News in POWHEG and MINLO

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[*]based mainly on [1206.3572, 1212.4504, 1309.0017]

- MiNLO: Multiscale Improved NLO

  [Hamilton,Nason,Zanderighi,1206.3572]

- NLOPS merging of X @ NLO and $X + 1j$ @ NLO

  [Hamilton,Nason,Oleari,Zanderighi,1212.4504]

- NNLOPS simulation of Higgs production

  [Hamilton,Nason,ER,Zanderighi,1309.0017]

- News in POWHEG BOX

MiNLO: Multiscale Improved NLO

- goal: method to a-priori choose scales in NLO computation
- relevant for processes with widely different scales (*e.g.* $X+$ jets close to Sudakov regions)

How?

- At LO, the CKKW procedure allows to take these effects into account: modify the LO weight $B(\mathbf{\Phi}_n)$ in order to include (N)LL effects.

$\Rightarrow$ "Use CKKW" on top of NLO computation that potentially involves many scales

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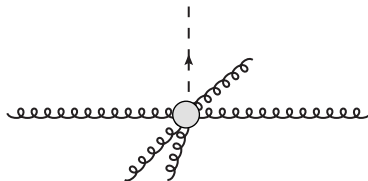
Next-to-Leading Order accuracy needs to be preserved

1. Scale dependence shows up at NNLO ["scale compensation"]:

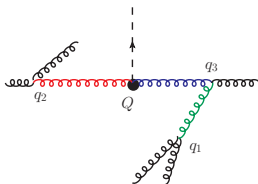$$O(\mu') - O(\mu) = \mathcal{O}(\alpha_{\mathrm{S}}^{n+2}) \quad \text{if} \quad O \sim \alpha_{\mathrm{S}}^n \quad \text{at LO}$$

2. Away from soft-collinear regions, exact NLO recovered:

$$O_{\mathrm{MiNLO}} = O_{\mathrm{NLO}} + \mathcal{O}(\alpha_{\mathrm{S}}^{n+2})$$

- Find "most-likely" shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$
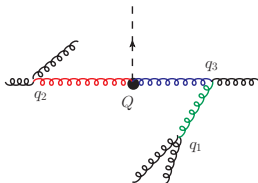
From CKKW to MiNLO

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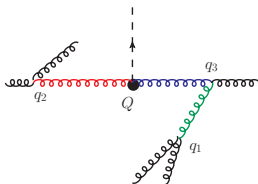

- Evaluate α_S at nodal scales

$$\alpha_S^n(\mu_R)B(\boldsymbol{\Phi}_n) \Rightarrow \alpha_S(q_1)\alpha_S(q_2)...\alpha_S(q_n)B(\boldsymbol{\Phi}_n)$$

* scale compensation requires $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$ in V

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- Sudakov FFs in internal and external lines of Born "skeleton"

$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n) \times \{\Delta(Q_0, Q)\Delta(Q_0, q_i)...\}$$

  * Upon expansion, $\mathcal{O}(\alpha_S^{n+1})$ (log) terms are introduced, and need to be removed

$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n)\Big(1 - \Delta^{(1)}(Q_0, Q) - \Delta^{(1)}(Q_0, q_i) + ...\Big)$$

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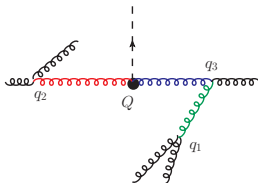* scale compensation requires $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$ in V

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✓ $X+$ jets cross-section finite without generation cuts
  $\Rightarrow \bar{B}$ with MiNLO prescription: ideal starting point for NLOPS (POWHEG) for $X+$ jets

MiNLO: example

Example, in 1 line: $H + 1$ jet

- Pure NLO:

$$d\sigma = \bar{B} \, d\mathbf{\Phi}_n = \alpha_{\mathrm{s}}^3(\mu_R) \Big[B + \alpha_{\mathrm{s}} V(\mu_R) + \alpha_{\mathrm{s}} \int d\Phi_{\mathrm{rad}} R \Big] \, d\mathbf{\Phi}_n$$

