Chiral Condensate and Topological Susceptibility for twisted mass fermion at maximally twisted mass

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Motivation

1 Very important properties of QCD are the **chiral symmetry and its spontaneous breaking**, which is manifested by a non zero value of the chiral condensate, the order parameter of chiral symmetry. In order to study the **low energy properties of QCD**, such as chiral symmetry breaking, we need a non perturbative regularization. In our case we use **Lattice QCD** which is characterized by the path integral representation of Green functions in a Euclidean space time discretized lattice. Thus an observable is represented by:

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int DA_\mu \, D\psi D\bar{\psi} \mathcal{O} \, e^{-S_{\text{gauge}}+S_{\text{ferm}}} \qquad (1)$$

2 The **Banks-Casher relation** connects the chiral condensate Σ with the density of eigenmodes at the origin of the spectrum and thus with the infrared properties of the Dirac operator.

$$\frac{\Sigma}{\pi} = \lim_{\lambda\to 0} \lim_{m\to 0} \lim_{V\to\infty} \rho(\lambda, m) \qquad (2)$$

$$\Sigma = -\lim_{m\to 0} \lim_{V\to\infty} \langle \bar{u}u \rangle \, , \qquad \rho(\lambda, m) = \frac{1}{V} \sum_{k=1}^{\infty} \langle \delta(\lambda - \lambda_k) \rangle \qquad (3)$$

Only when the eigenvalue density $\rho(\lambda, m)$ is non-zero at the origin, the chiral condensate does not vanish and hence the infrared properties of the Dirac operator are directly related to the mechanism of chiral symmetry breaking.

3 We calculate the **chiral condensate through the study of the mode number** ν, a more convenient quantity to compute and directly related to the spectral density.

$\nu \rightsquigarrow$ average number of eigenmodes of $D_m^\dagger D_m$ with $\lambda \le M^2$.

$$\nu(M, m) = V \int_{-\Lambda}^{\Lambda} d\lambda \rho(\lambda, m), \qquad \Lambda = \sqrt{M^2 - m^2} \qquad (4)$$

Spectral Projector method

1 A new method was introduced by [Giusti & Lüscher, 2009] to compute $\nu(M, m)$ decreasing considerably the computational effort required.

$$\nu(M, m_q) = \langle \text{Tr}\{\mathbb{P}_M\} \rangle \qquad (5)$$

In this way $\nu(M, m)$ can be defined as the trace of $\mathbb{P}_M$ which can be approximated by a rational function of $D^\dagger D$.

Specifically an approximation to $\mathbb{P}_M$ can be given by the function $h(\mathbb{X})$ which approximates the step function $\theta(-x)$ in the interval $[-1, 1]$ as $\mathbb{P}_M \approx h(\mathbb{X})^4$

$$h(x) = \frac{1}{2}\{1 - xP(x^2)\}, \qquad \mathbb{X} = 1 - \frac{2M_\star^2}{D_m^\dagger D_m + M_\star^2} \qquad (6)$$

2 We introduce a set of pseudo-fermions fields η_k which is generated randomly for each configuration.

$$\nu(M, m_q) = \langle \mathcal{O}_N \rangle \, , \quad \mathcal{O}_N = \frac{1}{N} \sum_{k=1}^{N} (\eta_k, \mathbb{P}_M \eta_k) \qquad (7)$$

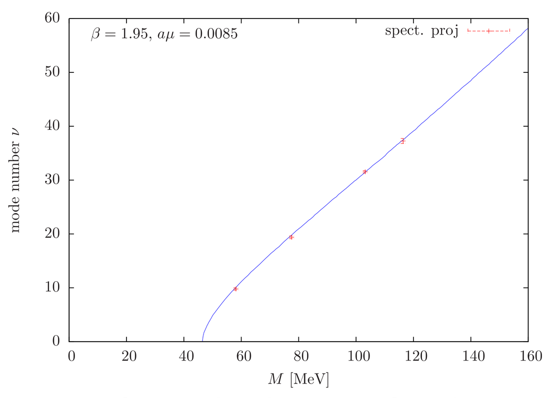
3 Finally we apply the following equation to extract the chiral condensate.

