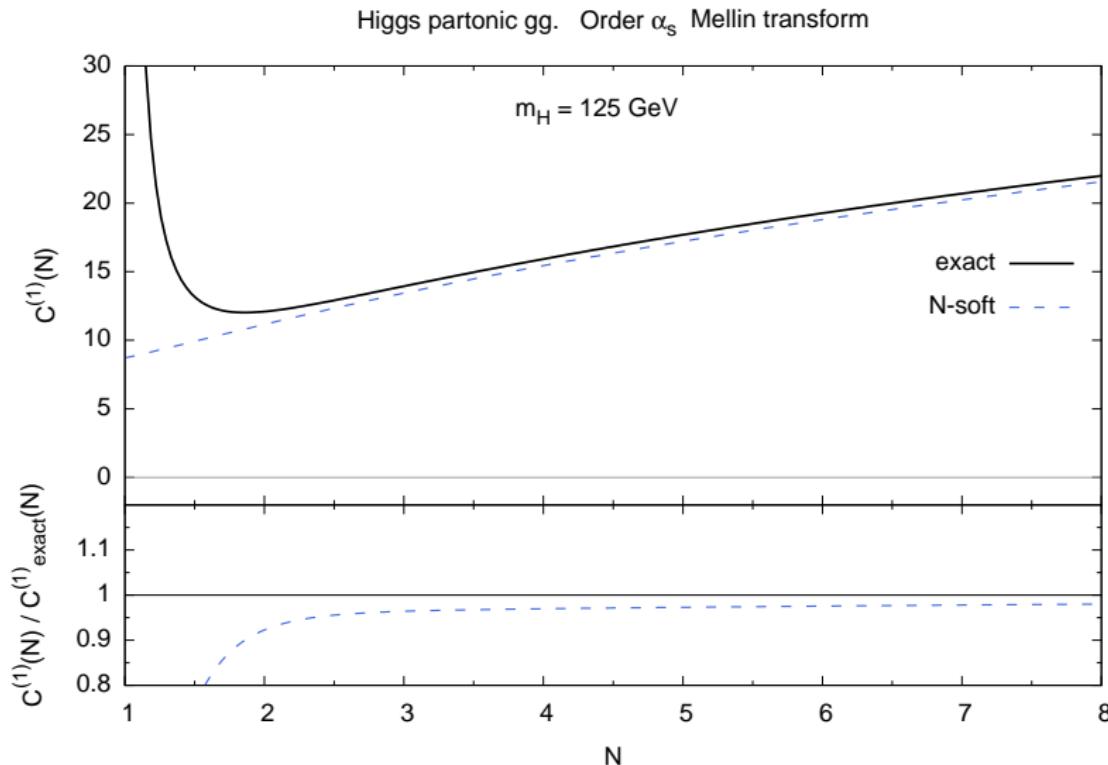
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- to prevent double counting with  $C_{\text{high-energy}}$ , we expand  $P_{gg}$  about  $z = 1$