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- MiNLO:

$$\bar{B} = \alpha_{\rm s}^2(M_H)\alpha_{\rm s}(q_T)\Delta_g^2(q_T, M_H)\Big[B\Big(1 - 2\Delta_g^{(1)}(q_T, M_H)\Big) + \alpha_{\rm s}V(\bar{\mu}_R) + \alpha_{\rm s}\int d\Phi_{\rm rad}R\Big]$$
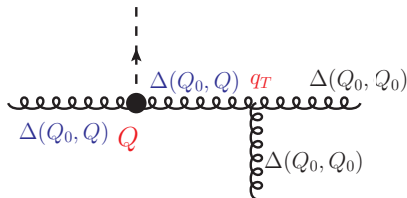
$$^\star \ \bar{\mu}_R = (M_H^2 q_T)^{1/3}$$

$$^\star \ \log\Delta_{\rm f}(q_T, Q) = -\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2}\frac{\alpha_{\rm s}(q^2)}{2\pi}\Big[A_f \log\frac{Q^2}{q^2} + B_f\Big]$$

$$^\star \ \Delta_{\rm f}^{(1)}(q_T, Q) = -\frac{\alpha_{\rm s}}{2\pi}\Big[\frac{1}{2}A_{1,{\rm f}}\log^2\frac{Q^2}{q_T^2} + B_{1,{\rm f}}\log\frac{Q^2}{q_T^2}\Big]$$

$$^\star \ \mu_F = Q_0(= q_T)$$

- Accuracy of `BJ+MiNLO` for inclusive observables carefully investigated
- `BJ+MiNLO` describes inclusive boson observables at relative order α_S wrt $B + 0j$ at LO
- However, to reach genuine NLO, higher terms must be order α_S^2, *i.e.*

$$O_{\mathrm{VJ+MiNLO}} = O_{\mathrm{V@NLO}} + \mathcal{O}(\alpha_S^2)$$

  if O is inclusive. "Original MiNLO" contains ambiguous $\mathcal{O}(\alpha_S^{3/2})$ terms
- Possible to improve `BJ+MiNLO` such that NLO $B + 0j$ is recovered, without spoiling NLO accuracy for $B + 1j$.
  - proof based on careful comparisons of general resummation formula with MiNLO ingredients
  - need to include B_2 in Sudakovs
  - need to evaluate $\alpha_S^{(\mathrm{NLO})}$ in `BJ+MiNLO` at scale q_T, and $\mu_F = q_T$

Effectively it is like if we merged $\mathrm{NLO}^{(0)}$ and $\mathrm{NLO}^{(1)}$ samples, without merging different samples (no merging scale used).

- Resummation formula

$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0 \frac{d}{dq_T^2}\Big\{[C_{ga} \otimes f_a](x_A, q_T) \times [C_{gb} \otimes f_b](x_B, q_T) \times \exp S(q_T, Q)\Big\} + R_f$$

- NLO$^{(0)}$ if $C_{ij}^{(1)}$ included and R_f is LO$^{(1)}$
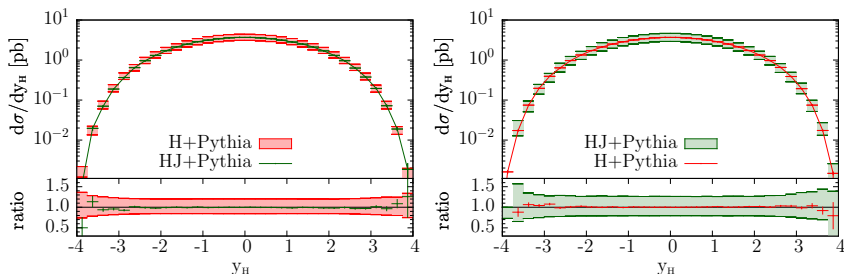
- Take derivative, then compare with `MiNLO`:

$$\sim \sigma_0 \frac{1}{q_T^2}[\alpha_{\rm S}, \alpha_{\rm S}^2, \alpha_{\rm S}^3, \alpha_{\rm S}^4, \alpha_{\rm S}L, \alpha_{\rm S}^2 L, \alpha_{\rm S}^3 L, \alpha_{\rm S}^4 L]\exp S(q_T, Q) \quad L = \log(Q^2/q_T^2)$$

