Yangian Symmetry from Amplitude – Correlator – Wilson Loop Triality in $\mathcal{N}=4$ SYM

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Motivation

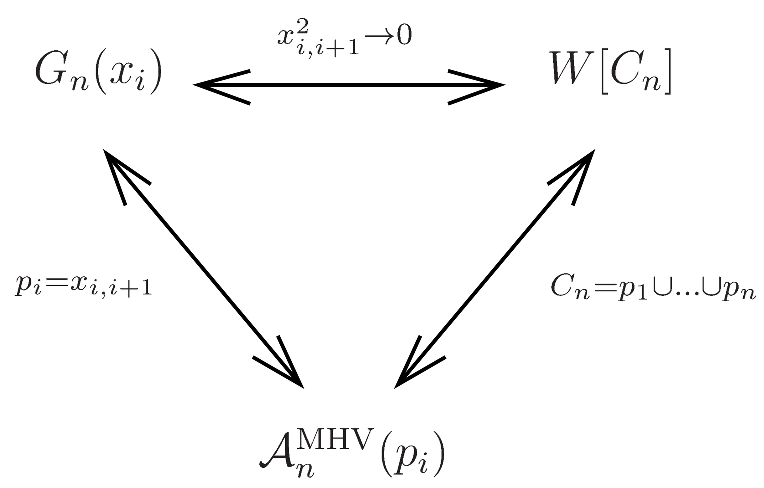
Scattering amplitudes are central observables in quantum field theories as they provide direct contact to collider experiments but the calculation of amplitudes is in general a very hard problem. Fortunately the development of novel on-shell methods in recent years has led to important progress in our capability of computing them. Scattering amplitudes are strongly constrained by the global symmetries of the theory (at minimum Poincaré). But even if we study $\mathcal{N}=4$ SYM, which admits a higher amount of symmetry than QCD the manifest symmetry is not sufficient to fix the observables.

There is more than meets the eye:

- Uncovering hidden symmetries at quantum level is fruitful to constrain observables.
- AdS/CFT relates $\mathcal{N}=4$ SYM to a IIB string theory living on $AdS_5 \times S^5$.
- This connection gives rise to the T-duality ⇔ Yangian picture.

Methods

The Amplitude – Wilson loop – Correlation function triality



$A_n^{MHV}(p_i)$ - Scattering depending on momenta p_i

$W[C_n]$ - Wilson loop depending on light-like contour C_n

$G_n(x_i)$ - Correlation function depending on region momenta x_i

This relation turned out to be very useful in finding hidden symmetries on the amplitude side because they are manifest on another side.

Previously

Superamplitudes in on-shell chiral superspace:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\,\delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 12\rangle\langle 23\rangle \ldots \langle n1\rangle}\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

with $\langle ij\rangle = \epsilon_{\alpha\beta}\lambda^\alpha\lambda^\beta$. E.g. $\mathcal{P}_n^{\mathsf{MHV}} = 1$.

Superamplitudes are superconformal $\mathfrak{psu}(2,2|4)$ invariant:

$$j^a \circ \mathbb{A}_n^{\mathsf{tree}} = 0 \quad j^a = \sum_{i=1}^n j_i^a \quad [j^a, j^b] = i\, f^{ab}{}_c\, j^c$$

with $j^a \in \mathfrak{psu}(2,2|4) = \{p^\mu, m^{\mu\nu}, r^{ab}; k^\mu, d;\, q^a_\alpha, s^a_\alpha\}$

$$\text{e.g.}\quad p^\mu = \sum_{i=1}^n p_i^\mu\,, \quad k^\mu = \sum_{i=1}^n 2\,p_i\cdot\partial_{p_i}\partial^\mu_{p_i} - p_i^\mu\,\partial^2_{p_i} + \partial^\mu_{p_i}$$

The Wilson loop – amplitude relation

$$\mathbb{A}_n/\mathbb{A}_n^{\mathsf{MHV\text{-}tree}}(p_i) = W(x_i) = \left\langle \mathrm{Tr}P\exp\oint dx^\mu A_\mu\right\rangle,\ x^2_{ii+1} = 0$$

uncovered a hidden dual superconformal symmetry leading to a Yangian symmetry which is not obviously related to an invariance of the Lagrangian[1].

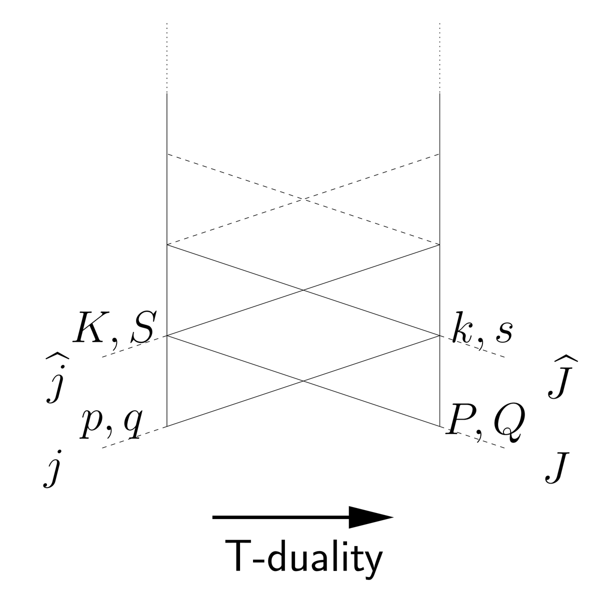
$$\widehat{j}^a \circ \mathbb{A}_n^{\mathsf{tree}} = 0 \quad \widehat{j}^a = \sum_{i<j} f^a_{bc}\, j_i^b\, j_j^b \quad [j^a, \widehat{j}^b] = i\, f^{ab}{}_c\,\widehat{j}^c$$

