



Construction of explicit de Sitter vacua in type IIB flux compactifications

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Outline:

1. Introduction: De Sitter vacua in type IIB
2. The complex structure moduli of $\mathbb{CP}_{11169}$ [18]
3. Scanning all vacua with paramotopy
4. Kähler uplifted de Sitter vacua
5. Conclusions

1. Introduction: De Sitter vacua in type IIB

Motivation: Construct explicit de Sitter vacua in type IIB/F-theory

Cosmology:

- ▶ Acceleration of the universe on large scales is observed.
- ▶ Simplest explanation: dS space with small cosmological constant.

The setup we use:

- ▶ Non-perturbative effects $W = A_i e^{-a_i T_i}$
[Kachru,Kallosh,Linde,Trivedi '03]
- ▶ Leading α' -corr. to the Kähler pot. $K = -2 \ln(\hat{\mathcal{V}}(T_i) + \alpha'^3 \hat{\xi}(\tau))$
[Becker,Becker,Haack,Louis '02]
- ▶ Quantized RR and NS-NS fluxes $\int F_3, H_3 \in \mathbb{Z}$
 $\Rightarrow$ **SUSY AdS vacua for the U_a and τ** [Giddings,Kachru,Polchinski '01]

Moduli stabilization in the large volume limit

$\mathcal{N} = 1$, 4D effective potential:

$$V = e^K \left(K^{\alpha\bar{\beta}} D_\alpha W \overline{D_\beta W} - 3|W|^2 \right)$$

For $\hat{\xi} \ll \hat{\mathcal{V}}$ this is to 0-th order the positive semi-definite potential

[Balasubramanian, Berglund, Conlon, Quevedo'05], [Westphal, MR'11]

$$V_{flux} = e^K \left(K^{\tau\bar{\tau}} |D_\tau W_0|^2 + K^{a\bar{b}} D_a W_0 \overline{D_b W_0} \right) + \mathcal{O}\left(\frac{\hat{\xi}}{\hat{\mathcal{V}}}\right)$$

due to no-scale structure [Cremmer, Ferrara, Kounnas, Nanopoulos'83]

- ▶ Every SUSY extremum for the τ , U_a is a minimum for $\hat{\mathcal{V}} \gg \hat{\xi}$!
- ▶ Separation of scales $\Rightarrow$ Stabilize U_a first!

2. The complex structure moduli of $\mathbb{CP}_{11169}$ [18]

$$\mathbb{CP}_{11169} : (x_1, x_2, x_3, x_4, x_5) \sim (\lambda x_1, \lambda x_2, \lambda x_3, \lambda^6 x_4, \lambda^9 x_5)$$

$$\text{e.g. } x_1^{18} + x_2^{18} + x_3^{18} + x_4^3 + x_5^2 = 0 \quad (h^{1,1} = 2 \text{ and } h^{2,1} = 272)$$

Complex structure moduli stabilization of $\mathbb{CP}_{11169}$ [18] (I)

Need to find $f = \int F_3 \in \mathbb{Z}$ and $h = \int H_3 \in \mathbb{Z}$ with

$$D_a W = 0, a = 1, \dots, h^{2,1} = 272 \text{ [Giddings, Kachru, Polchinski'01]}$$

Effective theory completely determined by $\mathcal{N} = 2$ prepotential:

$$\mathcal{G}(U_1, \dots, U_{h^{2,1}}) = \sum_{i+j \leq 3} c_{ij} U_a^i U_b^j + \xi + \mathcal{G}_{\text{instanton}}(e^{-2\pi U_1}, \dots, e^{-2\pi U_{h^{2,1}}})$$

- ▶ $W_0 = \int (F_3 - \tau H_3) \wedge \Omega(U_a) = (f - \tau h) \cdot (\mathcal{G}_{U_a}, U_a)$ [Gukov, Vafa, Witten'00]
- ▶ $K_{\text{CS}} = -\ln [i(\bar{U}_a \mathcal{G}_{U_a} - U_a \bar{\mathcal{G}}_{U_a})]$
- ▶ **Large complex structure limit:** $\left| \frac{\mathcal{G}_{\text{instanton}}}{\mathcal{G}_{\text{cubic}}} \right| \leq \epsilon_{\text{LCS}}$

Complex structure moduli stabilization of $\mathbb{CP}_{11169}$ [18] (II)