- can be shown that

$$\int^{Q^2} \frac{dq_T^2}{q_T^2} L^m \alpha_{\rm S}^{\,n}(q_T) \exp S \sim \big(\alpha_{\rm S}(Q^2)\big)^{n-(m+1)/2}$$

- if I drop B_2 in `MiNLO` Δ_g, I miss a term $(1/q_T^2)\alpha_{\rm S}^2 B_2 \exp S$

- upon integration, violate NLO$^{(0)}$ by a term $\mathcal{O}(\alpha_{\rm S}^{3/2})$

- "wrong" scale in $\alpha_{\rm S}^{(\rm NLO)}$ in `MiNLO` produces again same error

Alternative proof also available in the paper.

- "H+Pythia": standalone POWHEG ($gg \to H$) + PYTHIA (PS level) [7pts band, $\mu = m_H$]
- "HJ+Pythia": HJ-MiNLO* + PYTHIA (PS level) [7pts band, μ from MiNLO]
- ✓ very good agreement (both value and band)

Notice: band is $\sim 20 - 30\%$
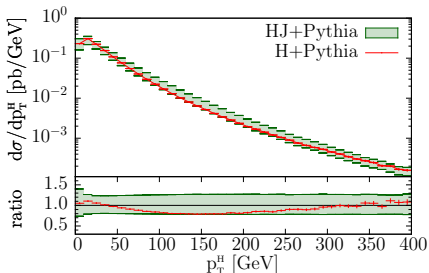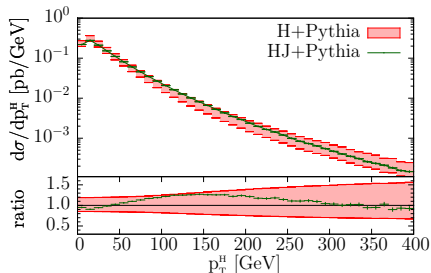
- Good agreement
- At high p_T, bands as expected (LO vs NLO)
  (POWHEG $(gg \to H)$ with hfact $= m_H/1.2$, YR1)
- Low p_T shape difference: different NNLL terms in MiNLO Sudakovs
- Bands at low p_T: "H+Pythia" band spurious (S-events, i.e. inherit property of full $\bar{B}$)

- `HJ-MiNLO`* differential cross section $(d\sigma/dy)_{\mathrm{HJ-MiNLO}}$ is NLO accurate

$$W(y) = \frac{\left(\frac{d\sigma}{dy}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\mathrm{HJ-MiNLO}}} = \frac{c_2\alpha_{\mathrm{S}}^2 + c_3\alpha_{\mathrm{S}}^3 + c_4\alpha_{\mathrm{S}}^4}{c_2\alpha_{\mathrm{S}}^2 + c_3\alpha_{\mathrm{S}}^3 + d_4\alpha_{\mathrm{S}}^4} \simeq 1 + \frac{c_4 - d_4}{c_2}\alpha_{\mathrm{S}}^2 + \mathcal{O}(\alpha_{\mathrm{S}}^3)$$

- thus, reweighting each event with this factor, we get NNLO+PS
  * obvious for y_H, by construction
  * α_{S}^4 accuracy of `HJ-MiNLO`* in 1-jet region not spoiled, because $W(y) = 1 + \mathcal{O}(\alpha_{\mathrm{S}}^2)$
  * if we had $\mathrm{NLO}^{(0)} + \alpha_{\mathrm{S}}^{3/2}$, 1-jet region spoiled because

$$[\mathrm{NLO}^{(1)}]_{\mathrm{NNLOPS}} = \mathrm{NLO}^{(1)} + \mathcal{O}(\alpha_{\mathrm{S}}^{4.5})$$

