Predictivity of models with spontaneously broken discrete flavour symmetries

Maximilian Fallbacher

in collaboration with Mu-Chun Chen, Yuji Omura, Michael Ratz and Christian Staudt.

Chen, MF, Ratz and Staudt, *Phys. Lett.* B **718** (2012), arXiv: 1208.2947 [hep-ph].

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Technische Universität München
Department Physik

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Neutrino mixing

Experimental success

- Measurements of neutrino mixing angles reach precision phase:

	θ_{12}	θ_{13}	θ_{23}
Best fit values ($\pm 1\sigma$):	$\left(33.6^{+1.1}_{-1.0}\right)^\circ$	$\left(8.93^{+0.46}_{-0.48}\right)^\circ$	$\left(38.4^{+1.4}_{-1.2}\right)^\circ$

Fogli et al. (2012)

$\Updownarrow$

Theory?

- Explanation for the (size of the) mixing.
- Predictions for the yet unknown parameters.

$\Rightarrow$ **Much work to do.**

Outline

1. Discrete symmetries and flavour model building

2. Kähler corrections to lepton flavour mixing

3. Conclusion

Discrete symmetries and flavour model building

Discrete non-abelian symmetry G_F acting on flavour space:

$$L_i \to U_{ij}\, L_j\,, \qquad R_i \to V_{ij}\, R_j\,.$$

e.g. $A_4,\ S_3,\ S_4,\ T',\ \Delta(3\,n^2),\ \ldots$

- So-called **flavon** fields are charged under G_F but are SM singlets.
- Symmetry G_F is spontaneously broken by the **flavons**.

  $\Rightarrow$ Mass terms in superpotential generated as effective operators.
- Here: focus on Majorana neutrinos in supersymmetric models.

An A_4 example

- The tetrahedral group A_4 has 4 representations: $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{3}$. Altarelli et al. (2006) Ma (2004)

- The leptons:

$$\boxed{L \sim \mathbf{3}, \quad R_i \sim \mathbf{1}, \mathbf{1}'', \mathbf{1}'}$$

- The flavons and their VEVs:

$$\begin{array}{ccc} \Phi_\nu \sim \mathbf{3} & \xi_\nu \sim \mathbf{1} & \Phi_e \sim \mathbf{3} \\ \langle \Phi_\nu \rangle = (v, v, v) & \langle \xi_\nu \rangle = w & \langle \Phi_e \rangle = (v', 0, 0) \end{array}$$

$$W_\nu = \frac{\lambda_1}{\Lambda \underbrace{\Lambda_\nu}_{\text{see-saw scale}}} \left\{ [(L\, H_u) \otimes (L\, H_u)]_{\mathbf{3}_s} \otimes \Phi_\nu \right\}_{\mathbf{1}} + \frac{\lambda_2}{\Lambda\, \Lambda_\nu} [(L\, H_u) \otimes (L\, H_u)]_{\mathbf{1}}\, \xi_\nu \,,$$

$$W_e = \frac{h_e}{\underbrace{\Lambda}_{\text{cut-off scale}}} (L \otimes \Phi_e)_{\mathbf{1}}\, H_d\, e_R + \frac{h_\mu}{\Lambda} (L \otimes \Phi_e)_{\mathbf{1}'}\, H_d\, \mu_R + \frac{h_\tau}{\Lambda} (L \otimes \Phi_e)_{\mathbf{1}''}\, H_d\, \tau_R \,.$$

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$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} .$$

Harrison et al. (2002)

	θ_{12}	θ_{13}	θ_{23}
TBM prediction:	$\approx 35.3^\circ$	0	45°
Best fit values ($\pm 1\sigma$):	$\left(33.6^{+1.1}_{-1.0}\right)^\circ$	$\left(8.93^{+0.46}_{-0.48}\right)^\circ$	$\left(38.4^{+1.4}_{-1.2}\right)^\circ$

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- Higher order corrections in the superpotential:

$$W_\nu \to W_\nu + \mathcal{O}\left(\frac{1}{\Lambda^2}\right),$$

$$W_e \to W_e + \mathcal{O}\left(\frac{1}{\Lambda^2}\right).$$

The Kähler potential

- Most often only the superpotential is discussed.
- However, there are also higher order terms in the Kähler potential:

Canonical terms

$$K = \boxed{{L_i}^\dagger\, L_i + {R_i}^\dagger\, R_i} + \boxed{{L_i}^\dagger\, (\Delta\mathcal{K}_L)_{ij}\, L_j + {R_i}^\dagger\, (\Delta\mathcal{K}_R)_{ij}\, R_j}$$

Correction terms
Contain contractions with the flavons like
$\frac{1}{\Lambda^2}\,(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}}(L \otimes \Phi_\nu)_{\mathbf{3}}$.

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$\frac{1}{\Lambda^2}\,(L \otimes \Phi_\nu)^{\dagger}_{\mathbf{3}}(L \otimes \Phi_\nu)_{\mathbf{3}}$.

