

Kaon Decays in the Standard Model

Helmut Neufeld

University of Vienna

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Review: Kaon Decays in the Standard Model

V. Cirigliano, G. Ecker, H. N., A. Pich, J. Portoles, Rev. Mod. Phys. 84 (2012) 399

Discussion of all decay modes with BRs $> 10^{-11}$

- ▶ Theoretical framework
- ▶ Estimates of low energy constants
- ▶ Leptonic and semileptonic decays
- ▶ Nonleptonic decays
- ▶ Rare and radiative decays

Theoretical Framework

Short-distance description

SM $\rightarrow$ strangeness-changing transitions via W exchange

$M_K \ll M_W \rightarrow$ integrate out W $\rightarrow$ effective Lagrangian at $E = M_K$

- ▶ W exchange between quark current and leptonic current

$$\mathcal{L}_{\text{eff}}^{\text{semilept.}} = -\frac{G_F}{\sqrt{2}} S_{\text{EW}}^{1/2} \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell \bar{u}_i \gamma^\mu (1 - \gamma_5) V_{ij} d_j + h.c.$$

- ▶ W exchange between two quark currents

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu)$$

e.g. $Q_2 = \bar{s} \gamma^\mu (1 - \gamma_5) u \bar{u} \gamma_\mu (1 - \gamma_5) d$

gluonic corrections $\rightarrow Q_1, Q_3\text{-}Q_6$

virtual electromagnetic interactions $\rightarrow Q_7\text{-}Q_{10}$

Theoretical Framework

Short-distance description ctd.

- ▶ leptons in final state $\rightarrow Q_{11}-Q_{13}$

$$Q_{13} \equiv Q(\bar{\nu}\nu) = \bar{s}\gamma^\mu(1-\gamma_5)d \bar{\nu}\gamma_\mu(1-\gamma_5)\nu$$

- ▶ exchange of two W bosons (box diagram)

$$\mathcal{L}_{\text{eff}}^{\Delta S=2} = -\frac{G_F^2 M_W^2}{(4\pi)^2} C^{\Delta S=2}(\mu) Q^{\Delta S=2}(\mu)$$

$$Q^{\Delta S=2} = \bar{s}\gamma^\mu(1-\gamma_5)d \bar{s}\gamma_\mu(1-\gamma_5)d$$

Theoretical Framework

Chiral Perturbation Theory

Low energy effective theory of the SM

Construct the **most general** QFT consistent with the symmetries of the SM in terms of the asymptotic fields relevant at low energies

- ▶ chiral symmetry of QCD in the limit $m_u = m_d = m_s = 0$

$$SU(3)_L \times SU(3)_R \xrightarrow{\text{SSB}} SU(3)_V$$

→ 8 Goldstone bosons π, K, η

interaction of the pseudoscalars **vanishes** for $p \rightarrow 0$

- ▶ $m_u, m_d, m_s \neq 0 \rightarrow M^2 \sim Bm_q$
- ▶ chiral counting: orders of $p^2 \sim M^2 \rightarrow \text{LECs}$
- ▶ add additional degrees of freedom: photon, light leptons
- ▶ LECs **not** restricted by chiral symmetry

Chiral Lagrangian

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}_{p^4}^{\text{odd}}(0) + \mathcal{L}_{G_8 p^2}^{\Delta S=1}(1) + \mathcal{L}_{G_{27} p^2}^{\Delta S=1}(1)$ $+ \mathcal{L}_{G_8 e^2 p^0}^{\text{emweak}}(1) + \mathcal{L}_{e^2 p^0}^{\text{em}}(1) + \mathcal{L}_{\text{kin}}^{\text{lepton}}(0)$	$L = 0$
$+ \mathcal{L}_{p^4}(10) + \mathcal{L}_{p^6}^{\text{odd}}(23) + \mathcal{L}_{G_8 p^4}^{\Delta S=1}(22) + \mathcal{L}_{G_{27} p^4}^{\Delta S=1}(28)$ $+ \mathcal{L}_{G_8 e^2 p^2}^{\text{emweak}}(14) + \mathcal{L}_{e^2 p^2}^{\text{em}}(13) + \mathcal{L}_{e^2 p^2}^{\text{lepton}}(5)$	$L \leq 1$
$+ \mathcal{L}_{p^6}(90)$	$L \leq 2$

