Renormalization in Perturbative Quantum Gravity

Motivation

- Perturbatively quantized Einstein gravity is famously nonrenormalizable due to negative mass dimension of its coupling constant $G_N \sim M_{\text{Planck}}^{-2}$.
- Nevertheless, it can be treated as an **effective field theory**

$$S_{grav} = \frac{1}{16\pi G_N} \int \mathrm{d}^d x \sqrt{-g} \left\{ 2\Lambda + R + c_1 G_N R^2 + c_2 G_N R_{\mu\nu} R^{\mu\nu} + \mathcal{O}(G_N^2) \right\}$$

  keeping only terms up to a fixed order in G_N.
- **Question in this framework:** Does inclusion of gravity alter the running of the Standard Model couplings, i. e. $g_{\text{Maxwell}}(\mu), g_{\text{YM}}(\mu), y_{\text{Yukawa}}(\mu), \lambda_{\varphi^4}(\mu)$?

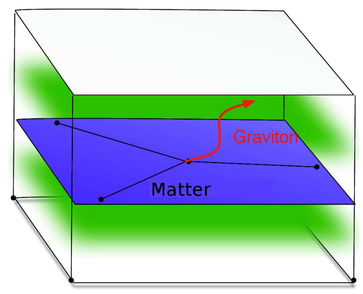
Some Details: Large Extra Dimensions

- Space Time topology $\mathcal{M} = \mathbb{R}^{1,d-1} \times T^\delta$
- Actual energy scale of gravitational interactions

$$M_\star^{d+\delta-2} = \frac{M_{\text{Pl}}^d}{(2\pi R)^\delta}$$

  $M_\star \sim 1\text{TeV}$ possible
- Gravitational Corrections

$$\left(\frac{p^2_{\text{typ.}}}{M_{\text{Pl}}}\right)^n \longrightarrow \left(\frac{p^2_{\text{typ.}}}{M_\star}\right)^n \gg \left(\frac{p^2_{\text{typ.}}}{M_{\text{Pl}}}\right)^n$$



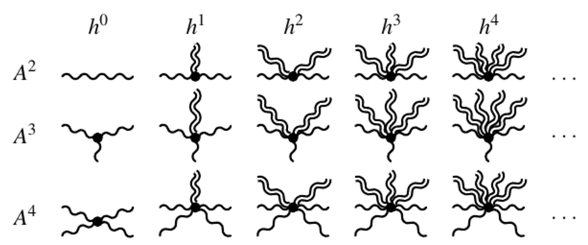
Methods and Results

- **Regularization.** Split the loop integrals at sliding scale μ

$$\int_{|k|\le\Lambda} \mathrm{d}^d k_E = \int_{|k|\le\mu} \mathrm{d}^d k_E + \int_{\mu<|k|\le\Lambda} \mathrm{d}^d k_E$$

- **Renormalization.**
  - Absorb the "divergent" part of the loop integral $\int_{\mu<|k|\le\Lambda} \mathrm{d}^d k_E$ by counterterms $\delta_\bullet(\mu,\Lambda)$
  - Determine the μ dependence of the renormalized coupling $g(\mu)$, from the counterterms via $g(\Lambda) = g(\mu) + \delta_g(\mu,\Lambda)$

- Expansion in the field variables yields Feynman rules for vertices of arbitrary order in h:



- The Yang-Mills β function is free of gravitational contributions

$$\beta_{\text{YM}}\big|_{G_N}^{4+\delta} = 0$$

- The β functions of Yukawa and φ^4 interactions receive gravitational correction, but these do not act in the direction of asymptotic freedom

$$\beta_\lambda\Big|_{G_N}^{4+\delta} = \frac{\lambda}{(4\pi)^{1+\delta/2}\Gamma\left(\frac{\delta}{2}+2\right) M_\star^{\delta+2}} \left(\frac{12(\delta+4)}{\delta+2} \mu^{2+\delta} + \frac{2\left(\delta^2+50\delta-32\right)}{\delta+2} m_\varphi^2 \mu^\delta \right)$$

$$\beta_y\Big|_{G_N}^{4+\delta} = \frac{y}{(4\pi)^{1+\delta/2}\Gamma\left(\frac{\delta}{2}+2\right) M_\star^{\delta+2}} \left(\frac{15}{2} \mu^{2+\delta} + \left\{ \frac{\delta^2+26\delta+16}{2(\delta+2)} m_\varphi^2 - \frac{19\delta^2-198\delta+128}{8(\delta+2)} m_\psi^2 \right\} \mu^\delta \right)$$

Recent Results

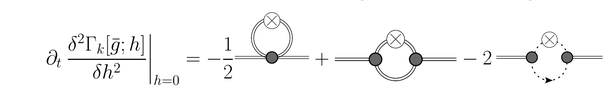
- **Beyond Perturbation Theory.** Quantum gravity can be defined as a non-perturbative quantum field theory within the asymptotic safety scenario. While the ultraviolet fixed point is very well-established, much less is known about the infrared behavior which is usually plagued by divergences.
- **Scale-dependent Einstein–Hilbert action.** We approximate the effective action by

