GRADUIERTENKOLLEG
Masse-Spektrum-Symmetrie

First Mode Excitations of Bosonic Strings in $AdS_5 \times S^5$ – Finding the Konishi Anomalous Dimension at $\lambda \gg 1$

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Motivation

$\mathcal{N}=4$ SYM: maximally sypersymmetric Yang-Mills theory in 4D with gauge group $SU(N_c)$
- toy-model for massless QCD
- (super) CFT with superconformal group $PSU(2,2|4) \overset{\text{bos}}{\supset} SO(2,4) \times SO(6)_R$
  $\Rightarrow$ higher-point correlators from **two-** and **three-point correlators** only

$\langle \mathcal{O}_a(x_1)\mathcal{O}_b(x_1)\rangle = \delta_{a\,b}\,(x_1 - x_2)^{-2\Delta_a}$ with Δ_a **scaling dimension** of $\mathcal{O}_a$

AdS/CFT correspondence: $\mathcal{N}=4$ SYM $\overset{\text{dual}}{\Leftrightarrow}$ IIB superstring in $AdS_5 \times S^5$
scaling dimensions Δ $\overset{\text{dual}}{\Leftrightarrow}$ **string energy spectrum** E

- planar limit, $N_c \to \infty$: 't Hooft coupling $\lambda = N_c\, g^2_{YM} = 4\pi N_c\, g_s$
  $\Rightarrow$ best description: *weak* coupling, $\lambda \ll 1$: $\mathcal{N}=4$ SYM; *strong* coupling, $\lambda \gg 1$: IIB superstring
- long/protected operators well understood $\to$ simplest **short unprotected** operator:

Konishi operator: $\mathcal{K} = \mathrm{Tr}\,(\phi_I \phi_I)$, with ϕ_I the six scalars of $\mathcal{N}=4$ SYM

Konishi anomalous dimension $\gamma = \Delta - \Delta_0$: known to 5 loops in $\mathcal{N}=4$ SYM regime [1]

GOAL: Find γ at *strong* coupling, $\lambda \gg 1$, from String Theory

Previous Approaches and Results

Combining algebraic curve with numerical results from Algebraic/Thermodynamic Bethe Ansatz and the Y-system led to the strong coupling prediction for the Konishi anomalous dimension [2]

$$\gamma = E - \Delta_0 = 2\,\lambda^{1/4} + 2\,\lambda^{-1/4} + \left(\frac{1}{2} - 3\,\zeta(3)\right)\lambda^{-3/4} + \mathcal{O}(\lambda^{-5/4}) \;,$$

with dual string state at **first excited level** in spectrum.
Methods however highly rely on the conjectured **integrability** of $\mathcal{N}=4$ SYM.

3 approaches to calculate the Konishi anomalous dimension γ directly from *String Theory*:
- Green-Schwarz string in light-cone gauge [3]: specific **λ-scaling of zero modes** identified but **ordering ambiguities** prevent calculating the first quantum corrections at order $\mathcal{O}(\lambda^{-1/4})$
- Extrapolating semiclassical strings (quantum numbers $Q \sim \sqrt{\lambda}$) to non-semiclassical regime [4]: Different semiclassical results coincide when extrapolated to $Q \sim 1$, yielding γ up to $\mathcal{O}(\lambda^{-1/4})$
- Pure Spinor Approach [5]: $\mathcal{O}(\lambda^{-1/4})$ from expectation values of operators **quartic** in **bosonic** Phase Space Variables (**PSVs**) only

General Bosonic String Theory

Phase space Polyakov action for bosonic strings in $AdS_5 \times S^5$: $X^\mu = (T, \vec{Z}, \varphi, \vec{Y})$

$$\mathcal{S}_{\text{bos}} = \int \frac{d^2\sigma}{2\pi}\Big(P_\mu\,\dot{X}^\mu - \xi_{\mathcal{K}}\,\underbrace{\frac{1}{2\sqrt{\lambda}}(G^{\mu\nu}\,P_\mu P_\nu + \lambda\,G_{\mu\nu}\,X'^{\,\mu}X'^{\,\nu})}_{\equiv\mathcal{K}} - \xi_{\mathcal{V}}\,\underbrace{P_\mu X'^\mu}_{\equiv\mathcal{V}}\Big) \;,$$

with $$G_{\mu\nu}(X^\rho) = \mathrm{diag}\Bigg(\underbrace{-\left(\frac{1+\vec{Z}^2/4}{1-\vec{Z}^2/4}\right)^2, \frac{1}{(1-\vec{Z}^2/4)^2}}_{AdS_5}, \underbrace{-\left(\frac{1-\vec{Y}^2/4}{1+\vec{Y}^2/4}\right)^2, \frac{1}{(1+\vec{Y}^2/4)^2}}_{S^5}\Bigg)$$

Canonical Transformation: $X^\mu = \lambda^{-1/4}\,x^\mu$, $P_\mu = \lambda^{1/4}\,p_\mu + \delta_{\mu 0}\,E + \delta_{\mu 5}\,J$

