The Wasteland of Random Supergravities

Timm Wrase

Bad Honnef

March 20, 2013

Based on:

David Marsh, Liam McAllister, TW 1112.3034

Thomas Bachlechner, David Marsh, Liam McAllister, TW 1207.2763

David Marsh, Liam McAllister, Enrico Pajer, TW 1304.xxxx

Motivation

- Generic string compactifications from 10D to 4D have many scalars

$$N \approx O(10^2 - 10^4)$$

- These scalar fields correspond to the size and shape of the internal space
- Fluxes, D-branes, O-planes, non-perturbative effects,… lead to a complicated scalar potential in 4D:

$$V(\phi^I),\ I = 1,\ldots,N$$

Motivation

We can only search under the lamppost

Even there we can only analyze

$$V(\phi^I), N \approx O(10)$$

Can we say something about the generic string landscape?

Motivation

We can only search under the lamppost

Even there we can only analyze

$V(\phi^I), N \approx O(10)$

Can we say something about the generic string landscape?

$\boxed{\text{Yes, if } V(\phi^I) \text{ is generic} \Rightarrow \text{ statistics}}$

The landscape

A simple toy model:

Susskind 2003

$$V(\phi^I) = \sum_{I=1}^{N} f_I(\phi^I)$$

$$V'(\phi^I) = 0 \;\Leftrightarrow\; f_I'(\phi^I) = 0 \;\; \forall I$$

$$V''(\phi^I) = \begin{pmatrix} f_1''(\phi^1) & & 0 \\ & \ddots & \\ 0 & & f_N''(\phi^N) \end{pmatrix}$$

The landscape

A simple toy model:

Susskind 2003

$$V(\phi^I) = \sum_{I=1}^{N} f_I(\phi^I)$$

$f_I(\phi^I)$ has α^I extrema

\# crit. points:

$$\prod_{I=1}^{N} \alpha^I \equiv \tilde{\alpha}^N$$

$$V'(\phi^I) = 0 \quad \Leftrightarrow \quad f_I'(\phi^I) = 0 \quad \forall I$$

$$V''(\phi^I) = \begin{pmatrix} f_1''(\phi^1) & & 0 \\ & \ddots & \\ 0 & & f_N''(\phi^N) \end{pmatrix}$$

The landscape

A simple toy model:

Susskind 2003

$$V(\phi^I) = \sum_{I=1}^{N} f_I(\phi^I)$$

$f_I(\phi^I)$ has α^I extrema

crit. points:

$$\prod_{I=1}^{N} \alpha^I \equiv \tilde{\alpha}^N$$

$$V'(\phi^I) = 0 \iff f_I'(\phi^I) = 0 \;\; \forall I$$

$$V''(\phi^I) = \begin{pmatrix} f_1''(\phi^1) & & 0 \\ & \ddots & \\ 0 & & f_N''(\phi^N) \end{pmatrix}$$

stability: $\boxed{\frac{1}{2^N}}$

The landscape

$\left(\frac{\tilde{\alpha}}{2}\right)^N$ for $\tilde{\alpha} = 5$ and $N = 500$ we have 10^{200} vacua!

A theory with that many vacua is consistent with a cosmological constant

$$\Lambda = 10^{-120} M_{Pl}^4$$

The landscape

$\left(\frac{\tilde{\alpha}}{2}\right)^N$ for $\tilde{\alpha} = 5$ and $N = 500$ we have 10^{200} vacua!

A theory with that many vacua is consistent with a cosmological constant

$$\Lambda = 10^{-120} M_{Pl}^4$$

$\Longrightarrow$ The cosmological constant or any other known fine tuning of parameters poses no challenge for string theory

Random non-susy landscape

Generically there are cross couplings

$V(\phi^I)$ is a random function

$$M^2 = \begin{pmatrix} \frac{\partial^2 V}{\partial \phi_1^2} & \cdots & \frac{\partial^2 V}{\partial \phi_1 \partial \phi_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 V}{\partial \phi_1 \partial \phi_N} & \cdots & \frac{\partial^2 V}{\partial \phi_N \partial \phi_N} \end{pmatrix}$$

is random matrix $\Rightarrow$ Random Matrix theory

Wasteland of Random SUGRA

Timm Wrase

Random Matrix Theory

Modelling gap-size distribution of parked cars using random-matrix theory

A.Y. Abul-Magd

Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt

We apply the random-matrix theory to the car-parking problem. For this purpose, we adopt a Coulomb gas model that associates the coordinates of the gas particles with the eigenvalues of a random matrix. The nature of interaction between the particles is consistent with the tendency of the drivers to park their cars near to each other and in the same time keep a distance sufficient for manoeuvring. We show that the recently measured gap-size distribution of parked cars in a number of roads in central London is well represented by the spacing distribution of a Gaussian unitary ensemble.

PACS: 05.40; 05.20.Gg; 02.50.r; 68.43.-h

Keywords: Car parking; Coulomb gas; Gaussian unitary ensemble

Random Matrix Theory

Modelling gap-size distribution of parked cars using random-matrix theory

A.Y. Abul-Magd
Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt

We apply the random-matrix theory to the car-parking problem. For this purpose, we adopt a Coulomb gas model that associates the coordinates of the gas particles with the eigenvalues of a random matrix. The nature of interaction between the particles is consistent with the tendency of the drivers to park their cars near to each other and in the same time keep a distance sufficient for manoeuvring. We show that the recently measured gap-size distribution of parked cars in a number of roads in central London is well represented by the spacing distribution of a Gaussian unitary ensemble.