Estimates of Low-Energy Constants

Matching between CHPT and the underlying SM

- ▶ LECs from low-energy data
- ▶ Resonance saturation
- ▶ Lattice determinations

Leptonic and Semileptonic Decays

- ▶ $K_{\ell 2}$ (and $\pi_{\ell 2}$) decays
 - ▶ Electromagnetic corrections
 - ▶ Extraction of V_{us}/V_{ud}
 - ▶ The ratio $R_{e/\mu}^{(K,\pi)}$
- ▶ $K_{\ell 2\gamma}$
- ▶ $K_{\ell 3}$
 - ▶ Electromagnetic effects in $K_{\ell 3(\gamma)}$ decays
 - ▶ Quark mass ratios and $K_{\ell 3}$ decays
 - ▶ Form factors and phase space integrals
 - ▶ SU(3) breaking effects in $f_+^{K^0\pi^-}(0)$
 - ▶ Determination of V_{us} and CKM unitarity tests
 - ▶ T violation in $K_{\mu 3}$ decays
- ▶ $K_{\ell 3\gamma}$
- ▶ $K_{\ell 4}$
- ▶ K_{e5}

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 - ▶ T violation in $K_{\mu 3}$ decays
- ▶ $K_{\ell 3\gamma}$
- ▶ $K_{\ell 4} \rightarrow \pi\pi$ scattering lengths from K_{e4} data
- ▶ K_{e5}

Nonleptonic decays

- ▶ $K \rightarrow \pi\pi$
 - ▶ $\pi\pi$ phase shifts from $K \rightarrow \pi\pi$ decays
 - ▶ ϵ'/ϵ
 - ▶ CP violation in K^0 - $\overline{K}^0$ mixing
- ▶ $K \rightarrow 3\pi$
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 - ▶ $\pi\pi$ scattering lengths from $K \rightarrow 3\pi$ decays near threshold

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Rare and radiative decays

- ▶ $K \rightarrow \pi \nu \bar{\nu}$
- ▶ $K \rightarrow \pi \pi \nu \bar{\nu}$
- ▶ $K \rightarrow \gamma^{(*)} \gamma^{(*)}$
 - ▶ $K_S \rightarrow \gamma \gamma$
 - ▶ $K_L \rightarrow \gamma \gamma$
 - ▶ $K_S \rightarrow \gamma \ell^+ \ell^-$
 - ▶ $K_L \rightarrow \gamma \ell^+ \ell^-$
 - ▶ $K_L \rightarrow \gamma \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$
- ▶ $K \rightarrow \ell^+ \ell^-$
 - ▶ $K_S \rightarrow \ell^+ \ell^-$
 - ▶ $K_L \rightarrow \ell^+ \ell^-$

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- ▶ $K_L \rightarrow \gamma \ell^+ \ell^-$

- ▶ $K_L \rightarrow \gamma \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$

- ▶ $K \rightarrow \ell^+ \ell^-$

- ▶ $K_S \rightarrow \ell^+ \ell^-$

- ▶ $K_L \rightarrow \ell^+ \ell^-$

Rare and radiative decays ctd.

- ▶ $K \rightarrow \pi\pi\gamma^{(*)}$

- ▶ $K^+ \rightarrow \pi^+\pi^0\gamma$

- ▶ $K_L \rightarrow \pi^+\pi^-\gamma$

- ▶ $K_S \rightarrow \pi^+\pi^-\gamma$

- ▶ $K \rightarrow \pi\pi\ell^+\ell^-$

- ▶ Other decays

- ▶ $K^0 \rightarrow \gamma\gamma\gamma$

- ▶ $K_L \rightarrow \gamma\gamma\ell^+\ell^-$

- ▶ $K_L \rightarrow \gamma\nu\bar{\nu}$

- ▶ $K_S \rightarrow \ell_1^+\ell_1^-\ell_2^+\ell_2^-$

- ▶ $K_L, K_S \rightarrow \pi^0\pi^0\gamma$

- ▶ $K_L, K_S \rightarrow \pi^0\pi^0\gamma\gamma$

- ▶ $K \rightarrow 3\pi\gamma$

$K_{\ell 2}$ (and $\pi_{\ell 2}$) decays

V_{us}/V_{ud} from decay widths and lattice data, V. Cirigliano, H. N., PLB 700 (2011) 7

$$\frac{\Gamma_{K_{\ell 2}(\gamma)}}{\Gamma_{\pi_{\ell 2}(\gamma)}} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{F_K^2}{F_\pi^2} \frac{M_{K^\pm} (1 - m_\ell^2/M_{K^\pm}^2)^2}{M_{\pi^\pm} (1 - m_\ell^2/M_{\pi^\pm}^2)^2} (1 + \delta_{\text{EM}} + \delta_{\text{SU}(2)})$$