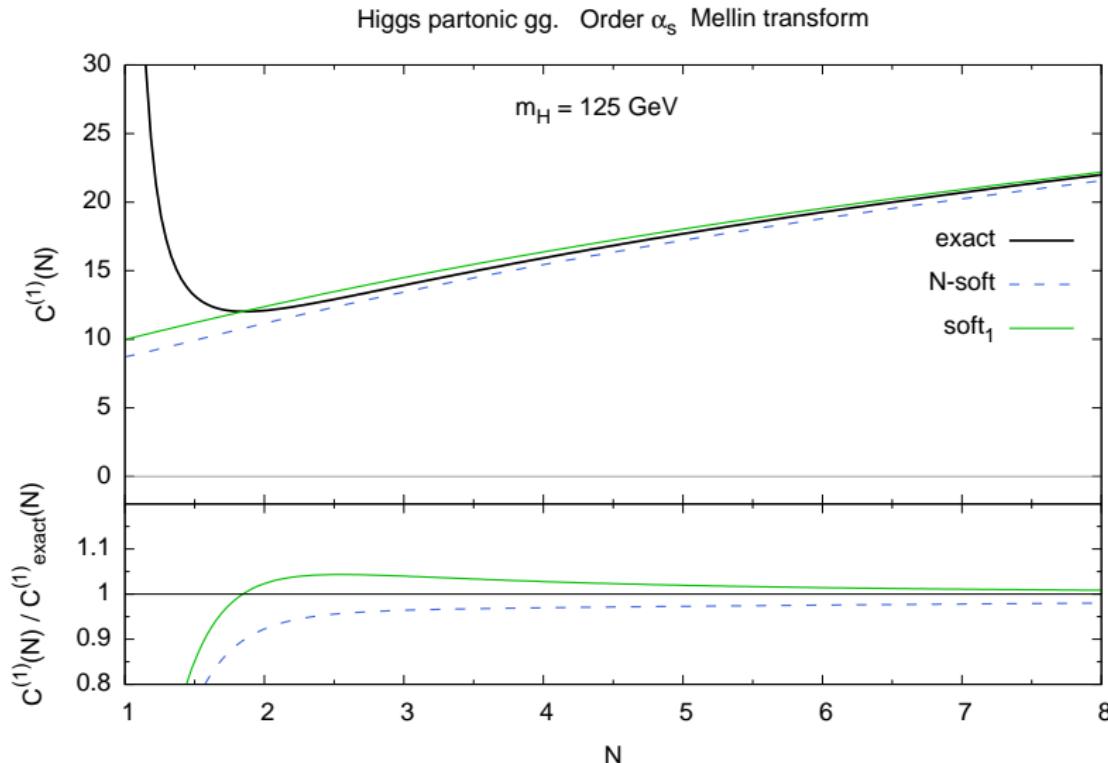
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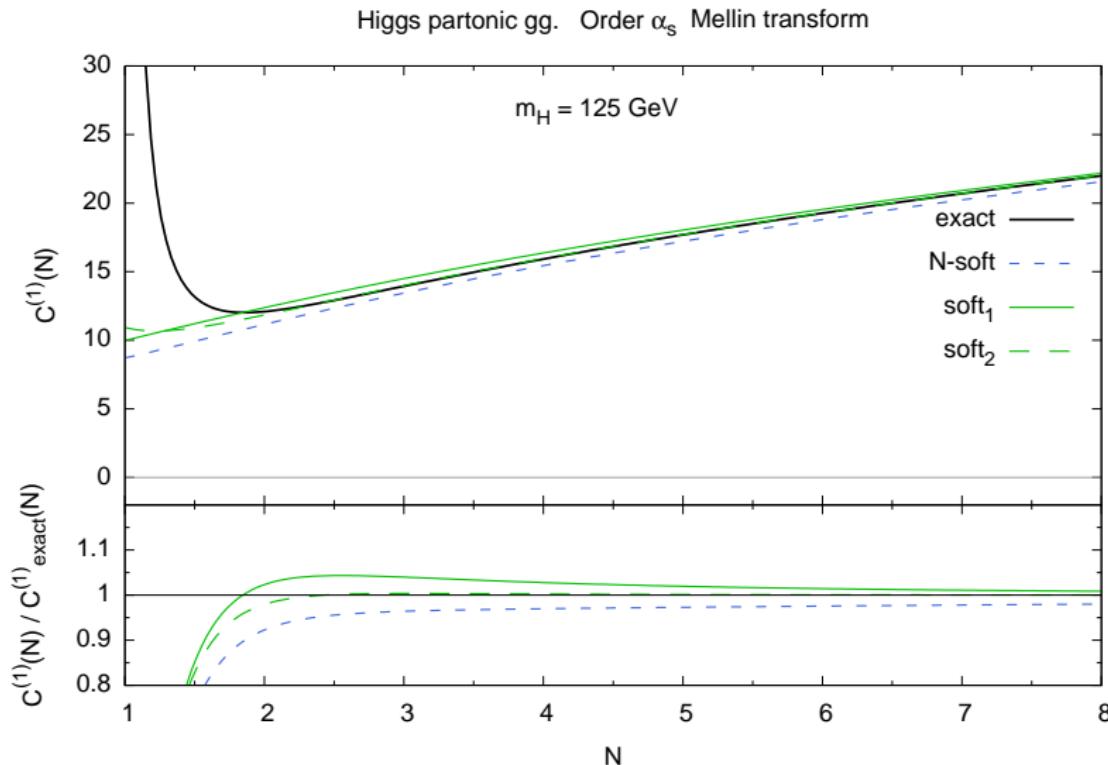
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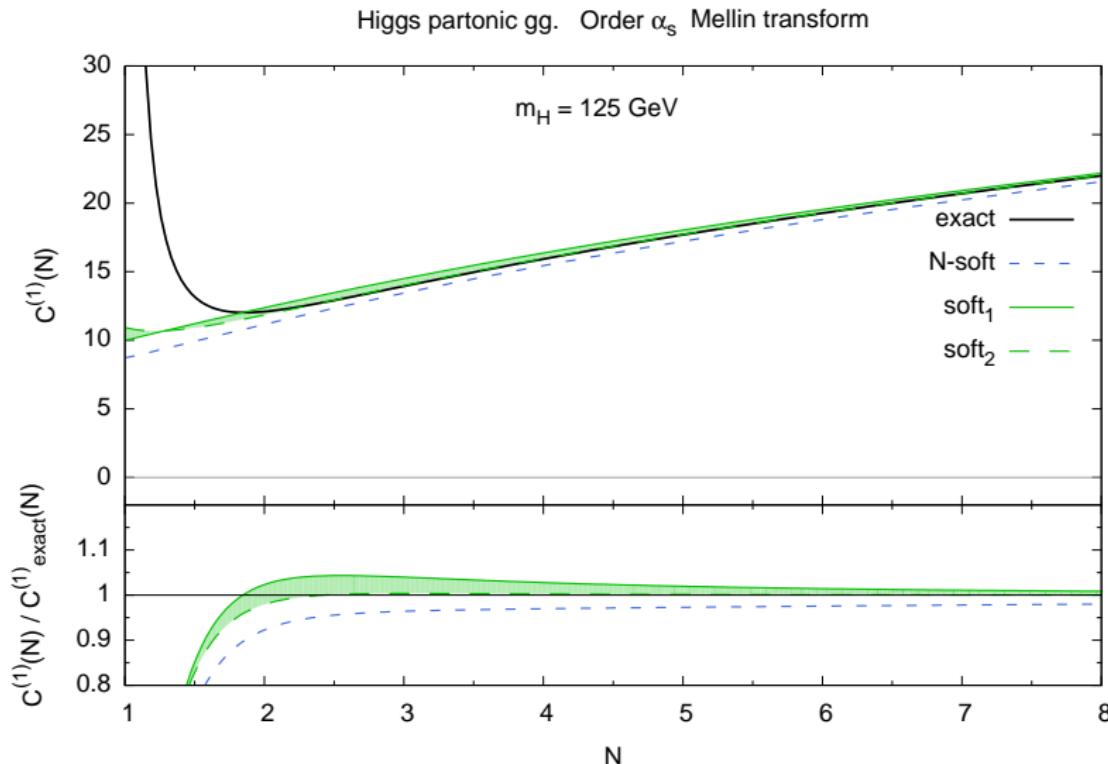
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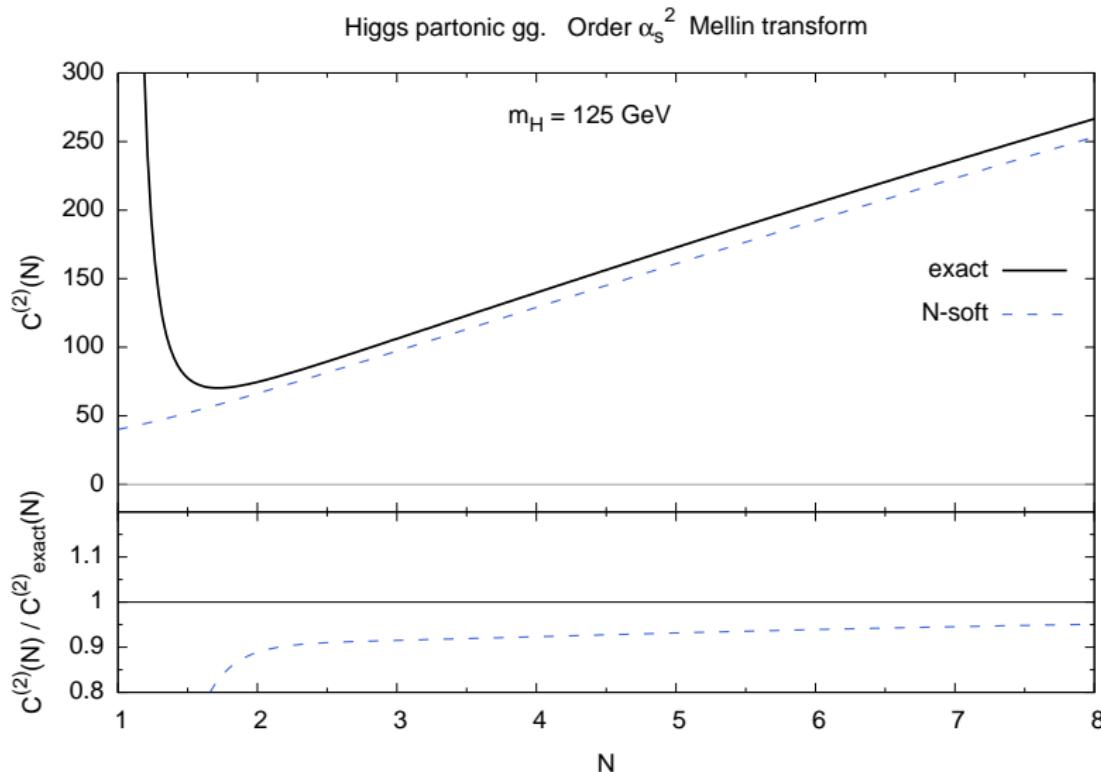
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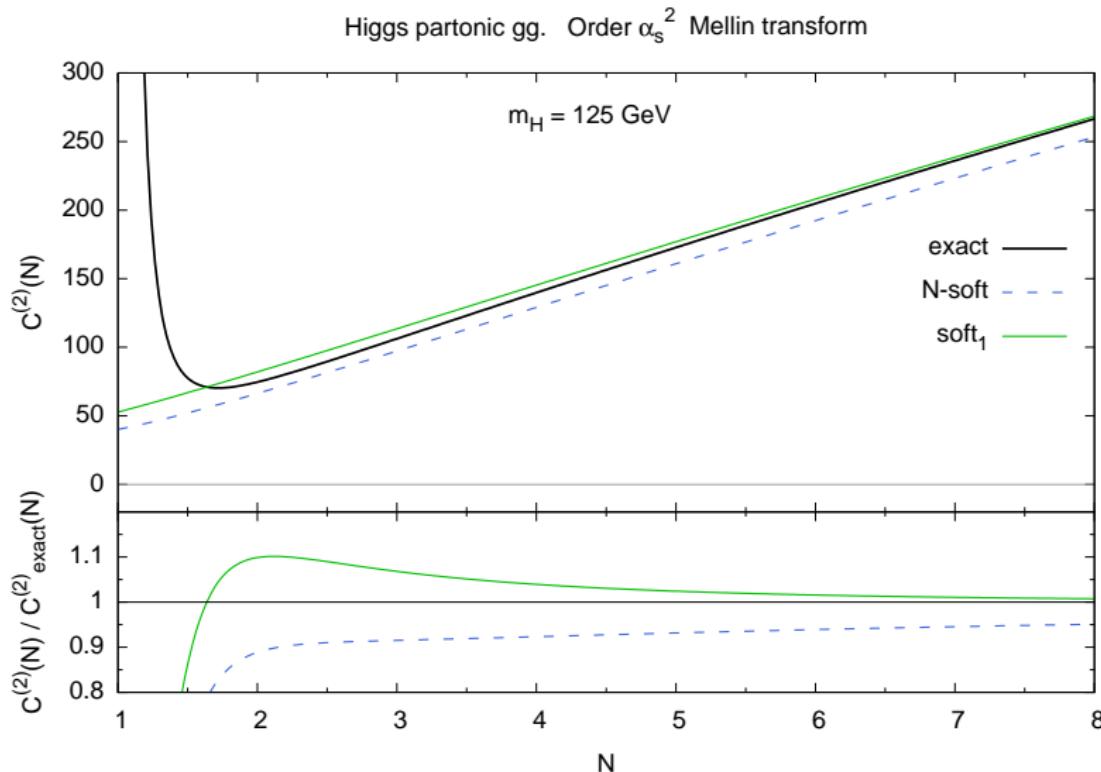
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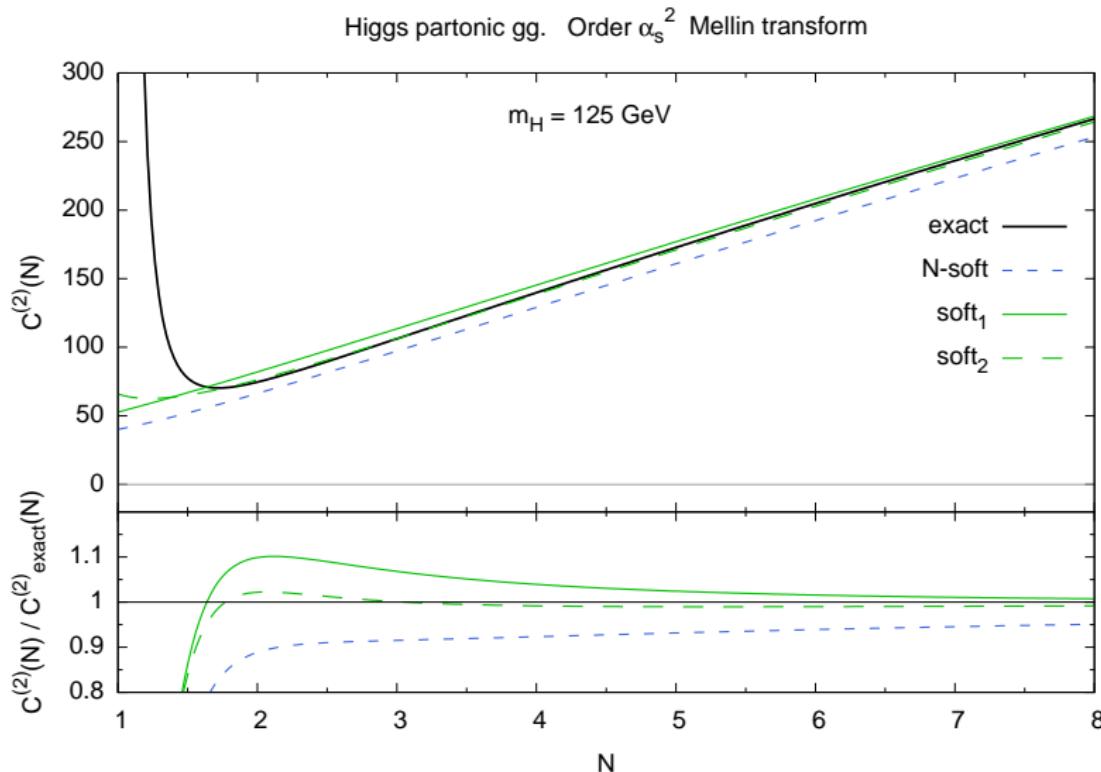
