Extremal black branes in gauged supergravity

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Motivation

- Gauge/Gravity correspondence:

Gauge theory at finite temperature and density	$\longleftrightarrow$	Charged black brane in AdS

- Mainly studied Reissner-Nordström (RN) black branes
- Problem: RN black branes have finite entropy density at T=0, in conflict with the 3rd law of thermodynamics (*Nernst* law)
- Task: Look for black branes with vanishing entropy density at T=0 (*Nernst* branes)

Outline

- AdS/CFT at finite T and charge density
- Review Reissner Nordström black holes
- Extremal black holes (T=0)
- Nernst brane solution

Short review of AdS/CFT at finite T and charge density

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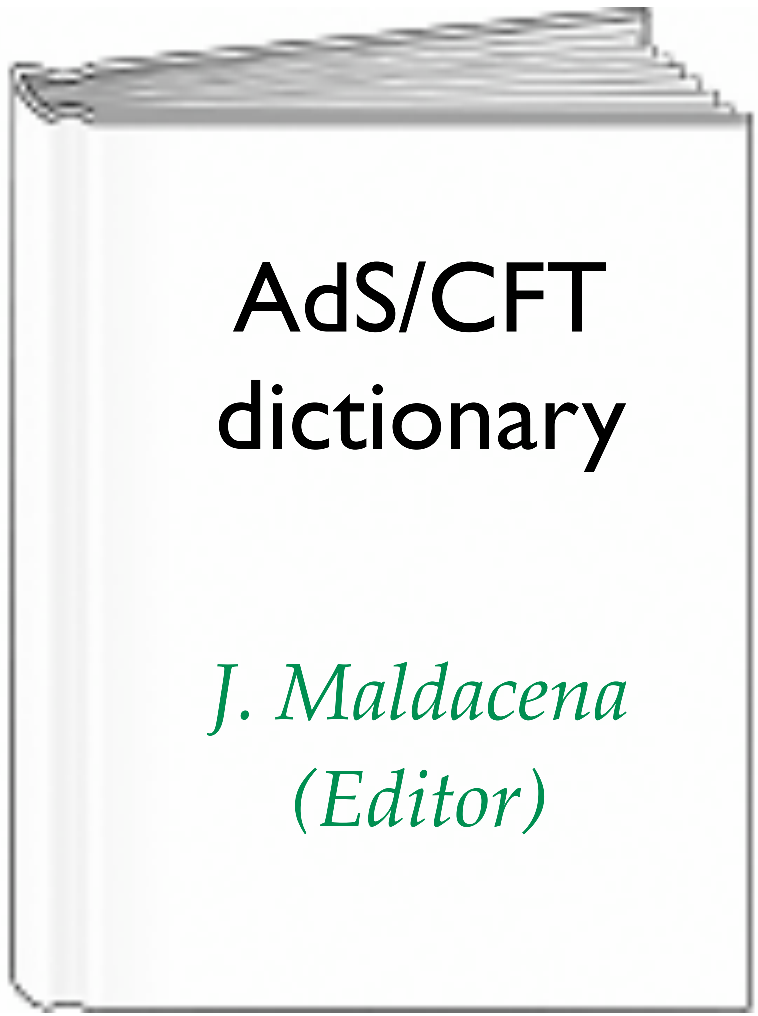
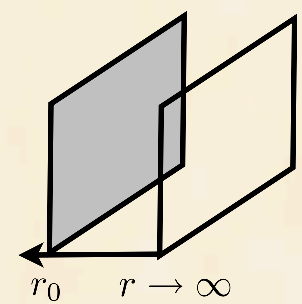
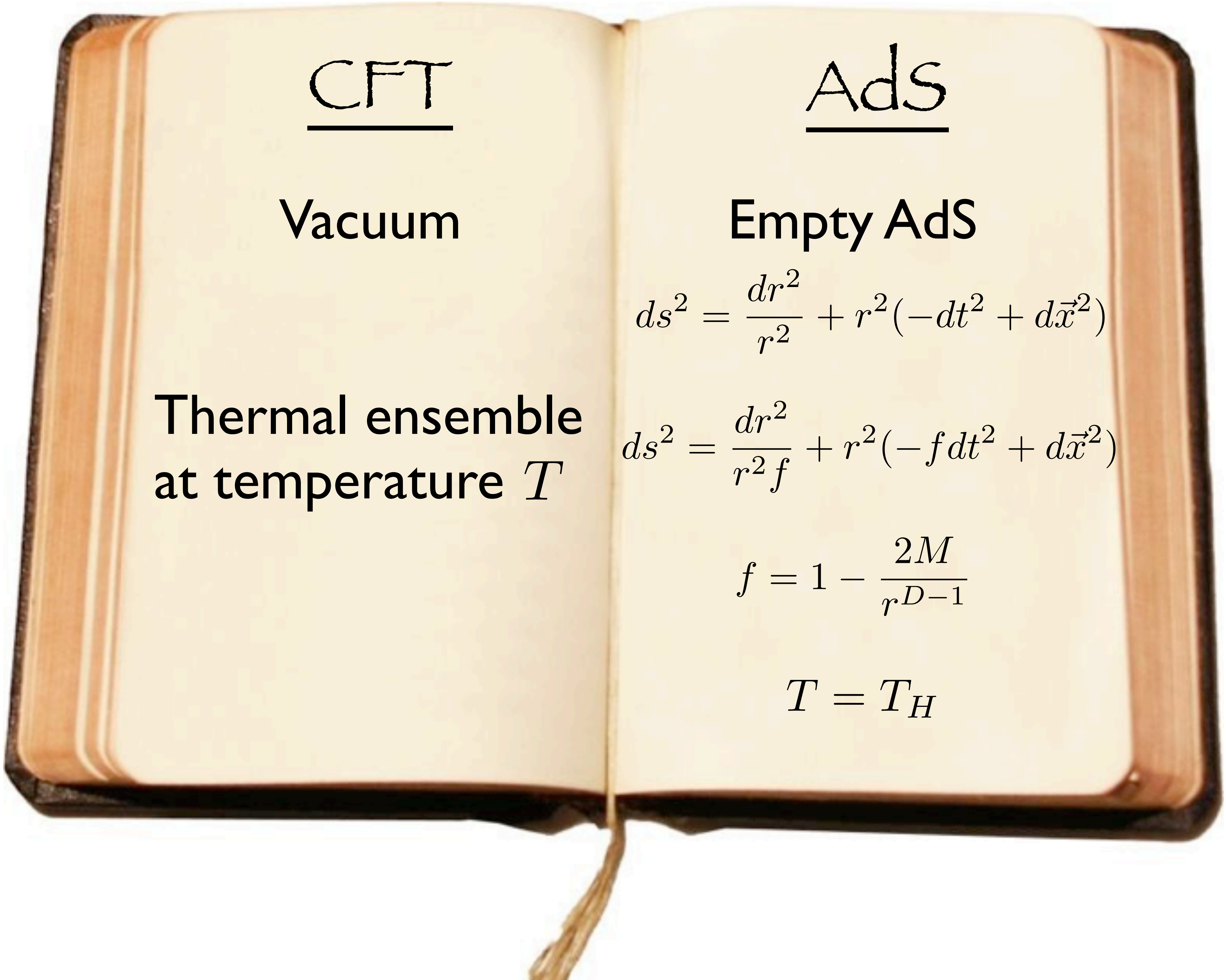