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* Variants for W are possible: with

$$W(y, p_T) = h(p_T)\frac{\int d\sigma^{\mathrm{NNLO}}\delta(y - y(\boldsymbol{\Phi})) - \int d\sigma_B^{\mathrm{MiNLO}}\delta(y - y(\boldsymbol{\Phi}))}{\int d\sigma_A^{\mathrm{MiNLO}}\delta(y - y(\boldsymbol{\Phi}))} + (1 - h(p_T))$$

$$d\sigma_A = d\sigma\ h(p_T), \qquad d\sigma_B = d\sigma\ (1 - h(p_T)), \qquad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}$$

we get exactly $(d\sigma/dy)_{\mathrm{NNLOPS}} = (d\sigma/dy)_{\mathrm{NNLO}}$ (no α_{S}^5 terms)
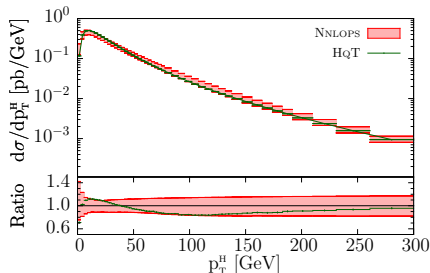* h essentially controls where the NNLO/NLO K-factor is spread.

- NNLO with $\mu = m_H/2$, HJ-MiNLO "core scale" m_H
- events reweighted at the LH level, then showered with PYTHIA (PS level)
- (7×3) pts scale var. in NNLOPS, 7pts in NNLO
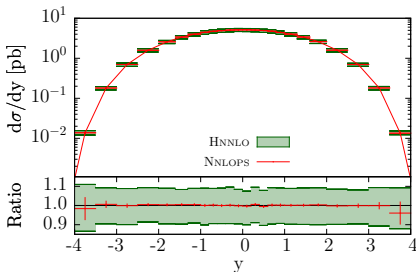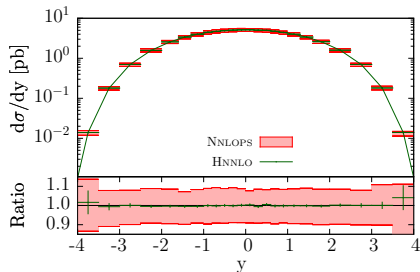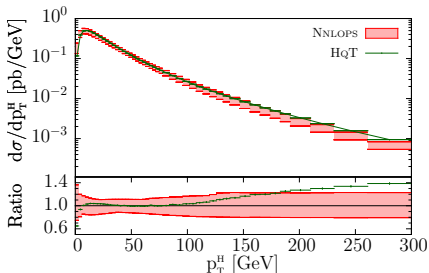

Notice: band is 10%

[Until and including $\mathcal{O}(\alpha_S^4)$, PS effects don't affect y_H (first 2 emissions controlled properly at $\mathcal{O}(\alpha_S^4)$ by MiNLO+POWHEG)]
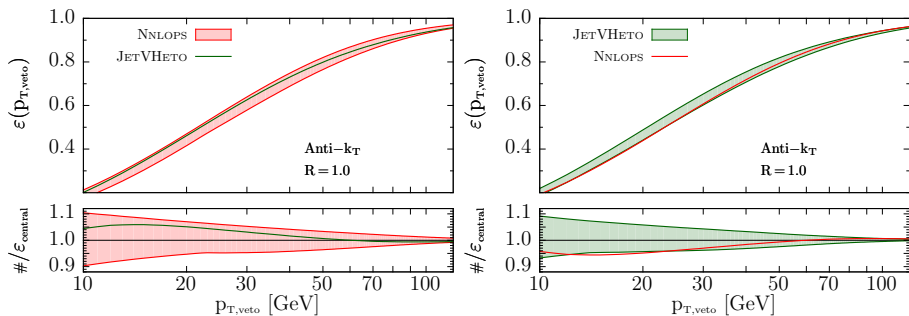
NNLO+PS (p_T^H)