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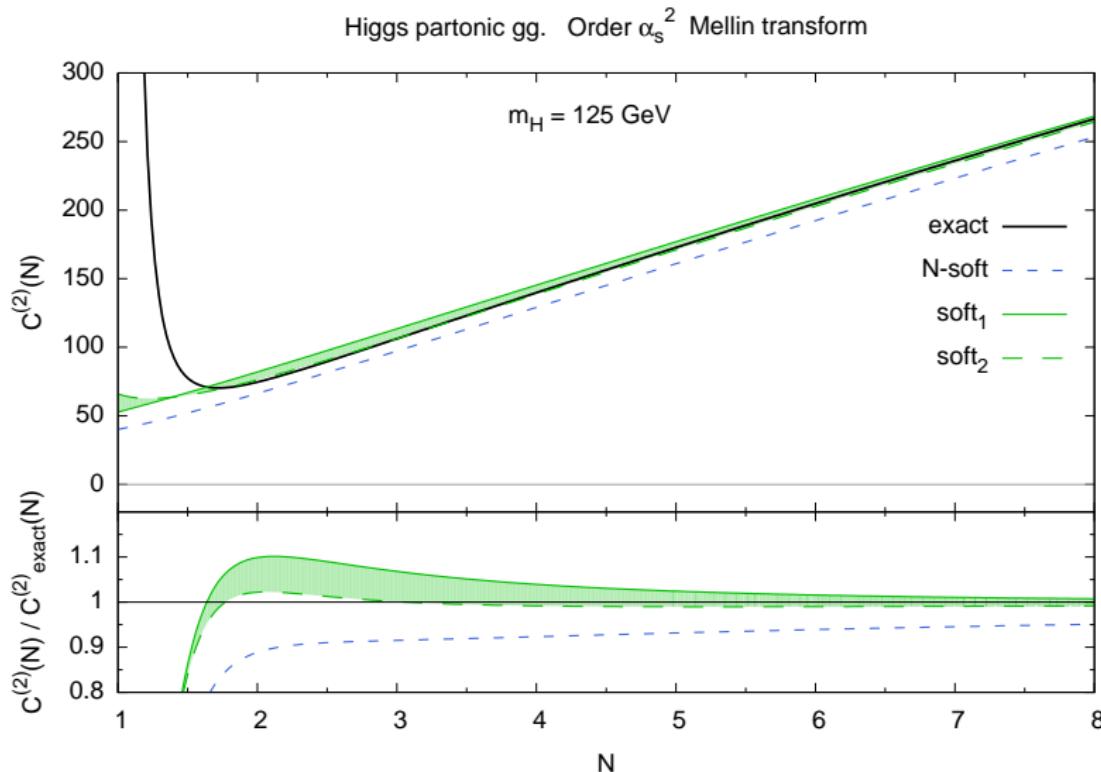
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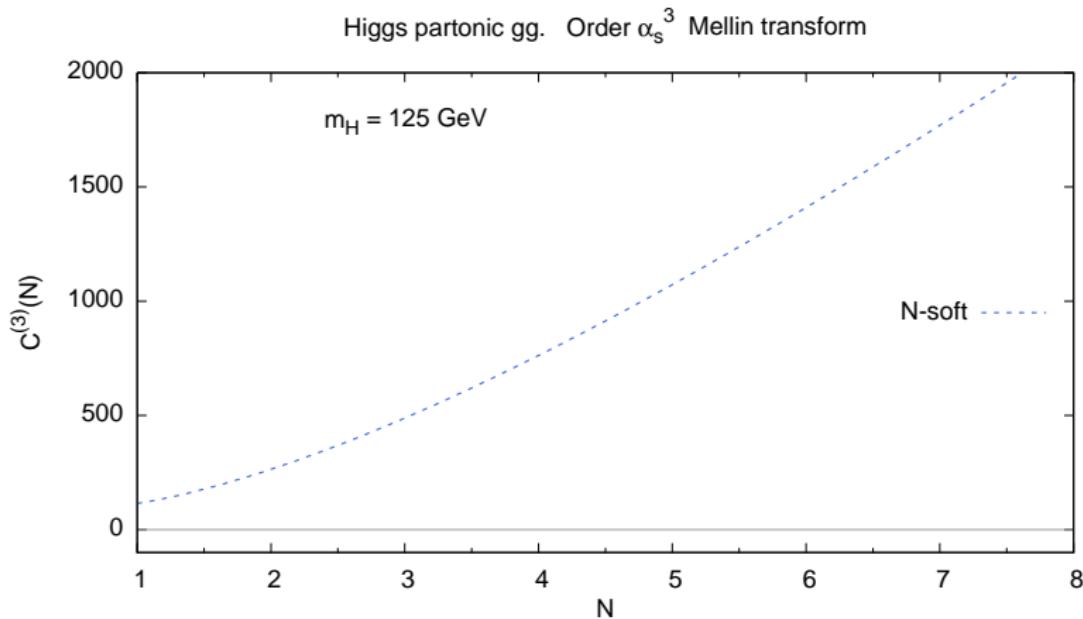
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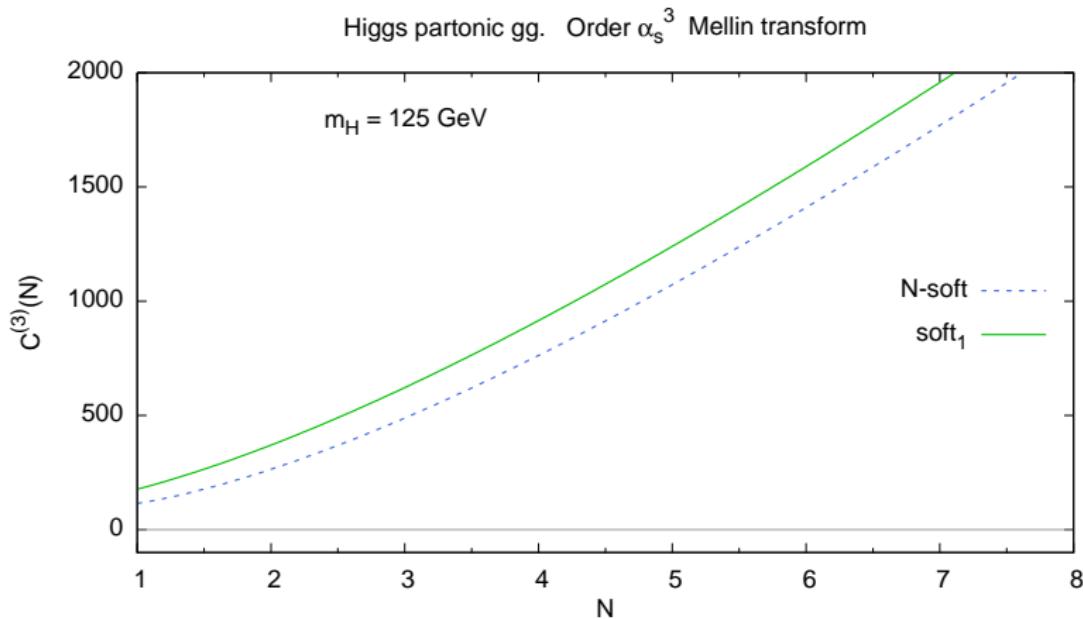
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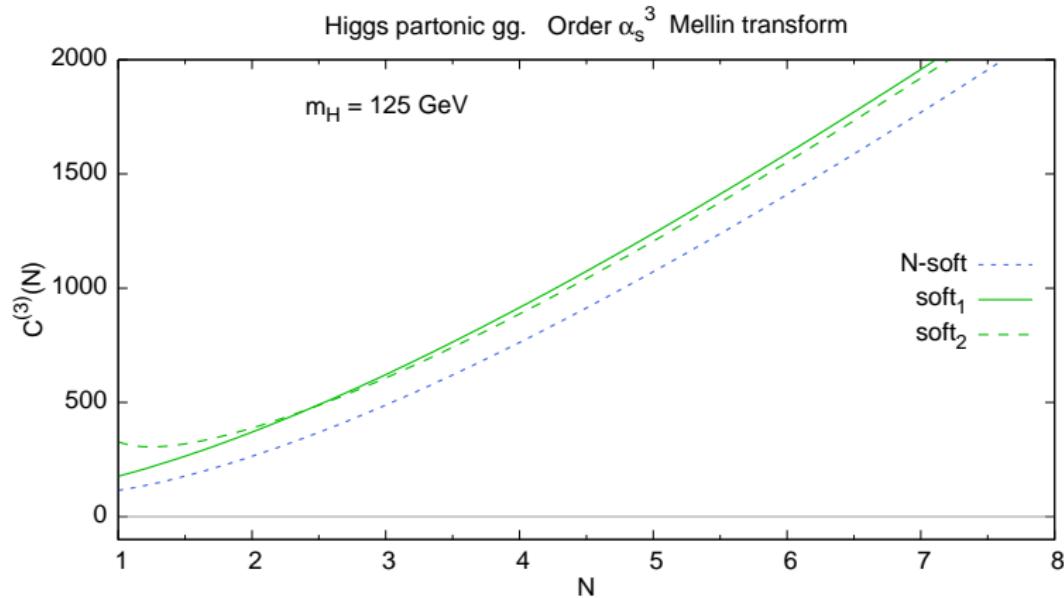
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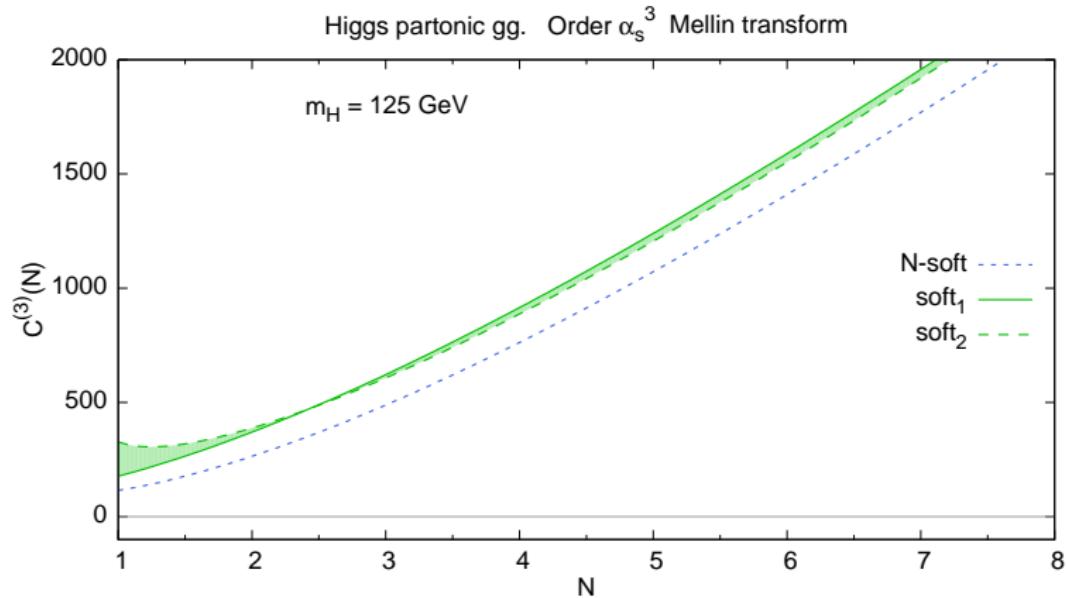
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## High-energy part: $C_{\text{high-energy}}$