F_K/F_π in the isospin limit ($m_u = m_d$, $e = 0$)

$$\delta_{\text{EM}} = \frac{\alpha}{\pi} \left(F(m_\ell/M_K) - F(m_\ell/M_\pi) + \frac{3-Z}{4} \ln \frac{M_K^2}{M_\pi^2} \right)$$

$$= -0.0069(17), \quad M_{\pi^\pm}^2 - M_{\pi^0}^2 = 8\pi\alpha Z F_0^2$$

$$\delta_{\text{SU}(2)} = \sqrt{3} \varepsilon \left[-\frac{4}{3} (F_K/F_\pi - 1) + \frac{1}{3(4\pi)^2 F_0^2} \left(M_K^2 - M_\pi^2 - M_\pi^2 \ln \frac{M_K^2}{M_\pi^2} \right) \right]$$

$$= -0.0044(5)(11),$$

$$\varepsilon = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \bar{m}} = 0.0116(13), \quad F_K/F_\pi = 1.193(6) \text{ from FLAG}$$

Update of $|V_{us}/V_{ud}|$

$$\begin{aligned}\frac{|V_{us}|}{|V_{ud}|} \frac{F_K}{F_\pi} &= \left(\frac{\Gamma_{K\ell 2(\gamma)} M_{\pi^\pm}}{\Gamma_{\pi\ell 2(\gamma)} M_{K^\pm}} \right)^{1/2} \frac{1 - m_\ell^2/M_{\pi^\pm}^2}{1 - m_\ell^2/M_{K^\pm}^2} (1 - \delta_{\text{EM}}/2 - \delta_{\text{SU}(2)}/2) \\ &= 0.23922(25) \times \left(\frac{\Gamma_{K\ell 2(\gamma)}}{\Gamma_{\pi\ell 2(\gamma)}} \right)^{1/2} \\ &= 0.2765(5)\end{aligned}$$

Experimental input:

$$\Gamma_{\pi\mu 2(\gamma)} = 38.408(7) (\mu\text{s})^{-1}, \quad \Gamma_{K\mu 2(\gamma)} = 51.33(13) (\mu\text{s})^{-1} \quad \text{PDG 2012}$$

Lattice input:

$$F_K/F_\pi = 1.193(6) \text{ (FLAG)} \rightarrow |V_{us}/V_{ud}| = 0.2318(12)$$

Isospin Breaking on the Lattice

G. M. de Divitiis et al. (RM123 Collaboration), JHEP (2012) 1204

Method: expand path integral in powers of $m_d - m_u$

- ▶ kaon masses
- ▶ kaon decay constant
- ▶ $K_{\ell 3}$ form factor
- ▶ neutron-proton mass splitting

$\delta_{\text{SU}(2)}^{\text{lattice}} = -0.0078(7)$ to be compared with $\delta_{\text{SU}(2)}^{\text{CHPT}} = -0.0044(12)$

The ratio $R_{e/\mu}^K$

V. Cirigliano, I. Rosell, JHEP 10 (2007) 005; Phys. Rev. Lett. 99 (2007) 231801

$$\begin{aligned} R_{e/\mu}^K &= \frac{\Gamma_{K_{e2}(\gamma)}}{\Gamma_{K_{\mu 2}(\gamma)}} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{M_K^2 - m_e^2}{M_K^2 - m_\mu^2}\right)^2 (1 + \delta_{\text{EM}}) \\ &= 2.477(1) \times 10^{-5} \end{aligned}$$

- ▶ Two-loop calculation in CHPT with virtual photons and leptons
- ▶ Large- N_C calculation of an associated counterterm
- ▶ Summation of leading logarithms $\alpha^n \ln^n(m_\mu/m_e)$ (Marciano, Sirlin 1993)