Stroung Coupling Expansion: Expand $G_{\mu\nu}$ and hence $K \equiv \int \frac{d\sigma}{2\pi}\mathcal{K} \overset{!}{=} 0$ in $\lambda \gg 1$

$$\frac{E^2 - J^2}{2\sqrt{\lambda}} = K_{n\neq 0} + K_{n=0} + \frac{1}{\sqrt{\lambda}}\,\delta K \;,$$

$K_{n\neq 0}$ ($K_{n=0}$) quadratic in non-zero (zero) mode PSVs, δK higher orders $\to$ **QM pert. theory**

Dual String State – First excitation in AdS_5 only: $|\Psi_0\rangle = a^i_{-1}\tilde{a}^j_{-1}|0\rangle$, $i,j = 1,\dots,4$
$\Rightarrow$ **different scaling** for non-zero and (spatial AdS) zero modes ($M^2_z = E^2/\lambda$) as in [3]:

$$x^\mu_{n\neq 0},\; p_{\mu,n\neq 0} \sim \lambda^0 \quad \forall\,\mu = 0,\dots,9 \qquad \textbf{BUT} \qquad z^i_0 \sim \lambda^{1/8} \;\Leftrightarrow\; p_{i,0} \sim \lambda^{-1/8} \quad i = 1,\dots,4$$

$\Rightarrow$ expansion in $\lambda \neq$ expansion in PSVs! $\Rightarrow$ $K_{n=0} \sim \lambda^{-1/4}$, δK complicated when ordered in λ

According to [5]: Neglecting zero-point energy, **bosonic string theory** up to sub-leading order in $\lambda \gg 1$ expansion plus **setting only** $L_0 + \tilde{L}_0 \propto K \equiv \mathcal{K}_0 = 0$ (otherwise *no gauge fixing*) suffices to get the first quantum corrections, $\mathcal{O}(\lambda^{-1/4})$.

We find the *same bosonic operators quartic in PSVs* as in [5] but run into **PROBLEMS**:
- our *numeric and analytic* results for the first quantum corrections, $\mathcal{O}(\lambda^{-1/4})$, **disagree** with [5]
- quartic operators also contribute at **2nd order perturb. theory** $\Rightarrow$ **gauge fixing needed**
- also **operators sextic in PSVs**, $(a^\mu_{n\neq 0})^2\,(z^i_0)^4 \sim \sqrt{\lambda}$, contribute at $\mathcal{O}(\lambda^{-1/4})$ in E
- no reason why **ordering ambiguities** coming from expansion of $G_{\mu\nu}$ should be under control

Alternative Route: First Mode Excitations

Observation: Intricate **mixing of first and zero modes**, while **higher modes** ($|n| \geq 2$) **decouple** at least to $\mathcal{O}(\lambda^{-1/2})$ in E

Take a **Minisuperspace approach**:
- Phase space Polyakov action in stereographic $AdS_5 \times S^5$ coordinates and temporal gauge [6]: ($\mathcal{K} = \mathcal{H} - E^2/2\sqrt{\lambda} \overset{!}{=} 0$)

$$E^2 \overset{!}{=} (1+Z^2)\left[P_Z^2 + (P_Z\cdot Z)^2 + \frac{(1+Y^2)^2}{4}\,P_Y^2 + \lambda\left(Z'^2 - \frac{(Z\cdot Z')^2}{1+Z^2} + \frac{4\,Y'^2}{(1+Y^2)^2}\right)\right]$$

and $\mathcal{V} = P_Z\cdot Z' + P_Y\cdot Y' \overset{!}{=} 0$.
- Neglect all but **first and zero modes**, first in AdS_5 only:

$$Z = Z_0 + Z_+e^{i\sigma} + Z_-e^{-i\sigma}\;,\quad Z'_0 = Z'_+ = Z'_- = Y' = 0\;,$$
$$P_Z = P_0 + P_+e^{i\sigma} + P_-e^{-i\sigma}\;,\quad P'_0 = P'_+ = P'_- = P'_Y = 0\;,$$

- Solve conformal constraints **classically** $\Rightarrow$ E^2 operator reads:

$$E^2 = (1 + Z_0^2 + q^2)\Big[(P_0^2 + p^2) + (P_0\cdot Z_0 + p\,q)^2 + \lambda\,q^2 + M^2\Big]$$

with $q \equiv |Z_+|/\sqrt{2}$ and $p \equiv |P_+|/\sqrt{2}$, M^2 the Casimir of S^5
- classical EoM's solvable up to one implicit integral for q
- apart from potential $\lambda\,q^2$ this is a **Particle in AdS_6**
- respects isometries $SO(2,4)$ **quantum mechanically**

- **Quantize** the **remaining phase space** as in [7]:
  - Laplace-Beltrami: **ordering ambiguities under control**
  - diagonalizing E^2 up to first order perturbation theory gives the final result for the **Konishi anomalous dimension**

$$\gamma_{\text{bos}} = E - \Delta_0 = 2\,\lambda^{1/4} + 4\,\lambda^{-1/4} + \mathcal{O}(\lambda^{-3/4}) \;,$$

with $\Delta_0 = 2$ and no momentum in S^5

Mismatch due to lack of fermions? $\Rightarrow$ **Include Fermions!**

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