Leading Log poles in  $C_{gg}(N, \alpha_s)$ :

$$\frac{\alpha_s^k}{(N-1)^k} \left( \alpha_s^k \frac{\log^{k-1} z}{z} \right)$$

$$C_{\text{high-energy}}(N, \alpha_s) = \sum_{k_1, k_2 \geq 0} c_{k_1, k_2} \left( \frac{m_H}{m_t} \right) [\gamma_+]^{k_1} [\gamma_+]^{k_2}$$

$\gamma_+(N)$ : DGLAP anomalous dimension (largest eigenvalue)

[Marzani, Ball, Del Duca, Forte, Vicini 2008]

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- accurate at LL (with running coupling effects)
- momentum conservation  $C_{\text{high-energy}}(N=2, \alpha_s) = 0$ , but grows at  $N \rightarrow \infty$

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Leading Log poles in  $C_{gg}(N, \alpha_s)$ :

$$\frac{\alpha_s^k}{(N-1)^k} \left( \alpha_s^k \frac{\log^{k-1} z}{z} \right)$$

$$C_{\text{high-energy}}(N, \alpha_s) = \sum_{k_1, k_2 \geq 0} c_{k_1, k_2} \left( \frac{m_H}{m_t} \right) [\gamma_+]^{k_1} [\gamma_+]^{k_2}$$

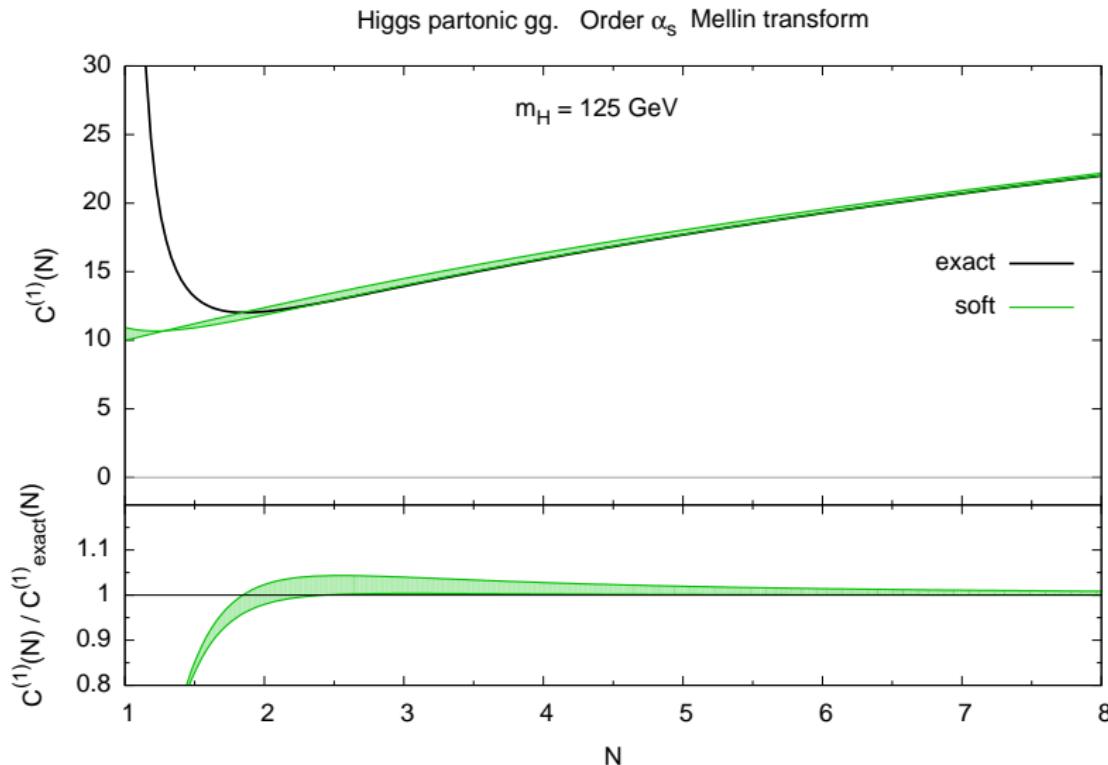
$\gamma_+(N)$ : DGLAP anomalous dimension (largest eigenvalue)

[Marzani, Ball, Del Duca, Forte, Vicini 2008]

- accurate at LL (with running coupling effects)
- momentum conservation  $C_{\text{high-energy}}(N=2, \alpha_s) = 0$ , but grows at  $N \rightarrow \infty$
- we use an expansion of  $\gamma_+(N)$  to NLL (removes the growth)
- we enforce mom. cons. adding subdominant terms (poles at  $N \leq 0$ )

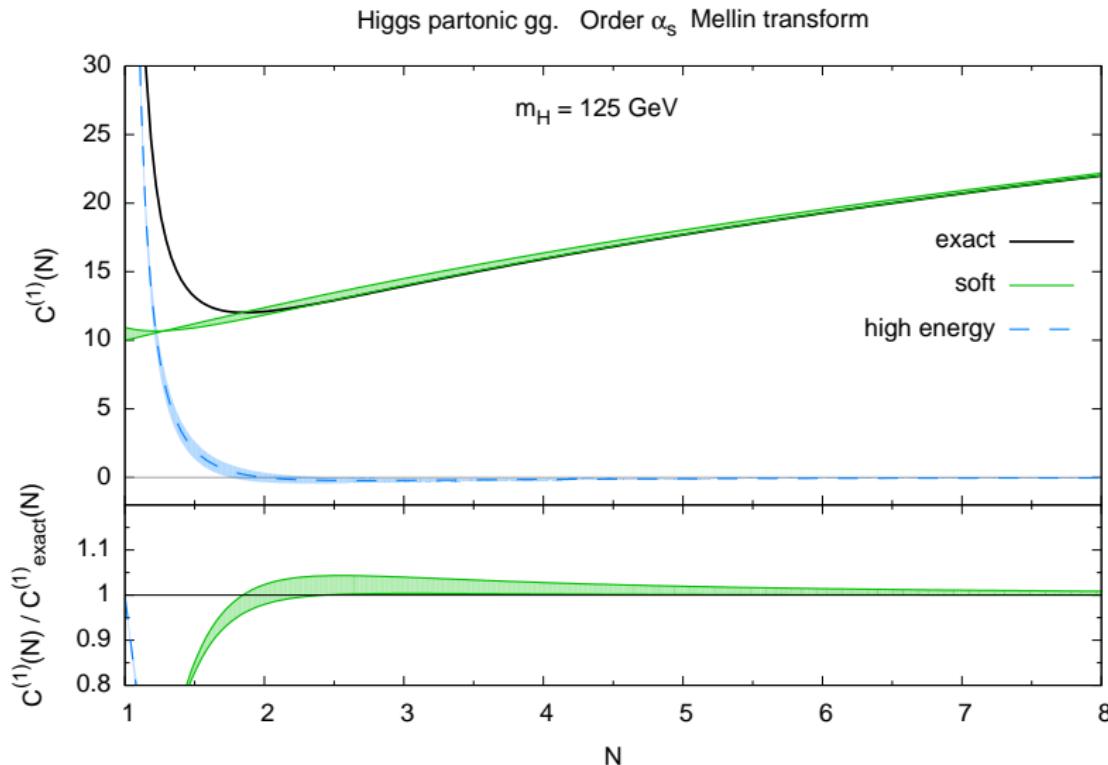
# Full approximation: soft + high-energy

$$C_{gg}(N, \alpha_s) = 1 + \alpha_s C^{(1)}(N) + \alpha_s^2 C^{(2)}(N) + \alpha_s^3 C^{(3)}(N) + \dots$$



