Superconformal Quantum Field Theories in String — Gauge Theory Dualities

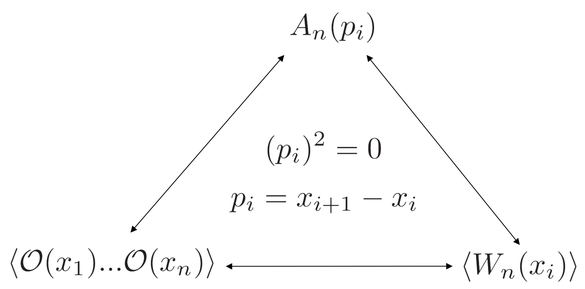
Motivation

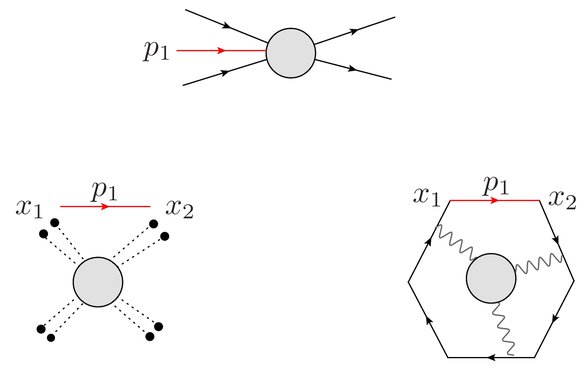
The understanding of processes at the LHC requires a precise understanding of QCD scattering amplitudes. N=4 Super Yang-Mills theory has many properties that are similar to QCD, but at the same time it has a number of features that make it considerably simpler, especially conformal symmetry and supersymmetry.

This thesis [1] reviews recent developments on the perturbative quantum field theory side of so-called AdS/CFT dualities [6,7], which state the equivalence of certain conformal field theories (CFT) and their dual string theories living on anti-de Sitter space (AdS). The research performed in this thesis is summarized in the following and was published in [1-5]. It extends these developments, strongly relying on conformal symmetry methods.

A milestone in the understanding of corrections to scattering amplitudes in the duality pair N=4 Super Yang-Mills theory and IIB superstring theory on AdS5 x S5 was the discovery of the dualities between scattering amplitudes, light-like polygonal Wilson loops and correlators illustrated on the right-hand side.

"Triality of Amplitudes, Wilson Loops and Correlators"





Important advantages of dual descriptions of scattering amplitudes

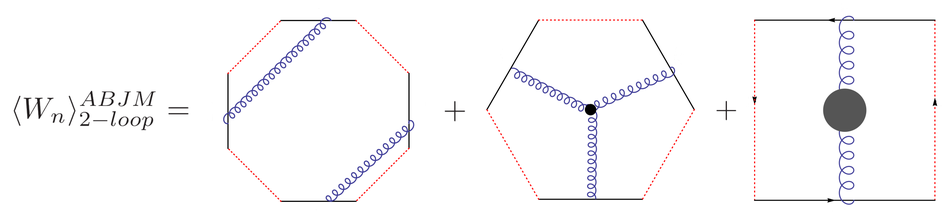
- hidden symmetries are manifest in dual objects, e.g. dual conformal symmetry
- Wilson loops are easier to calculate and constrained by all-order Ward identities
- the integrand of correlation functions is invariant under new permutation symmetry of external and internal (integration) points, allows to go to high loop orders easily

Wilson Loop / Amplitude Duality in ABJM Theory

Motivation and previous results

The amplitude / Wilson loop duality is very well studied in N=4 SYM theory, however its origin is not completely clear. Therefore, the question is natural, whether it might be present in any other theories, e.g. N=6 super Chern-Simons (ABJM) theory, the gauge theory counterpart in the second example of an AdS/CFT correspondence [7]. Therefore, we studied light-like polygonal Wilson loops in pure Chern-Simons and ABJM theory. We found [2], that the expectation value of the Wilson loop vanishes at one-loop order, just as the one-loop correction to the four-point function. Later it was shown that the correlation function in the light-like limit vanishes as well [8], thus establishing the triality mentioned above at one-loop order.

Two-Loop Calculation of the Wilson loop in ABJM theory



$$= \left(\frac{N}{k}\right)^2 \left[-\frac{1}{2}\sum_{i=1}^{n} \frac{(-x^2_{i,i+2}\mu'^2)^{2\epsilon}}{(2\epsilon)^2} + \mathcal{F}^{\mathrm{WL}}_n(x^2_{ij}) + r'_n \right]$$

$$\langle W_n \rangle^{\mathcal{N}=4\ \mathrm{SYM}}_{\text{1-loop}} = \frac{g^2 N}{8\pi^2} \left[-\frac{1}{2}\sum_{i=1}^{n} \frac{(-x^2_{i,i+2}\mu^2)^{\epsilon}}{\epsilon^2} + \mathcal{F}^{\mathrm{WL}}_n(x^2_{ij}) \right]$$

Results:

- 4-sided Wilson loop at two-loop order calculated analytically using anomalous conformal Ward identities [2]
- fascinatingly, the two-loop result in ABJM is identical to the 1-loop result in N=4 SYM
- following to this work, the 4-point amplitude at two loops was shown to agree with this result [9]
- we generalized the result to n points and find again equivalence with the 1-loop result in N=4 SYM [4]

Three-Point Functions of Twist-Two Operators in N=4 Super Yang-Mills Theory

Why Three-Point Functions?