- ▶ Discrete symmetry $\mathbb{Z}_6 \times \mathbb{Z}_{18}$: U_a , $a = 1, \dots, h_{\text{inv.}}^{2,1} = 2$ and $\tilde{U}_a$, $a = 3, \dots, 272$ [Greene,Plesser'89], [Candelas,Font,Katz,Morrison'94]
- ▶ Switch on flux only on $h_{\text{inv.}}^{2,1} = 2$ [Giryavets,Kachru,Tripathy,Trivedi '03]
- ▶ Symmetry $\Rightarrow D_{\tilde{U}_a} W_0 = 0$ at $\tilde{U}_a = 0$ for $a = 3, \dots, 272$
- ▶ $\Rightarrow$ **Only need to solve $D_\phi W|_{\tilde{U}_a=0} = 0$ for $\phi = \tau, \mathbf{U}_1, \mathbf{U}_2$**
- ▶ $\Rightarrow V \sim |D_a W|^2$ ensures stable minimum for all 272 U_a in the large volume limit!

3. Scanning all vacua with paramotopy

Paramotopy

see e.g. [Li '03], [Sommese, Wampler '05]

Have to solve polynomial system $P(x) = (p_1(x), \dots, p_m(x))^T = 0$ with $x = (x_1, \dots, x_m)^T$

- ▶ Maximal number of isolated solutions in $\mathbb{C}^m$: $\prod_{i=1}^m d_i$, with d_i the degree of the i th polynomial. (*Classical Bézout bound*)
- ▶ Construct homotopy $H(x, t) = \gamma(1 - t)Q(x) + t P(x)$, with e.g. $Q(x) = (x_1^{d_1} - 1, \dots, x_m^{d_m} - 1) \Rightarrow$ **Easy to solve.**
- ▶ Follow paths $H(x, t) = 0$ for $0 \leq t \leq 1$ for all solutions to $Q(x) = 0$

$\Rightarrow$ **Will find all solutions to $P(x) = 0$**

The scan

Use to solve $D_\phi W(\mathbf{f}) = 0$ for all fluxes $\mathbf{f}$ with $L \sim \mathbf{f}^2 \leq L_{\max}$

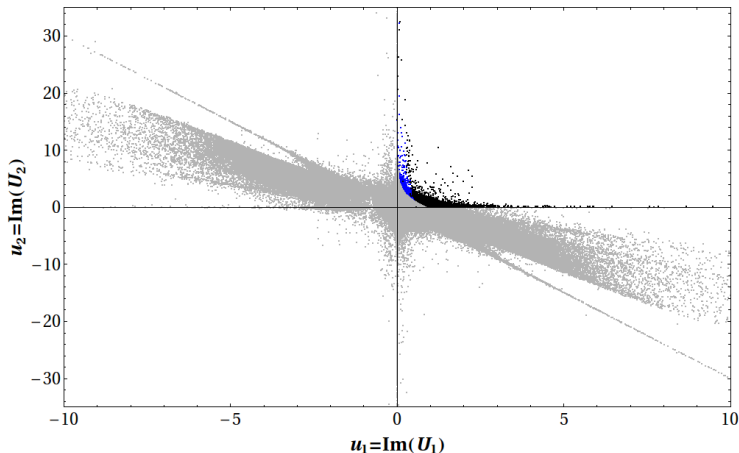
[Martinez-Pedrera, Mehta, Westphal, MR '12]

- ▶ The 10D IIB action is $SL(2, \mathbb{Z})$ invariant: $\tau \rightarrow \frac{a\tau + b}{c\tau + d}$,

$$G_3 = F_3 - \tau H_3 \rightarrow \frac{G_3}{c\tau + d} \quad \text{with} \quad a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1$$

- ▶ Make sure to only consider physically inequivalent configurations!
- ▶ Our scan: ~ 50.000 parameter points ($L \leq 35$).

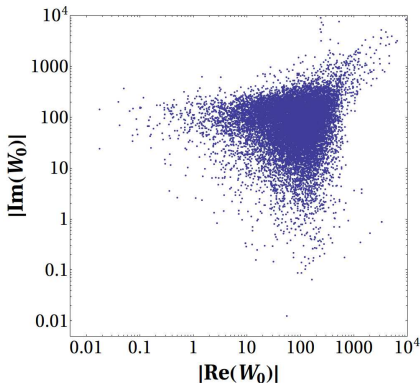
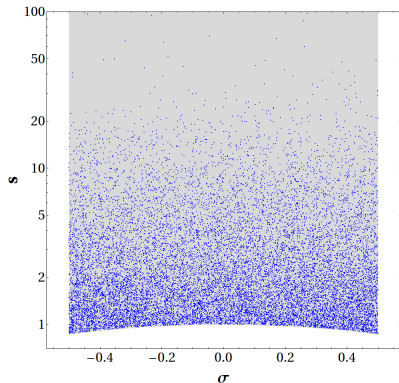
Scan results: The limit of large complex structure



