Tensors and symmetry-breaking vacua in F-theory

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Introduction and Motivation

Motivating 6d effective theories

- Six-dimensional theories are ideal to probe concepts of string theory
  - chiral representations for spinors: strong constraints from anomalies
  - supersymmetric theories

  - **(1,0) - eight supercharges** analog "heterotic string"
    ⇒ gauge theory: spontaneous compactification + susy breaking:
    N=1, 4d string phenomenology

  - **(2,0) - sixteen supercharges** analog "Type IIB string"
    ⇒ tensor theory: two forms with self-dual field strength
    non-Abelian version mysterious since discovery in '96 [Witten]

  - Two main examples:

F-theory on elliptically fibered Calabi-Yau threefolds

world-volume theory of multiple M5-branes

Goals of this talk:

- Part I: Discuss 6d effective theories arising in F-theory compactifications using M-theory
  - 6d pseudo-action and anomalies
  - 6d Chern-Simons terms and landscape analysis
  - α' - corrections

- Part II: Study vacua of 6d gauged supergravity theories from F-theory
  - G-fluxes and SU(3) structure geometry
  - spontaneous compactification: 6d → 4d
  - supersymmetry breaking: 8 supercharges → 4 supercharges

- Message: F-theory via M-theory is ideal to study 6d theories with both applications: 6d non-Abelian tensor theories, 4d N=1 phenomenology

Formulating six-dimensional effective theories

F-theory compactifications

- Type IIB has non-perturbative $Sl(2,\mathbb{Z})$ symmetry rotating $\tau = C_0 + ie^{-\phi}$

  $\Rightarrow$ interpret τ as complex structure of a two-torus (2 auxiliary dimensions)

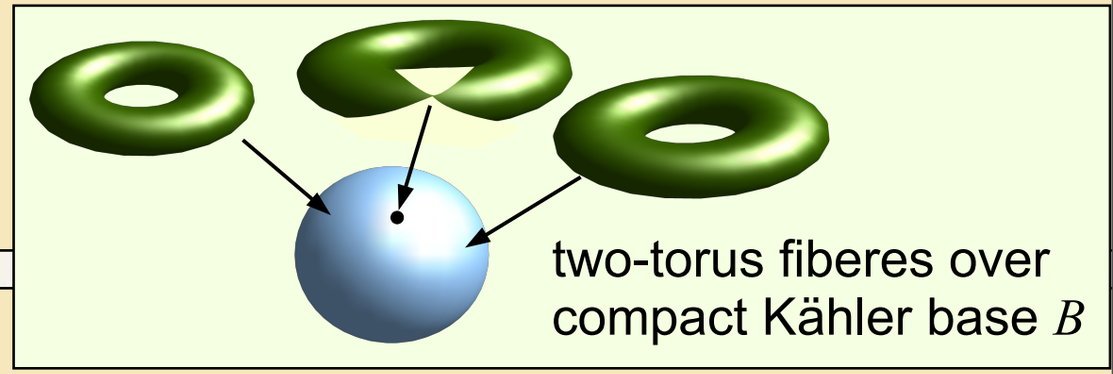
  [Vafa] [Morrison,Vafa]

- minimally supersymmetric F-theory compactifications:

  - Part I: F-theory on torus fibered Calabi-Yau threefold Y_3

    $\Rightarrow$ 6d (1,0) supergravity theory

    $\Rightarrow$ base B_2 is a Kähler manifold



  - Part II: Y_3 SU(3)-structure threefold

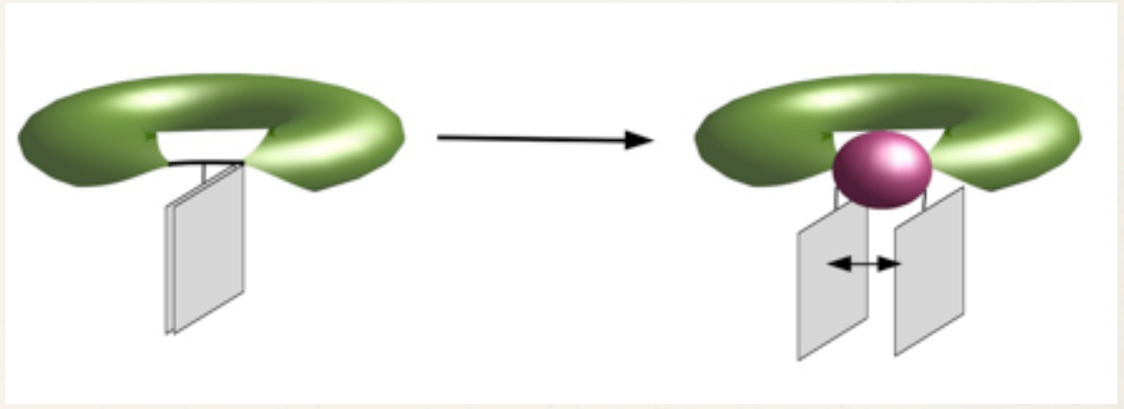
    but B_2 remains Kähler $\Rightarrow$ 6d (1,0) sugra with gauged shift symmetries

