Light-Cone Averaging and Dispersion of the $d_L - z$ Relation

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DESY

1202.1247, 1207.1286, 1209.4326, 1302.0740
+ Work in Progress

IBD, M. Gasperini, G. Marozzi, F. Nugier, G. Veneziano

Bad Honnef XXV Workshop

Main stream Cosmology

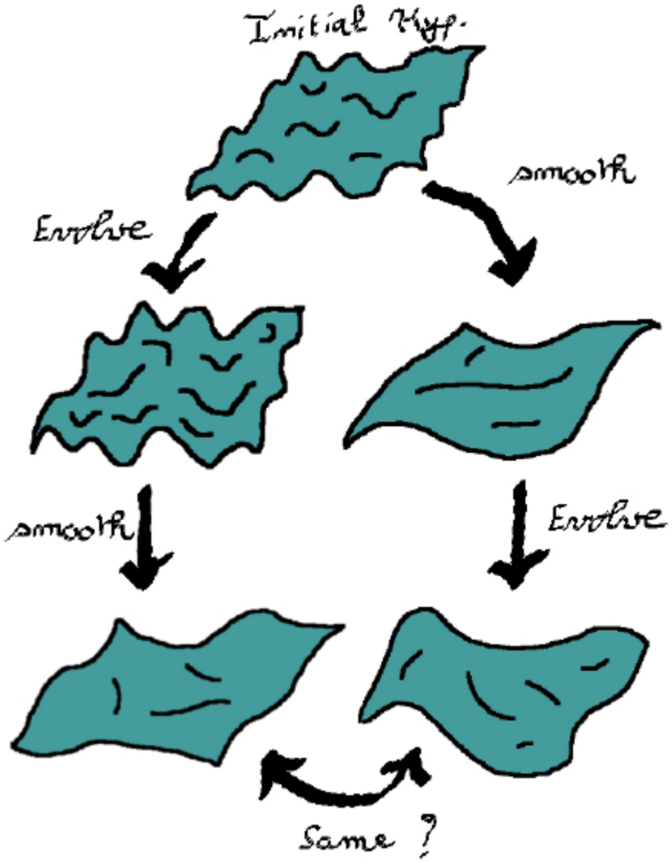
- Assume FLRW = homogeneous and isotropic metric.

$\Rightarrow$ Implicit averaging

- Modeling the energy momentum tensor as a perfect fluid.
- Pert. give rise to structure, highly non-linear at some scale. Background unchanged.

$\Rightarrow$ Implicitly neglected the possibility of backreaction.

Importance of averaging

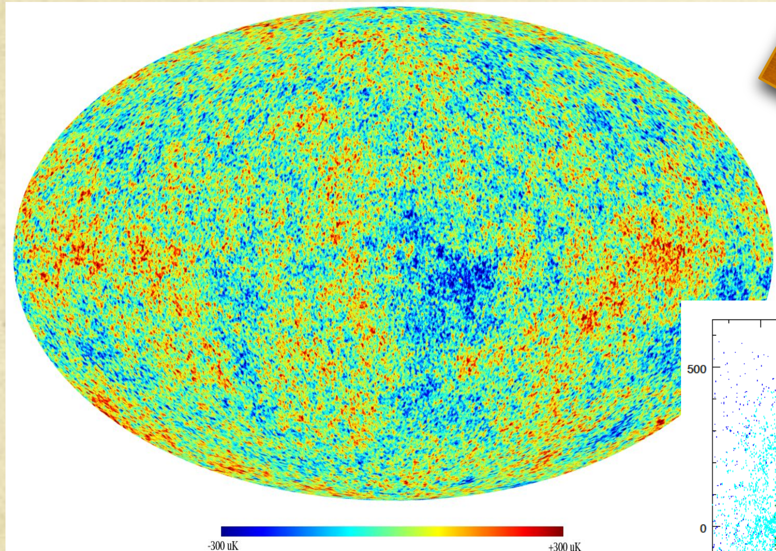


The evolution of an inhomogeneous spacetime after averaging differs from the evolution of its averaged spacetime.

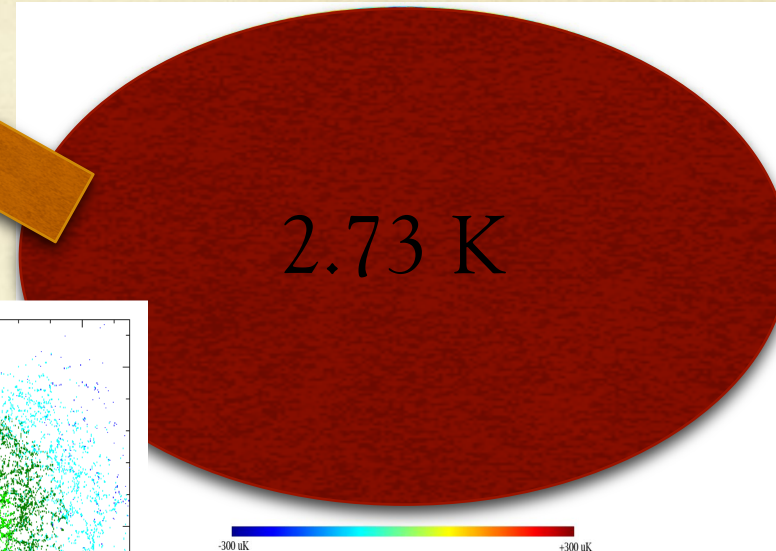
Few questions:

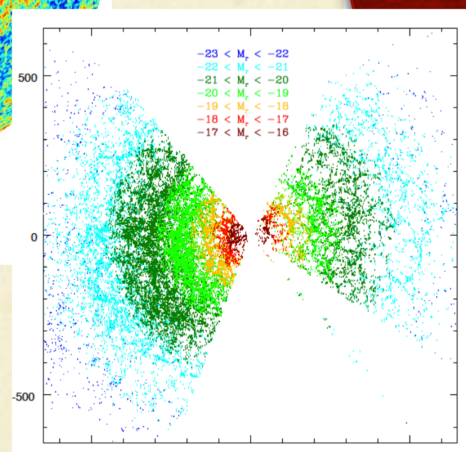
- Can smoothing of structure contribute to an acceleration term (DE)?
  Is there an effect from small scales to large scales ?
  $\Rightarrow$ Nice way out of the coincidence problem? (T. Buchert)
- Consequences on cosmological parameters? (C. Clarkson, J. Larena)
  Negligible for a physical reason?
- What is the scale of homogeneity in the Universe? $100Mpc$?
- Fitting Problem : What is the best-fit FLRW model to a given lumpy Universe?
- How Einstein field equations transform after a coarse-graining procedure? How do we average vectors and tensors?

Prelude

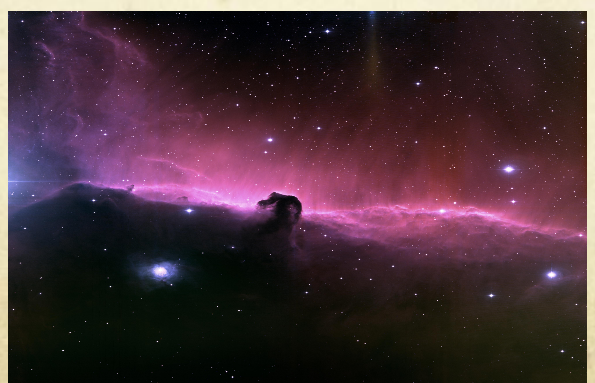


WMAP





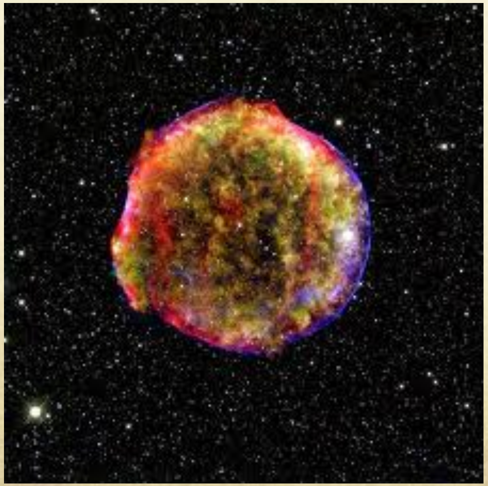
SDSS



Perturbations at Low Redshift

- Measurements of SNIa ➔ Mostly neglected, naively argued as irrelevant ~10^{-10}
- The concordance model of cosmology:

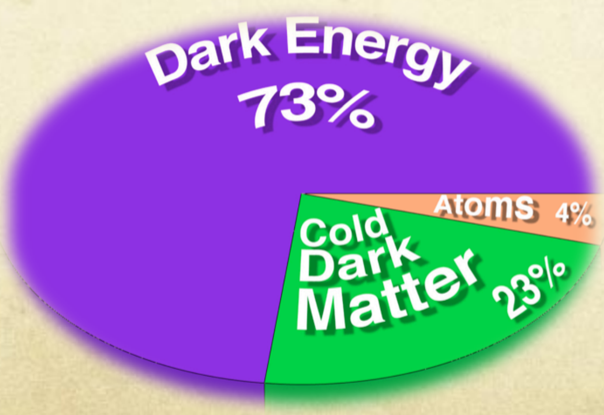
~73% of the critical energy density is not accounted for by known matter, dark matter or curvature.



Universe Composition Today

$$\frac{H^2}{H_0^2} = \Omega_{R0}(1+z)^4 + \Omega_{K0}(1+z)^2 + \Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0}; \quad \Phi = \frac{L}{4\pi d_L^2}$$

- CC/DE becomes relevant only at $z \tilde{} 1$, Coincidence Problem?
- Based on CMB, LSS and SNIa observations.