PACS: 05.40; 05.20.Gg; 02.50.r; 68.43.-h
Keywords: Car parking; Coulomb gas; Gaussian unitary ensemble

The statistical properties of the city transport in Cuernavaca (Mexico) and Random matrix ensembles

Milan Krbálek [1,3] and Petr Šeba [2,3]
[1] *Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Trojanova 13, Prague Czech republic*
[2] *Institute of Physics, Czech Academy of Sciences, Cukrovarnicka 10, Prague, Czech Republic*
[3] *Department of Physics, Pedagogical University, Vita Nejedleho 573, Hradec Kralove, Czech Republic*

We analyze statistical properties of the city bus transport in Cuernavaca (Mexico) and show that the bus arrivals display probability distributions conforming those given by the Unitary Ensemble of random matrices.

Random non-susy landscape

Generically there are cross couplings

$V(\phi^I)$ is a random function

$$M^2 = \begin{pmatrix} \frac{\partial^2 V}{\partial \phi_1^2} & \cdots & \frac{\partial^2 V}{\partial \phi_1 \partial \phi_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 V}{\partial \phi_1 \partial \phi_N} & \cdots & \frac{\partial^2 V}{\partial \phi_N \partial \phi_N} \end{pmatrix}$$

is random matrix $\Rightarrow$ Random Matrix theory

Random non-susy landscape

Assume M^2 is a random symmetric matrix:

$$M^2 = A + A^\dagger$$

Wasteland of Random SUGRA

Timm Wrase

Motivation

Non-SUSY

N=1 SUGRA

Inflation

Conclusion

Outlook

Random non-susy landscape

Assume M^2 is a random symmetric matrix:

$$M^2 = A + A^\dagger$$

A independent, identically distributed entries

Gaussian distribution

$$A_{ij} \in \Omega\left(0, \frac{1}{\sqrt{N}}\right)$$

mean standard deviaton

Studied by E. Wigner in the 50's

Random non-susy landscape

Joint probability distribution

$$\underbrace{P}_{\text{probability}}(\underbrace{\lambda_1,\ldots,\lambda_N}_{\text{eigenvalues}}) = c_N \, e^{-\frac{1}{2}\sum_I \lambda_I^2} \prod_{J<K} |\lambda_J - \lambda_K|$$

Random non-susy landscape

Joint probability distribution

$$P(\lambda_1,\ldots,\lambda_N) = c_N\, e^{-\frac{1}{2}\sum_I \lambda_I^2} \prod_{J<K} |\lambda_J - \lambda_K|$$

N one dimensional charged particles in a quadratic potential

$$P(\lambda_I = \lambda_J) = 0$$

Eigenvalue repulsion!