$\beta = \infty$ (W indep. of p_T) $\beta = 1/2$

- HqT: NNLL+NNLO, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\rm res} \equiv m_H/2$
- $\beta = 1/2 \ \& \ \infty$: uncertainty bands of HqT (not shown) contain NNLOPS at low-/moderate p_T
- $\beta = 1/2$: NNLOPS tail → NLOPS tail [$W(y, p_T \gg m_H) \to 1$] larger band (affected just marginally by NNLO, so it's ∼ genuine NLO band)
- $\beta = 1/2$: HqT tail harder than NNLOPS tail ($\mu_{\rm HqT} < "\mu_{\rm MiNLO}"$)
- $\beta = 1/2$: very good agreement with HqT resummation

$$\varepsilon\left(p_{\mathrm{T,veto}}\right) = \frac{\Sigma(p_{\mathrm{T,veto}})}{\sigma^{\mathrm{tot}}} = \frac{1}{\sigma^{\mathrm{tot}}} \int d\sigma \; \theta\left(p_{\mathrm{T,veto}} - p_{\mathrm{T}}^{\mathrm{j_1}}\right)$$

- JetVHeto: NNLL resum, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\mathrm{res}} \equiv m_H/2$, (a)-scheme only
- nice agreement, differences never more than 5-6 %

POWHEG BOX: news and technical improvements

Number of processes still increasing (~ 30)

Automation:

- Interface to MadGraph 4 [Frederix]: automatically builds subprocesses list, B, B_{ij}, $B^{\mu\nu}$, R and large-N Born color structures.
  - Used to build the code for Hj and Hjj [Campbell, Ellis, Frederix, Nason, Oleari], with virtuals from MCFM.
- interface to GoSam [Luisoni, Nason, Oleari, Tramontano]: automatically write the code for 1-loop amplitudes, and interface it via BLHA - Used to study VH and VHj

PDF and scale uncertainties:

- Generate MC samples for different scale choices, and, even more, for different PDFs, is very time consuming
- Primitive reweighting facility now superseded by new mechanism   [Hamilton,Nason,ER]

V2 ready and under testing; MiNLO will also be the default for X+jets processes.

Conclusions

1. **MiNLO:**
   - assign scales and Sudakov FF in $B + n$ jets NLO computations
   - well-behaved in Sudakov regions
   - NLO away from Sudakov regions
   - ideal as starting point for `POWHEG`

2. **Improved MiNLO:**
   - $B + 1$ jet improved MiNLO allows to merge $\text{NLO}^{(0)}$ and $\text{NLO}^{(1)}$ samples, without merging (no merging scale used)
   - merging for higher multiplicity requires further study, it'll take some time

3. **NNLOPS:**
   - MiNLO allows to define a procedure to reach NNLOPS
   - Shown results for Higgs production

- Progress in POWHEG BOX:
   - list of processes steadily increasing
   - automation (via interfaces to MG4 / GoSam)
   - several technical improvements to keep up with theoretical and experimental needs
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Thanks for your attention!

- Start from ME weight: $B(\mathbf{\Phi}_n)$
- Find "most-likely" shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$
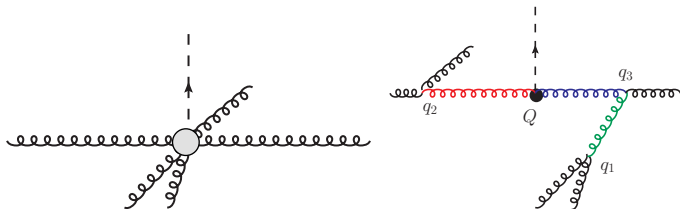
- New weight:

$$\alpha_{\rm S}^5(Q) B(\mathbf{\Phi}_3) \quad \rightarrow \quad \alpha_{\rm S}^2(Q) B(\mathbf{\Phi}_3) \frac{\Delta_g(Q_0, Q)}{\Delta_g(Q_0, q_2)} \frac{\Delta_g(Q_0, Q)}{\Delta_g(Q_0, q_3)} \frac{\Delta_g(Q_0, q_3)}{\Delta_g(Q_0, q_1)}$$
$$\Delta_g(Q_0, q_2) \Delta_g(Q_0, q_2) \Delta_g(Q_0, q_3) \Delta_g(Q_0, q_1) \Delta_g(Q_0, q_1)$$
$$\alpha_{\rm S}(q_1) \alpha_{\rm S}(q_2) \alpha_{\rm S}(q_3)$$

where typically

$$\log \Delta_{\rm f}(q_T, Q) = - \int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_{\rm S}(q^2)}{2\pi} \Big[A_{1,\rm f} \log \frac{Q^2}{q^2} + B_{1,\rm f} \Big]$$

- Fill phase space below Q_0 with vetoed shower

Backup (2)

MiNLO: All α_S in Born term are chosen with CKKW (local) scales $q_1, ..., q_n$

$$\alpha_S^n(\mu_R)B \Rightarrow \alpha_S(q_1)\alpha_S(q_2)...\alpha_S(q_n)B$$

- Normal NLO structure ($\mu = \mu_R$):

$$\sigma(\mu) = \underbrace{\alpha_S^n(\mu)B}_{\text{Born}} + \underbrace{\alpha_S^{n+1}(\mu)\Big(C + nb_0\log(\mu^2/Q^2)B\Big)}_{\text{Virtual}} + \underbrace{\alpha_S^{n+1}(\mu)R}_{\text{Real}}$$

- Explicit μ dependence of virtual term as required by RG invariance:

$$\alpha_S^n(\mu')B = \Big[\alpha_S(\mu) \,\boxed{-nb_0\alpha_S^{n+1}(\mu)\log(\mu'^2/\mu^2)}\,\Big]B + \mathcal{O}(\alpha_S^{n+2})$$

$$\text{Virtual}(\mu') = \text{Virtual}(\mu) \,\boxed{+\alpha_S^{n+1}(\mu)nb_0\log(\mu'^2/\mu^2)}\, B + \mathcal{O}(\alpha_S^{n+2})$$

$$\Rightarrow \sigma(\mu') - \sigma(\mu) = \mathcal{O}(\alpha_S^{n+2})$$

- In MiNLO "scale compensation" kept if

$$\Big(C + nb_0\log(\mu_R^2/Q^2)B\Big) \Rightarrow \Big(C + nb_0\log(\bar{\mu}_R^2/Q^2)B\Big)$$

with $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$

Few technicalities for original MiNLO:

- $\mu_F = Q_0$ (as in CKKW)
- Cluster with CKKW also V and R kinematics
  - Actual implementation uses FKS mapping for first cluster of $\mathbf{\Phi}_{n+1}$
  - Ignore CKKW Sudakov for 1st clustering of $\mathbf{\Phi}_{n+1}$ (inclusive on extra radiation)
- Some freedom in choice of $\alpha_S^{(n+1)}$ (entering V, R and $\Delta^{(1)}$) (not free for MiNLO merging)
- Used full NLL-improved Sudakovs (A_1, B_1, A_2)

p_T^H spectrum:

- "$\mu_{\text{HJ-MiNLO}} = m_H, m_H, p_T$"
- At high p_T, $\mu_{\text{HJ-MiNLO}} = p_T$
- If $\beta = 1/2$, NNLOPS $\rightarrow$ HJ-MiNLO at high p_T
- NNLO/NLO ~ 1.5, because HNNLO with $\mu = m_H/2$, $\mu_{\text{HJ-MiNLO,core}} = m_H$