Most recent NA62 result: $R_{e/\mu}^K = 2.488(10) \times 10^{-5}$

Full 2007 data set: article to be submitted in 2012, G. Ruggiero, CKM 2012

$K \rightarrow \pi \ell \nu$ ($K_{\ell 3}$) decays

$$\Gamma_{K_{\ell 3}(\gamma)} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{128 \pi^3} S_{\text{EW}} |f_+^{K^0 \pi^-}(0)|^2 I_{K\ell}^{(0)}(\lambda_i) \left(1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU}(2)}^{K\pi}\right)$$

$$C_K = 1 \text{ for } K_{\ell 3}^0, \quad C_K = 1/\sqrt{2} \text{ for } K_{\ell 3}^+$$

$I_{K\ell}^{(0)}$ phase space integral

$$\delta_{\text{SU}(2)}^{K\pi} = (f_+^{K\pi}(0)/f_+^{K^0 \pi^-}(0))^2 - 1$$

$$\text{Long-distance EM corrections} \quad \delta_{\text{EM}}^{K\ell} = \delta_{\text{EM}}^{K\ell}(\mathcal{D}_3) + \delta_{\text{EM}}^{K\ell}(\mathcal{D}_{4-3})$$

$$\text{Short-distance factor} \quad S_{\text{EW}} = 1 + \frac{2\alpha}{\pi} \left(1 - \frac{\alpha_s}{4\pi}\right) \ln \frac{M_Z}{M_\rho} + \mathcal{O}\left(\frac{\alpha\alpha_s}{\pi^2}\right) = 1.0223(5)$$

A. Sirlin, 1977; 1981; W.J. Marciano, A. Sirlin, 1993

Electromagnetic effects in $K_{\ell 3}(\gamma)$ decays

V. Cirigliano, M. Giannotti, H. N., JHEP 11 (2008) 006

	$\delta_{\text{EM}}^{K\ell}(\mathcal{D}_3)(\%)$	$\delta_{\text{EM}}^{K\ell}(\mathcal{D}_{4-3})(\%)$	$\delta_{\text{EM}}^{K\ell}(\%)$
K_{e3}^0	0.50	0.49	0.99 ± 0.22
$K_{e3}^{\pm}$	-0.35	0.45	0.10 ± 0.25
$K_{\mu 3}^0$	1.38	0.02	1.40 ± 0.22
$K_{\mu 3}^{\pm}$	0.007	0.009	0.016 ± 0.25

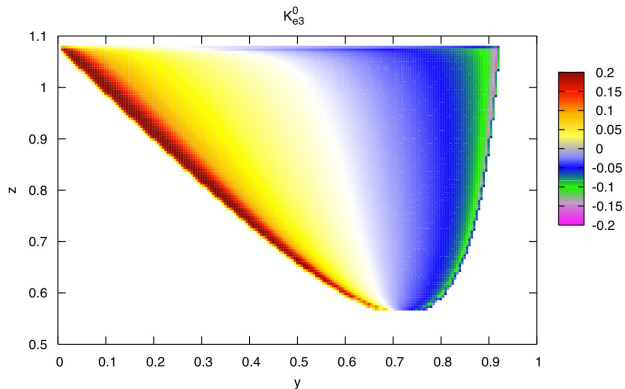
Electromagnetic effects in $K_{\ell 3}(\gamma)$ decays ctd.

Differential decay distribution

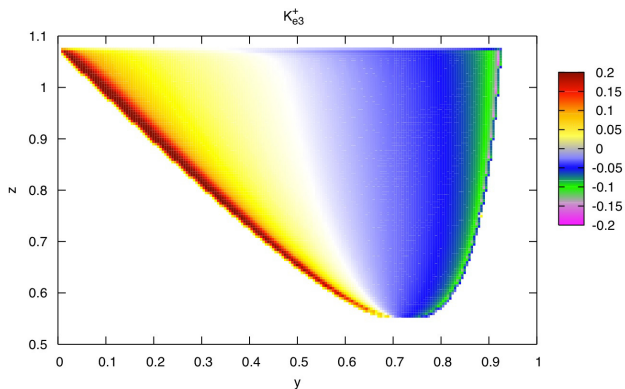
$$\frac{d^2\Gamma}{dy\,dz} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{128\pi^3} S_{\text{EW}} |f_+^{K\pi}(0)|^2 \left[\bar{\rho}^{(0)}(y, z) + \delta\bar{\rho}^{\text{EM}}(y, z) \right]$$

$$y = 2p_K \cdot p_\ell / M_K^2 = 2E_\ell / M_K, \quad z = 2p_\pi \cdot p_K / M_K^2 = 2E_\pi / M_K$$