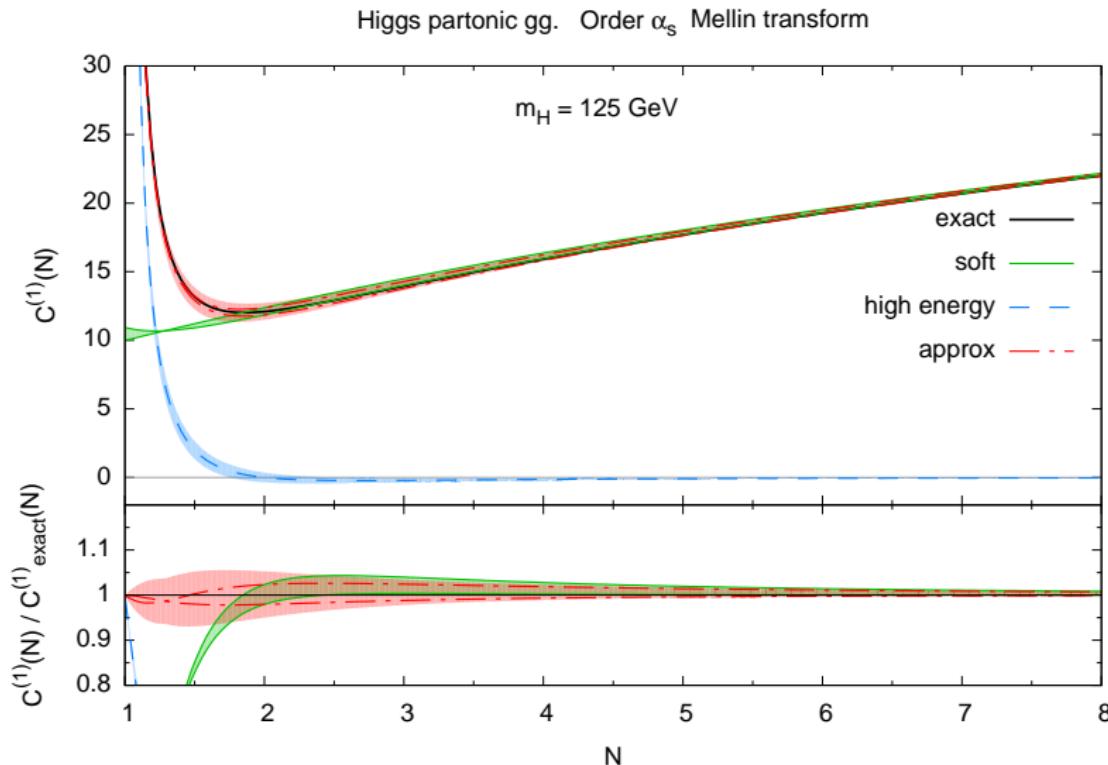
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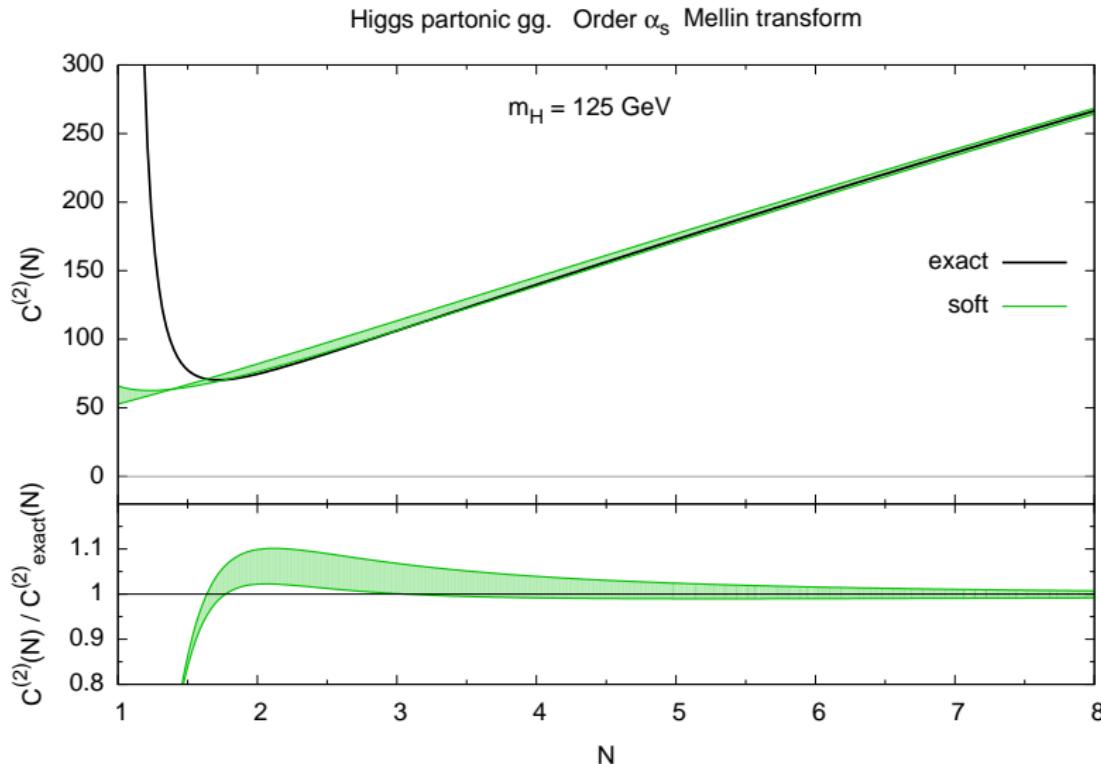
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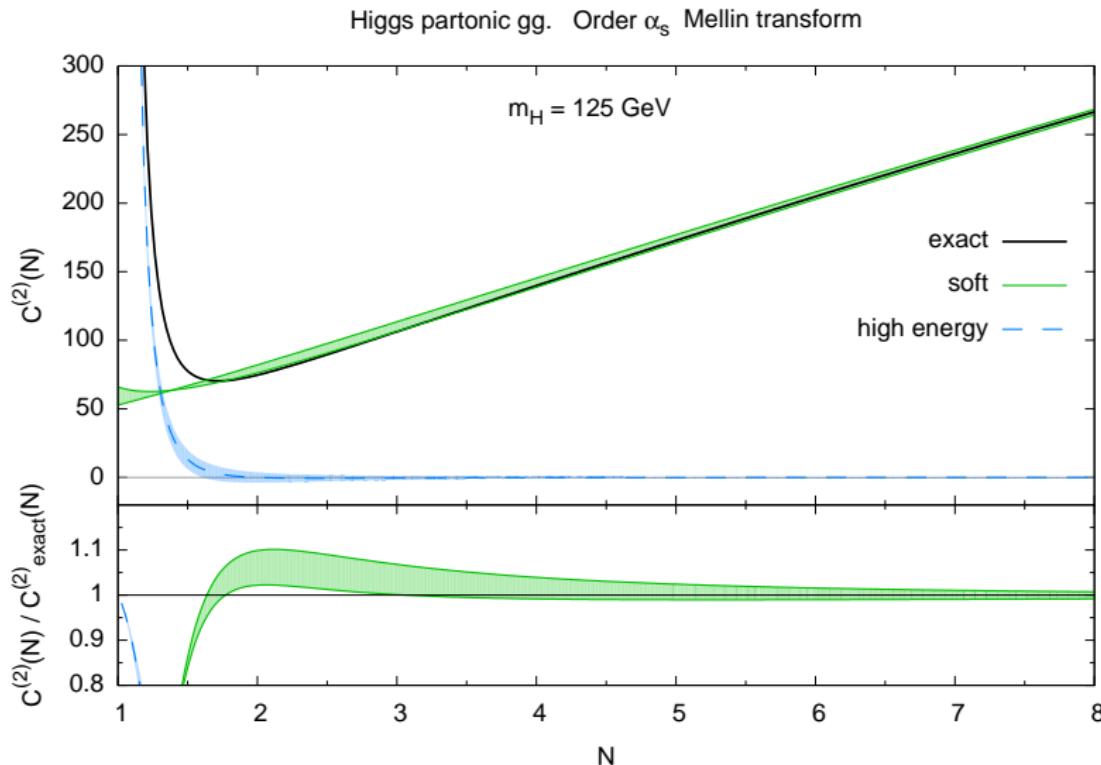
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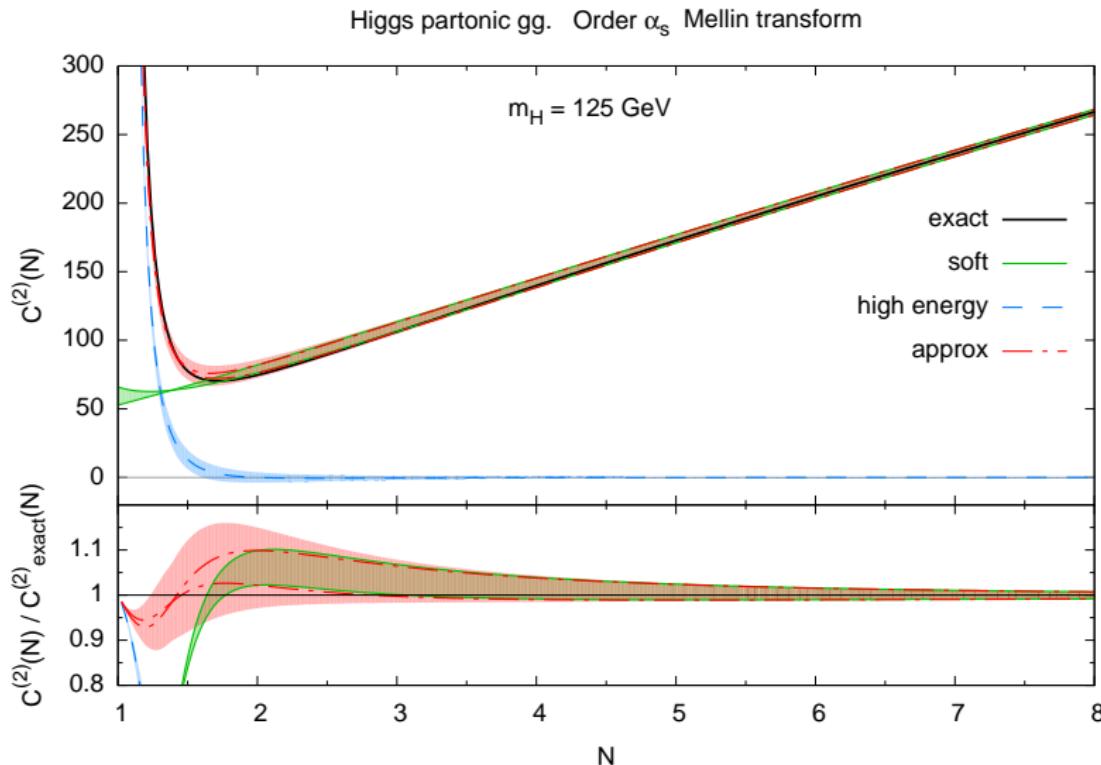
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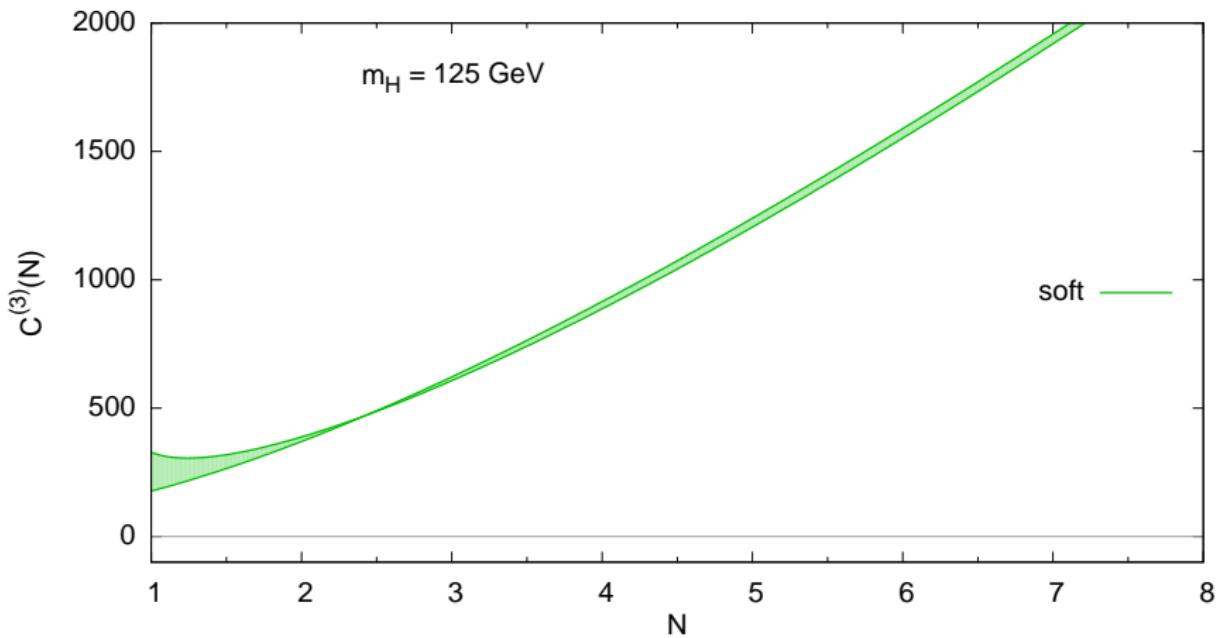
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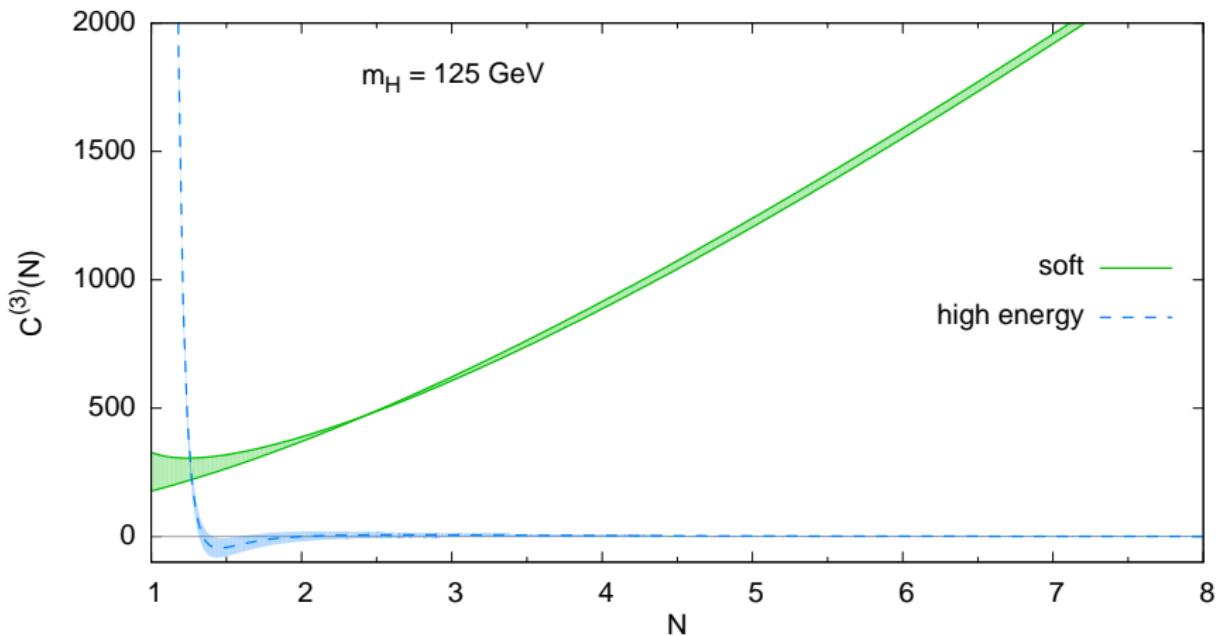
Higgs partonic gg. Order  $\alpha_s^3$  Mellin transform