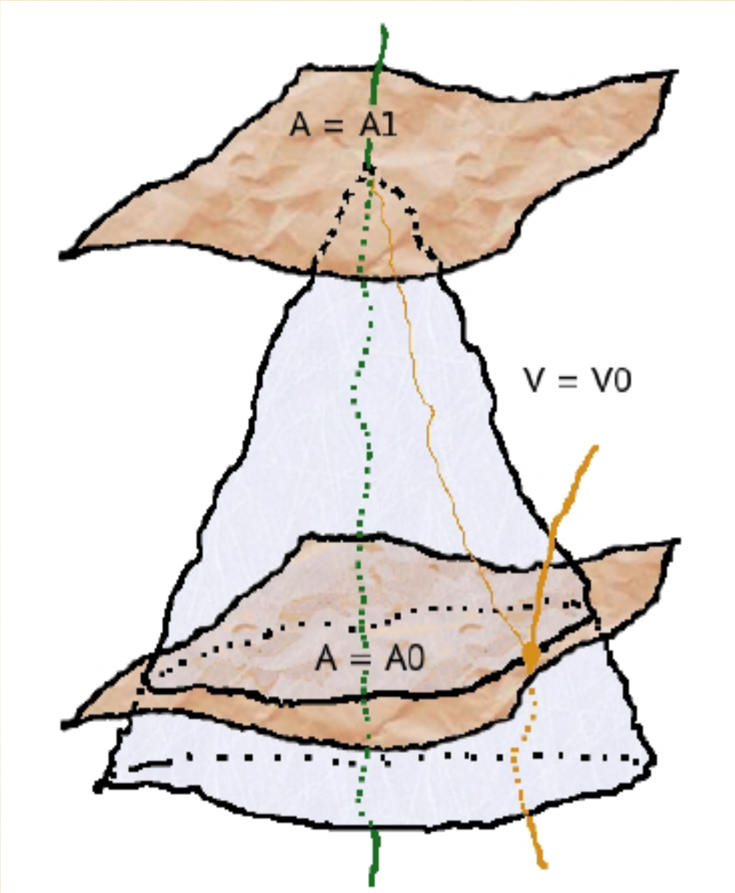
Outline

- Gauge invariant light-cone (LC) averaging formalism
- Application: luminosity-distance (d_L) redshift (z) relation for a perturbed FLRW Universe.
- Ingredients: FLRW + subhorizon pert., properly averaged and calculate the d_L-z relation in CDM, LCDM and for different functions of d_L

$$
\begin{aligned}
d_L^{FLRW}(z_s) &= (1+z_s)a_0 \int_{\eta_s}^{\eta_0} d\eta = (1+z_s)\int_0^{z_s} \frac{dz}{H(z)} \\
&= \frac{1+z_s}{H_0}\int_0^{z_s} dz \left[\sum_n \Omega_{n0}(1+z)^{3(1+w_n)}\right]^{-1/2}
\end{aligned}
$$

LC Averaging

- Hyper-surfaces using meaningful physical quantities: Redshift, temperature etc.
- Observations are made on the light-cone.
- Light travels on geodesics.
- Past attempts: Coley 0905.2442; Rasanen 1107.1176, 0912.3370



Light Cone Averaging 1104.1167

- A-priori - the averaging is a geometric procedure, does not assume a specific energy momentum tensor.

$$I(S; V_0, A_0; -) = \int d^4x \sqrt{-g}\, \delta(V_0 - V)\delta(A - A_0)|\partial_\mu V \partial^\mu A| S(x)$$

$$k_\mu \equiv \partial_\mu V$$

$$\langle S \rangle_{V_0, A_0} = \frac{I(S; V_0, A_0; -)}{I(1; V_0, A_0; -)}$$

The prescription is gauge inv., {field reparam. A->A'(A), V->V'(V)} and invariant under general coordinate transformation. A(x) is a time-like scalar, V(x) is null.

This gives a procedure for general space-times.

GLC Metric and Averages

$$ds^2_{GLC} = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\tilde{\theta}^a - U^a dw)(d\tilde{\theta}^b - U^b dw).$$

- Ideal Observational Cosmology – Ellis et al.
- Evaluating scalars at a constant redshift for a geodetic observer.

$$I(S, w_0, z) = \int d^2\tilde{\theta} \sqrt{\gamma(w_0, z, \tilde{\theta}^a)} S(w_0, z, \tilde{\theta}^a);$$

$$\langle S \rangle = \frac{I(S, w_0, z)}{I(1, w_0, z)};$$

$$1 + z = \frac{\Upsilon_O}{\Upsilon_S}$$

GLC Metric

- FLRW

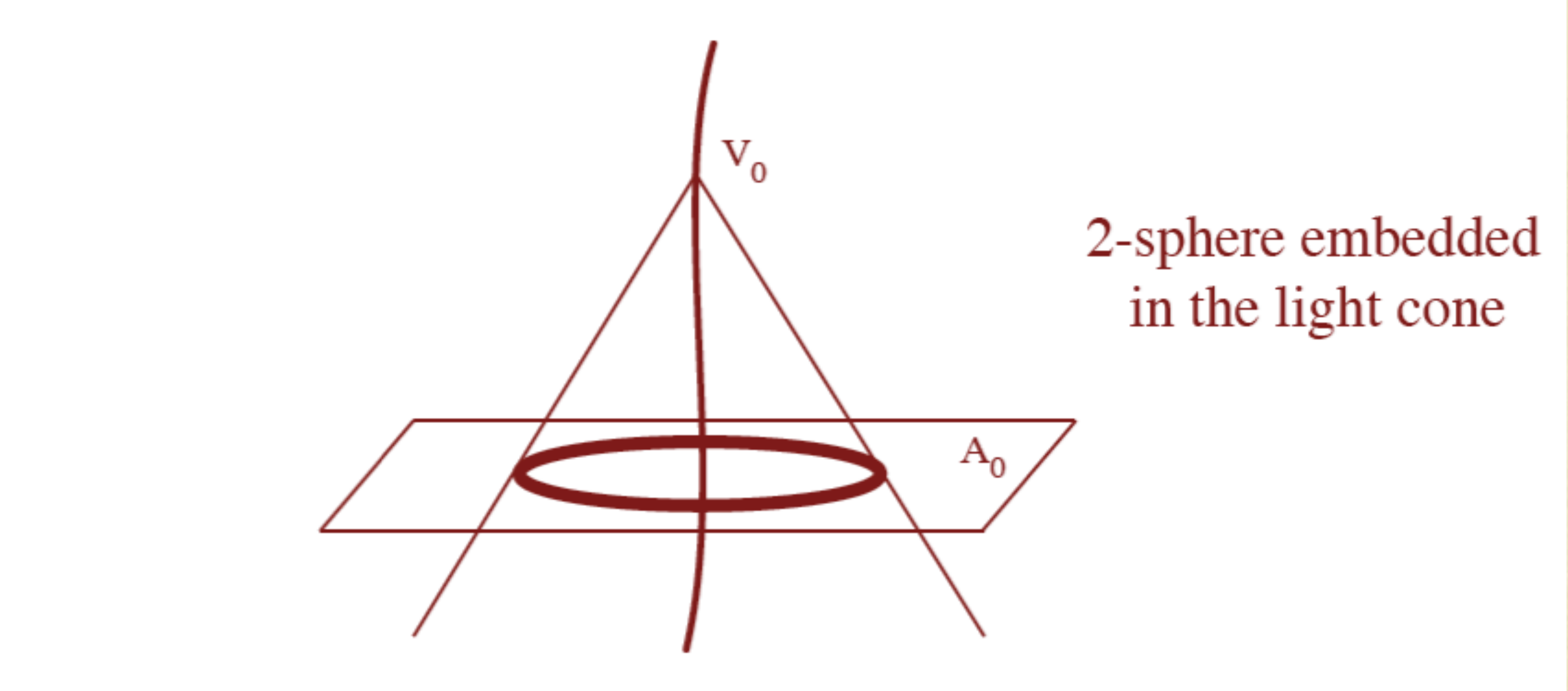
$$w = r + \eta, \qquad \tau = t, \qquad \Upsilon = a(t), \qquad U^a = 0,$$
$$\gamma_{ab} d\theta^a d\theta^b = a^2(t) r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

- τ can be identified as the time coordinate in the synchronous gauge of arbitrary space-time.

$$g^{t\mu}_{SG} = \{-1, \vec{0}\} = -\left[\partial_\tau + \Upsilon^{-1}(\partial_w + U^a \partial_a)\right] X^\mu = -u^\nu \partial_\nu X^\mu = -\frac{dX^\mu}{d\lambda},$$

IBD et al. '12

Average d_L on the past LC at constant redshift



A= redshift, V= light-cone coordinate

Averaged d_L at Constant Redshift

- Novelty: In principle, exact treatment of the geodesic equations and the averaging hyper-surface for any spacetime and any DE model, as long as the geodesic equation is unchanged.
- Previous attempts are limited to perturbations about FLRW and had to solve order by order: Vanderveld et al. – post Newtonian, Barausse et al. 2005, Kolb et al. 2006 – SG superhorizon, Pyne at al 2005…
- Rebuttal : Hirata et al., Geshnizjani et al.
- We work up to 2nd order in perturbation theory in the Poisson Gauge.

Averaged d_L at Constant Redshift

- Ethrington's Reciprocity Law, for any spacetime:

$$\Phi = \frac{L}{4\pi d_L^2}; \quad d_L(z) = (1+z)^2 d_S; \quad d_S^2 \equiv \frac{dS}{d\Omega_O} = \frac{\sqrt{\gamma}}{\sin\tilde{\theta}}$$

$$\langle d_L \rangle_{w_0,z} = (1+z)^2 \frac{\int d^2\theta \sqrt{|\gamma(w_0,\tau(z,w_0,\theta^a),\theta^a)|}\, d_s(w_0,\tau(z,w_0,\theta^a),\theta^a)}{\int d^2\theta \sqrt{|\gamma(w_0,\tau(z,w_0,\theta^a),\theta^a)|}} \,,$$

Measure of Integration

Fluctuations in scalar

Functions of d_L

- Overbars and {...} denote ensemble average, <..> denote LC average.

$$\left\langle \left\{ F(S) \right\} \right\rangle \neq F\left\langle \left\{ S \right\} \right\rangle$$

- Averages of different functions of scalars receive different contributions.

$$\boxed{\begin{aligned} &\mathbf{\Phi} \sim \left\langle \left\{ d_L^{-2} \right\} \right\rangle = (d_L^{FLRW})^{-2}[1 + f_\Phi(z)] \\ &\left\langle \left\{ d_L \right\} \right\rangle = d_L^{FLRW}[1 + f_d(z)] \end{aligned}}$$

The Optimal Observable - Flux

- LC average of flux for any space-time amounts to the area of the 2-sphere!