Wasteland of Random SUGRA

Timm Wrase

Wasteland of Random SUGRA

Timm Wrase

Motivation

Non-SUSY

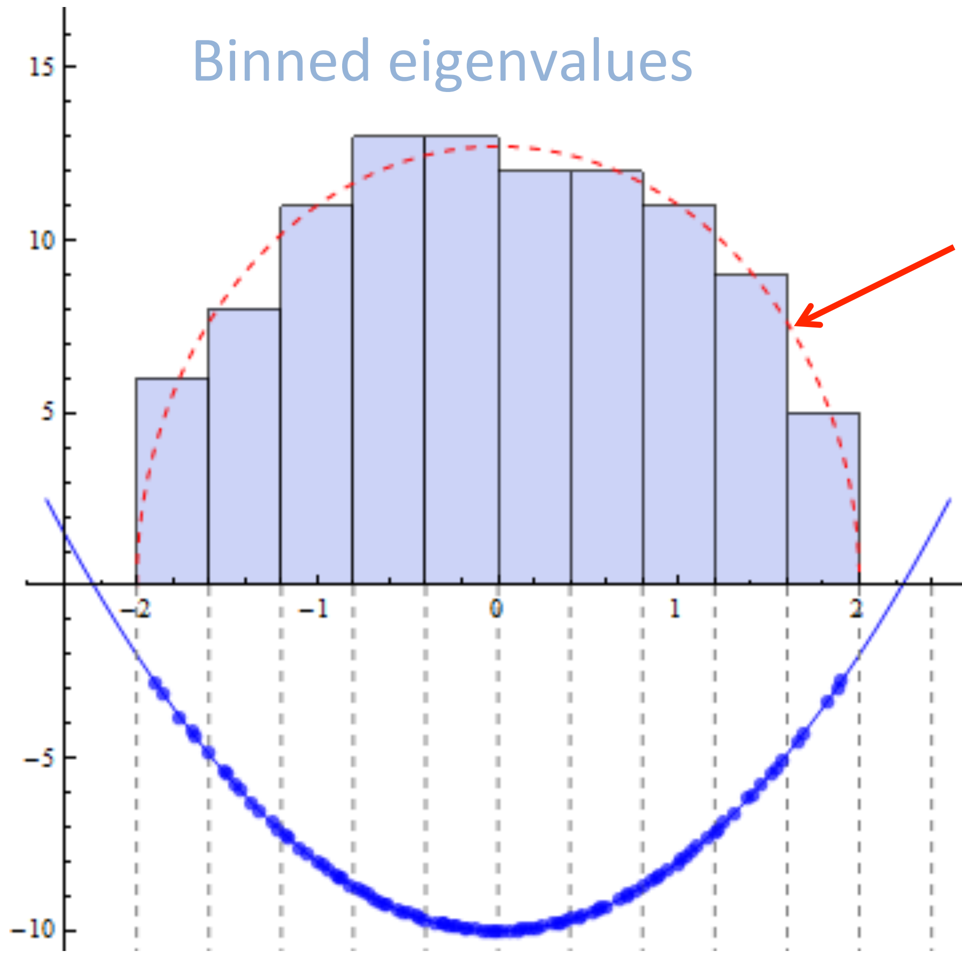
N=1 SUGRA

Inflation

Conclusion

Outlook

Random non-susy landscape



Wigner semi-circle

$$PDF(\lambda) = \frac{1}{2\pi}\sqrt{4-\lambda^2}$$

1D charged particles in quadratic potential

Random non-susy landscape

Wasteland of Random SUGRA

Timm Wrase

Motivation

Non-SUSY

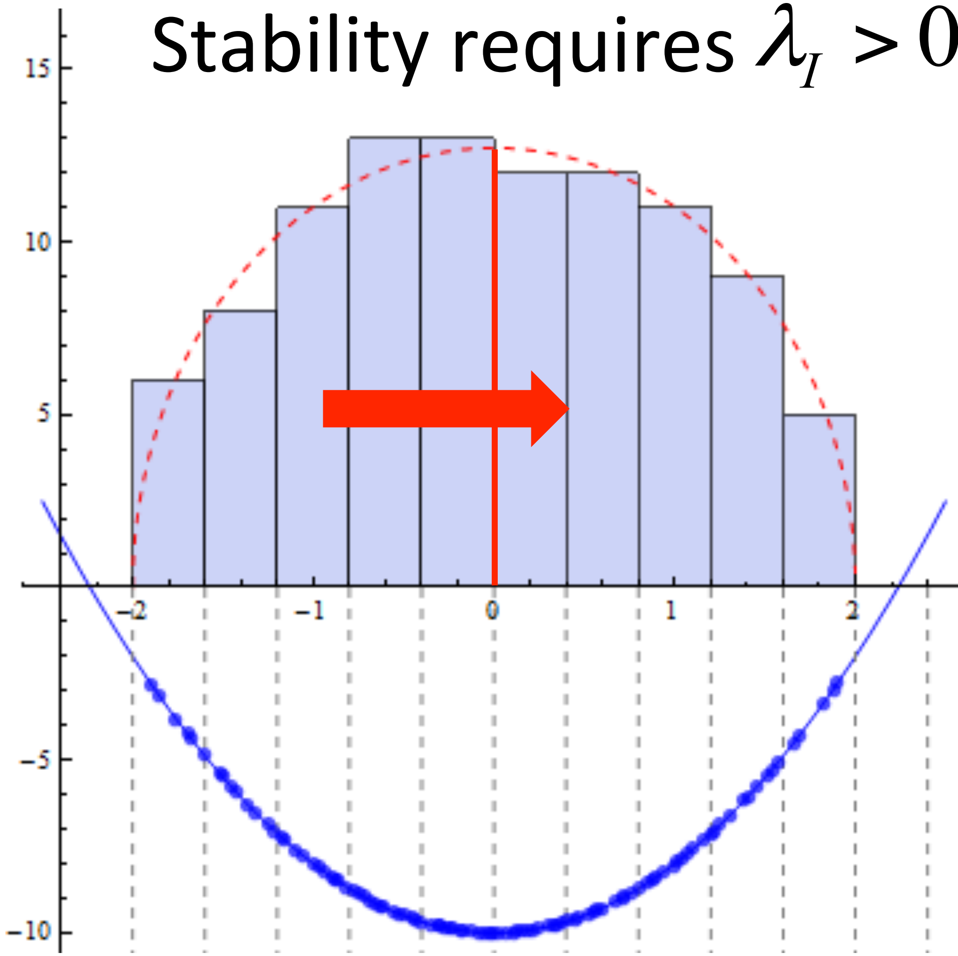
N=1 SUGRA

Inflation

Conclusion

Outlook

Stability requires $\lambda_I > 0, \forall I$



1D charged particles in quadratic potential

Wasteland of Random SUGRA

Timm Wrase

Random non-susy landscape

Eigenvalue repulsion makes stability extremely unlikely

Aazami, Easther hep-th/0512050
Dean, Majumdar cond-mat/0609651
Borot, Eynard, Majumdar, Nadal 1009.1945
Chen, Shiu, Sumitomo, Tye 1112.3338

$$\boxed{P(\lambda_{\min} > 0) = e^{-\frac{Log(3)}{4}N^2 + O(N)}}$$

Random non-susy landscape

Eigenvalue repulsion makes stability extremely unlikely

Aazami, Easther hep-th/0512050
Dean, Majumdar cond-mat/0609651
Borot, Eynard, Majumdar, Nadal 1009.1945
Chen, Shiu, Sumitomo, Tye 1112.3338

$$\boxed{P(\lambda_{\min} > 0) = e^{-\frac{Log(3)}{4}N^2 + O(N)}}$$

$$P(\lambda_{\min} > 0) \approx e^{-\frac{Log(3)}{4}(500)^2} \approx 10^{-30,000}$$

Is there a landscape of stable vacua?

Random supergravities: SUSY

The scalar potential $V(\phi^I)$ might be less random

Study N=1 SUGRA with F-term potential

$$V(\phi^I, \bar{\phi}^{\bar{I}}) = e^K \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2 \right)$$

Random supergravities: SUSY

The scalar potential $V(\phi^I)$ might be less random

Study N=1 SUGRA with F-term potential

$$V(\phi^I, \bar{\phi}^{\bar{I}}) = e^K \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2 \right)$$

