GRADUIERTEN KOLLEG
Masse-Spektrum-Symmetrie

Non-perturbative investigation of current correlators in twisted mass lattice QCD

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Motivation

Twisted mass lattice QCD (tmLQCD) allows for the calculation of 2-point current correlators from first principles. We study them in detail to extract the hadronic leading order muon anomalous magnetic moment, a_μ^{hlo}, the charm quark mass and QCD coupling, m_c and α_s. By virtue of the 3σ discrepancy between theory and experiment a_μ^{hlo} is a gold-plated candidate in the search for physics beyond the standard model. m_c enters the latter as well via fits of CKM matrix elements and α_s is relevant for virtually any perturbative calculation of QCD matrix elements at small distances.

Methods

tmLQCD is a non-perturbative regularization of QCD on a discrete and finite space-time lattice with a chirally rotated quark mass matrix, leading to the automatic $\mathcal{O}(a)$ improvement of physical observables. Via the path-integral approach observables become statistical estimators based on ensembles of gauge field configurations generated by the ETMC. Covering a range of quark masses, lattice volumes and lattice spacings, they allow us to investigate all relevant systematic errors. For a_μ^{hlo} this work for the first time includes non-perturbative estimates of quark-disconnected diagrams.

Some details

(I) Starting from the current correlators in Euclidean space

$$\int d^4x \, \langle J_\mu(x) \, J_\nu(0) \rangle \, \mathrm{e}^{iqx} = (q_\mu q_\nu - \delta_{\mu\nu} \, q^2) \, \Pi(q^2) \qquad (1)$$

with e.g. the electromagnetic current $J_\mu = +2/3 \, \bar{u} \, \gamma_\mu \, u - 1/3 \, \bar{d} \, \gamma_\mu \, d$ in the light quark sector, we extract the polarization function $\Pi(q^2)$. The finite lattice extension L restricts knowledge of Π to a discrete set of momenta above $2\pi/L$. A smooth parametrization gives access to $\Pi(q^2)$ for $q^2 < (2\pi/L)^2$ and in particular to $\Pi(0)$. Based on that we estimate a_μ^{hlo} from the integral relation

$$a_\mu^{\mathrm{hlo}} = 4 \, \alpha_{\mathrm{QED}}^2 \int_0^\infty \frac{dq^2}{q^2} \, w(q^2/m_\mu^2) \, \left[\Pi(q^2) - \Pi(0) \right] \qquad (2)$$

with a weight function $w(q^2)$ known from leading order QED perturbation theory.

(II) Using the charm vector current $J_\mu^c = \bar{c} \, \gamma_\mu \, c$, perturbative QCD (pQCD) provides expansions of derivatives of Π at $q^2 = 0$

$$\left. \frac{d^n \, \Pi}{d \, q^{2\,n}} \right|_{q^2=0} = \sum_n \alpha_s^n \, C_n(m_c) \propto \sum_{t,\vec{x}} t^{2n+2} \, \langle J_i(t,\vec{x}) \, J_i(0) \rangle \, . \qquad (3)$$

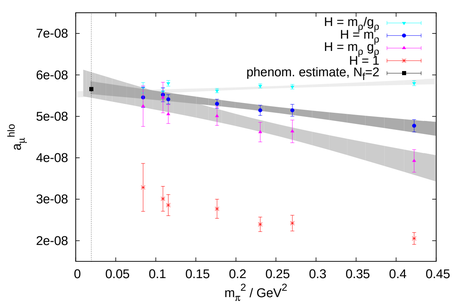
Matching results for $d\Pi/dq^{2n}$ from lattice temporal moments of the current correlator to continuum pQCD then allows for estimates of α_s and m_c.

Recent Results

(I) Based on vector meson dominance we proposed a family of parametrizations for Π valid in the complete momentum regime $2\pi/L \leq \sqrt{q^2} \lesssim \pi/a$. We introduced a generalized class of estimators for a_μ^{hlo} (lattice units indicated by a hat),

$$a_{\hat{\mu}}^{\mathrm{hlo}} = 4 \, \alpha_{\mathrm{QED}}^2 \int_0^\infty \frac{d\hat{q}^2}{\hat{q}^2} \, w(\hat{q}^2/m_\mu^2 \cdot H^2/\hat{H}^2) \, \left[\Pi(\hat{q}^2) - \Pi(0) \right] \qquad (4)$$

labeled by a lattice scale function H. An optimized choice of H leads to a better controllable dependence on the pion mass and even allows for a linear extrapolation in m_π^2. Existing methods are generalized to yield estimates of a_l^{hlo} using $H = m_\rho$ for all three standard model leptons, which are in agreement with our phenomenological estimates for 2 quark flavors.