$$\begin{aligned}\langle d_L^{-2}\rangle(w_o,z_s) &= (1+z_s)^{-4}\frac{\int dS \frac{d\Omega_0}{dS}}{\int dS} = (1+z_s)^{-4}\frac{\int d\Omega_0}{\int dS} = (1+z_s)^{-4}\frac{4\pi}{\mathcal{A}(w_o,z_s)} \ ,\\ \mathcal{A}(w_o,z_s) &= \int_{\Sigma(w_o,z)} d^2\xi\sqrt{\gamma}\ .\end{aligned}$$

$$\sqrt{\gamma} = \rho^2 \sin\theta$$

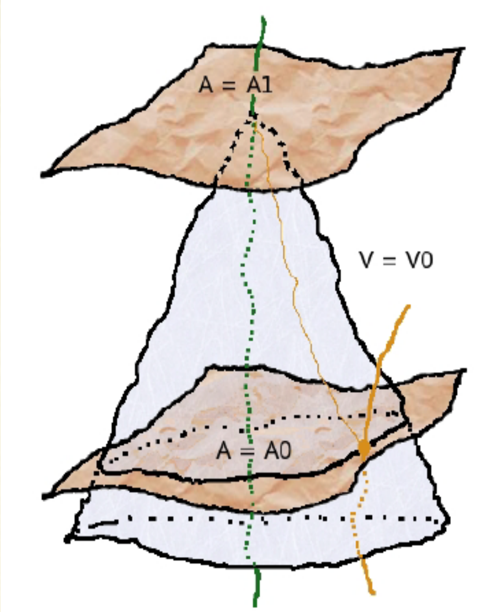
$$d_s \equiv \rho = \sum_{l,m} a_{lm}(w_0,z_s)Y_{lm}(\theta,\varphi)$$

$$\int d^2\theta\sqrt{\gamma} = \int d^2\theta\rho^2\sin\theta = \sum_{l,m}\left|a_{lm}(w_0,z_s)\right|^2 > a_{00}^2$$

Anisotropies always “mimic” acceleration!

The Perturbed Quantities

- We work up to 2nd order in perturbation theory in the Poisson Gauge.
- Both the area distance and the measure of integration are expressed in terms of the gravitational potential and its derivatives. Vector and tensor pert. do not contribute.



$$d_L = d_L^{(0)}[1 + d_L^{(1)}(\Psi, \partial\Psi \ldots) + d_L^{(2)}(\Psi, \partial\Psi \ldots) + \ldots]$$

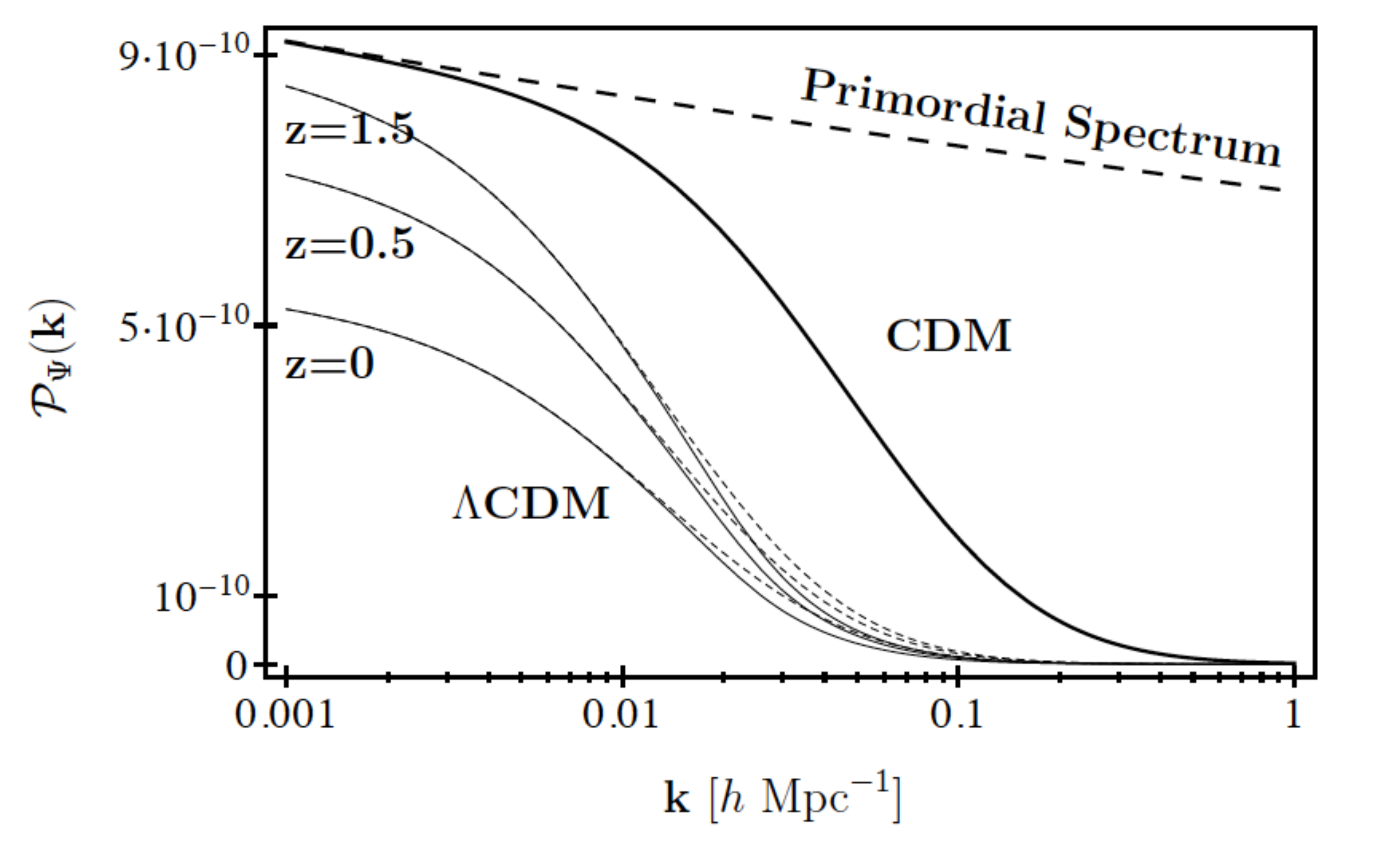
$$\int d^2\tilde{\theta}\sqrt{\gamma} = \int d\Omega[1 + \mu^{(1)}(\Psi, \partial\Psi \ldots) + \mu^{(2)}(\Psi, \partial\Psi \ldots) + \ldots]$$

Statistical Properties

- In principle, we can now calculate <d_L>(z) to first order in the gravitational potential ~ void model. $\overline{\Psi} = 0, \overline{\Psi^2} \neq 0$
- In order not to resort to a specific realization we need LC+statistical/ensemble average. If perturbations come from primordial Gaussian fluc. (inflation)

$$\{\langle d_L \rangle\} = d_L^{(0)}\left[1 + \{\langle \mu^{(1)} d_L^{(1)} \rangle\} + \{\langle d_L^{(2)} \rangle\} + \ldots\right]$$

$$(Var \frac{d_L}{d_L^{FLRW}}) = \{\langle (d_L^{(1)})^2 \rangle\}$$



$9 \cdot 10^{-10}$
$5 \cdot 10^{-10}$
10^{-10}
0
$\mathcal{P}_{\Psi}(\mathbf{k})$
Primordial Spectrum
z=1.5
z=0.5
z=0
CDM
ΛCDM
0.001
0.01
0.1
1
$\mathbf{k}$ [h Mpc^{-1}]

Interpretation & Analysis

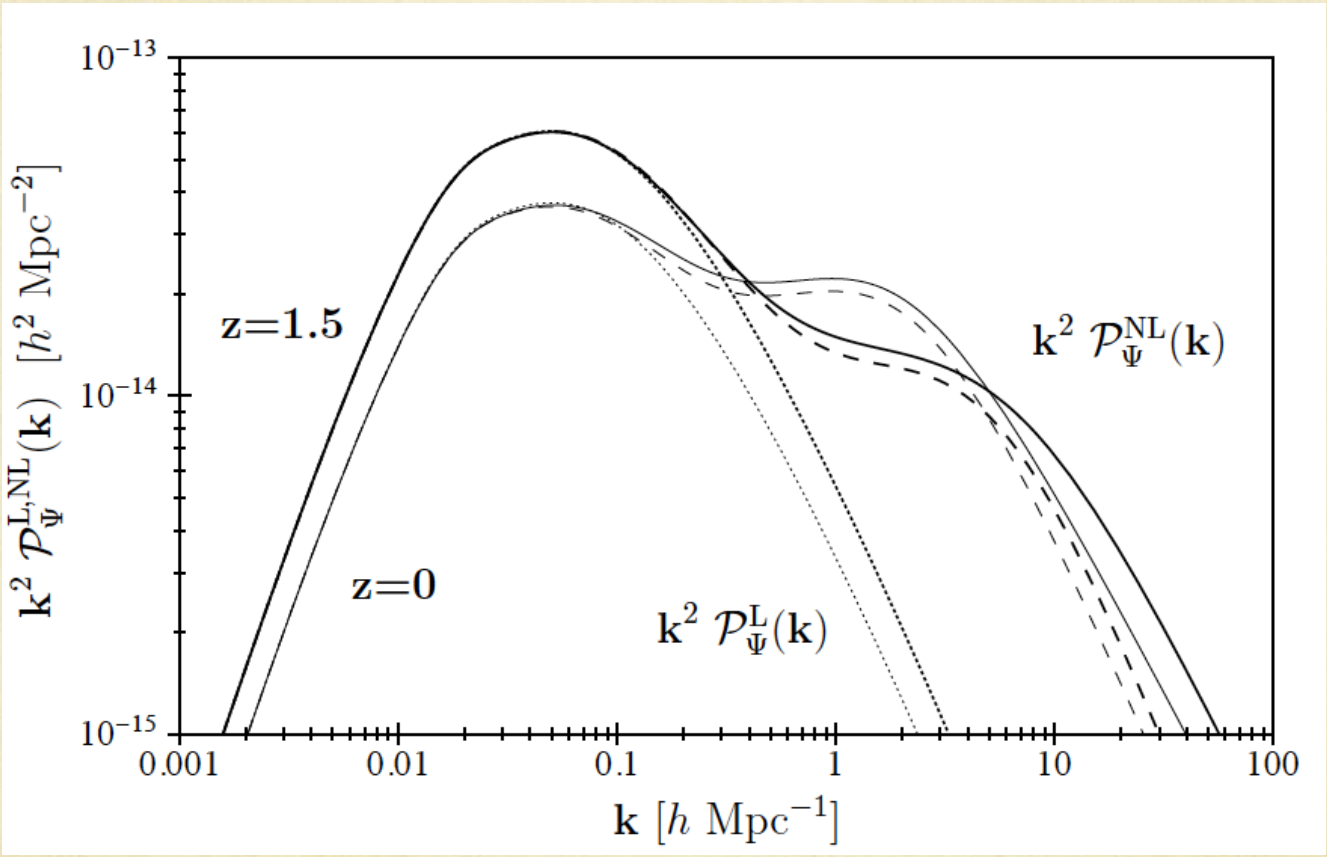
- d_L is a stochastic observable – mean, dispersion, skewness…

$$f_\Phi(z) = \left[\widetilde{f}_{1,1}(z) + \widetilde{f}_2(z)\right] \int_0^\infty \frac{dk}{k} \left(\frac{k}{\mathcal{H}_0}\right)^2 \mathcal{P}(k),$$

In the flux - The dominant contribution are Doppler terms $\sim k^2$

- Any other function of d_L gets also k^3 contributions – lensing contribution, dominates at large redshift, $z>0.3$
- Linear treatment $k<0.1\text{-}1\ \mathrm{Mpc}^{-1}$ and non-linear treatment $k<21\ \mathrm{Mpc}^{-1}$

No Divergences!