CFT	AdS
Vacuum	Empty AdS $$ds^2 = \frac{dr^2}{r^2} + r^2(-dt^2 + d\vec{x}^2)$$
Thermal ensemble at temperature T	AdS black brane




CFT
AdS
Vacuum
Empty AdS
$$ds^2 = \frac{dr^2}{r^2} + r^2(-dt^2 + d\vec{x}^2)$$
Thermal ensemble at temperature T
$$ds^2 = \frac{dr^2}{r^2 f} + r^2(-f dt^2 + d\vec{x}^2)$$
$$f = 1 - \frac{2M}{r^{D-1}}$$
$$T = T_H$$

CFT

Thermal ensemble at temperature T and charge density ρ

AdS

Charged black brane with gauge field

$$A_t \sim \frac{\rho}{r^{D-3}}$$

Usually Reissner-Nordström (RN):

$$f = 1 - \frac{2M}{r^{D-1}} + \frac{Q^2}{r^{2(D-2)}}$$

with $Q \sim \rho$

RN black hole

(4D, asymptotically flat, spherical horizon)

- Charged black hole solution of

$$\int d^4x\sqrt{-g}[R - F_{\mu\nu}F^{\mu\nu}]$$

- $$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} + r^2 d\Omega^2$$

- $$A = \frac{Q}{r}dt$$

- RN metric can be written as

$$ds^2 = -\frac{\Delta}{r^2}dt^2 + \frac{r^2}{\Delta}dr^2 + r^2 d\Omega^2$$

with

$$\Delta = (r - r_+)(r - r_-)$$

where

$$r_\pm = M \pm \sqrt{M^2 - Q^2}$$

- r_+ : event horizon
- Black hole for $M \geq |Q|$
- $T_H = \frac{\sqrt{M^2 - Q^2}}{2Mr_+^2} \xrightarrow{|Q| \to M} 0$

Extremal black holes

- $|Q| = M$
- $T_H = 0$
- $r_+ = r_- = M$
- $ds^2 = -\left(1 - \frac{M}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1 - \frac{M}{r}\right)^2} + r^2 d\Omega^2$
- Distance to the horizon at $r = M$

$$\int_{M+\epsilon}^{R} \frac{dr}{1 - \frac{M}{r}} \xrightarrow{\epsilon \to 0} \infty$$

- Near horizon geometry: Introduce $r = M(1+\lambda)$

$$ds^2 \sim (-\lambda^2 dt^2 + M^2 \lambda^{-2} d\lambda^2) + M^2 d\Omega^2$$

to lowest order in λ

$AdS_2 \times S^2$

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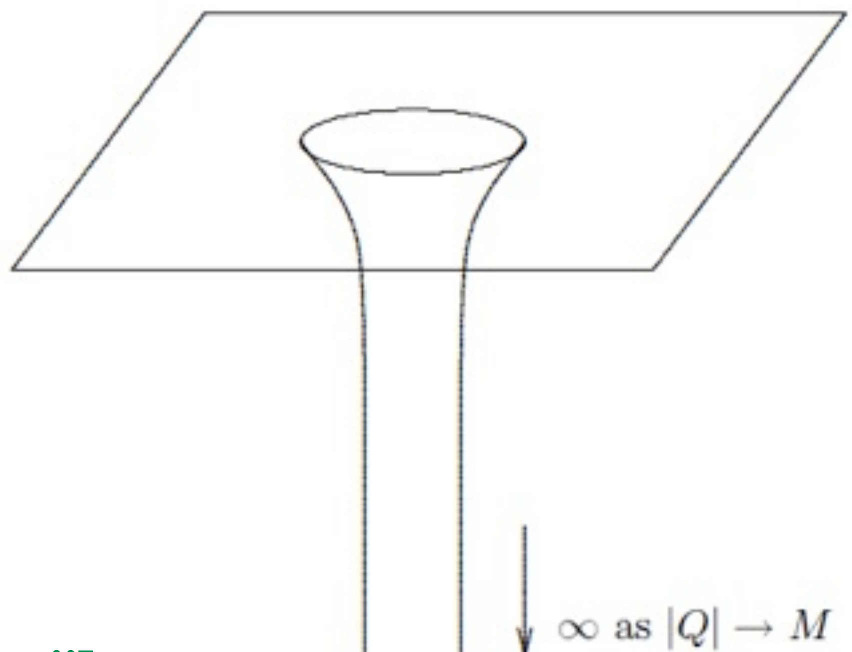
- Infinite throat:



[From: Townsend "Black Holes"]

- Entropy: $S \sim A_H$

  Horizon area

- Extremal RN: $A_H = 4\pi M^2$, i.e. non-vanishing entropy!