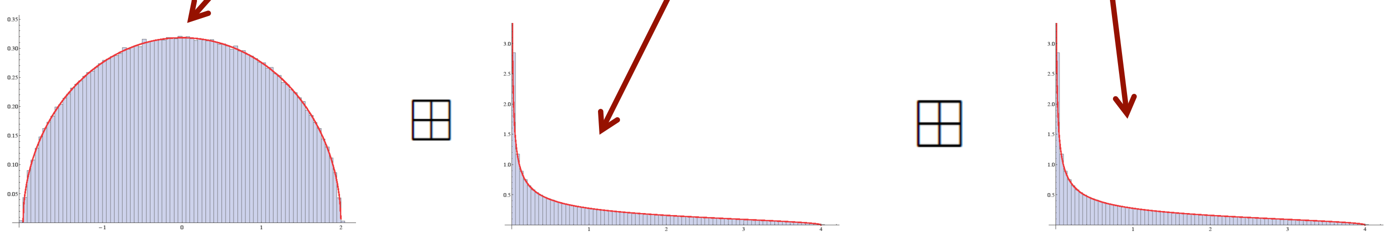
$$M^2 \equiv M^2(W, D_I W, D_I D_J W, D_I D_J D_K W)$$

Mass matrix has special structure

Random supergravities: ~~SUSY~~

Sum of 1 Wigner, 2 Wishart and constant shift:

$$M^2 = (A + A^\dagger) \;+\; B \cdot B^\dagger \;+\; C \cdot C^\dagger \;+\; c\mathbb{1}$$

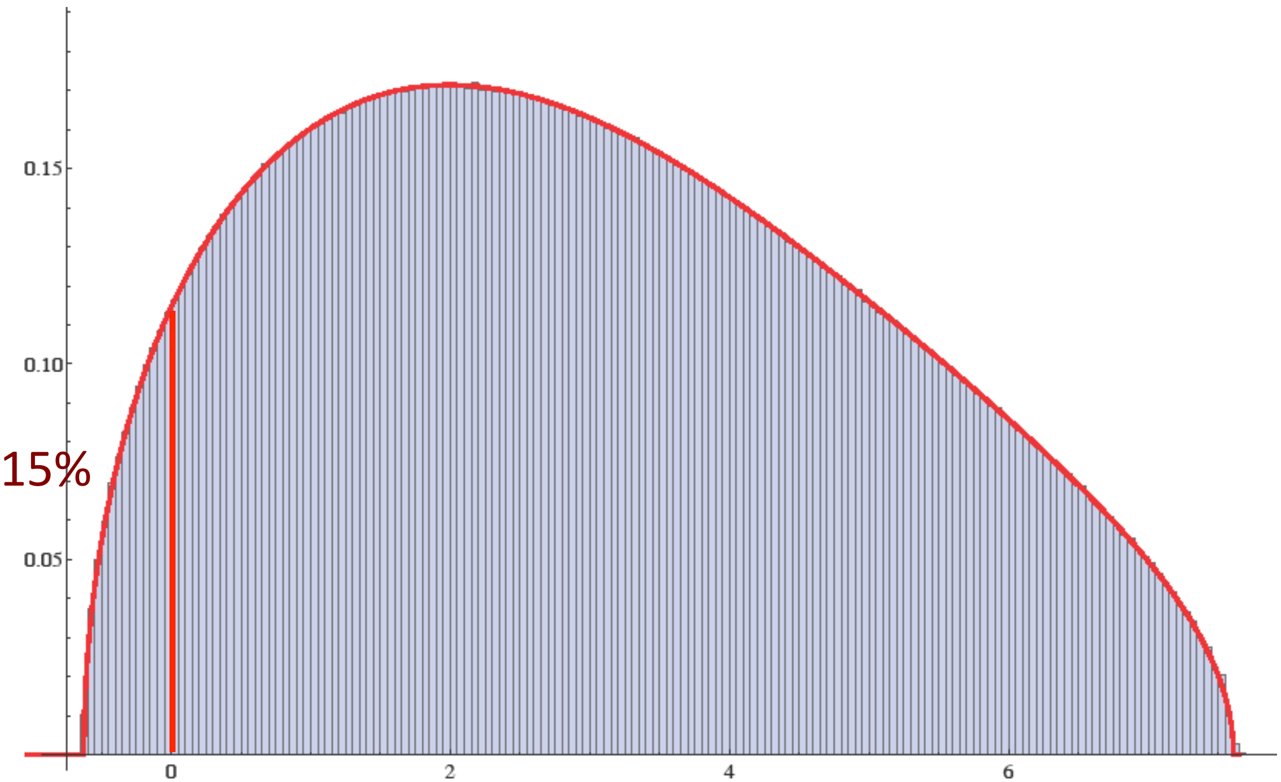


We can do a free convolution ("boxplus")

Voiculescu, Dykema, Nica 1992
Rao, Edelman math/0601389

Random supergravities: ~~SUSY~~

Compare with full simulation :