10^{-13}
10^{-14}
10^{-15}
0.001
0.01
0.1
1
10
100
$\mathbf{k}^2\ \mathcal{P}_\Psi^{\mathrm{L,NL}}(\mathbf{k})\ [h^2\ \mathrm{Mpc}^{-2}]$
$\mathbf{k}\ [h\ \mathrm{Mpc}^{-1}]$
z=1.5
z=0
$\mathbf{k}^2\ \mathcal{P}_\Psi^{\mathrm{NL}}(\mathbf{k})$
$\mathbf{k}^2\ \mathcal{P}_\Psi^{\mathrm{L}}(\mathbf{k})$

Interpretation & Analysis

- Superhorizon scales are subdominant.
- At small enough scales, the transfer function wins.
- At intermediate scales, the phase space factor competes with the initial small amplitude.
- In the non-linear regime we use a fit from simulations.

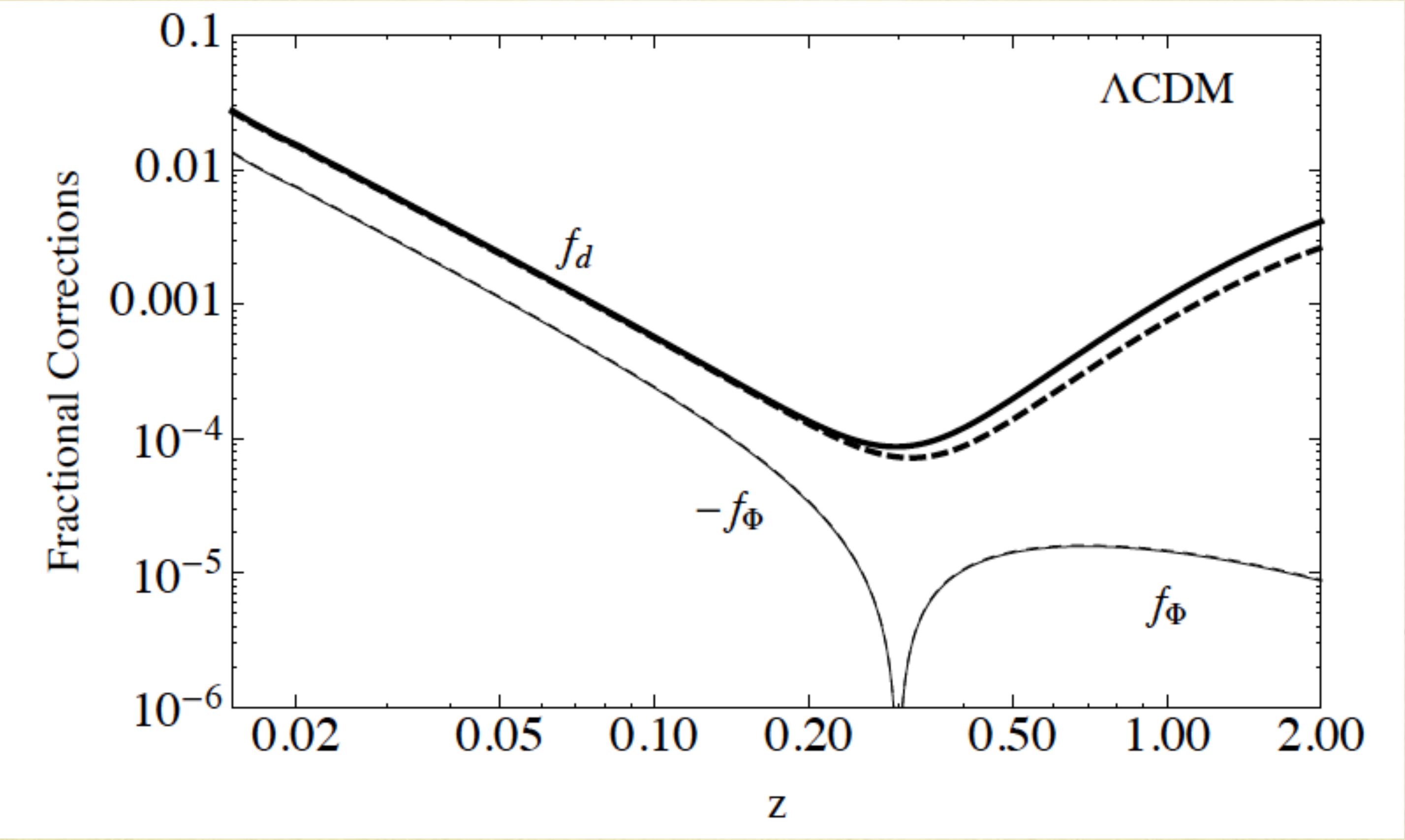
$$P_k = P_{prim.} T^2$$

$$\sqrt{\left(Var \frac{d_L}{d_L^{FLRW}} \right)} = \int \frac{dk}{k} P_k h(k,z)$$

$$T^2(k << k_{eq}) \sim 1$$

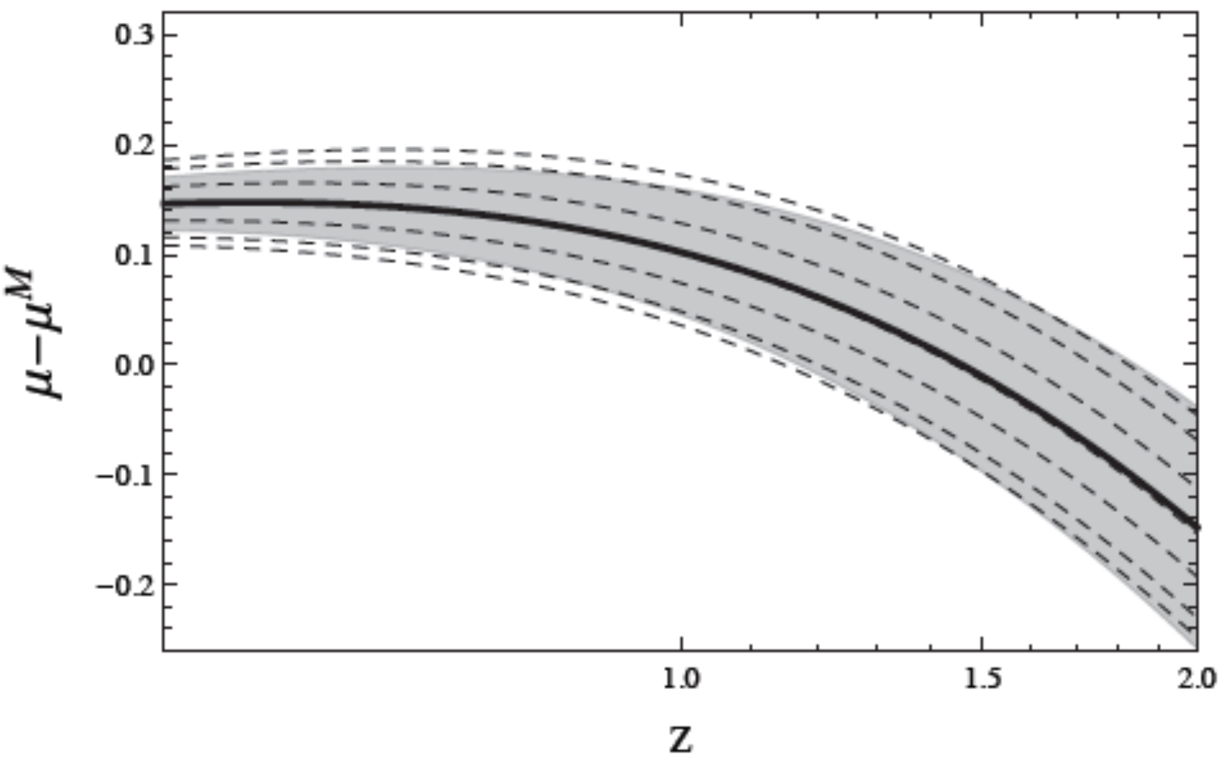
$$T^2(k >> k_{eq}) \sim \ln^2 k \big/ k^4$$