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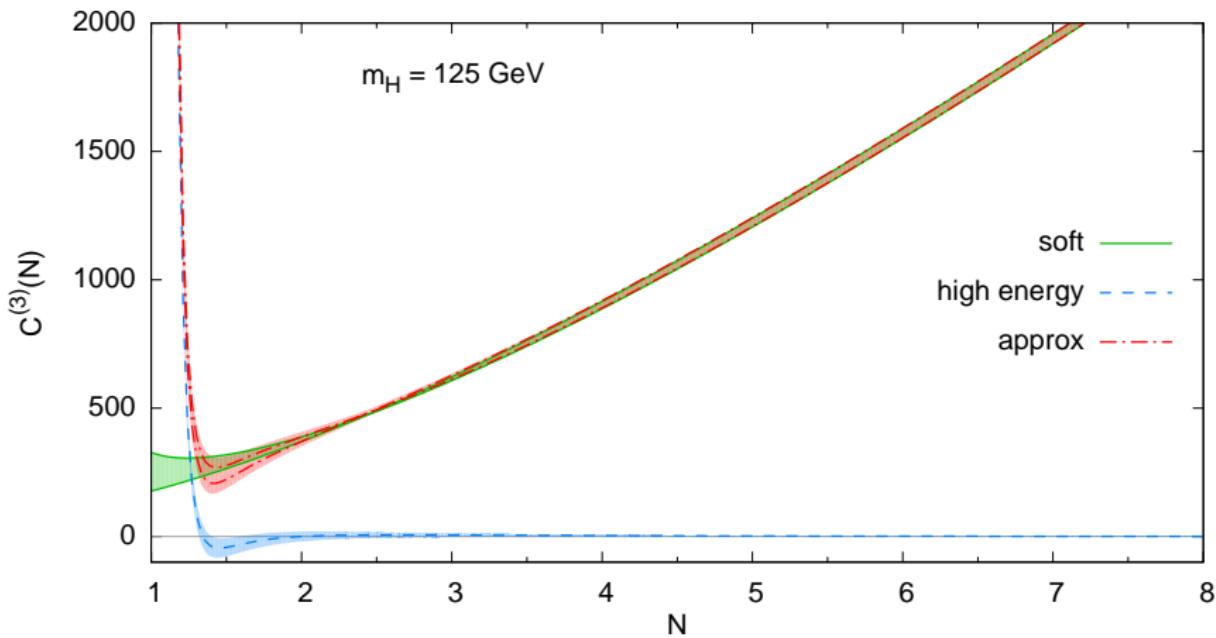
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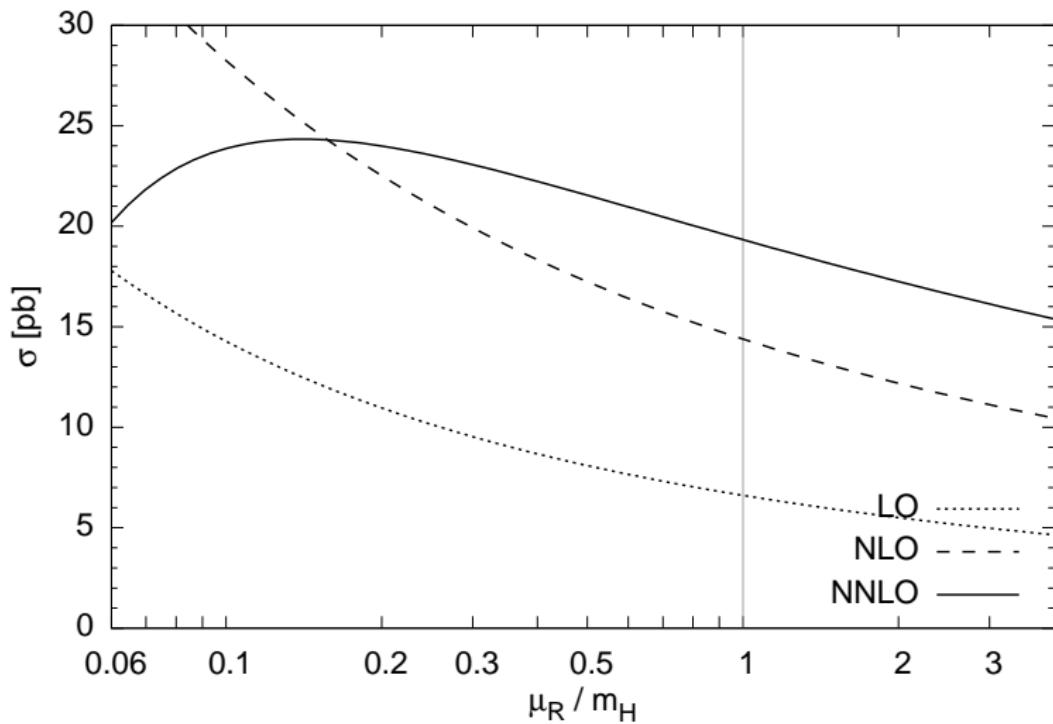
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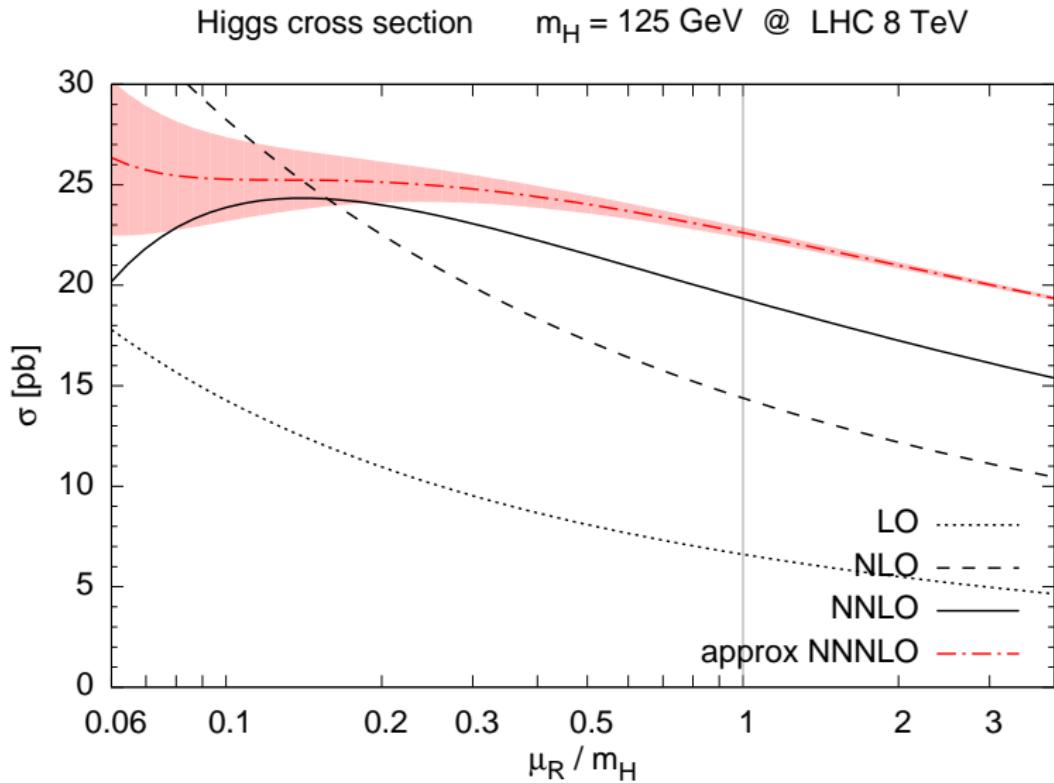