5%-15%

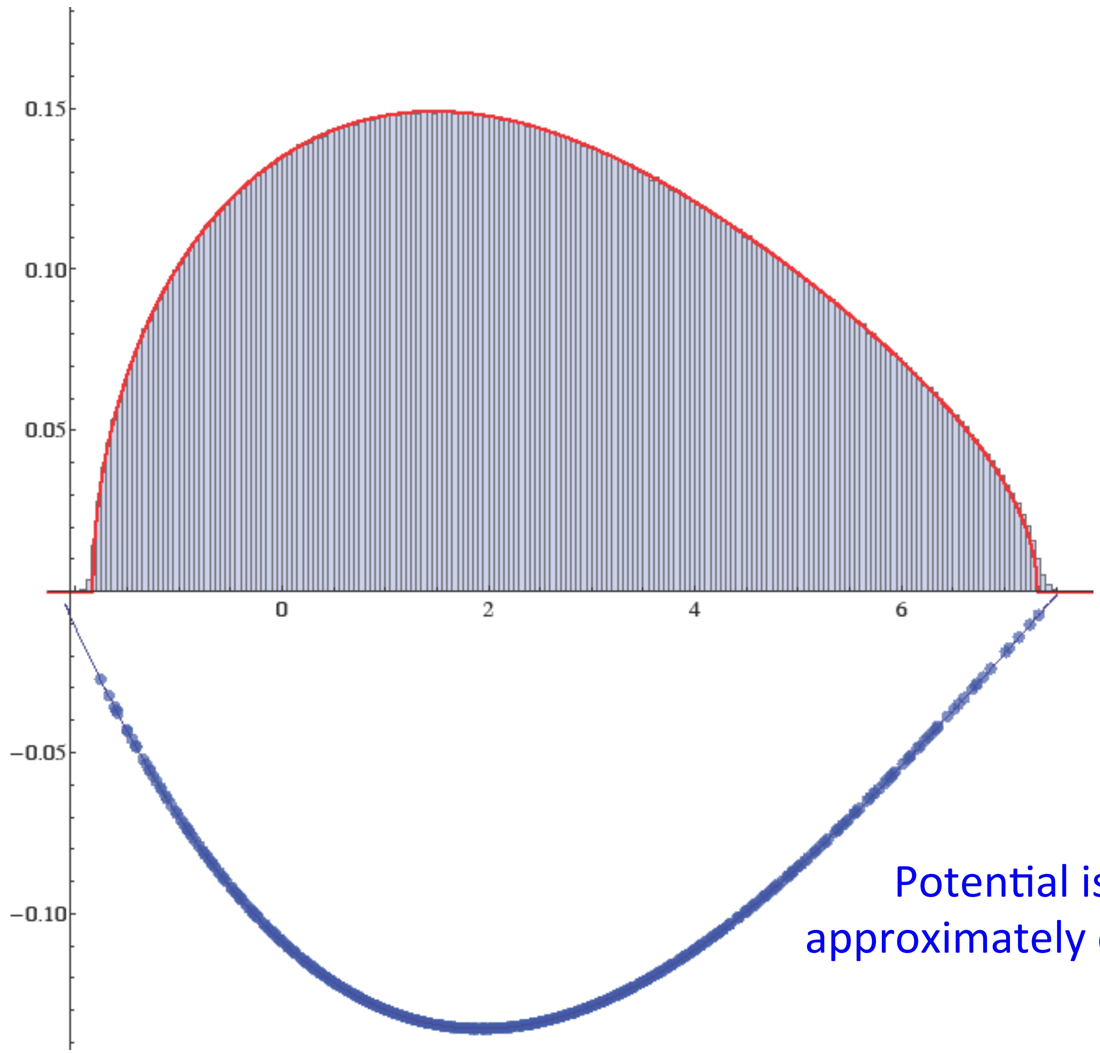
Generic Minkowski/dS critical point is unstable!