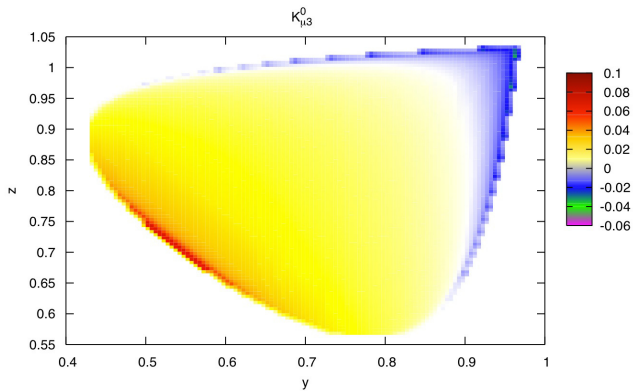
Ratio $\delta\bar{\rho}^{\text{EM}}(y, z)/\bar{\rho}^{(0)}(y, z)$ for K_{e3}^0



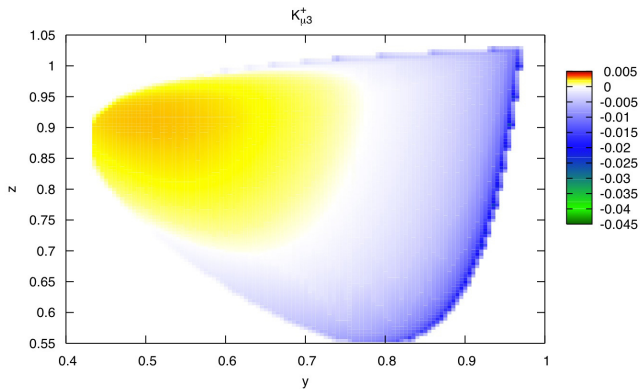
Ratio $\delta\bar{\rho}^{\text{EM}}(y, z)/\bar{\rho}^{(0)}(y, z)$ for K_{e3}^+



Ratio $\delta\bar{\rho}^{\text{EM}}(y, z)/\bar{\rho}^{(0)}(y, z)$ for $K_{\mu 3}^0$



Ratio $\delta\bar{\rho}^{\text{EM}}(y, z)/\bar{\rho}^{(0)}(y, z)$ for $K_{\mu 3}^+$



Isospin Violation in $K_{\ell 3}$ decays

CHPT (quark mass ratios & deviation from Dashen's limit):

$$\delta_{\text{SU}(2)}^{K^{\pm}\pi^0} = 0.058(8)$$

A. Kastner, H. N., 2008

electromagnetic LECs from B. Ananthanarayan, B. Moussallam, 2004

Combine recent $K_{\ell 3}$ data with

$$\delta_{\text{SU}(2)}^{K^{\pm}\pi^0} = \frac{2\Gamma_{K_{\ell 3}^+}}{\Gamma_{K_{\ell 3}^0}} \frac{I_{K^0\ell}}{I_{K^+\ell}} \left(\frac{M_{K^0}}{M_{K^+}} \right)^5 - 1 - \left(\delta_{\text{EM}}^{K^+\ell} - \delta_{\text{EM}}^{K^0\ell} \right)$$