$$Integrand \sim A \times T^2 \times \left(\frac{k}{H_0} \right)^p$$

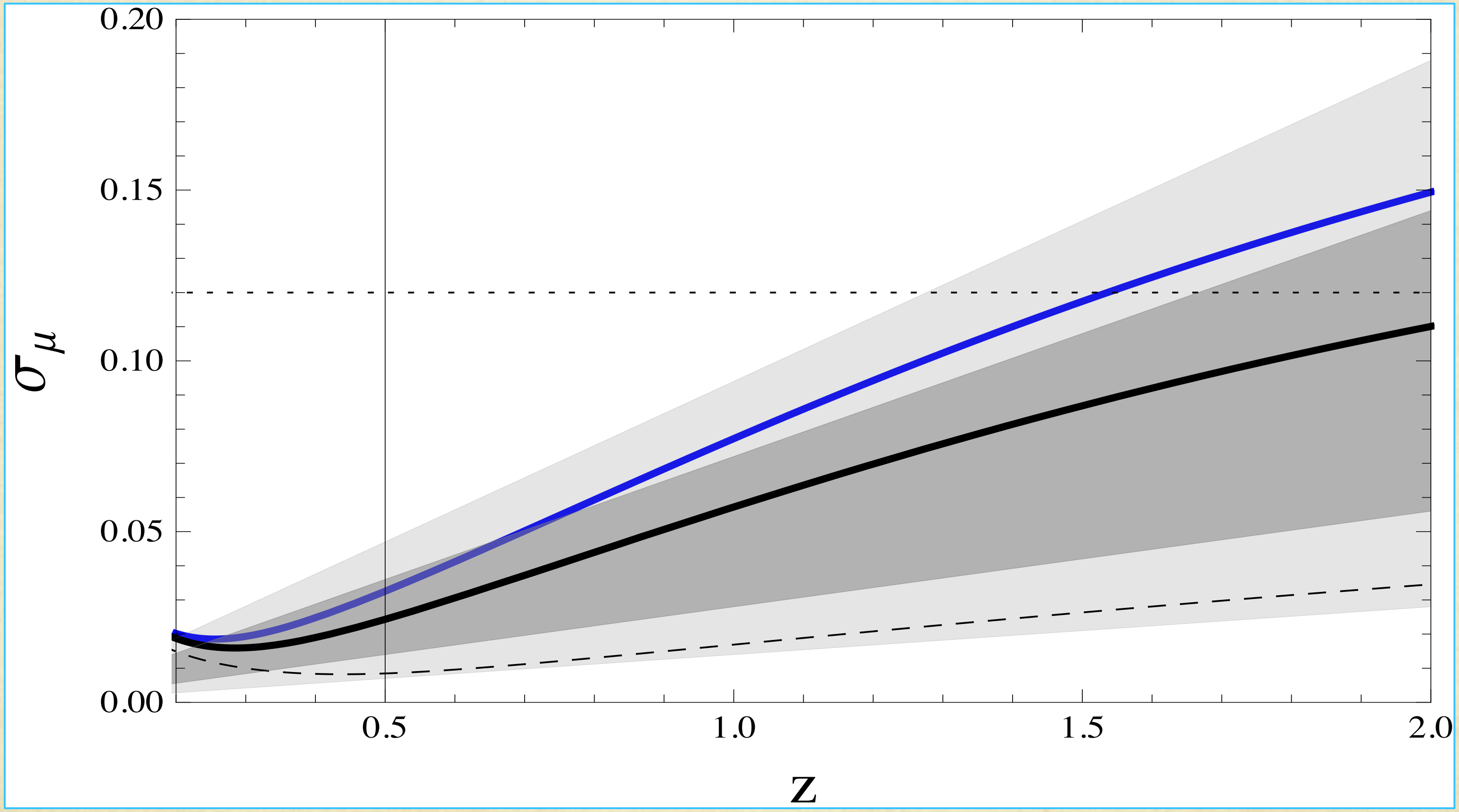


0.1
0.01
0.001
10^{-4}
10^{-5}
10^{-6}
0.02
0.05
0.10
0.20
0.50
1.00
2.00
z
Fractional Corrections
ΛCDM
f_d
$-f_\Phi$
f_Φ

$$\Delta(m - M) = 5\log_{10}\left[\overline{\langle d_L\rangle}\right] - 5\log_{10}\left[\frac{(2+z_s)z_s}{2H_0}\right].$$



Lensing Dispersion



Results and Lessons

- Unlike volume averages: No divergences
- The contribution from inhomogeneities is several orders of magnitude larger than the naïve expectations due to the large phase space factor.
- The size of the contribution ($f_{d,} f_{\Phi}$) is strongly dependent on the quantity whose average is considered.
- Our approach is useful whenever dealing with information carried by light-like signals travelling along our past light-cone.

Conclusions

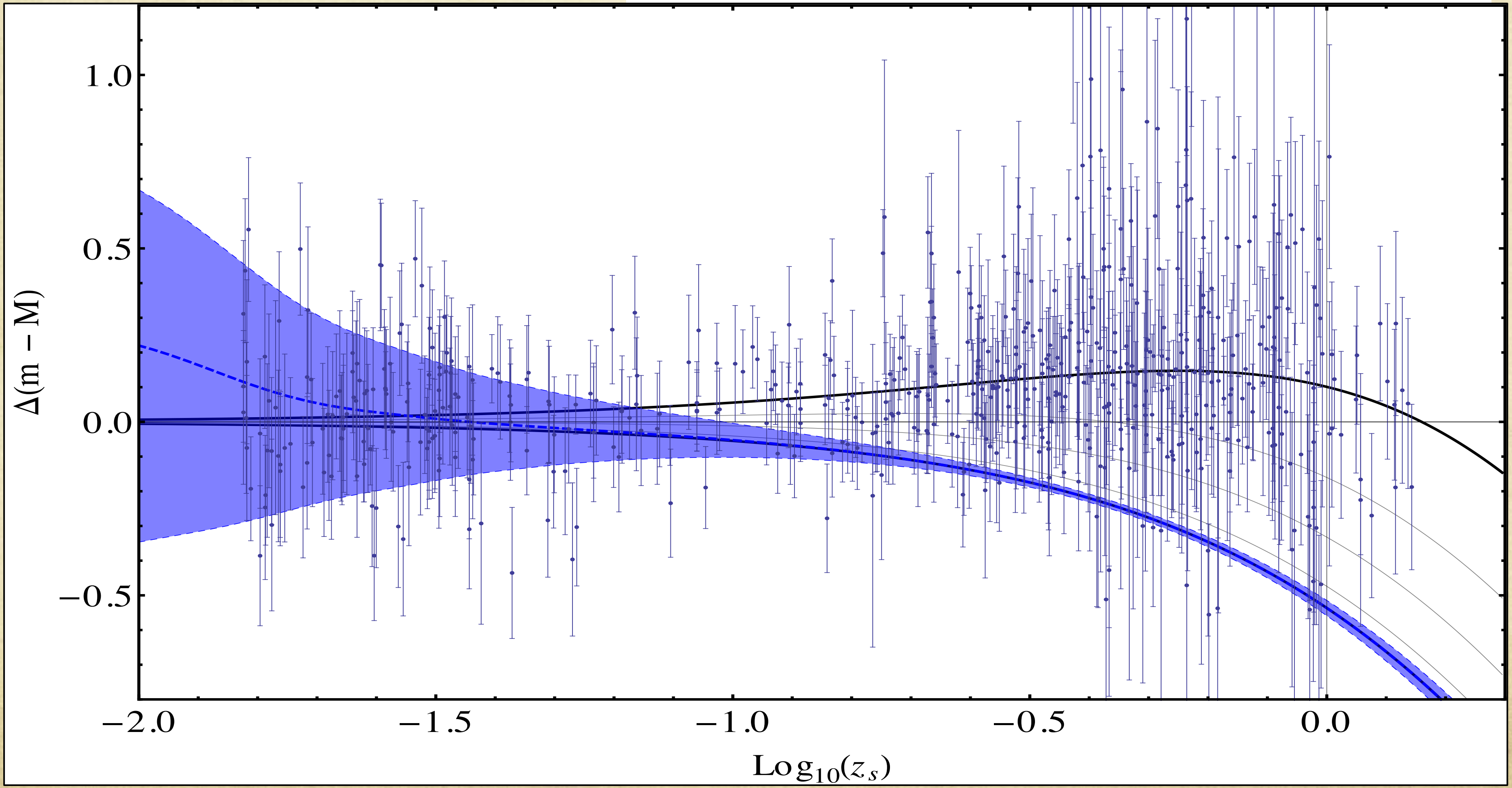
- Flux is the optimal observable. Different bias or "subtraction" mechanisms, in order to extract cosmological parameters.
- Irreducible Scatter - The dispersion is large ~ 2-10% ΛCDM, of the critical density depending on the spectrum. Scatter is mostly from the LC average. It gives *theoretical* explanation to the intrinsic scatter of SN measurements.
- The effect is too SMALL AND has the WRONG z dependence to simulate observable CC!

Open Issues/Future Prospects

1. Using the lensing dispersion to constrain the power spectrum.
2. Matching the effect to other probes: CMB, LSS
3. Applying LC averaging to cosmic shear, BAO, kSZ, strong lensing etc.
4. Other applications, averaging of EFE ....Many open theoretical and pheno. problems.

k_{max}=0.1 Mpc^{-1}, CDM

$$\Delta(m - M) = 5\log_{10}\left[\overline{\langle d_L \rangle}\right] - 5\log_{10}\left[\frac{(2+z_s)z_s}{2H_0}\right].$$



Prescription Properties

- Dynamical properties: Generalization of Buchert-Ehlers commutation rule:

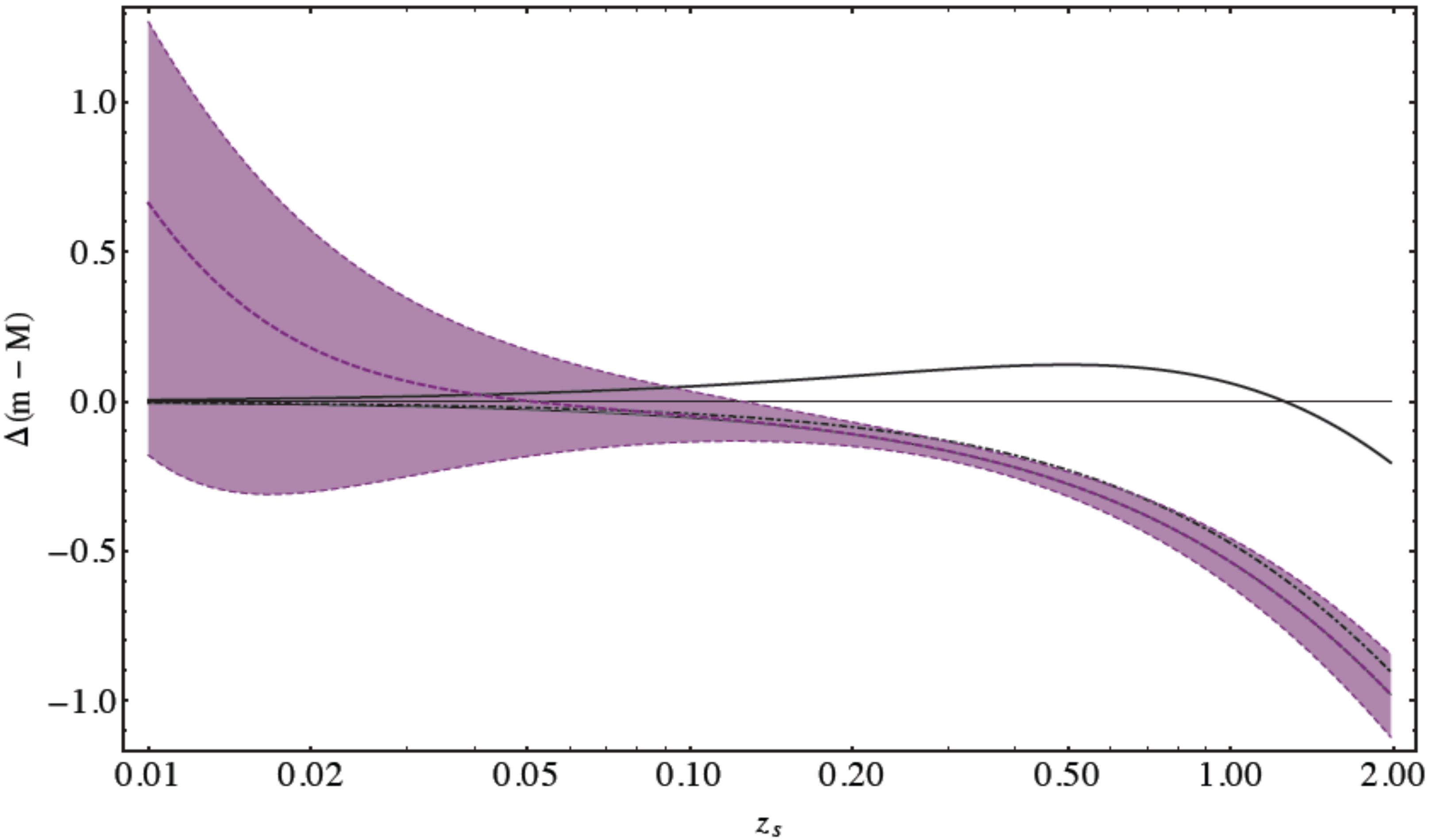
$$\frac{\partial}{\partial A_0}\langle S\rangle_{V_0,A_0} = \left\langle \frac{k\cdot\partial S}{k\cdot\partial A}\right\rangle_{V_0,A_0} + \left\langle \frac{\nabla\cdot k}{k\cdot\partial A}S\right\rangle_{V_0,A_0} - \left\langle \frac{\nabla\cdot k}{k\cdot\partial A}\right\rangle_{V_0,A_0}\langle S\rangle_{V_0,A_0}$$

- For actual physical calculations, use EFE/ energy momentum tensor for evaluation. Example: which gravitational potential to use in evaluating the dL-z relation.

- Averages of different functions give different outcome

$$\left\langle \overline{F}(S)\right\rangle \neq F\left\langle \overline{S}\right\rangle$$

k_{max}=1 Mpc^{-1}



Results

- From z=0.03 correlated term gives a contribution of 1 part in 10000 to the mean of d_L

<u>However</u> it is still 6 orders of magnitude bigger than the naive expectation.

- The dispersion is very large~ 10% of the critical density. Scatter is mostly from angular average.
- The lensing term squared also appears in the 2nd order term of the mean, the term exists independently of the averaging prescription. Barring cancellations, this term will dominate over the correlated contribution $d_L^{(2)}$, $(d_L^{(1)})^2$?

Work In Progress

- Calculation of the averaged d_L and variance to second order in perturbation theory – LCDM+ tensors and vectors
- Actually measuring flux! The area distance is factored out! Unique! Any other power of d_L will have significant deviations.

$$\langle d_L^{-2}\rangle(z_s, w_0) = (1+z_s)^{-4}\left[\int \frac{d^2\tilde{\theta}}{4\pi}\gamma^{\frac{1}{2}}(w_0, \tau_s(z_s, \tilde{\theta}^a), \tilde{\theta}^b)\right]^{-1}$$

- …ence, only the variance does => Similar results!

Summary & Conclusions

- Application of light cone averaging formalism to the d_L-z relation.
- INHOMOGENEITIES CANNOT FAKE DARK ENERGY AT THE OBSERVABLE LEVEL!
- The effect of averaging can, in principle, be distinguished from the homogeneous contribution of CC/DE.

$$\overline{\langle (d_L/d_L^{FLRW})^{\alpha} \rangle} - \alpha \overline{\langle d_L/d_L^{FLRW} \rangle} = 1 - \alpha + \frac{\alpha(\alpha-1)}{2} \overline{\langle \sigma_1^2 \rangle}.$$

- Variance gives *theoretical* explanation to the intrinsic scatter of SN measurements.

Open Issues/Future Prospects

1. Estimates of the non-linear regime, especially 0.1 $\mathrm{Mpc}^{-1} < k < 1\ \mathrm{Mpc}^{-1}$.

2. Cosmological parameter analysis.

3. Matching the effect to other probes: CMB, LSS

4. Applying LC averaging to BAO, kSZ and many more.

5. Other applications, averaging of EFE ….Many open theoretical and pheno. problems.

GLC Metric

- FLRW

$$w = r + \eta, \qquad \tau = t, \qquad \Upsilon = a(t), \qquad U^a = 0,$$
$$\gamma_{ab} d\theta^a d\theta^b = a^2(t) r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

- τ can be identified as the time coordinate in the synchronous gauge of arbitrary space-time.

$$g^{t\mu}_{SG} = \{-1, \vec{0}\} = -\left[\partial_\tau + \Upsilon^{-1}(\partial_w + U^a \partial_a)\right] X^\mu = -u^\nu \partial_\nu X^\mu = -\frac{dX^\mu}{d\lambda},$$

IBD et al. '12

GLC to FLRW NG 1st Order

$$g^{\mu\nu}_{NG} = a^{-2}(\eta)\,\mathrm{diag}\left(-1+2\Phi, 1+2\Psi, (1+2\Psi)\gamma_0^{ab}\right).$$

$$\begin{aligned}
\tau &= \int_{\eta_{in}}^{\eta} d\eta' a(\eta')\left[1+\Psi(\eta', r, \theta^a)\right] \ , \\
w &= \eta_+ + \int_{\eta_+}^{\eta_-} dx\, \hat{\Psi}(\eta_+, x, \theta^a) \ , \\
\widetilde{\theta}^a &= \theta^a + \frac{1}{2}\int_{\eta_+}^{\eta_-} dx\, \hat{\gamma}_0^{ab}(\eta_+, x, \theta^a)\int_{\eta_+}^{x} dy\, \partial_b \hat{\Psi}(\eta_+, y, \theta^a) \ ,
\end{aligned}$$

$$\hat{\Psi}(\eta_+, \eta_-, \theta^a) \equiv \Psi(\eta, r, \theta^a)$$

$$\hat{\gamma}_0^{ab}(\eta_+, \eta_-, \theta^a) \equiv \gamma_0^{ab}(\eta, r, \theta^a) = diag(r^{-2}, r^{-2}\sin^{-2}\theta)$$

$$\eta_\pm = \eta \pm r$$

LC Calculation and LCDM

- Pure FLRW:

$$d_L^{FLRW}(z_s) = (1+z_s)a_0 \int_{\eta_s}^{\eta_0} d\eta = (1+z_s)\int_0^{z_s} \frac{dz}{H(z)}$$
$$= \frac{1+z_s}{H_0}\int_0^{z_s} dz \left[\sum_n \Omega_{n0}(1+z)^{3(1+w_n)}\right]^{-1/2}$$

$$\frac{d_L(z_s,\theta^a)}{(1+z_s)a_0\Delta\eta} \equiv \frac{d_L(z_s,\theta^a)}{d_L^{FLRW}(z_s)} = 1 - \Psi(\eta_s,\eta_0-\eta_s,\theta^a) + 2\Psi_{\rm av} + \left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)J - J_2.$$

- Perturbed:

Measure of Integration

- The de $$\int d^2\widetilde{\theta}\sqrt{\gamma} = \int d\phi \sin\theta \, d\theta a^2 r^2 (1 - 2\Psi)$$

For futu $$\mu_1 = -2\Psi_s + 4\Psi_{\rm av} + 2\left(1 - \frac{1}{\mathcal{H}_s \Delta\eta}\right) J(z_s, \theta^a)$$

- Subhorizon fluc. $H_0<k$. Superhorizon fluc. are subdominant.
- No UV (k➔∞) or IR (z ➔0, k ➔ 0) divergences.

Statistical Properties

$$\boxed{\left\{\left\langle d_L \right\rangle\right\} = d_L^{(0)}\left[1 + \left\{\left\langle \mu^{(1)} d_L^{(1)} \right\rangle\right\} + \left\{\left\langle d_L^{(2)} \right\rangle\right\} + \ldots\right]}$$

- In principle, we can now calculate <d_L>(z) to first order in the gravitational potential ~ void model. $\overline{\Psi} = 0,\ \overline{\Psi^2} \neq 0$
- In order not to resort to a specific realization we need LC+statistical/ensemble average. If perturbations come from primordial Gaussian fluc. (inflation)

$$\mu = \sum_i \mu_i, \qquad \sigma = \sum_i \sigma_i,$$

$$\langle S \rangle_\Sigma = \frac{\int_\Sigma d^2\mu\, S}{\int_\Sigma d^2\mu}$$

$$d^2\mu = (d^2\mu)^{(0)}(1+\mu), \qquad S = S^{(0)}(1+\sigma),$$

BR of Statistical and LC Averaging

- The mean of a scalar:

$$\overline{\langle S/S^{(0)}\rangle} = 1 + \overline{\langle \sigma_2 \rangle} + \mathrm{IBR}_2 + \overline{\langle \sigma_3 \rangle} + \mathrm{IBR}_3 + \ldots$$

$$\begin{aligned} \mathrm{IBR}_2 &= \overline{\langle \mu_1\sigma_1\rangle} - \overline{\langle \mu_1\rangle\langle\sigma_1\rangle}, \\ \mathrm{IBR}_3 &= \overline{\langle \mu_2\sigma_1\rangle} - \overline{\langle \mu_2\rangle\langle\sigma_1\rangle} + \overline{\langle \mu_1\sigma_2\rangle} - \overline{\langle \mu_1\rangle\langle\sigma_2\rangle} - \overline{\langle \mu_1\rangle\langle\mu_1\sigma_1\rangle} + \overline{\langle \mu_1\rangle\langle\mu_1\rangle\langle\sigma_1\rangle}, \end{aligned}$$

=> Effects are second order, but we have the full backreaction of the inhomogeneities of the metric at this order!

