

$$\begin{aligned}
\mathcal{F}(s, \kappa, w) &= \frac{1}{2} \sum_{\gamma \in SL(2; \mathbb{Z}) / \Gamma_\infty} (c\tau + d)^{-w} \mathcal{M}_{s, w}(-\kappa \operatorname{Im} \gamma \cdot \tau) e^{-2\pi i \kappa \operatorname{Re}(\gamma \cdot \tau_1)} \\
&= \frac{1}{2} \sum_{(c, d)=1} (c\tau + d)^{-w} \mathcal{M}_{s, w} \left( -\frac{\kappa \tau_2}{|c\tau + d|^2} \right) \exp \left\{ -2\pi i \kappa \left( \frac{a}{c} - \frac{c\tau_1 + d}{c|c\tau + d|^2} \right) \right\}
\end{aligned}$$