$$\Sigma_R = \frac{\pi}{2V} \sqrt{1 - \left(\frac{m_R}{M_R}\right)^2} \frac{\partial}{\partial M_R} \nu_R(M_R, m_R) \qquad (8)$$

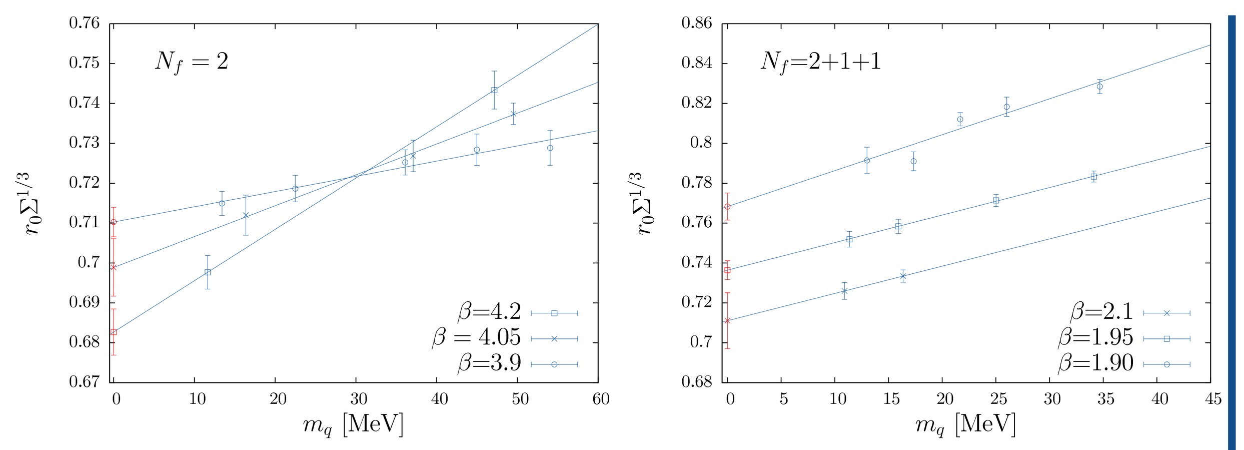
which is chosen to match the chiral condensate to leading order of chiral perturbation theory.

Chiral Condensate Σ

1 First of all we need to **test** our implementation and the method itself. We should observe a linear behavior in the mode number for the chosen range of M.

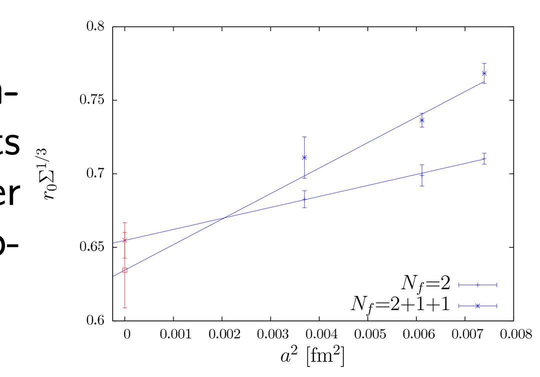


2 We can directly compare our result with the result obtained by computing the eigenvalues and counting, a less efficient method but equally valid. In this figure we can see the result of both methods and the required **linear behavior** for the values of M that we chose.



3 We have computed the chiral condensate for different values of the quark mass, therefore we can extrapolate the value in the **chiral limit** for different values of the lattice spacing a and for $N_f = 2$ and $N_f = 2+1+1$ dynamical flavors of maximally twisted mass fermions.