When $\Phi_\nu \to$ VEV, this re-normalises the kinetic terms.

$$\xrightarrow{\text{flavons}\to\text{VEV}} \boxed{L_i^{\dagger}\, (\mathbb{1} + \Delta\mathcal{K}_L|_{\text{VEV}})_{ij}\, L_j + R_i^{\dagger}\, (\mathbb{1} + \Delta\mathcal{K}_R|_{\text{VEV}})_{ij}\, R_j}$$

non-diagonal matrices

⇒ Canonical normalisation leads to change of the mixing.

Chen et al. (2012)
Antusch et al. (2009)
Antusch et al. (2008)
Espinosa et al. (2004)

Correction terms in the A_4 model

- Terms linear in a specific flavon can always be forbidden by additional abelian symmetries.

$$\cancel{(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}} L}, \qquad \cancel{(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}} (L \otimes \Phi_e)_{\mathbf{3}}}.$$

- Quadratic terms cannot be forbidden this way.

$$(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}} (L \otimes \Phi_\nu)_{\mathbf{3}} \,.$$

- An A_4 example: Chen et al. (2012)

$$K \supset \frac{\kappa}{\Lambda^2} (L \otimes \Phi_\nu)^\dagger_{\mathbf{3}_s} (L \otimes \Phi_\nu)_{\mathbf{3}_a} + \mathrm{h.\,c.}$$

$$\xrightarrow{\Phi_\nu \to \langle \Phi_\nu \rangle = (v,v,v)} \kappa \frac{v^2}{\Lambda^2} \frac{3\sqrt{3}}{2} L^\dagger \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix} L + \mathrm{h.\,c.}$$

Analytic formulae

Input
Masses Holomorphic angles and phases

→

Output
Corrected angles and phases

$$K_L = L^\dagger L \to \boxed{K_L = L^\dagger\,(\mathbb{1} - 2\,x\,P)\,L}$$

infinitesimal (x) Hermitian (P)

- Canonical normalisation: $L \to (\mathbb{1} + x\,P)\,L$.
- This changes the mass terms in the superpotential, e.g.

$$\begin{aligned} W_\nu &= \frac{1}{2}\,L^T\,m_\nu\,L \\ &\to \frac{1}{2}\,L^T\,\left(\mathbb{1} + x\,P^T\right)\,m_\nu\,(\mathbb{1} + x\,P)\,L \\ &\simeq \frac{1}{2}\,L^T\,\left[m_\nu + x\,\left(P^T\,m_\nu + m_\nu\,P\right)\right]\,L\,. \end{aligned}$$

Analytic formulae

Input		**Output**
Masses Holomorphic angles and phases	→	Corrected angles and phases

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infinitesimal (x) Hermitian (P)

- Canonical normalisation: $L \to (\mathbb{1} + x\,P)\,L$.
- This changes the mass terms in the superpotential, e.g.

$$W_\nu \simeq \frac{1}{2}\,L^T\,\left[m_\nu + x\,\left(P^T\,m_\nu + m_\nu\,P\right)\right]\,L\,.$$

- This leads, for infinitesimal x, to the differential equation

$$\frac{\mathrm{d}m_\nu}{\mathrm{d}x} = P^T\,m_\nu + m_\nu\,P\,.$$

- Analogous to RGEs. Chen et al. (2013) Antusch et al. (2005) Antusch et al. (2003)

The MATHEMATICA package

- The package contains the analytic formulae.
- Several functions are provided to simplify their use.
- The package can be found here:
  http://einrichtungen.ph.tum.de/T30e/codes/KaehlerCorrections/

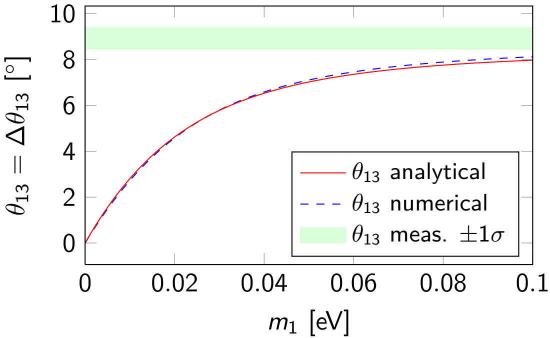
Example I

- Kähler correction of the form:

$$\Delta K = \kappa \frac{v^2}{\Lambda^2} 3\sqrt{3} \cdot L^\dagger \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix} L\,.$$

- This changes θ_{13} by:

$$\Delta\theta_{13} \simeq \kappa \frac{v^2}{\Lambda^2} \; 3\sqrt{6} \; \frac{m_1}{m_1 + m_3}$$



$$\kappa \frac{v^2}{\Lambda^2} = 1 \cdot (0.2)^2$$

$$m_3 = \sqrt{m_1^2 + \Delta m_{13}^2}$$

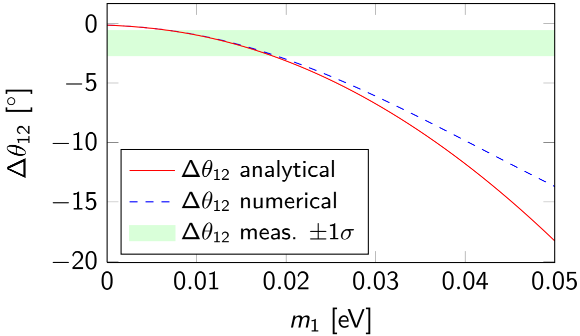
Example II

- Kähler correction of the form:

$$\Delta K = \kappa' \frac{v'^2}{\Lambda^2} \cdot L^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} L \, .$$