- singularities of the fibration are crucial to encode 7-brane physics

  $\Rightarrow$ pinching torus indicates presence of 7-branes magn. charged under τ

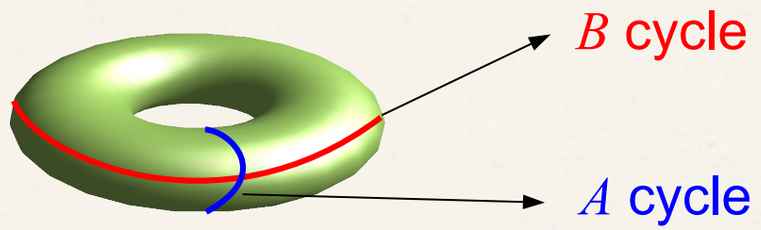
F-theory / M-theory geometries

- F-theory geometries can be constructed and analyzed
  - singularities of elliptic fibration induce non-Abelian gauge symmetry
  - singularity resolution:
    (resolution at each codimension)



- Unification of brane and bulk physics on **resolved CY manifold**

- **M-theory to F-theory limit:** M-theory on



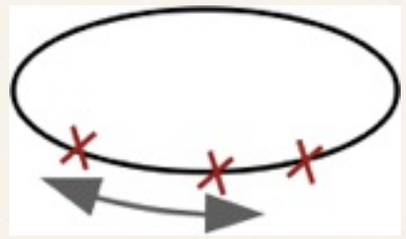
(1) A-cycle: if small than M-theory becomes Type IIA
(2) B-cycle: T-duality $\Rightarrow$ Type IIA becomes Type IIB
(3) grow extra dimension: send T^2- volume T-dual $\Rightarrow$ B-cycle becomes large

$\Rightarrow$ M-theory to F-theory limit connects 6d and 5d effective theories

M-theory on resolved CY manifolds

- physical interpretation of resolution only possible in M-theory
  - moving branes apart on the B-circle:

    Coulomb branch of the lower-dimensional gauge theory:



$$G \;\to\; U(1)^{\mathrm{rank}G}$$

- Massive states from M2 branes on geometric 2-cycles:
  - M2-branes on resolution $\mathbb{P}^1$'s over generic points of S
    $\Rightarrow$ massive `W-bosons' of G-breaking
  - M2-branes on resolution $\mathbb{P}^1$'s over intersection
    $\Rightarrow$ massive matter multiplets
  - M2-branes on the elliptic fiber $\Rightarrow$ massive Kaluza-Klein modes

- All massive states have to be integrated out to determine Wilsonian effective action $\Rightarrow$ in circle compactification also KK-modes are crucial!!

6d F-theory effective actions via M-theory

- effective actions can be computed via M-theory / 11-dimensional supergravity on the resolved Calabi-Yau threefolds

F-theory on singular Y_3

6d, (1,0) effective theory with non-Abelian gauge symmetry G

circle compactification

5d, N=2 effective theory pushed to 5d Coulomb branch: $U(1)^{\mathrm{rk}G}$

M-theory on resolved $\tilde{Y}_3$

5d, N=2 effective theory with only abelian gauge symmetries

compare

- explicit: (1,0) characteristic data determining the action

6d/5d: [Ferrara,Minasian,Sagnotti], [Antoniadis,Ferrara,Minasian,Narain] [Bonetti,TG] [Bonetti,TG,Hohenegger]

gauged sugra: [TG,Pugh]

Part I: Systematics for six-dimensional theories

Classical geometric data

[Bonetti,TG]

- general form of N=1 pseudo-action

$$S = \int \tfrac{1}{2} g_{\alpha\beta}\, G^\alpha \wedge *G^\beta + g_{\alpha\beta}\, dj^\alpha \wedge *dj^\beta + j^\alpha \Omega_{\alpha\beta} \big(a^\beta \mathrm{tr}(R \wedge *R) + b_A^\beta \mathrm{tr}(F^A \wedge *F^A) \big)$$
$$+ B_2^\alpha \Omega_{\alpha\beta} \wedge \big(a^\beta \mathrm{tr}(R \wedge R) + b_A^\beta \mathrm{tr}(F^A \wedge F^A) \big) + \text{hypers}$$

(anti-) self-duality has to be imposed on the level of e.o.m.