Random supergravities: ~~SUSY~~

Wasteland of Random SUGRA

Timm Wrase

- Motivation
- Non-SUSY
- N=1 SUGRA
- Inflation
- Conclusion
- Outlook



Potential is approximately cubic

Random supergravities: ~~SUSY~~

Wasteland of Random SUGRA

Timm Wrase

Motivation

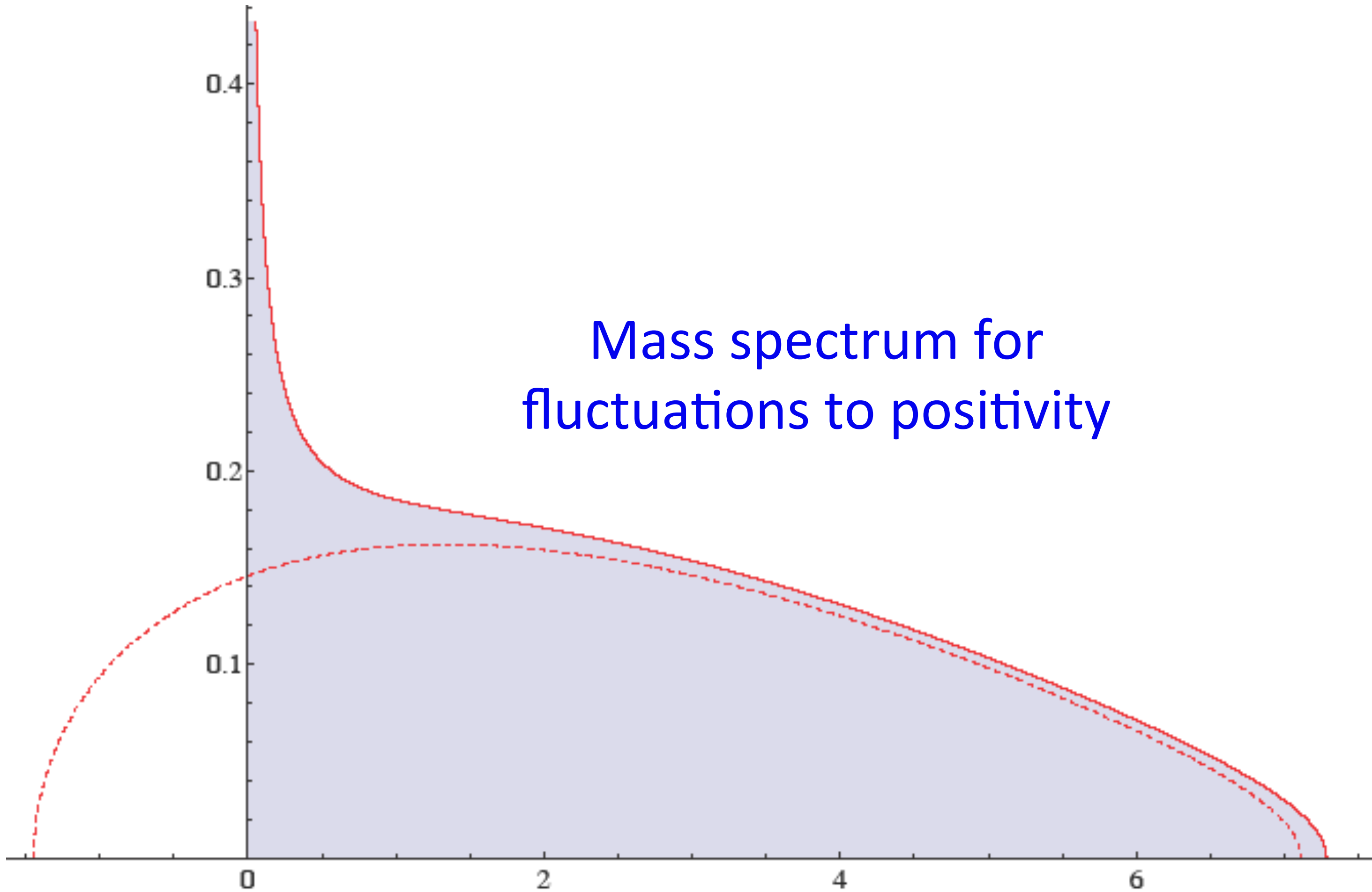
Non-SUSY

N=1 SUGRA

Inflation

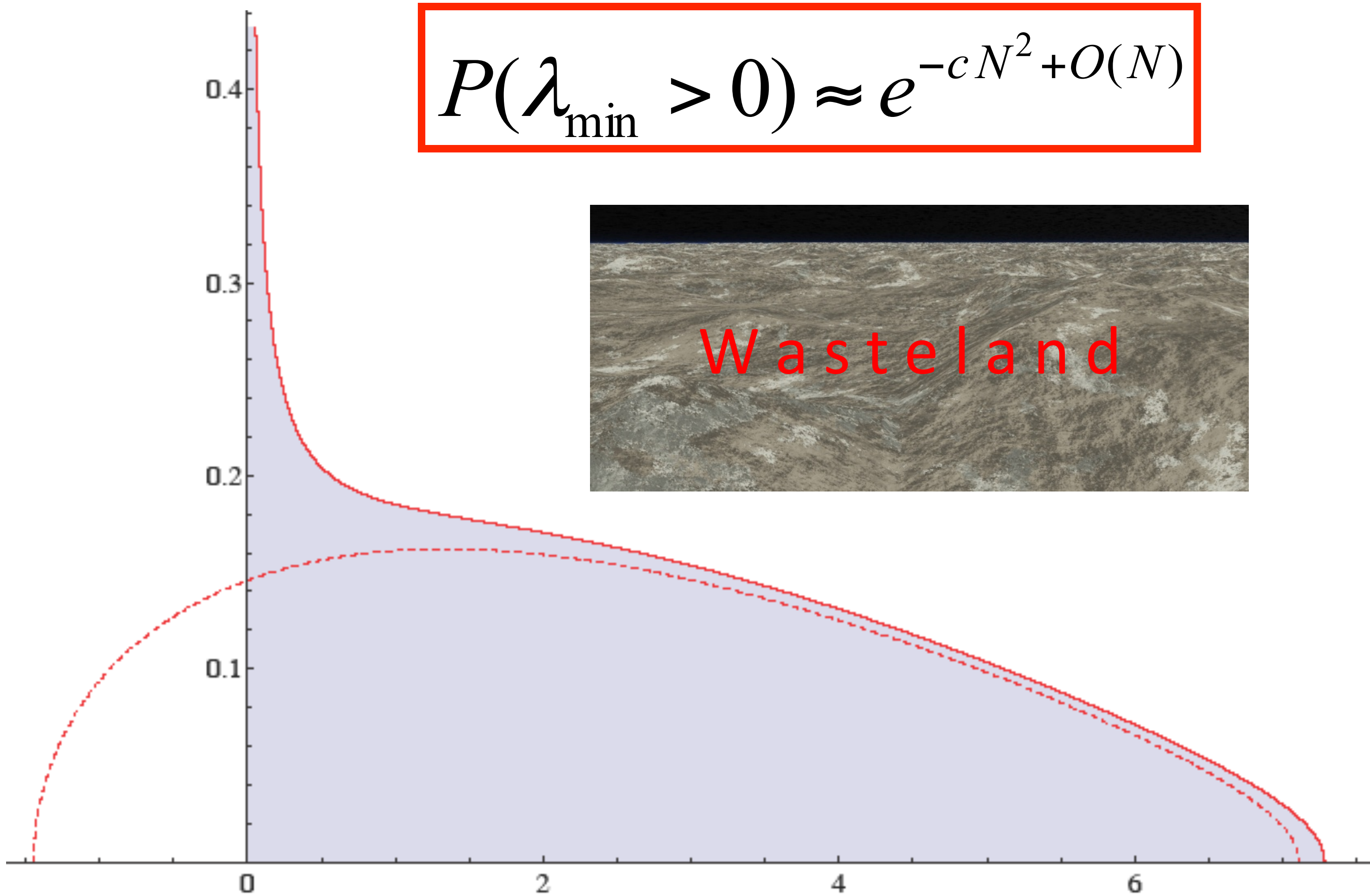
Conclusion

Outlook



Random supergravities: ~~SUSY~~

$$\boxed{P(\lambda_{\min} > 0) \approx e^{-cN^2+O(N)}}$$



Applications

- Applicable to generic Kähler sector of KKLT
- Can evade conclusion there and more generically:

Applications

- Applicable to generic Kähler sector of KKLT
- Can evade conclusion there and more generically:

Assume there are 2 sectors:

- Sector 1: preserves SUSY and all fields have large positive SUSY masses (e.g. complex structure)
- Sector 2: few fields with DSB (e.g. Kähler sector)

Applications

- Applicable to generic Kähler sector of KKLT
- Can evade conclusion there and more generically:

Assume there are 2 sectors:

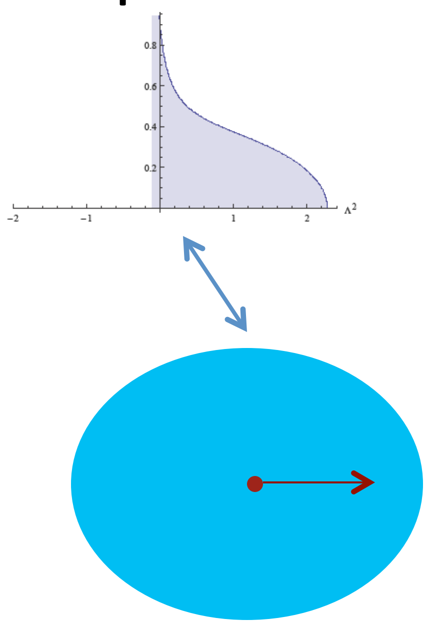
- Sector 1: preserves SUSY and all fields have large positive SUSY masses (e.g. complex structure)
- Sector 2: few fields with DSB (e.g. Kähler sector)

Number of vacua: $\alpha_{(1)}^{N_{(1)}} >> 1$ (e.g. many complex structure moduli)

Minima if: $\alpha_{(2)}^{N_{(2)}} P(\lambda_{\min}^{(2)} > 0) \approx 1$ (e.g. few Kähler moduli)

Multifield Inflation

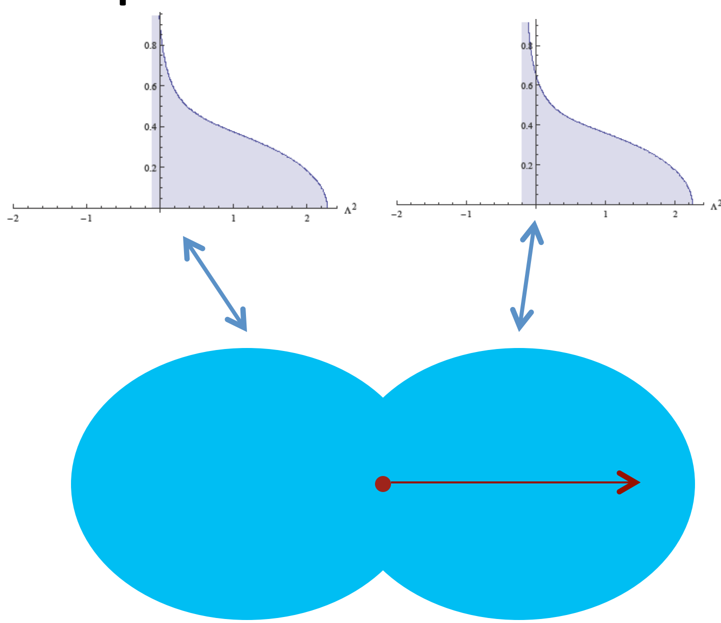
Generate V locally and evolve from patch to patch