$$\rightarrow \delta_{\text{SU}(2)}^{K^{\pm}\pi^0} = 0.054(8)$$

Determination of V_{us} and CKM unitarity tests

Experimental average: $|V_{us}|f_+(0) = 0.2163(5)$

Recent update M. Moulson, CKM 2012, Cincinnati

Lattice (RBC/UKQCD 2010): $f_+(0) = 0.9599(34)(^{+31}_{-47})(14)$

$$\rightarrow |V_{us}| = 0.2255(5)_{\text{exp}}(12)_{\text{th}}$$

Fit of V_{us} , V_{ud}

V. Cirigliano, H. N., PLB 700 (2011) 7

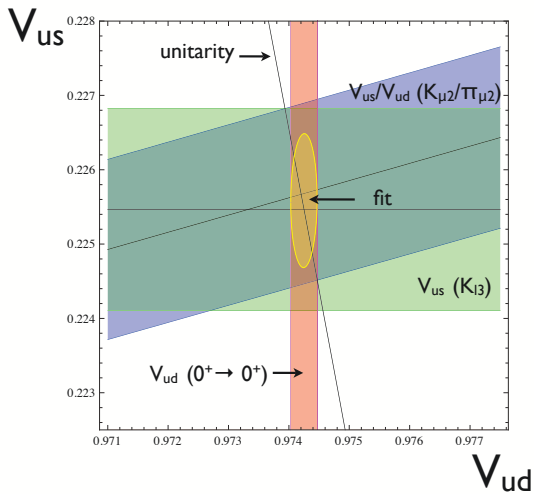
Input:

$|V_{ud}| = 0.97425(22)$ from $0^+ \rightarrow 0^+$, J.C. Hardy, I.S. Towner, 2009

$|V_{us}/V_{ud}|$ from $K_{\ell 2}/\pi_{\ell 2}$

$|V_{us}|_{K_{\ell 3}} = 0.2255(5)_{\text{exp}}(12)_{\text{th}}$ from $K_{\ell 3}$ decays and $f_+(0)$ from lattice

$$\rightarrow |V_{ud}| = 0.97425(22) , \quad |V_{us}| = 0.2256(9)$$



CKM unitarity test

$$\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.0001(6)$$

→ strong bound on effective scale Λ of dimension-six operators violating quark-lepton universality of charged current weak interactions: $\Lambda > 11 \text{ TeV}$ (90% CL)

V. Cirigliano, J.P. Jenkins, M. González-Alonso, 2010

$K \rightarrow \pi\pi\ell\nu$ ($K_{\ell 4}$) decays

$\pi\pi$ scattering lengths from K_{e4} data

$\pi\pi$ s-wave scattering lengths:

$$a_0 = \underbrace{\frac{7M_\pi^2}{32\pi F_\pi^2}}_{\text{tree}} = 0.159 \rightarrow \underbrace{0.200}_{\text{one-loop}} \rightarrow \underbrace{0.216}_{\text{two-loop}} \rightarrow \underbrace{0.220(5)}_{\text{disp. th. + CHPT}}$$

$$a_2 = \underbrace{-\frac{M_\pi^2}{16\pi F_\pi^2}}_{\text{tree}} = -0.0454 \rightarrow \underbrace{-0.0445}_{\text{one-loop}} \rightarrow \underbrace{-0.0445}_{\text{two-loop}} \rightarrow \underbrace{-0.0444(10)}_{\text{disp. th. + CHPT}}$$

most precise prediction: G. Colangelo, J. Gasser, H. Leutwyler, 2001

Results from NA48/2 K_{e4} data

J. R. Batley et al., 2010

two-parameter fit:

$$a_0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}(37)_{\text{th}} \quad a_2 = -0.0432(86)(34)(28)$$

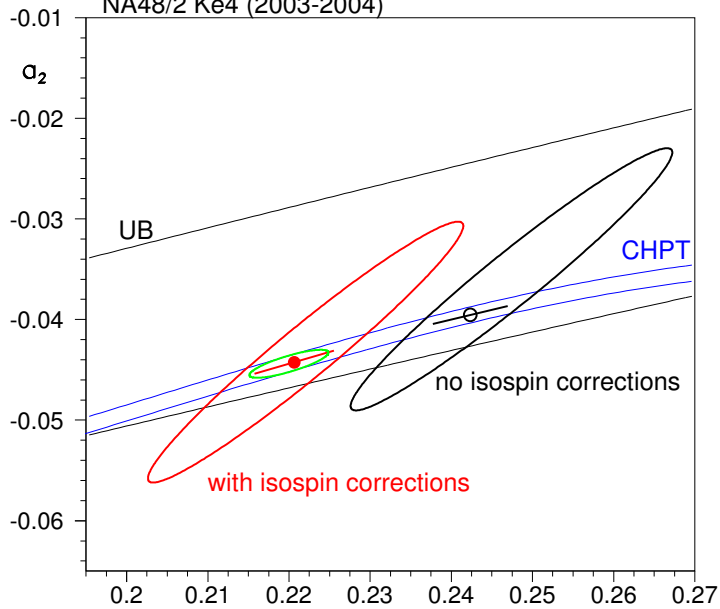
alternative: using additional theoretical constraint