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- For black branes

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}dr^2 + e^{2A(r)}d\vec{x}^2$$

entropy density

$$s \sim e^{(D-2)A(r_0)}$$

Horizon radius

Possible solution

Generically RN not the zero temperature ground state when coupled to

(i) charged scalars (*holographic superconductors*)

[Gubser; Hartnoll, Herzog, Horowitz]

(ii) neutral scalars (*dilatonic black branes*)

[Goldstein, Kachru, Prakash, Trivedi; Cadoni, D'Appollonio, Pani; Charmousis, Gouteraux, Kim, Kiritsis, Meyer]

(iii) magnetic field in presence of Chern-Simons term (in 5d)

[D'Hoker, Kraus]

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no embedding into string theory

(iii) magnetic field in presence of Chern-Simons term (in 5d)

[D'Hoker, Kraus]

Strategy

- Look for extremal Nernst branes (with AdS asymptotics) in N=2 gauged supergravity with vector multiplets

  (i) Contains neutral scalars

  (ii) Straightforward embedding into string theory

  (iii) Attractor mechanism

- Attractor mechanism: Values of scalar fields at the horizon are fixed, independent of their asymptotic values

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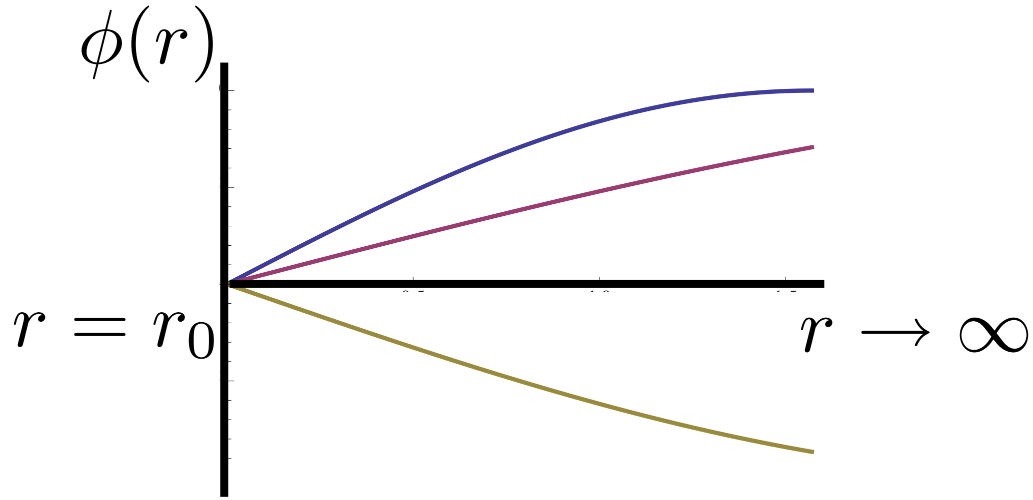

- First found for N=2 supersymmetric, asymptotically flat black holes in 4D [Ferrara, Kallosh, Strominger]
- Consequence of infinite throat of extremal BH
- Half the number of d.o.f. $\Longrightarrow 1^{\mathrm{st}}$ order equations
- Compare domain walls in fake supergravity

[Freedmann, Nunez, Schnabl, Skenderis; Celi, Ceresole, Dall'Agata, Van Proyen, Zagermann; Zagermann; Skenderis, Townsend]

$\mathcal{N} = 2, D = 4$ Supergravity

- $\frac{1}{2}R - N_{IJ}\,\mathcal{D}_\mu X^I\,\mathcal{D}^\mu \bar{X}^J + \frac{1}{4}\mathrm{Im}\mathcal{N}_{IJ}\,F^I_{\mu\nu}\,F^{\mu\nu J}$
$$-\frac{1}{4}\mathrm{Re}\mathcal{N}_{IJ}\,F^I_{\mu\nu}\,\tilde{F}^{\mu\nu J} - V(X,\bar{X})$$

- $N_{IJ}, \mathcal{N}_{IJ}, V$ can be expressed in terms of holomorphic prepotential $F(X)$

- E.g.

$$V(X,\bar{X}) = \left[N^{IJ} - 2X^I\bar{X}^J\right]\left(h^K F_{KI} - h_I\right)\left(h^K \bar{F}_{KJ} - h_J\right)$$

$F_{KI} = \dfrac{\partial^2 F}{\partial X^I \partial X^K}$

$1^{\rm st}$ order equations

- Ansatz

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}dr^2 + e^{2A(r)}(dx^2 + dy^2)$$

$$F^I_{tr} \sim \left[(\mathrm{Im}\,\mathcal{N})^{-1}\right]^{IJ} Q_J \quad , \quad F^I_{xy} \sim P^I$$