local patch in field space

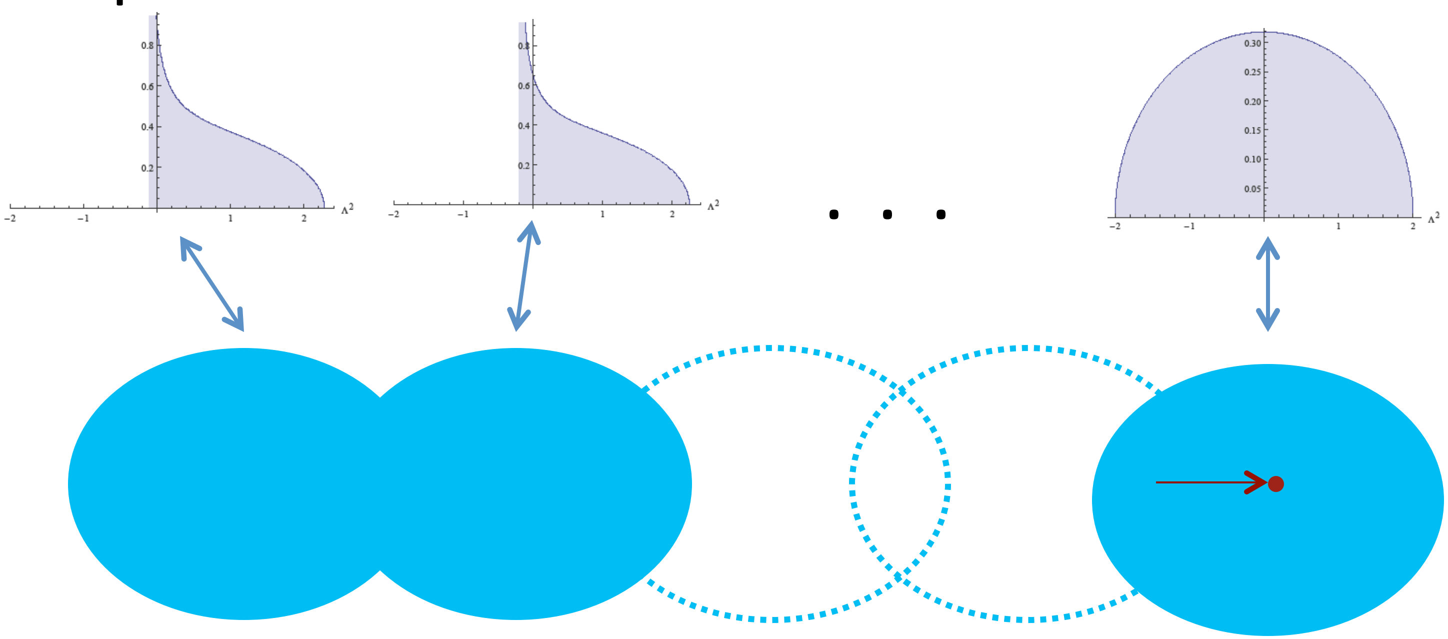
Multifield Inflation

Generate V locally and evolve from patch to patch



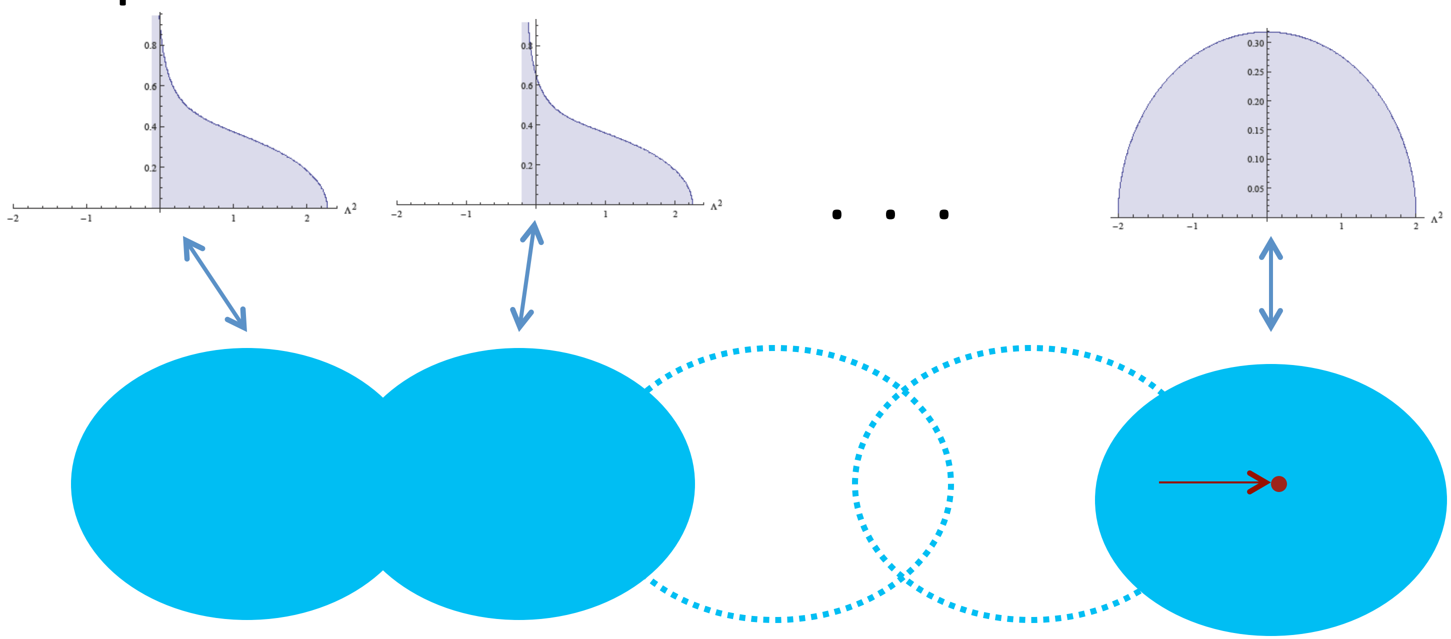
Multifield Inflation

Generate V locally and evolve from patch to patch



Multifield Inflation

Generate V locally and evolve from patch to patch



+ Desired mass spectrum automatically build in

Conclusion

- Unstable critical points are exponentially more likely than stable vacua
- Wasteland contains still many stable vacua

Conclusion

- Unstable critical points are exponentially more likely than stable vacua
- Wasteland contains still many stable vacua

We just have to search in the right place

Outlook

- Add more structure to the mass matrix

Chen, Shiu, Sumitomo, Tye 1112.3338

Sumitomo, Tye 1204.5177, 1209.5086, 1211.6858

Outlook

- Add more structure to the mass matrix

Chen, Shiu, Sumitomo, Tye 1112.3338
Sumitomo, Tye 1204.5177, 1209.5086, 1211.6858

- Search for other regions of increased stability

Denef, Douglas hep-th/0411183
David Marsh, Liam McAllister, TW 1112.3034
Bausch 1208.2691

Outlook

- Add more structure to the mass matrix

  Chen, Shiu, Sumitomo, Tye 1112.3338
  Sumitomo, Tye 1204.5177, 1209.5086, 1211.6858

- Search for other regions of increased stability

  Denef, Douglas hep-th/0411183
  David Marsh, Liam McAllister, TW 1112.3034
  Bausch 1208.2691

- Study`random' multifield inflation:

  Pedro, Westphal 1303.3224
  Marsh, McAllister, Pajer, TW 1304.xxxx

  - Generate local potential using RMT
  - Can study $N = O(100)$ fields

Outlook

- Add more structure to the mass matrix

Chen, Shiu, Sumitomo, Tye 1112.3338
Sumitomo, Tye 1204.5177, 1209.5086, 1211.6858

- Search for other regions of increased stability

Denef, Douglas hep-th/0411183
David Marsh, Liam McAllister, TW 1112.3034
Bausch 1208.2691

- Study`random' multifield inflation:

Pedro, Westphal 1303.3224
Marsh, McAllister, Pajer, TW 1304.xxxx

  - Generate local potential using RMT
  - Can study $N = O(100)$ fields

Thank you!