- But symmetry anomalous at loop-level[2]:
  With $\log(\mathbb{A}_n/\mathbb{A}_n^{\mathsf{tree}}) = D_n + F_n$ ($D_n \sim 1/\epsilon^2$: divergent, F_n: finite pieces) one has

$$\hat{p}^\mu \circ F_n = \Gamma_{\mathsf{cusp}}(\lambda)\sum_i p_i^\mu \log\left(\frac{p_i\cdot p_{i+1}}{p_{i-1}\cdot p_{i+1}}\right)$$

Absorption of anomaly in deformed $\hat{p}^\mu$?

Yangian symmetry $Y[\mathfrak{psu}(2,2|4)]$: Both symmetries can be combined into a infinite tower[3].



Infinite dimensional symmetry algebra for tree level amplitudes ⇒ Hint at integrability also in amplitude sector!

Next Episode

Amplitudes versus correlation functions:

$$G_n = \langle \mathcal{O}(x_1)\ldots\mathcal{O}(x_n)\rangle = \text{off-shell information}$$
$$\mathbb{A}_n(p_1\ldots p_n) = \text{on-shell S-matrix}$$

- Amplitudes are undefined in D dimensions (IR-divergencies require regulator). Breaking of conformal and Yangian symmetries by IR-regulator
- Correlation functions are well defined in $D=4$. Should inherit *all* symmetries of the theory.

Surprising relation in the light-like limit [4]:

$$\lim_{p_i^2\to 0} \ln G_n(x_i) = 2\ln A_n^{\mathsf{MHV}}(p_i)\,, \quad p_i = x_i - x_{i+1}\,.$$

The supercorrelator can be expressed in analytic superspace

$$\mathbb{G}_n = \langle \mathcal{T}(x_1, \theta_1^+, \bar{\theta}_1^-, u), \ldots, \mathcal{T}(x_n, \theta_n^+, \bar{\theta}_n^-, u)\rangle$$

and is invariant under the dual $\mathfrak{psu}(2,2|4)$: $J^a \circ \mathbb{G}_n^{\mathsf{tree}} = 0$

Restricting the correlator to the chiral sector yields the expansion of the stress-tensor multiplet

$$\mathcal{T}(x, \theta^+, 0, u) = \mathcal{T}_0 + \theta^+\mathcal{T}_1 + (\theta^+)^2\mathcal{T}_2 + (\theta^+)^3\mathcal{T}_3 + (\theta^+)^4\mathcal{T}_4$$

with the scalar operator $\mathcal{T}_0 = \mathrm{tr}(\phi^{++}\phi^{++})$ in the bottom. In this chiral subspace the supersymmetric generalisation of the correlator–amplitude duality is possible

$$\lim_{x^2_{ii+1}\to 0} \mathbb{G}_n(x_i)/G_n(x_i) = \left(\mathbb{A}_n(p_i)/\mathbb{A}_n^{\mathsf{MHV\text{-}tree}}(p_i)\right)^2.$$

Exploiting this duality the supercorrelator seems to be a natural off-shell regularization of IR-divergences for scattering amplitudes.

Goals:

- Establish Yangian symmetry for $\mathbb{G}_n$ in chiral and non-chiral case.
- Does deformed representation in $x^2_{ii+1}\to 0$ limit provide us with an exact Yangian symmetry for amplitudes at loop-level?
- Study related questions for super-Wilson loops.

References

- [1] J. M. Drummond, J. Henn, G. P. Korchemsky and E. Sokatchev, "Dual superconformal symmetry of scattering amplitudes in N=4 super-Yang-Mills theory," Nucl. Phys. B **828** (2010) 317 [arXiv:0807.1095 [hep-th]].
- [2] N. Beisert, J. Henn, T. McLoughlin, J. Plefka, "One-Loop Superconformal and Yangian Symmetries of Scattering Amplitudes in $\mathcal{N}=4$ Super Yang-Mills," JHEP **1004** (2010) 085. [arXiv:1002.1733 [hep-th]].
- [3] J. M. Drummond, J. M. Henn and J. Plefka, "Yangian symmetry of scattering amplitudes in N=4 super Yang-Mills theory," JHEP **0905** (2009) 046 [arXiv:0902.2987 [hep-th]].
- [4] B. Eden, G. P. Korchemsky and E. Sokatchev, "From correlation functions to scattering amplitudes," JHEP **1112** (2011) 002 [arXiv:1007.3246 [hep-th]];

Further Scientific Activity

- G. Jorjadze, J. Plefka and J. Pollok, "Bosonic String Quantization in Static Gauge," J. Phys. A **45** (2012) 485401 [arXiv:1207.4368 [hep-th]].

Profit from the GK

- Travel funding: 6th Parma International School of Theoretical Physics Scattering Amplitudes in QCD, Supersymmetric Gauge Theories and Supergravity
- GK courses: Introduction to supersymmetry, Introduction to string theory

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