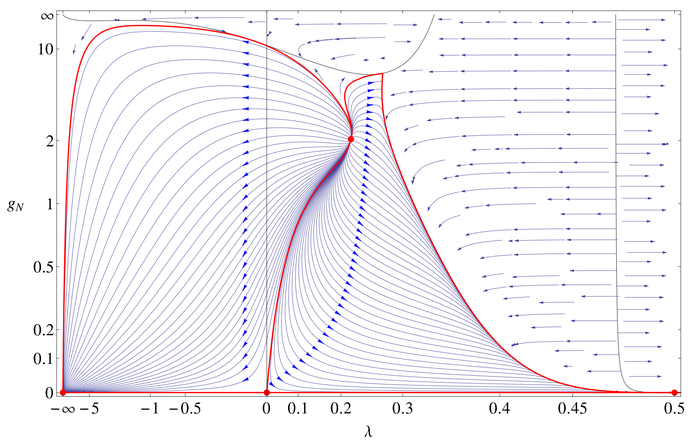
$$\Gamma_k[\bar{g}; h, \bar{C}, C] = \frac{1}{16\pi G_{N,k}} \int \sqrt{|g|} \left(R(g) - 2\Lambda_k\right) + S_{\text{gf}}[\bar{g}; h] + S_{\text{gh}}[\bar{g}; h, \bar{C}, C].$$

  Dimensionless couplings $g_N \equiv k^2 G_{N,k}$ and $\lambda \equiv \Lambda_k / k^2$.
- **The Functional Renormalization Group** is governed by its exact flow equation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + \mathcal{R}_k} \partial_t \mathcal{R}_k \,.$$

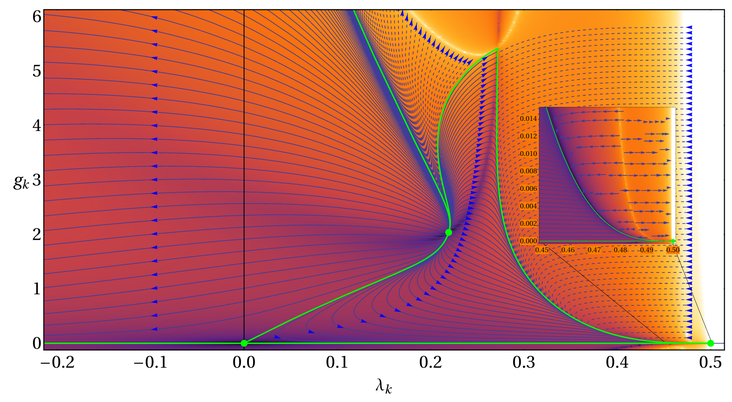
- We computed the RG flow of the graviton propagator





Fixed points of the RG flow

$$(g^*, \lambda^*)^{\text{Gauss}} = (0,0) \qquad (g^*, \lambda^*)^{\text{UV}} = (2.03, 0.22)$$
$$(g^*, \lambda^*)^{\text{IR, AdS}} = (0, -\infty) \qquad (g^*, \lambda^*)^{\text{IR, dS}} = (0, 1/2)$$



Publications

1. D. Ebert, J. Plefka, A. Rodigast, "Absence of gravitational contributions to the running Yang-Mills coupling," *Phys. Lett. B* **660** (2008) 579, arXiv: 0710.1002 [hep-th].
2. D. Ebert, J. Plefka, A. Rodigast, "Gravitational Contributions to the Running Yang-Mills Coupling in Large Extra-Dimensional Brane Worlds," *J. High Energy Phys.* **0902** (2009) 028, arXiv: 0809.0624 [hep-ph].
3. A. Rodigast, T. Schuster, "No Lee-Wick Fields out of Gravity", *Phys. Rev. D* **79** (2009) 125017, arXiv: 0903.3851 [hep-ph].
4. A. Rodigast, T. Schuster, "Gravitational Corrections to Yukawa and ϕ^4 Interactions," *Phys. Rev. Lett.* **104** (2010) 081301, arXiv: 0908.2422 [hep-th].
5. A. Rodigast, T. Schuster, "Gravitational corrections to non-gauge interactions," *Nucl. Phys. Proc. Suppl.* **216**, 263-264 (2011).
6. N. Christiansen, D. F. Litim, J. M. Pawlowski and A. Rodigast, "Fixed points and infrared completion of quantum gravity," arXiv: 1209.4038 [hep-th].

Selected Talks

- 03/2008 XXth Workshop Beyond the Standard Model, Bad Honnef
- 06/2010 Johannes Gutenberg-Universität Mainz
- 11/2010 Ruprecht-Karls-Universität Heidelberg

Profit from the GK

- exchange of ideas with scientists from different branches of high energy physics
- possibility to attend a plethora of lectures and seminars; scientific and soft skill
- travel expenses for schools and conferences

Contact Details and further Information

PhD student: Andreas Rodigast, a.rodigast@thphys.uni-heidelberg.de
PhD advisors: Jan Plefka, Dietmar Ebert

WWW: www.thphys.uni-heidelberg.de

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