$$\left| \frac{\mathcal{G}_{\text{instanton}}}{\mathcal{G}_{\text{cubic}}} \right| \leq \epsilon_{LCS} = \begin{cases} 10^{-1} & \text{(blue)} \Rightarrow 25.000 \text{ of } 500.000 \\ 10^{-2} & \text{(black)} \Rightarrow 15.000 \text{ of } \quad \quad \end{cases}$$

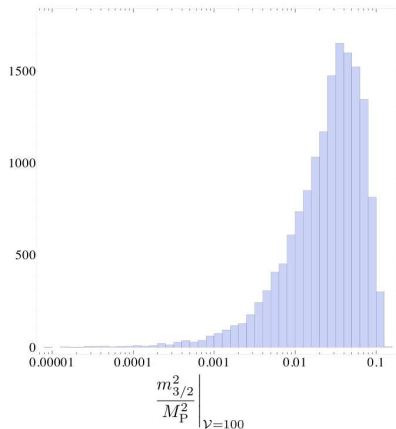
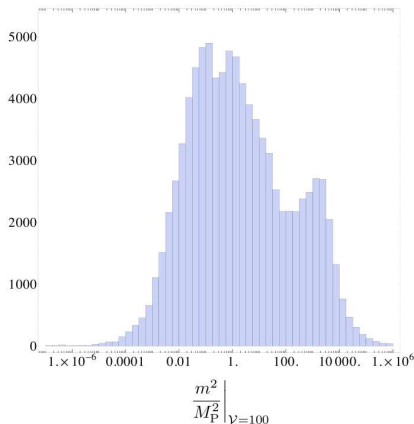
Scan results: $\tau = \sigma + i s$ and W_0

- ▶ $SL(2, \mathbb{Z})$ fundamental domain: $-\frac{1}{2} \leq \text{Re}(\tau) \leq \frac{1}{2}$ and $|\tau| > 1$
- ▶ Transformations: $\tau \rightarrow \tau + b$, $G_3 \rightarrow G_3$ and $\tau \rightarrow -1/\tau$, $G_3 \rightarrow G_3/\tau$



Scan results: Mass scales

Masses $\frac{M^2}{M_{\text{P}}^2} \Big|_{\hat{\nu}=100} \left(\frac{100}{\hat{\nu}} \right)^2$ for 6 real moduli fields and gravitino:

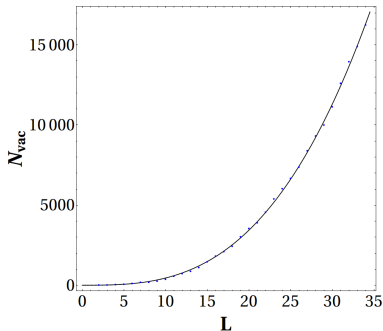


Scan results: Number of vacua N_{vac}

$$\begin{aligned} \blacktriangleright N_{vac}^{\text{stat}} &= \frac{(2\pi L)^3}{3!} \int \det(-\mathcal{R} - \mathbf{1} \cdot \omega) \\ &\simeq 0.03 L^3 \end{aligned}$$

[Ashok,Douglas'04],[Denef,Douglas,Florea'04]

$$\blacktriangleright N_{vac} \simeq (0.50 \pm 0.04) L^{2.94 \pm 0.03}$$



Scan results: Tuning of the cosmological constant

- ▶ Untuned cosmological constant (cc): $\Lambda \sim \frac{m_{3/2}^2}{\hat{v}}$
- ▶ Spacing in cc: $\frac{\Delta\Lambda}{\Lambda} \sim 2 \frac{\Delta m_{3/2}}{m_{3/2}} \sim \frac{C}{L^a (h_{\text{eff}}^{2,1} + 1)}$
- ▶ Fit, **Extrapolate** $\Rightarrow \frac{\Delta\Lambda}{\Lambda} \simeq (6.0 \pm 0.3) L^{-(0.95 \pm 0.005)} (h_{\text{eff}}^{2,1} + 1)$