# $N^3\text{LO}$ prediction for Higgs production

Higgs cross section       $m_H = 125 \text{ GeV}$  @ LHC 8 TeV

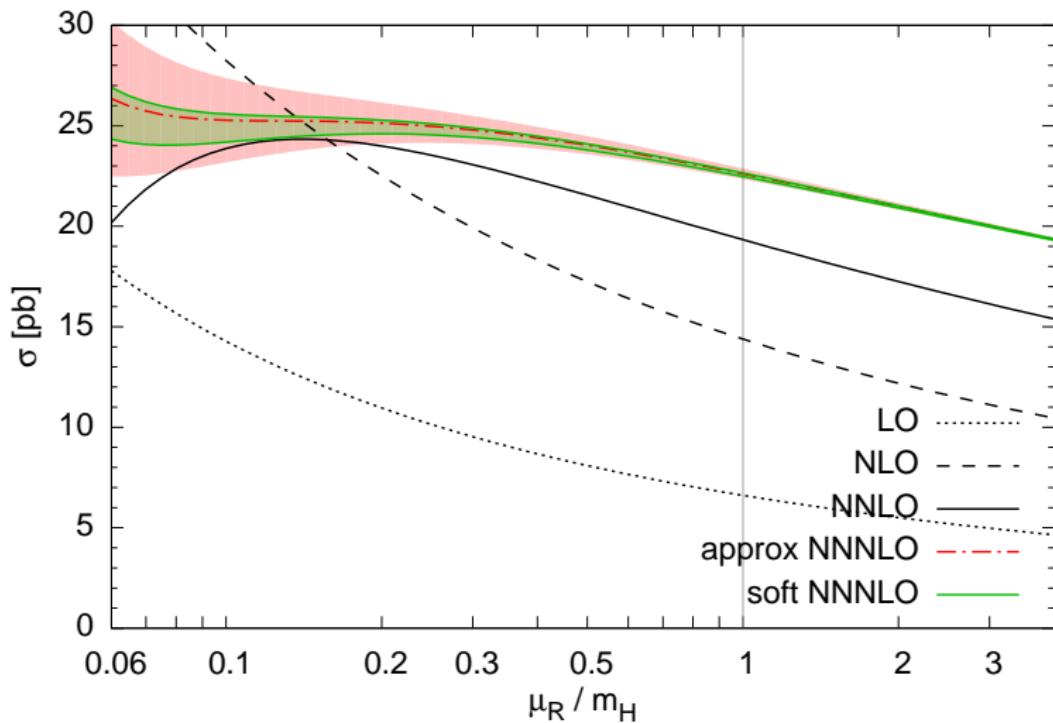


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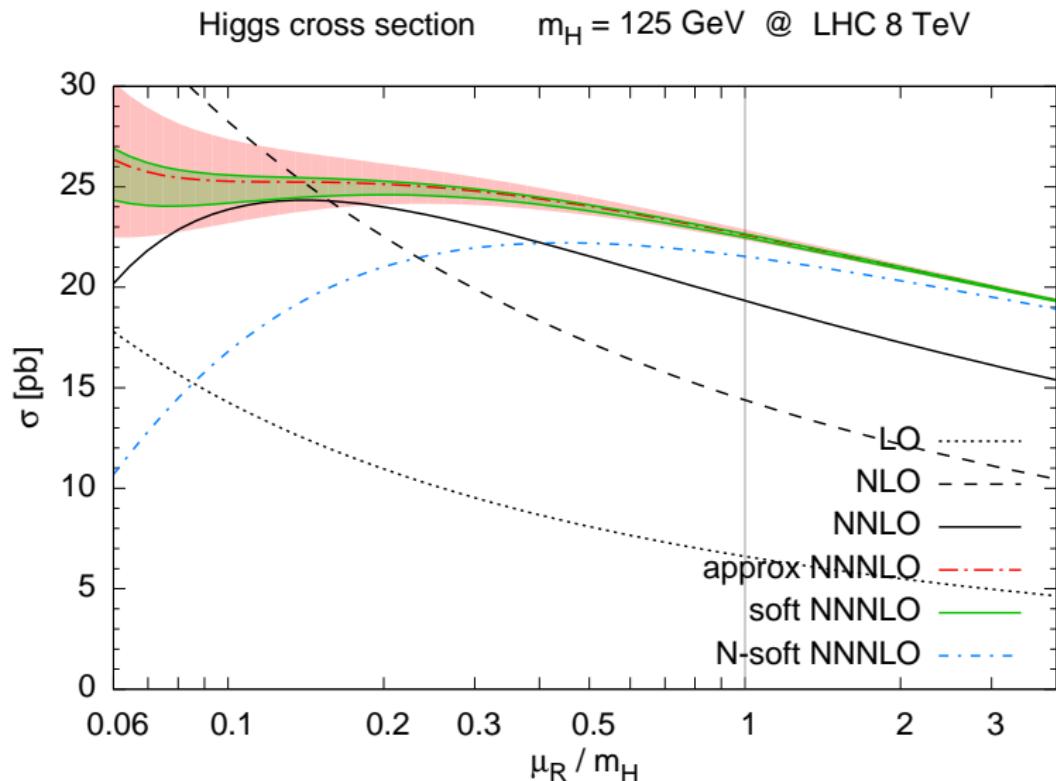


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# $N^3\text{LO}$ prediction for Higgs production



N-soft: [Moch, Vogt 2005], expanded resummation [de Florian, Grazzini 2012]

# Conclusions

- We are predicting the inclusive Higgs N<sup>3</sup>LO cross section using

$$C_{gg}^{(3)}(z) \simeq C_{\text{soft}}^{(3)}(z) + C_{\text{high-energy}}^{(3)}(z)$$

with:

- exact  $m_t$  dependence
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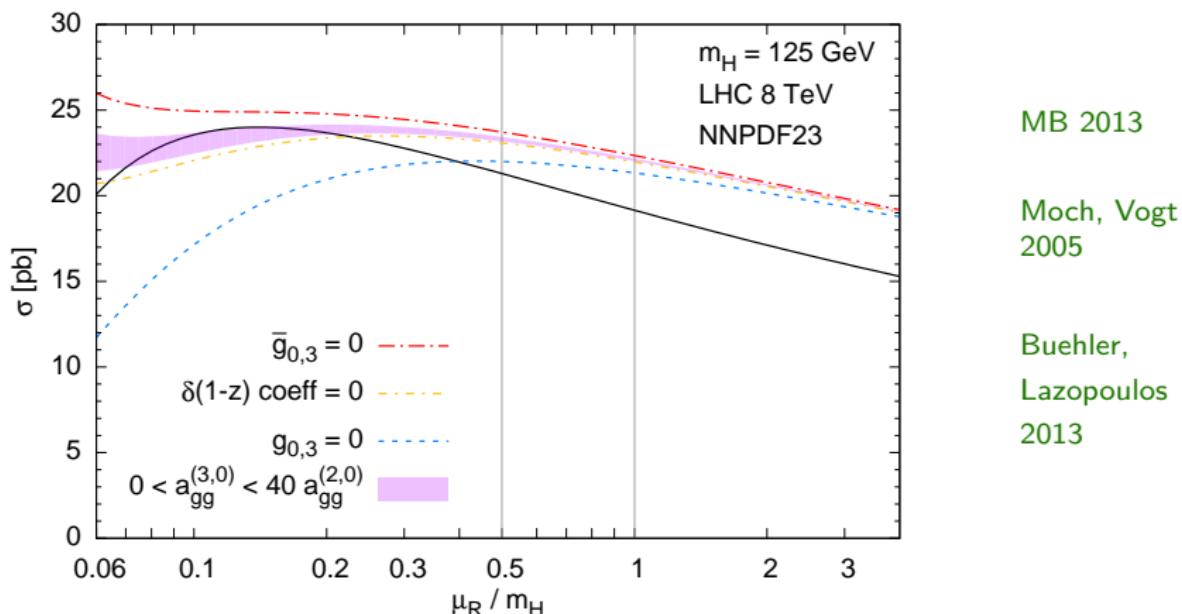
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  - What's next?
    - the  $\delta(1 - z)$  term at order  $\alpha_s^3$
    - subleading high-energy terms
    - other channels (at NNLO the  $qg$  channel gives a 3% contribution)
    - resummation

# Backup slides

# Uncertainty due to constant term

order $n$	$\delta(1-z)$ coeff	$\bar{g}_{0,n}$	+	$r_n$	=	$g_{0,n}$
1	4.9374	4.9374		3.7779		8.7153
2	8.94	10.92		29.18		40.10
3	unknown	unknown		114.7		unknown

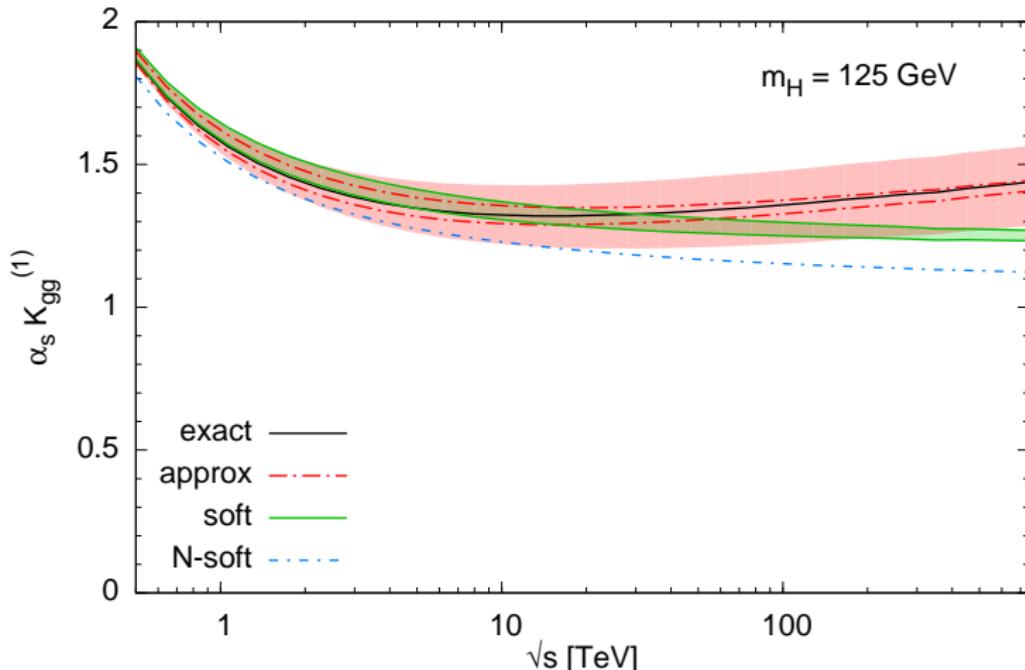
Higgs hadron-level cross section



# K-factors (NNLO pdfs)

$$\frac{\sigma_{gg}}{\sigma_{\text{LO}}} = 1 + \alpha_s K^{(1)} + \alpha_s^2 K^{(2)} + \alpha_s^3 K^{(3)} + \dots$$