- The variance to leading order: $\mathrm{Var}[S/S^{(0)}] = \overline{\langle \sigma_1^2 \rangle}.$

Dominant Terms

$J = I_+ - I_r.$

$$I_+ = \int_{\eta_+^s}^{\eta_-^s} dx\, \partial_+ \Psi(\eta_s^+, x, \theta^a) = \Psi_s - \Psi_o - 2\int_{\eta_s}^{\eta_0} d\eta\, \partial_r \Psi(\eta, r, \theta^a),$$

$$I_r = \int_{\eta_{in}}^{\eta_s} d\eta \frac{a(\eta)}{a(\eta_s)} \partial_r \Psi(\eta, r_s, \theta^a) - \int_{\eta_{in}}^{\eta_0} d\eta \frac{a(\eta)}{a(\eta_0)} \partial_r \Psi(\eta, 0, \theta^a).$$

$$\boxed{I_r = (\vec{v}_S - \vec{v}_0) \cdot \hat{n}}$$

$$\vec{v}_{s,o} = -\int_{\eta_{in}}^{\eta_{s,o}} d\eta' \frac{a(\eta')}{a(\eta_{s,0})} \vec{\nabla}\Psi(\eta', r, \theta^a)$$

- Doppler effect due to the perturbation of the geodesic.

- The Lensing Term:

$$J_2 = \frac{1}{\eta_0 - \eta_s} \int_{\eta_s}^{\eta_0} d\eta \frac{\eta - \eta_s}{\eta_0 - \eta} \left[\partial_\theta^2 + \cot\theta\, \partial_\theta + (\sin\theta)^{-2} \partial_\phi^2\right] \Psi(\eta', \eta 0 - \eta', \theta^a) \equiv \frac{1}{\eta_0 - \eta_s} \int_{\eta_s}^{\eta_0} d\eta \frac{\eta - \eta_s}{\eta_0 - \eta} \Delta_2 \Psi$$

Conclusions

- GR + Standard Pert. Theory + Averaging challenge the concordance model.
- Perturbations cannot be discarded as negligible. Any explanation of the cosmic acceleration will have to take them into account.

2011 Nobel Prize in Physics

The 2011 Nobel Prize in Physics is awarded "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae" with one half to Saul Perlmutter and the other half jointly to Brian P. Schmidt Adam G. Riess

$$\mathrm{IBR}_2 = \int_0^\infty \frac{dk}{k}\, P_\Psi(k) \sum_{i=1}^{4}\sum_{j=1}^{5} 2\Big[\mathcal{C}_{ij}(k,\eta_0,\eta_s) - \mathcal{C}_i(k,\eta_0,\eta_s)\,\mathcal{C}_j(k,\eta_0,\eta_s)\Big]$$

$$\left(\mathrm{Var}\left[\frac{d_L}{d_L^{FLRW}}\right]\right)^{1/2} = \sqrt{\overline{\langle\sigma_1^2\rangle}} = \left[\int_0^\infty \frac{dk}{k}\, P_\Psi(k) \sum_{i=1}^{5}\sum_{j=1}^{5} \mathcal{C}_{ij}(k,\eta_0,\eta_s)\right]^{1/2} .$$

$\overline{\langle A_iA_j\rangle}$	$\mathcal{C}_{ij}(k,\eta_0,\eta_s)$	$\mathcal{C}_{ij}$ for $k\Delta\eta \ll 1$	$\mathcal{C}_i\,\mathcal{C}_j$ for $k\Delta\eta \ll 1$
$\overline{\langle A_1A_1\rangle}$	1	1	$1-\frac{l^2}{3}$
$\overline{\langle A_1A_2\rangle}$	$-\frac{2}{l}\mathrm{SinInt}(l)$	$-2+\frac{l^2}{9}$	$-2+\frac{4}{9}l^2$
$\overline{\langle A_1A_3\rangle}$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left[1-\frac{\sin(l)}{l}\right]$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{6}$	$-\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{6}$
$\overline{\langle A_1A_4\rangle}$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{f_0}{\Delta\eta}\left[\cos l-\frac{\sin(l)}{l}\right]$	$-\frac{f_0}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	$-\frac{f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$
$\overline{\langle A_1A_5\rangle}$	$-2\left[1-\frac{\sin(l)}{l}\right]$	$-\frac{l^2}{3}$	0
$\overline{\langle A_2A_2\rangle}$	$\frac{8}{l^2}\left[-1+\cos l+l\,\mathrm{SinInt}(l)\right]$	$4-\frac{l^2}{9}$	$4-\frac{4}{9}l^2$
$\overline{\langle A_2A_3\rangle}$	0	0	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$
$\overline{\langle A_2A_4\rangle}$	$2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{f_0+f_s}{\Delta\eta}\left[1-\frac{\sin(l)}{l}\right]$	$\frac{f_0+f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	$\frac{f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{2}{3}l^2$
$\overline{\langle A_2A_5\rangle}$	$\frac{2}{3l^2}\left[-4+(4+l^2)\cos l+l\sin l+l^3\,\mathrm{SinInt}(l)\right]$	$\frac{l^2}{3}$	0
$\overline{\langle A_3A_3\rangle}$	$2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\left[1-\frac{\sin(l)}{l}\right]$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^2}{3}$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^4}{36}$
$\overline{\langle A_3A_4\rangle}$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{f_0-f_s}{\Delta\eta}\left[\cos l-\frac{\sin(l)}{l}\right]$	$-\frac{f_0-f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^2}{3}$	$\frac{f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^4}{18}$
$\overline{\langle A_3A_5\rangle}$	$-2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left[1-\frac{\sin(l)}{l}\right]$	$-\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	0
$\overline{\langle A_4A_4\rangle}$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\left[\frac{f_0^2+f_s^2}{\Delta\eta^2}\frac{l^2}{3}-\frac{2f_0f_s}{\Delta\eta^2}\left(2\cos l+(-2+l^2)\frac{\sin l}{l}\right)\right]$	$\left(\frac{f_0-f_s}{\Delta\eta}\right)^2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^2}{3}$	$\left(\frac{f_s}{\Delta\eta}\right)^2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^4}{9}$
$\overline{\langle A_4A_5\rangle}$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left[\frac{f_0+3f_s}{\Delta\eta}\cos l+\frac{f_0-f_s}{\Delta\eta}\frac{\sin l}{l}+\frac{(f_0+f_s)(-2+l\,\mathrm{SinInt}(l))}{\Delta\eta}\right]$	$\frac{f_0-f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	0
$\overline{\langle A_5A_5\rangle}$	$\frac{1}{15l^2}\left[-24+20l^2+(24-2l^2+l^4)\cos(l)+l(-6+l^2)\sin(l)+l^5\,\mathrm{SinInt}(l)\right]$	$\frac{l^2}{3}$	0

A_i	$\mathcal{C}_i(k,\eta_0,\eta_s)$
A_1	$\frac{\sin l}{l}$
A_2	$-\frac{2}{l}\mathrm{SinInt}(l)$
A_3	$-\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left(1-\frac{\sin l}{l}\right)$
A_4	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{f_s}{\Delta\eta}\left(\cos l-\frac{\sin l}{l}\right)$
A_5	0

$$l = k\Delta\eta$$

$$f_{0,s} \approx \frac{\eta_{0,s}}{3}$$

CC/ Dark Energy/ Modified Gravity/ Voids

- **(Averaging of) perturbations in a consistent way. Does it have any effects?**
- **In this work, we are NOT: assuming voids, modifying GR, adding scalar fields, having a CC….Doh!**
- **Application: FLRW + pert. , pure CDM, properly averaged and calculate the d_L-z relation.**

=> Strong evidence that perturbations induce changes in the estimated cosmological parameters

GLC to FLRW NG 1st Order

$$\Upsilon = a(\eta)\left[1+\hat{\Psi}(\eta_+,\eta_+,\theta^a) - \int_{\eta_+}^{\eta_-} dx\,\partial_+\hat{\Psi}(\eta_+,x,\theta^a)\right] + \int_{\eta_{in}}^{\eta} d\eta' a(\eta')\partial_r\Psi(\eta',r,\theta^a);$$

$$U^a = \frac{1}{2}\hat{\gamma}_0^{ab}\int_{\eta_+}^{\eta_-} dx\,\partial_b\hat{\Psi}(\eta_+,x,\theta^a) - \frac{1}{a(\eta)}\gamma_0^{ab}\int_{\eta_{in}}^{\eta} d\eta' a(\eta')\,\partial_b\Psi(\eta',r,\theta^a)$$

$$+\frac{1}{2}\int_{\eta_+}^{\eta_-} dx\,\partial_+\left[\hat{\gamma}_0^{ab}(\eta_+,x,\theta^a)\int_{\eta_+}^{x} dy\,\partial_b\hat{\Psi}(\eta_+,y,\theta^a)\right] - \frac{1}{2}\lim_{x\to\eta_+}\left[\hat{\gamma}_0^{ab}(\eta_+,x,\theta^a)\int_{\eta_+}^{x} dy\,\partial_b\hat{\Psi}(\eta_+,y,\theta^a)\right];$$

$$\gamma^{ab} = \frac{1}{a(\eta)^2}\left\{[1+2\Psi(\eta,r,\theta^a)]\,\gamma_0^{ab} + \frac{1}{2}\left[\hat{\gamma}_0^{ac}\int_{\eta_+}^{\eta_-} dx\,\partial_c\left(\hat{\gamma}_0^{bd}(\eta_+,x,\theta^a)\int_{\eta_+}^{x} dy\,\partial_d\hat{\Psi}(\eta_+,y,\theta^a)\right) + a\leftrightarrow b\right]\right\}.$$

LC Calculation and LCDM

- Pure FLRW:

$$d_L^{FLRW}(z_s) = (1+z_s)a_0 \int_{\eta_s}^{\eta_0} d\eta = (1+z_s)\int_0^{z_s} \frac{dz}{H(z)}$$
$$= \frac{1+z_s}{H_0}\int_0^{z_s} dz \left[\sum_n \Omega_{n0}(1+z)^{3(1+w_n)}\right]^{-1/2}$$