- This changes θ_{12} by:

$$\Delta\theta_{12} \simeq \kappa' \frac{v'^2}{\Lambda^2} \frac{1}{3\sqrt{2}} \frac{m_1 + m_2}{m_1 - m_2}$$



$$\kappa' \frac{v'^2}{\Lambda^2} = \frac{1}{4} \cdot (0.2)^2$$

$$m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}$$

Conclusion

- Kähler corrections cannot be avoided in models with discrete flavour symmetries.
- The corrections to the mixing angles can be sizable.
  - $\Rightarrow$ It might be premature to exclude models only because of their superpotential predictions.
- This introduces a considerable arbitrariness into flavour model building.
  - $\Rightarrow$ Difficult to achieve theoretical precision comparable with experimental precision without knowledge of the Kähler potential.

Thank You!

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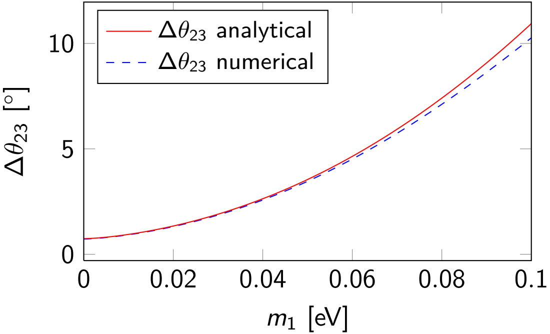
Example III

- Kähler correction of the form:

$$\Delta K = \kappa \frac{v^2}{\Lambda^2} \cdot L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} L\,.$$

- This changes θ_{23} by:

$$\Delta\theta_{23} \simeq \kappa \frac{v^2}{\Lambda^2} \; \frac{1}{12} \left(3 + \frac{2m_1}{m_3 - m_1} + \frac{4m_2}{m_3 - m_2} \right)$$



$$\kappa \frac{v^2}{\Lambda^2} = 1 \cdot (0.2)^2$$

A_4 basis

- The A_4 generators in the chosen basis are given by

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}\,, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}\,, \text{ with } \omega = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}}\,.$$

- The non-trivial contraction is $\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_\mathrm{s} \oplus \mathbf{3}_\mathrm{a}$ with

$$\begin{aligned}
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1}} &= a_1\, b_1 + a_2\, b_3 + a_3\, b_2\,, \\
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1}'} &= a_2\, b_2 + a_1\, b_3 + a_3\, b_1\,, \\
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1}''} &= a_3\, b_3 + a_1\, b_2 + a_2\, b_1\,, \\
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{3}_\mathrm{s}} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 2a_1\, b_1 - a_2\, b_3 - a_3\, b_2 \\ 2a_3\, b_3 - a_1\, b_2 - a_2\, b_1 \\ 2a_2\, b_2 - a_1\, b_3 - a_3\, b_1 \end{pmatrix}\,, \\
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{3}_\mathrm{a}} &= \mathrm{i}\sqrt{\frac{3}{2}} \begin{pmatrix} a_2\, b_3 - a_3\, b_2 \\ a_1\, b_2 - a_2\, b_1 \\ a_3\, b_1 - a_1\, b_3 \end{pmatrix}\,.
\end{aligned}$$

Contraction

$$
\begin{aligned}
&(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}_s} (L \otimes \Phi_\nu)_{\mathbf{3}_a} \\
&\qquad = \frac{\mathrm{i}\sqrt{3}}{2} \Big[\Big(2\, L_1^\dagger \Phi_{\nu\,1}^{\;\dagger} - L_2^\dagger \Phi_{\nu\,3}^{\;\dagger} - L_3^\dagger \Phi_{\nu\,2}^{\;\dagger} \Big) \left(L_2 \Phi_{\nu\,3} - L_3 \Phi_{\nu\,2} \right) \\
&\qquad\qquad + \Big(2 L_3^\dagger \Phi_{\nu\,3}^{\;\dagger} - L_2^\dagger \Phi_{\nu\,1}^{\;\dagger} - L_1^\dagger \Phi_{\nu\,2}^{\;\dagger} \Big) \left(L_1 \Phi_{\nu\,2} - L_2 \Phi_{\nu\,1} \right) \\
&\qquad\qquad + \Big(2 L_2^\dagger \Phi_{\nu\,2}^{\;\dagger} - L_1^\dagger \Phi_{\nu\,3}^{\;\dagger} - L_3^\dagger \Phi_{\nu\,1}^{\;\dagger} \Big) \left(L_3 \Phi_{\nu\,1} - L_1 \Phi_{\nu\,3} \right) \Big]
\end{aligned}
$$