	lattice	pheno
$a_e^{\mathrm{hlo}}(N_f = 2)$	$1.512\,(43)\,10^{-12}$	$1.553\,(34)\,10^{-12}$
$a_\mu^{\mathrm{hlo}}(N_f = 2)$	$5.69\,(15)\,10^{-8}$	$5.66\,(04)\,10^{-8}$
$a_\tau^{\mathrm{hlo}}(N_f = 2)$	$2.635\,(54)\,10^{-6}$	$2.681\,(76)\,10^{-6}$

(II) In a benchmark calculation we estimated the temporal moments of the charm vector current correlator for $n = 1, \ldots, 4$. We find good agreement with results from the dispersion relation using experimental data for the hadronic R-ratio ($e^+e^- \to \mathrm{photon} \to \mathrm{hadrons}$) and extract the charm quark mass in the $\overline{\mathrm{MS}}$ scheme at $\mu = 3\,\mathrm{GeV}$.

No.	$f_\pi \, \Pi^{(n)\,\frac{1}{2n}}$ (exp.)	$f_\pi \, \Pi^{(n)\,\frac{1}{2n}}$ (theo.)	$\bar{m}_c(\mu)/\mathrm{GeV}$
1	0.04107 (32)	0.04170 (25)	0.971 (09) (01)
2	0.08792 (48)	0.08810 (52)	0.981 (10) (02)
3	0.13081 (60)	0.13059 (68)	0.990 (10) (11)
4	0.17106 (70)	0.17098 (82)	1.014 (08) (35)

Both projects are presently continued.

Publications

X. Feng, G. Hotzel, K. Jansen, M. Petschlies and D. B. Renner, "Leading-order hadronic contributions to a_μ and α_{QED} from $N_f = 2+1+1$ twisted mass fermions," arXiv:1211.0828 [hep-lat].
D. B. Renner, X. Feng, K. Jansen and M. Petschlies, "Nonperturbative QCD corrections to electroweak observables," PoS(Lattice 2011)022
K. Jansen, M. Petschlies and C. Urbach, "Charm current-current correlators in twisted mass lattice QCD," PoS LATTICE **2011** (2011) 234
X. Feng, K. Jansen, M. Petschlies and D. B. Renner, "Two-flavor QCD correction to lepton magnetic moments at leading-order in the electromagnetic coupling," Phys. Rev. Lett. **107** (2011) 081802
D. B. Renner, X. Feng, K. Jansen and M. Petschlies, "Leading-order hadronic contribution to $g-2$ from lattice QCD," PoS ICHEP **2010** (2010) 371
F. Farchioni, G. Herdoiza, K. Jansen, A. Nube, M. Petschlies and C. Urbach, "Pseudoscalar decay constants from $N_f = 2+1+1$ twisted mass lattice QCD," PoS LATTICE **2010** (2010) 128
D. B. Renner *et al.* [ETM Collaboration], "Leading order hadronic contribution to $g-2$ from twisted mass QCD," PoS LATTICE **2010** (2010) 155
B. Blossier *et al.* [ETM Collaboration], "Average up/down, strange and charm quark masses with $N_f = 2$ twisted mass lattice QCD," Phys. Rev. D **82** (2010) 114513

Selected Talks

- 29th Int. Symposium on Lattice Field Theory, Squaw Valley, California USA, 06/2011, "Charm current-current correlators in twisted mass lattice QCD"
- 15. Arbeitstreffen des SFB/TR9, Berlin, 05/2011, "Moments of charm current-current correlators in twisted mass lattice QCD"
- 28th Int. Symposium on Lattice Field Theory, Sardinia Italy, 06/2010, "Charm current-current correlators in twisted mass lattice QCD"
- DPG Frühjahrstagung, Bonn, 03/2010, "Charm current-current correlators in twisted mass lattice QCD"
- GK block course, HU Berlin, 10/2009, "α_s, m_c and the muon $g-2$ from lattice QCD"
- 12. Arbeitstreffen des SFB/TR9, DESY/Zeuthen, 03/2009, "Charm quark mass from moments of correlators"

Collaborations

- European Twisted Mass Collaboration (ETMC), specifically HUB, DESY/Zeuthen, HISKP (Bonn), JLab (USA) an KEK (Japan)
- SFB/TR9 "Computational Particle Physics", Project A4

Profit from the GK

- participation in the block courses 10/2009, 03/2010, 10/2010 and 03/2011
- attendance of lecture "Physics at the LHC", WS 2009/10
- financial support for the participation in the XXVIII. and XXIX. International Symposium on Lattice Field Theory (2010, Villasimius, Sardinia Italy and 2011 Squaw Valley, California USA)

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