→ one-parameter fit: $a_0 = 0.2206(49)(18)(64)$

($a_2 = -0.0442$) from constraint

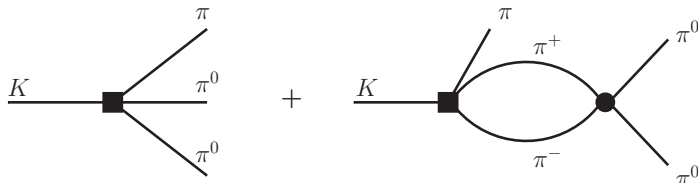
to be compared with theory: $a_0 = 0.220(5)$, $a_2 = -0.0444(10)$

NA48/2 Ke4 (2003-2004)



$\pi\pi$ scattering lengths from $K \rightarrow 3\pi$ decays near threshold

$\pi^+-\pi^0$ mass difference $\rightarrow$ **cusp** in the $M_{\pi^0\pi^0}$ distribution at $M_{\pi^0\pi^0} = 2M_{\pi^+}$



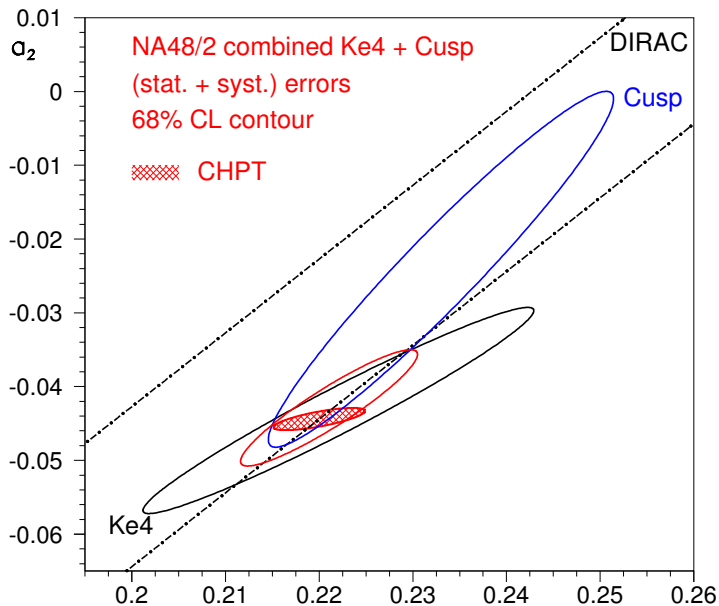
seen in:

- ▶ $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ J.R. Batley et al. (NA48/2), 2006,2009
- ▶ $K_L \rightarrow 3\pi^0$ E. Abouzaid et al. (KTeV), 2008

Cusp in $K \rightarrow 3\pi$ (theory)

$$a_0 - a_2 \sim A(\pi^+\pi^- \rightarrow \pi^0\pi^0)_{\text{threshold}}$$

- ▶ original idea: P. Budini, L. Fonda, 1961; N. Cabibbo, 2004
- ▶ unitarity analyticity $\rightarrow$ expansion of singular term in $M_{\pi^0\pi^0}$ distribution in powers of scattering lengths: N. Cabibbo, G. Isidori 2005
- ▶ unitarity, analyticity, CHPT: E. Gámiz, 2007
- ▶ two-loop dispersive representation of $K \rightarrow 3\pi$ amplitudes in the presence of isospin breaking: K. Kampf et al., 2009
- ▶ most advanced approach: nonrelativistic effective field theory (NRQFT)
 $K \rightarrow 3\pi$ amplitudes expanded in powers of the scattering lengths and of the pion momenta in the K rest frame: G. Colangelo et al., 2006;
M. Bissegger et al., 2007; J. Gasser et al., 2011
inclusion of electromagnetic corrections: M. Bissegger et al., 2009

