Remainder functions for self-crossing Wilson loops in $\mathcal{N} = 4$ super Yang-Mills theory

Motivation

Light-like polygonal Wilson loops in $\mathcal{N} = 4$ SYM theory have recently attracted a lot of interest due to their correspondence to maximally helicity violating (MHV) gluon scattering amplitudes in $\mathcal{N} = 4$ SYM. The relation between those two quantities is realized in terms of dual coordinates $p_i = (x_{i+1} - x_i)$. The amplitude is a function on the n external momenta p_i, while the corresponding Wilson loop is built out of a polygon with the n cusps at x_i and thus the edges are the corresponding momenta p_i. Then the polygon is closed because of momentum conservation. This duality implies that the amplitude has to obey the dual conformal Ward identities that are valid for light-like polygonal Wilson loops. Those Ward identities are solved by the so-called BDS Ansatz and include a remainder function. This function only depends on conformal invariants of the Wilson loop and its perturbative expansion starts at $\mathcal{O}(a^2)$. A lot of past and ongoing work is devoted to the study of the analytic structure of the remainder functions.

Methods

Our aim is to extract information about the remainder functions by studying self-crossing Wilson loops. Self-crossing Wilson loops have been studied in QCD and they can be multiplicatively renormalized using a $\mathcal{Z}$ matrix

$$\mathcal{W}_a \;=\; \mathcal{Z}_{ab}\, \mathcal{W}_b^{ren.} \; .$$

$\{\mathcal{W}_a\}$ is the set of mixing Wilson loop operators, $a \in \{1, 2\}$. The structure of the crossing anomalous dimension matrix is known. We insert the BDS Ansatz

$$\log \mathcal{W} = \Big[\text{BDS}\Big] + \mathcal{R}(\mu^2, \epsilon, \{s\})$$

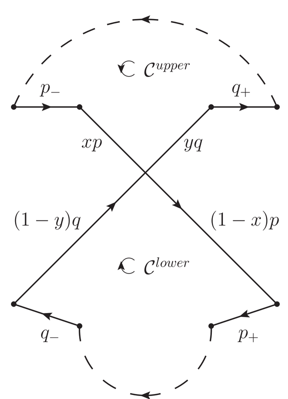
into the renormalization group equation (RGE) to calculate the leading divergences of the remainder function due to the crossing.

Some details

The Wilson loop is defined via

$$\langle \mathcal{W}(\mathcal{C})\rangle := \langle \frac{1}{N}\,\text{tr}\,\mathcal{P}\;\exp\left(ig\oint_{\mathcal{C}} A^{\mu}\text{d}x_{\mu}\right)\rangle \; ,$$

where $\mathcal{C}$ is the integration contour.



We are faced with a mixing of the Wilson loops

$$\mathcal{W}_1 := \langle \mathcal{W}(\mathcal{C})\rangle \; , \; \mathcal{W}_2 \overset{N\to\infty}{:=} \langle \mathcal{W}(\mathcal{C}_{\text{upper}})\rangle\langle W(\mathcal{C}_{\text{lower}})\rangle \; .$$

In the 't Hooft limit under consideration the $\mathcal{Z}$ -matrix has the upper-triangular form

$$\mathcal{Z} = \begin{pmatrix} \mathcal{Z}_{11} & \mathcal{Z}_{12} \\ 0 & \mathcal{Z}_{22} \end{pmatrix} \; .$$

Then also the anomalous dimension matrix

$$\Gamma := \mathcal{Z}^{-1}\mu\frac{d}{d\mu}\mathcal{Z}\,\Big|_{g_{\text{bare}}\text{ fixed}} \; ,$$

is upper triangular. The RGE to start with is

$$\mu\frac{\partial}{\partial\mu}\,\log \mathcal{W}_1^{\text{ren.}} \;=\; -\;\Gamma_{12}\,\frac{\mathcal{W}_2^{\text{ren.}}}{\mathcal{W}_1^{\text{ren.}}} \;-\; \Gamma_{11} \; .$$

Then, using minimal subtraction, we expand $\log \mathcal{W}_a$ in the coupling to express the renormalized Wilson loops in terms of the bare ones and the Z-factors, for example

$$\log \mathcal{W}_1^{\text{ren.}(2)} = \text{MS}\Big[\log \mathcal{W}_1^{(2)} + \mathcal{Z}_{12}^{(1)}\left(\mathcal{W}_1^{\text{ren.}(1)} - \mathcal{W}_2^{\text{ren.}(1)}\right)\Big] \; .$$