- Plug into action and rewrite it as sum of squares

$$S_{1d} = \int dr \sum_\alpha \Big(\phi'_\alpha - f_\alpha(\phi)\Big)^2 + \text{total derivative}$$

with $\{\phi_\alpha\} = \{U, A, X^I\}$

- $\phi'_\alpha - f_\alpha(\phi) = 0 \quad \Longrightarrow \quad \delta S_{1d} = 0$

- Supersymmetry preserving configurations solve:

[Barisch, Cardoso, Haack, Nampuri, Obers]

$$U' = \mathrm{e}^{-U-2A}\,\mathrm{Re}\left[X^I\,\hat{Q}_I\right] - \mathrm{e}^{-U}\,\mathrm{Im}\left[X^I\,\hat{h}_I\right]$$

$$A' = -\mathrm{Re}\left[X^I\,q_I\right]$$

$$Y'^I = \mathrm{e}^A\,N^{IJ}\,\bar{q}_J$$

[compare also: Gnecchi, Dall'Agata]

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- $$\hat{Q}_I = Q_I - F_{IJ}P^J \quad , \quad \hat{h}_I = h_I - F_{IJ}h^J$$

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- Constraint:

$$Q_I h^I - P^I h_I = 0$$

Nernst brane

[Barisch, Cardoso, Haack, Nampuri, Obers]

- STU-model, i.e. $X^I \ , \ I = 0, \ldots, 3$
- Consider $Q_0, h_1, h_2, h_3 \neq 0 \quad (Q_I h^I - P^I h_I = 0)$
- Nernst brane with Killing horizon at $r = 0$
- Near horizon: Infinite throat of unusual type (not $AdS_2 \times \mathbb{R}^2$)

$$ds^2 = -r^{5/2} dt^2 + r^{-5/2} dr^2 + r^{1/2} d\vec{x}^2$$

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Vanishing area (i.e. entropy) density

- Conformal to Lifshitz metric with $z = 3$

$$ds^2 = \tilde{r}^{-1} \left(-\tilde{r}^{2z} dt^2 + \tilde{r}^{-2} d\tilde{r}^2 + \tilde{r}^2 d\vec{x}^2 \right)$$

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- $ds_5^2 = e^{2\phi}ds_4^2 + e^{-4\phi}(dz + Cdt)^2$
- $\mathrm{Re}(X_{4d}) = e^{-2\phi}X_{5d}$
- X_{5d} finite, metric interpolates between Nernst throat and AdS_5

[Barisch, Cardoso, Haack, Nampuri]

- Curvature invariants well behaved, but diverging tidal forces at horizon [Horowitz, Way]

- Can be cured by including loop or α'-corrections [Harrison, Kachru, Wang; Cardoso et al, in progress]

- However: Corrected metric has finite entropy density again

Summary

- Extremal RN black brane has finite entropy (density), in tension with the third law of thermodynamics
- Ground state might be a different extremal black brane with vanishing entropy $\longrightarrow$ Nernst brane

  [Related top-down approach by Gauntlett in different collaborations with Donos, Pantelidou, Sonner, Withers]

- Applications to gauge/gravity correspondence?

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Gauge-gravity duality and condensed matter physics

05.08.2013 - 09.08.2013

The Arnold Sommerfeld Center for Theoretical Physics will host a PhD school on "Gauge-gravity duality and condensed matter physics" in the period August 5-9, 2013.

Lecturers:

- Joe Bhaseen (King's College London)
- Sean Hartnoll (Stanford University)
- John McGreevy (San Diego University)
- Subir Sachdev (Harvard University)
- Tadashi Takayanagi (Kyoto University)

Preparatory material

Organizers

Johanna Erdmenger, Michael Haack, Stefan Kehrein, Jan von Delft and Wilhelm Zwerger

Application

Please send a short email to **asc_school13@physik.lmu.de** indicating your affiliation, full name and professional status (postdoc, PhD student, ...), before July 1. There is no fee but you will have to arrange your trip and accomodation yourself.

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