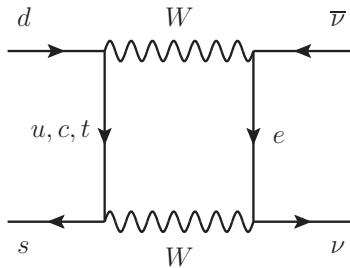
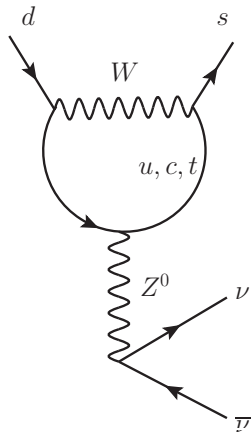


Results for the s-wave $\pi\pi$ scattering lengths

	$a_0 - a_2$	a_0	a_2
$K \rightarrow 3\pi$	0.2571(48)(25)(14)		-0.0241(129)(94)(18)
K_{e4}		0.2220(128)(50)(37)	-0.0432(86)(34)(28)
theory	0.264(4)	0.220(5)	-0.0444(10)

Impressive agreement between theory and experiment!

$$K \rightarrow \pi \nu \bar{\nu}$$



Z-penguin and box contributions to $K \rightarrow \pi \nu \bar{\nu}$

$K \rightarrow \pi \nu \bar{\nu}$ ctd.

$$A(s \rightarrow d \nu \bar{\nu}) \sim \frac{m_t^2}{M_W^2} \lambda_t + \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c} \lambda_c + \frac{\Lambda_{\text{QCD}}^2}{M_W^2} \lambda_u, \quad \lambda_q = V_{qd} V_{qs}^*$$

GIM mechanism:

- ▶ t contribution (large CP-violating phase): $\sim 68\%$
- ▶ c contribution: $\sim 29\%$
- ▶ u contribution: $\sim 3\%$

large suppression of the u -quark (long-distance) contribution and complete dominance of the CP-violating contribution by the t -quark term

$K \rightarrow \pi \nu \bar{\nu}$ decays are “short-distance” dominated

$K \rightarrow \pi \nu \bar{\nu}$ ctd.

low energies: dominated by $Q_{13} = \bar{s} \gamma^\mu (1 - \gamma_5) d \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu$

hadronic matrix element of Q_{13} can be related to $K_{\ell 3}$ form factors

→ very accurate theoretical prediction of $K \rightarrow \pi \nu \bar{\nu}$ rates

Contributions of many authors: M. Lu, M.B. Wise, 1994; G. Buchalla, A. Buras, 1994, 1996, 1998, 1999; W.J. Marciano, Z. Parsa, 1996; G. Buchalla, G. Isidori, 1998; M. Misiak, J. Urban, 1999; A. Falk, A. Lewandowski, A. Petrov, 2001; G. Isidori, F. Mescia, , C. Smith, 2005; A. Buras, M. Gorbahn, U. Haisch, U. Nierste, 2005, 2006; J. Brod, M. Gorbahn, 2008; F. Mescia, C. Smith, 2007; J. Brod, M. Gorbahn, E. Stamou, 2011

Present status of $K \rightarrow \pi \nu \bar{\nu}$

from: V. Cirigliano, Kaon Physics Working Group, Project X Physics Study, June 21 2012

mode	SM	experiment
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$7.81(75)(29) \times 10^{-11}$	$(1.73^{+1.15}_{-1.05}) \times 10^{-10}$ E787/949
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	$2.43(39)(6) \times 10^{-11}$	$< 2.6 \times 10^{-8}$ E391a

Summary

- ▶ Interplay of short- and long-distance physics
- ▶ EFT methods: operator product expansion and CHPT
- ▶ Resonance model and lattice calculations to determine LECs
- ▶ Excellent precision for semileptonic decays
- ▶ Electromagnetic contributions
- ▶ Strong isospin breaking
- ▶ Determination of V_{us} from $K_{\ell 3}$ decays
- ▶ CKM unitarity test

Summary ctd.

- ▶ Test of lepton universality in $R_{e/\mu}^K$
- ▶ Extraction of $\pi\pi$ scattering lengths from K_{e4} and $K \rightarrow 3\pi$
- ▶ Impressive interplay of high precision measurements and refined theoretical methods
- ▶ Short distance dominated decays $K \rightarrow \pi\nu\bar{\nu}$
- ▶ Very accurate theoretical predictions in the SM
- ▶ Sensitive probe for new physics