Using the BDS Ansatz we are able to compute $\text{MS}\Big[\mathcal{R}(\mu^2, \epsilon, \{s\})\Big]$. From the $\log\mu^2$ dependence we deduce the leading and next-to-leading divergences of $\mathcal{R}(\mu^2, \epsilon, \{s\})$. However, these expressions are not conformally invariant.
If we consider "almost crossing" Wilson loops the distance $z_\perp$ between the close edges will regulate the crossing divergences.
Then we relate the poles in ϵ to logarithms in $z_\perp$. Finally, we wish to relate $\log\left(\frac{1}{-z_\perp^2\mu^2}\right)$ to expressions that only involve conformal invariants. One can prove an identity of the form

$$\log\left(\frac{1}{-z_\perp^2\mu^2}\right) = f(\{u\}) + g(\{s\}) + \mathcal{O}(z_\perp^2) \; ,$$

such that approaching the self-crossing configuration $f(\{u\}) \to \infty$ and $g(\{s\})$ stays finite. Using this relation we generate conformally invariant and non conformally invariant terms. However, together with the non conformal parts of $\mathcal{R}(\mu^2, \epsilon, \{s\})$ the total expression becomes conformally invariant.

We use the same technique to obtain results up to three loops for a crossing between two legs (hexagon) and between two vertices (octagon). Also, our result for the self-crossing two-loop hexagon remainder is, after a suitable analytic continuation, in full agreement with results from the literature. In principle, one can also examine crossings between a leg and a vertex or even multiple intersections of the Wilson loop contour.

Recent Results

We calculate the divergent parts of $\mathcal{R}^{(3)}(\{u\})$ of the hexagon remainder function with the crossing momenta p and q. Depending on sgn(pq) we find

$$\mathcal{R}^{(3)}(\{u\})_{pq<0} = \mp\frac{7}{240}\pi i\;\log^5(u_2 - 1) - \frac{3}{16}\pi^2\;\log^4(u_2 - 1) + \mathcal{O}\left(\log^3(u_2 - 1)\right) \quad , \quad \mathcal{R}^{(3)}(\{u\})_{pq>0} = \pm\frac{7}{240}\pi i\;\log^5(1 - u_2) - \frac{1}{24}\pi^2\;\log^4(1 - u_2) + \mathcal{O}\left(\log^3(1 - u_2)\right) \; .$$

In an earlier publication we presented the calculation at two loops for the octagon remainder function for a crossing at two vertices.

Publications

- H. Dorn, G. Jorjadze and S. Wuttke,
  "On spacelike and timelike minimal surfaces in AdS(n),"
  JHEP **0905** (2009) 048, [arXiv:0903.0977 [hep-th]].
- H. Dorn, G. Jorjadze, C. Kalousios, L. Megrelidze and S. Wuttke,
  "Vacuum type space-like string surfaces in $AdS_3 \times S^3$,"
  J. Phys. A A **44** (2011) 025403, [arXiv:1007.1204 [hep-th]].
- H. Dorn and S. Wuttke,
  "Wilson loop remainder function for null polygons in the limit of self-crossing,"
  JHEP **1105** (2011) 114, [arXiv:1104.2469 [hep-th]].
- H. Dorn and S. Wuttke,
  "Hexagon Remainder Function in the Limit of Self-Crossing up to three Loops,"
  JHEP **1204** (2012) 023, [arXiv:1111.6815 [hep-th]].

Selected Talks

- DESY Theory Workshop on Quantum Field Theory, DESY Hamburg, 09/2010 "Space-like string surfaces in $AdS_3 \times S^3$ of vacuum type "
- Block Course des Graduiertenkollegs, Berlin, 10/2010 " Minimal surfaces in AdS/CFT "
- Recent Advances in Quantum Field and String Theory, Tbilisi, 09/2011 " Wilson loop remainder function for null polygons in the limit of self-crossing "

Profit from the GK

During my PhD studies I was a full member of the graduate school. Apart from the funding and travel money I took profit from the Block Courses offered by the graduate school. Also more experimentally oriented lessons and talks at the Block Courses were interesting and helpful to keep in touch with the experiments.

Contact Details and further Information

PhD student: Sebastian Wuttke, wuttke@physik.hu-berlin.de
PhD advisors: Harald Dorn, Jan Plefka

WWW: http://qft.physik.hu-berlin.de/
collaborators: Harald Dorn, George Jorjadze, Chrysostomos Kalousios

Supported by DFG Deutsche Forschungsgemeinschaft