- due to the relation of correlation functions to scattering amplitudes and Wilson loops described above, they are very important observables to study in N=4 SYM
- the simplest type of correlation functions are two-point functions, which are very well understood
- three-point functions are the next complicated objects to study and additionally they have a particular meaning due to their relation to the operator product expansion (OPE):

$$\langle \mathcal{O}_A(x_1)\mathcal{O}_B(x_2)\mathcal{O}_c(x_3)\rangle = \frac{C_{ABC}}{|x_{12}|^{\Delta_{AB;C}}|x_{23}|^{\Delta_{BC;A}}|x_{13}|^{\Delta_{AC;B}}}$$

$$\Delta_{AB;C} = \Delta_A + \Delta_B - \Delta_C$$

- OPE: $\lim_{x\to 0} \mathcal{O}_A(x)\mathcal{O}_B(0) = \sum_C \frac{C_{ABC}\mathcal{O}_C}{|x|^{\Delta_{AB;C}}}$
- with the knowledge of all structure constants C_{ABC} and all two-point functions (anomalous dimensions) one can in principle construct any higher-point correlation function!

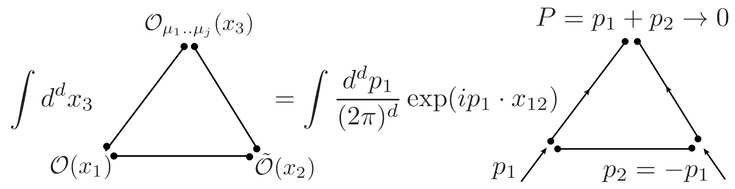
Twist-Two Operators

- simplest non-trivial infinite series of operators relevant for the OPE are the twist-two operators with arbitrary spin j

$$\mathcal{O}_{\mu_1..\mu_j}(x) = \mathrm{Tr}(Z(x)D_{\mu_1}..D_{\mu_j}Z(x)) + ...$$

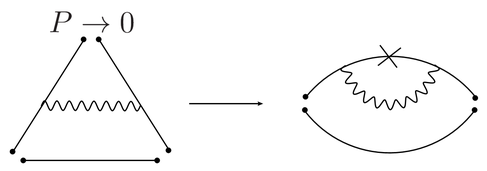
- simplest three-point function: two protected operators $\mathcal{O} = \mathrm{Tr}(\bar{Z}W), \tilde{\mathcal{O}} = \mathrm{Tr}(\bar{Z}\bar{W})$ and one twist-two operator

Method: Take Soft Limit P → 0



Advantages

- space-time structure reduces from 2^j to j terms
- one-loop mixing matrix drops out
- three-point integrals become simple two-point integrals, e.g.



Results and Outlook:

- method works very well, obtained the simple one-loop result

$$C'_{\mathcal{O}\tilde{\mathcal{O}}_j}(g) = C'^{(0)}_{\mathcal{O}\tilde{\mathcal{O}}_j}\left(1 + \frac{g^2 N}{8\pi^2}(2H_j(H_j - H_{2j}) - H_{j,2})\right)$$

$$H_j = \sum_{n=1}^{j}\frac{1}{n}, \quad H_{j,2} = \sum_{n=1}^{j}\frac{1}{n^2},$$

  which agrees with OPE extraction from four-point functions
- study more complicated three-point functions next
- generalize to higher loops, preferably using integrability methods
- obtain information about conformally invariant remainder function of the six-gluon MHV amplitude from correlation functions using OPE techniques

Publications and References

[1] *"Perturbative Methods for Superconformal Quantum Field Theories in String - Gauge Theory Dualities"*, K.Wiegandt, PhD-thesis Humboldt-Universität zu Berlin, arXiv:1212.5181

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[4] *"Equivalence of Wilson Loops in N=6 Super Chern-Simons and N=4 Super Yang-Mills Theory"*, K. Wiegandt, Phys.Rev. D84 (2011) 126015

[5] *"Three-Point Functions of Twist-Two Operators in N=4 Super Yang-Mills Theory at One Loop"*, J. Plefka, K. Wiegandt, JHEP 1210 (2012) 177

[6] *„The large N limit of superconformal field theories and supergravity"*, J. M. Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231-252

[7] *"N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals"*, O. Aharony, O. Bergman, D. L. Jafferis, J. Maldacena, JHEP 0810 (2008) 091

[8] *"From Correlators to Wilson Loops in Chern-Simons Matter Theories"*, M. S. Bianchi, M. Leoni, A. Mauri, S. Penati, C. A. Ratti, A. Santambrogio, **JHEP 1106 (2011) 118**

[9] „Scattering in ABJ theories", M. S. Bianchi, M. Leoni, A. Mauri, S. Penati, A. Santambrogio, JHEP 1112 (2011) 073; *"Dualities for Loop Amplitudes of N=6 Chern-Simons Matter Theory"*, W. Chen, Y. Huang, JHEP 1111 (2011) 057

Profit from the GK

- Soft skill courses at the HU graduate school
- Financial support for conferences and international schools
- Advanced GK lectures at the Humboldt-Universität zu Berlin and TU Dresden

Contact Details and further Information

PhD Student: Konstantin Wiegandt (HU), wiegandt@physik.hu-berlin.de

PhD Advisors: Prof. Jan Plefka (HU) / Prof. Matthias Staudacher (HU)

Current Position: Postdoctoral researcher at Queen Mary University of London Center for Research in String Theory

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