- Perturbed:

$$\frac{d_L(z_s,\theta^a)}{(1+z_s)a_0\Delta\eta} \equiv \frac{d_L(z_s,\theta^a)}{d_L^{FLRW}(z_s)} = 1 - \Psi(\eta_s, \eta_0-\eta_s, \theta^a) + 2\Psi_{\rm av} + \left(1 - \frac{1}{\mathcal{H}_s\Delta\eta}\right)J - J_2.$$

- Comparing by defining an effective redshift and averaging at constant redshift and w=w0

$$\frac{a(\bar{\eta}_s^{(0)})}{a(\eta_o)} = \frac{1}{1+z}$$

$$w_0 = \eta_+^s - 2\Delta\eta\Psi_{\rm av} = \eta_0,$$

$$\int_{\eta_+^s}^{\eta_-^s} dx\,\hat{\Psi}(\eta_+^s, x, \theta^a) = -2\int_{\eta_s}^{\eta_0} d\eta'\,\Psi(\eta', \eta_0-\eta', \theta^a) \equiv -2\Delta\eta\Psi_{\rm av}.$$

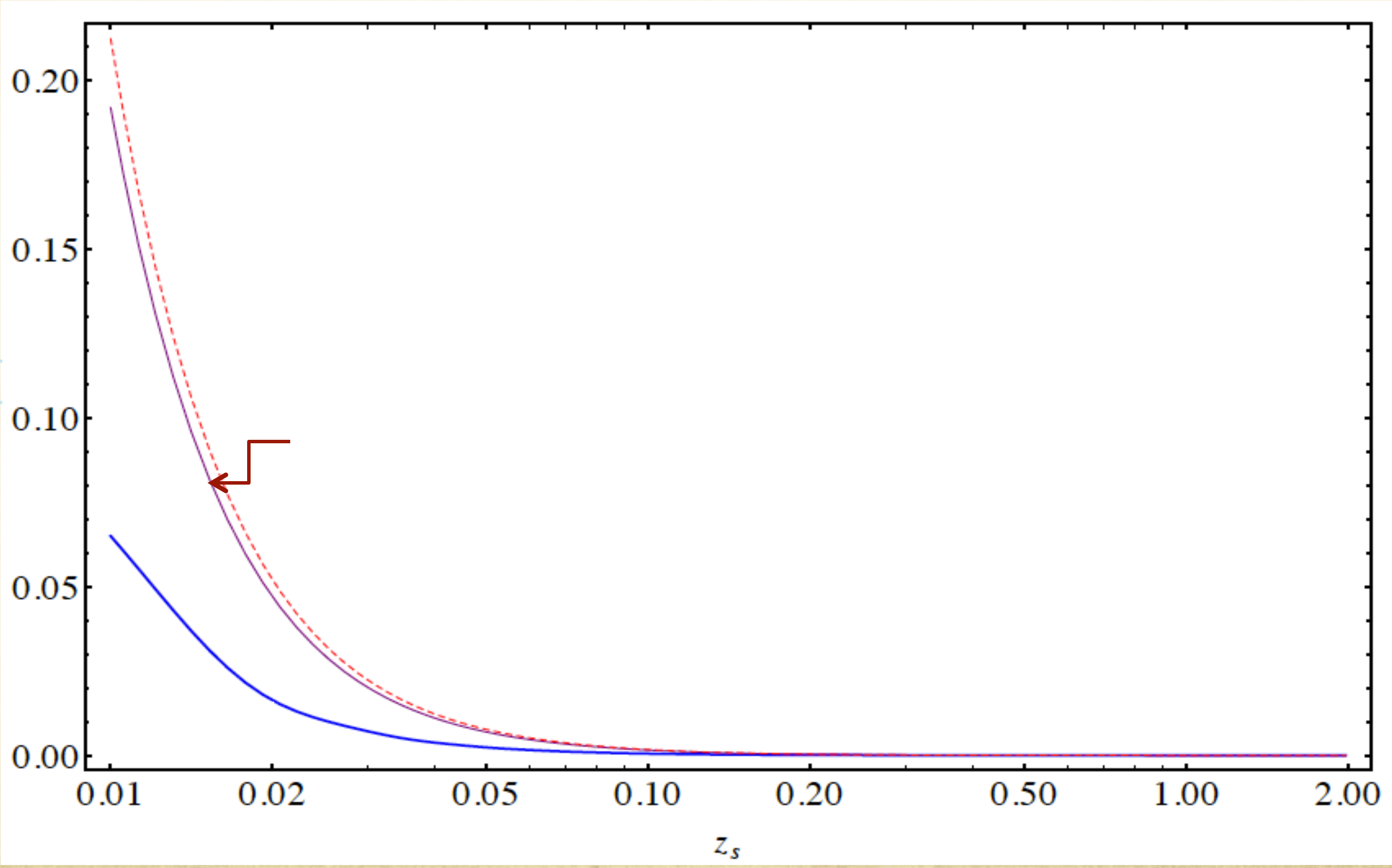
Power Spectrum

- We use the WMAP7 best fit value and the transfer function of Eisenstein & Hu 1997 for CDM.
- We are interested in the overall magnitude so we neglect the baryonic oscillations.

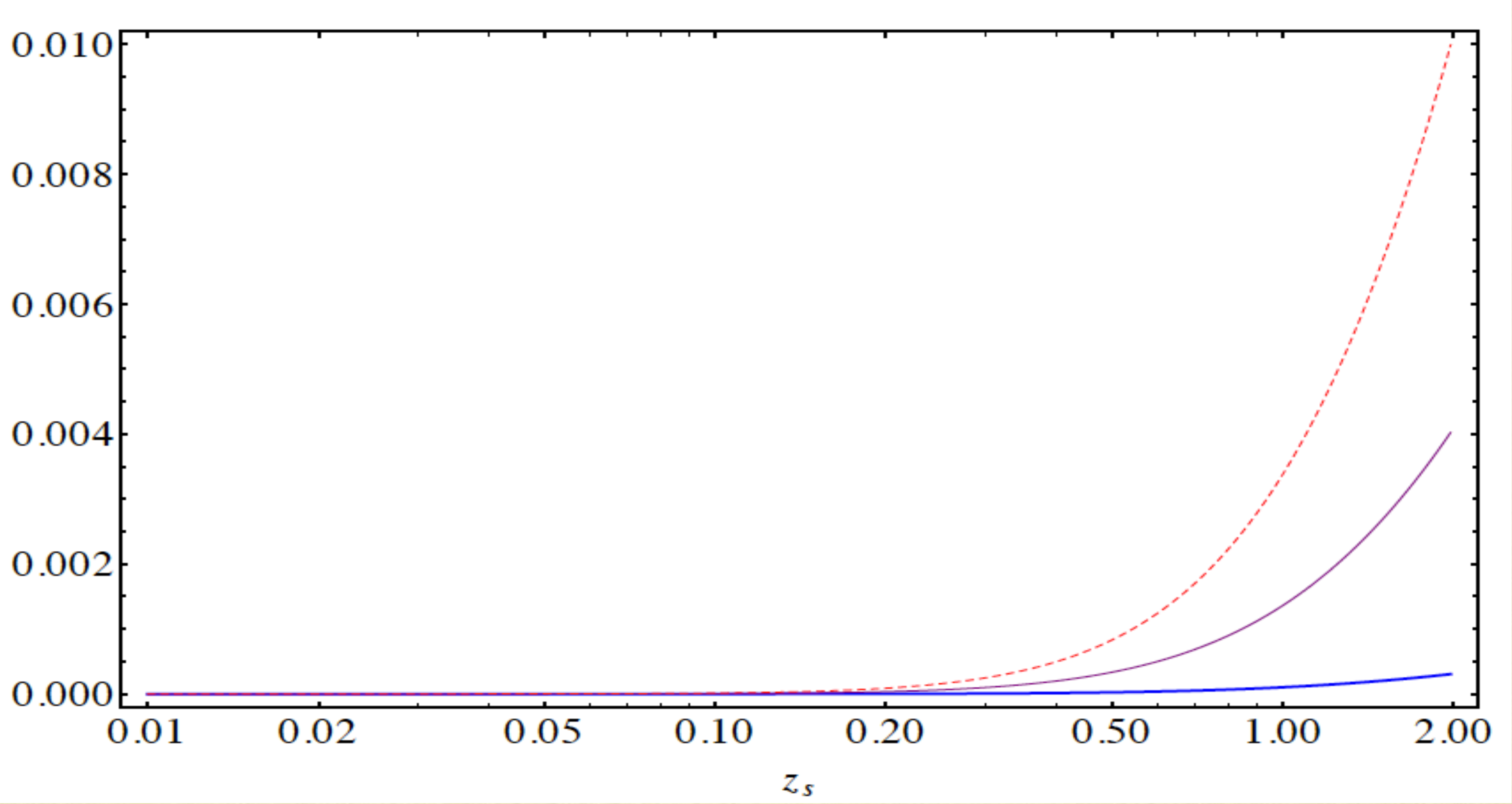
$$P_\Psi(k) \equiv \frac{k^3}{2\pi^2}|\Psi_k|^2 = \left(\frac{3}{5}\right)^2 \Delta_R^2 T(k)^2, \quad \Delta_R^2 = A\left(\frac{k}{k_0}\right)^{n_s-1}$$

- Only subhorizon fluc. $H_0<k$. Superhorizon fluc. are subdominant.
- No UV ($k \rightarrow \infty$) or IR ($z \rightarrow 0$, $k \rightarrow 0$) divergences.

"Doppler^2" term



Lensing^2 Term



Averaging in Physics

- Electromagnetism: Maxwell's equations are linear. Averaging and solving commute. Microscopic E&M is smoothly averaged to Macroscopic E&M.

- Fluid Mechanics/Turbulence: Navier-Stokes equation is non-linear. Averaging and solving do not commute – many open problems.

- GR is non-linear. Averaging and solving do not commute.

(Newtonian Cosmology treated by Ellis, amounts to a boundary term => simulations don't help much)

Non-Trivial Averages

- Off-Center LTB
- Anisotropic Models (Except Kantowski-Sachs)
- More general metrics.
- Perturbed FLRW

Application: calculating the averaged luminosity – distance redshift relation

Past attempts: Vanderveld et al. – post Newtonian, Barausse et al., Kolb et al. – SG superhorizon, Pyne at al.,