4 Finally we perform the **continuum limit**. These results that can be compare with other collaborations and methods obtaining compatible results.



$$r_0\Sigma^{1/3}|_{N_f=2+1+1} = 0.672(14) \qquad (9)$$

$$r_0\Sigma^{1/3}|_{N_f=2} = 0.654(12) \qquad (10)$$

Topological Susceptibility

1 Another application of the spectral projector method is the computation of the **topological susceptibility** χ_{top}.

2 This quantity is very difficult to compute due to the **short distance singularities**

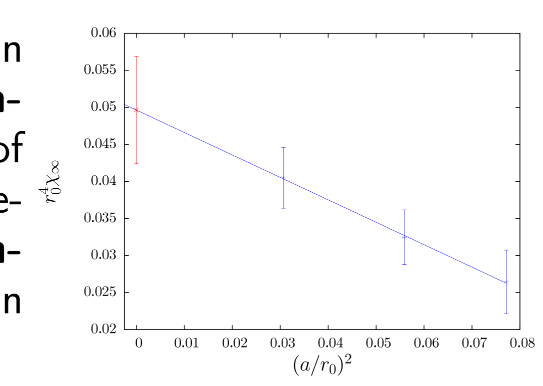
$$\chi_{top} = \frac{\langle Q_{top}^2 \rangle}{V} = \int d^4x \langle q(x) q(0) \rangle \qquad (11)$$

Over the years different definitions have been proposed, but none of them had a clear and well defined continuum limit or are too expensive to apply for large volumes. Following the work presented in [Rossi *et al*, 2001] Lüscher proposed in 2004 a definition of the topological susceptibility with a **well defined continuum limit** for chiral fermions which respect chiral symmetry.

3 A generalization of that definition for any kind of fermion was made through the spectral projector method and applied with Wilson fermions in [Lüscher & Palombi, 2010] and in this work we apply it to Twisted Mass fermions.

$$\chi_{top} = \frac{Z_S^2}{Z_P^2} \frac{\langle \text{Tr}\{\gamma 5 \mathbb{P}_M\} \, \text{Tr}\{\gamma 5 \mathbb{P}_M\} \rangle}{V} \qquad (12)$$

4 We have computed χ_∞ in the **quenched approximation** for different values of the lattice spacing a, therefore we can extract the **continuum limit** as shown in the figure on the right.



5 This result is compatible with other results, for example [Giusti *et al*, 2010] and allow us to test the Witten-Veneziano formula. We will include a fourth ensemble with a smaller lattice spacing in the near future.

Publications

- K. Cichy, V. Drach, E. Garcia Ramos, K. Jansen, C. Michael, K. Ottnad, C. Urbach, F. Zimmermann. "Properties of pseudoscalar flavour-singlet mesons from 2+1+1 twisted mass lattice QCD. ", hep-lat 1211.4497
- K. Cichy, V. Drach, E. Garcia-Ramos, G. Herdoiza, K. Jansen " Overlap valence quarks on a twisted mass sea: a case study for mixed action Lattice QCD. ", [hep-lat] 1211.1605 accepted for publication at Nucl.Phys.B
- K. Cichy, V. Drach, E. Garcia-Ramos, K. Jansen " Topological susceptibility and chiral condensate with Nf=2+1+1 dynamical flavors of maximally twisted mass fermions. ",PoS LATTICE2011 (2011) 102
- K. Cichy , V. Drach, E. Garcia-Ramos, G. Herdoiza, K. Jansen "Overlap Valence Quarks on a Twisted Mass Sea.", PoS LATTICE2010 (2010) 077

Selected Talks

- **Seminar at University of Liverpool**, UK — 10/12
  "Chiral condensate and topological susceptibility with maximally twisted mass fermions"
- **Strongnet Summer School**, University of Edinburgh, UK — 06/12
  "Women in physics"
- **Seminar at Laboratori Nazionale di Frascati**, INFN, Frascati,Italy — 01/12
  "Chiral condensate and topological susceptibility with twisted mass fermions"
- **XXIX International Symposium on Lattice Field Theory**, California, USA — 07/11
  "Chiral condensate and topological susceptibility with $N_f = 2+1+1$ dynamical flavors of maximally twisted mass fermion"

Collaborations

- European Twisted Mass Collaboration
- DFG Sonderforschungsbereich Transregio 9

Profit from the GK

- Informative and enjoyable lectures.
- Opportunity to meet students from other fields.
- Travel funding.
- Soft skills training courses.

Contact Details and further Information

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