$h_{\text{eff}}^{2,1}$	L	$\Delta\Lambda/\Lambda$
2	34	$7 \cdot 10^{-3} \pm 5 \cdot 10^{-4}$
2	500	$5 \cdot 10^{-5} \pm 4 \cdot 10^{-6}$
40	34	$3 \cdot 10^{-58} \pm 2 \cdot 10^{-58}$
40	500	$10^{-102} \pm 10^{-102}$

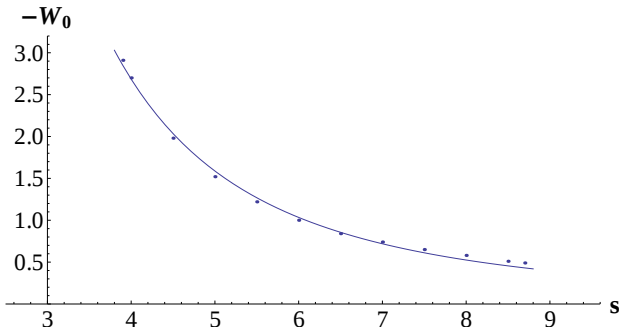
4. Kähler uplifted de Sitter vacua

Kähler uplifted de Sitter vacua (I)

Global model for Kähler moduli stabilization in a de Sitter vacuum on $\mathbb{CP}_{11169}$ [18] via non-perturbative effects: [Louis,Valandro,Westphal,MR '12]

$$V = e^K \left(K^{T_i \bar{T}_j} [W_{T_i} \overline{W_{T_j}} + W_{T_i} \cdot \overline{W K_{T_j}}] + 3 \hat{\xi} \frac{\hat{\xi}^2 + 7 \hat{\xi} \hat{\mathcal{V}} + \hat{\mathcal{V}}^2}{(\hat{\mathcal{V}} - \hat{\xi})(\hat{\xi} + 2 \hat{\mathcal{V}})^2} |W|^2 \right)$$

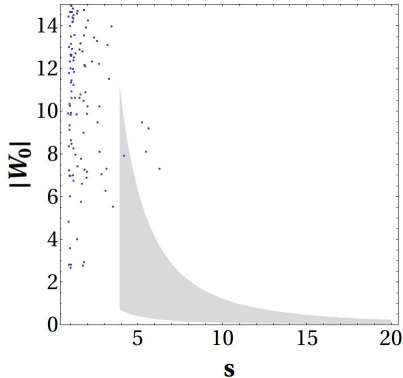
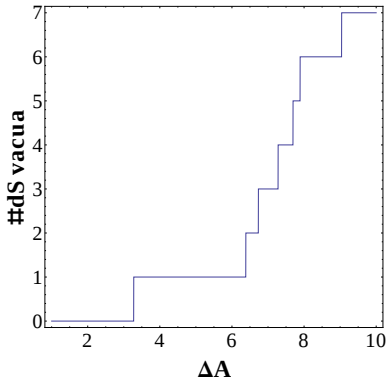
Numerically: Stable de Sitter solutions for $A_1, A_2 \sim 1$



Kähler uplifted de Sitter vacua (II)

Parametrize missing knowledge of A_1 , A_2 by ΔA with scaling relations:

$$W_0 \rightarrow W_0 \cdot \Delta A, \quad A_1 \rightarrow A_1 \cdot \Delta A, \quad A_2 \rightarrow A_2 \cdot \Delta A \quad \Rightarrow \quad V \rightarrow V \cdot \Delta A^2$$



(Uplifting can be applied in the shaded region for $\Delta A = 4$)

5. Conclusions

Conclusions

- ▶ All solutions to polynomial equations can be found using Paramotopy (highly parallelizable! $\Rightarrow$ 3000 days on 3000 cores $\Rightarrow$ 1 day!)
- ▶ $\Rightarrow$ All flux vacua of reduced moduli space have been constructed for given D3-tadpole ($L = 35$) in the large complex structure limit (LCS)
- ▶ $g_s \lesssim 1$ and $W_0 \sim \mathcal{O}(10^1 - 10^3)$ are preferred in our solutions
- ▶ N_{vac} and $\Delta\Lambda/\Lambda \sim 10^{-100}$ for $h_{\text{eff}}^{2,1} = 40$ and $L = 500$ consistent with semi-analytical predictions [Ashok,Douglas'04],[Denef,Douglas,Florea'04]
- ▶ Consistent global model of Kähler uplifting on $\mathbb{CP}_{11169}$ [18]
 $\Rightarrow \sim 10^{-4}$ flux vacua can be uplifted