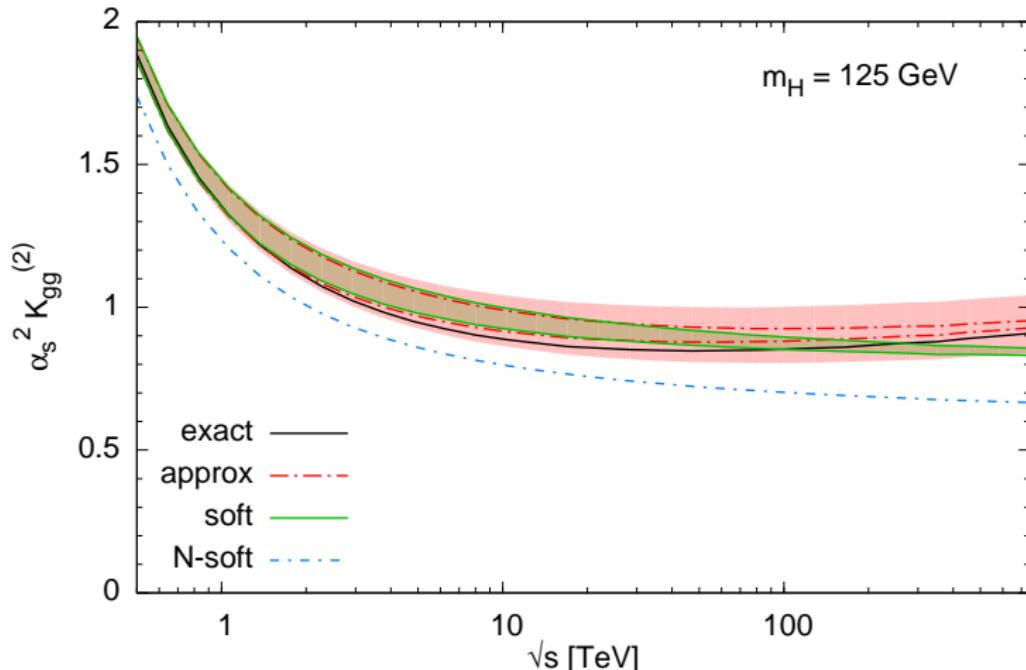
Higgs K-factor at NLO (NNLO PDFs)



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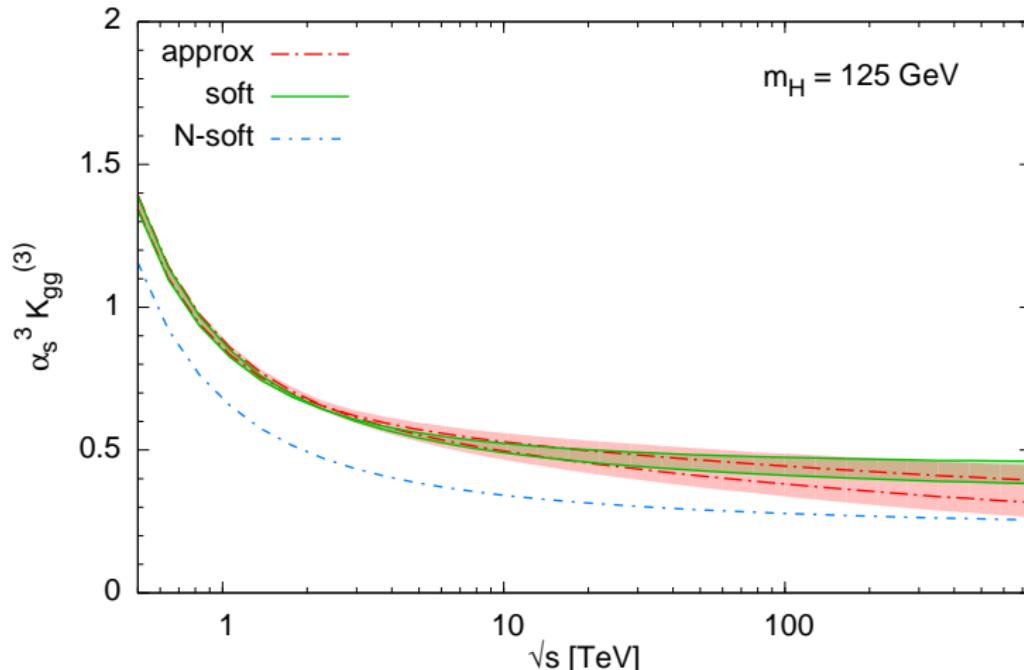
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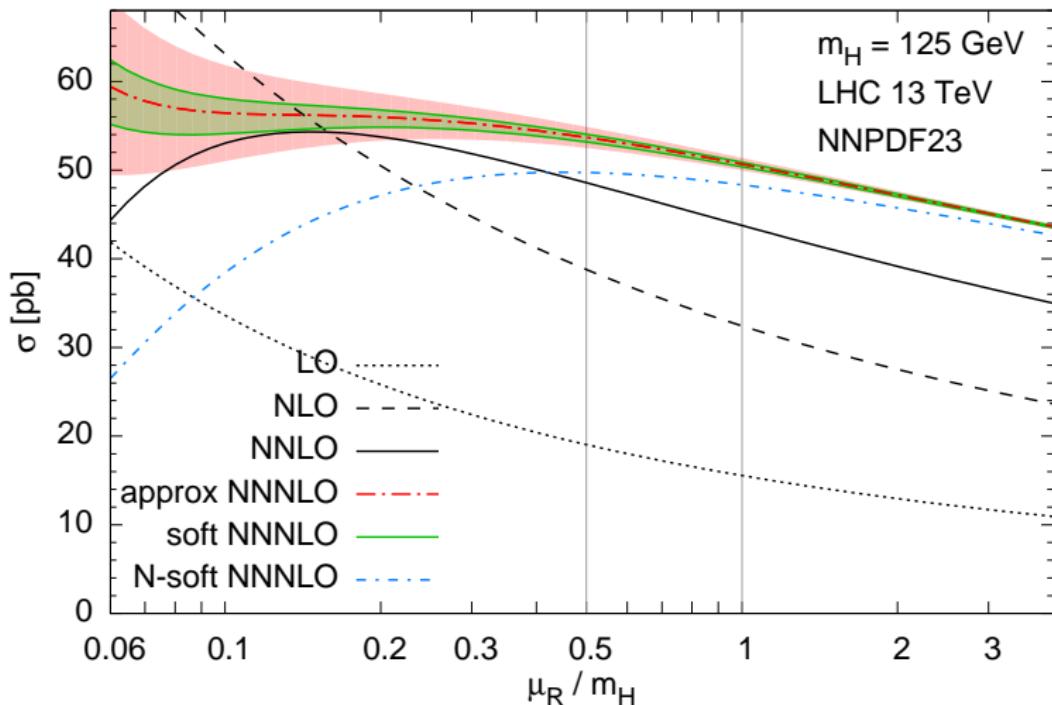
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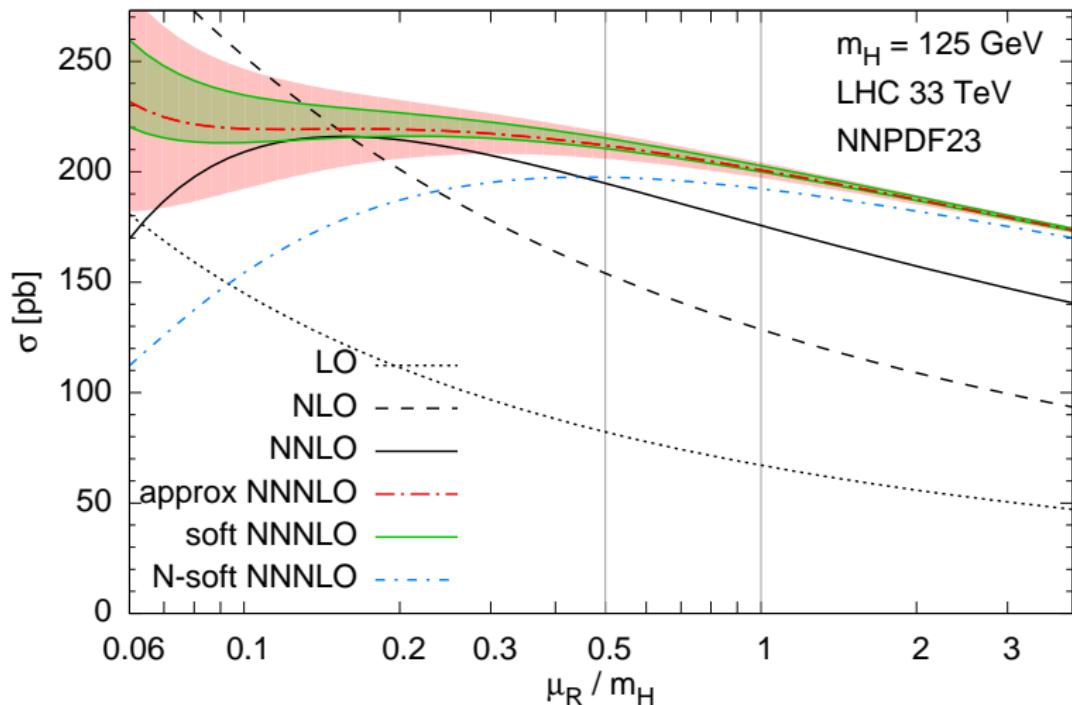
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# Perturbative expansion

$$C_{gg}(N, \alpha_s) = 1 + \alpha_s C^{(1)}(N) + \alpha_s^2 C^{(2)}(N) + \alpha_s^3 C^{(3)}(N) + \dots$$

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