- reduce on circle and move to 5d Coulomb branch:
  - **(1) comparison with M-theory:** determine const. a^α, b_A^α, $\Omega_{\alpha\beta}$ geometrically

e.g. $c_1(B_2) = -a^\alpha \omega_\alpha$ canonical class of B_2

$\Omega_{\alpha\beta} = \int_{B_2} \omega_\alpha \wedge \omega_\beta$ intersection numbers on B_2

  - **(2) crucial knowledge of** Kaluza-Klein action of tensor tower (and other fields)
  - **(3) integrate out massive Coulomb branch / Kaluza-Klein modes**

Higher curvature terms / α' - corrections

- determine α'- corrections to 6d F-theory effective action by dimensionally reducing known higher-curvature correction to 11d supergravity

$$S^{(11)}_{\text{curv}} = \frac{1}{l_M^9}\int *1\big(R^{(11)}_{\text{sc}} + l_M^6\mathcal{J}_0\big) + l_M^6 C_3\wedge\mathcal{I}_0$$

$$\mathcal{J}_0 = t_8 t_8 (R^{(11)})^4 - \frac{1}{4!}\epsilon_{11}\epsilon_{11}(R^{(11)})^4$$
$$\mathcal{I}_0 = \epsilon_{11}(\text{Tr}(\mathcal{R}^{(11)\,4}) - \tfrac{1}{4}\text{Tr}(\mathcal{R}^{(11)\,2})^2)$$

- 5d volume correction / higher curvature terms from $\mathcal{J}_0,\ \mathcal{I}_0$ on $\hat{Y}_3$

$$S^{(5)}_{\text{curv}} = \int *1\big(\tilde{\mathcal{V}}_3\ R^{(5)}_{\text{sc}} + \tilde{\mathcal{V}}_2\ \text{Tr}(\mathcal{R}^{(5)}\wedge *\mathcal{R}^{(5)})\big) + c_\Sigma\, A^\Sigma\wedge\text{Tr}(\mathcal{R}^{(5)}\wedge\mathcal{R}^{(5)}))$$

$$\tilde{\mathcal{V}}_3 = \tfrac{1}{3!}\int J^3 + \chi(Y_3) \qquad \tilde{\mathcal{V}}_2 = \int c_2(Y_3)\wedge J \qquad c_\Sigma = \int c_2(Y_3)\wedge\omega_\Sigma$$

$\Rightarrow$ corrections survive partly in the F-theory limit $\rightarrow$ 6d α' - corrections
$\Rightarrow$ non-surviving corrections correspond to 1-loop terms in 5d!

- volume corrections in 4d N=1 Kähler potential $\rightarrow$ volume D7 $\cap$ O7- curve

[TG,Weissenbacher,Savelli] $\rightarrow$ R. Savelli's talk

5d perspective on tensor actions

- action for Kaluza-Klein tensor tower

$$\hat{B} = \sum_{n\in\mathbb{Z}} e^{iny}\left[B_n + A_n \wedge (dy + A^0)\right]$$

$$S^{\text{KK}}_{\text{tensor}} = \int -\tfrac{1}{2} r^{-1}\mathcal{F}\wedge *\mathcal{F} + \tfrac{1}{2} c A^0\wedge\mathcal{F}\wedge\mathcal{F} + \sum_{n=1}^{\infty}\int -r^{-1}\bar{B}_n\wedge *B_n + i\,n\,c\,\bar{B}_n\wedge\mathcal{D}B_n$$

A^0 Kaluza-Klein vector

$\mathcal{D}B_n = dB_n - inA^0\wedge B_n$

[Townsend etal.]

[Bonetti,TG,Hohenegger 1206]

- proposal for non-Abelian generalization
  - zero modes become non-Abelian gauge potentials, e.g. YM $\mathcal{F} = dA + \frac{1}{2}[A,A]$
  - massive tensor modes are gauged $\mathcal{D}B_n = dB_n + [A, B_n] - inA^0\wedge B_n$

$$S^{\text{KK}}_{\text{tensor}} = \sum_{n=1}^{\infty}\int -r^{-1}\text{Tr}(\bar{B}_n\wedge *B_n) + i\,n\,c\,\text{Tr}(\bar{B}_n\wedge\mathcal{D}B_n) + \text{fermions} + \ldots$$

  - proposed 5d, N=2 superconformal action for KK-spectrum of (2,0) theory
    $\Rightarrow$ How can we test the correctness of this proposal?

[Bonetti,TG,Hohenegger 1209] → F. Bonetti's talk

6d anomalies and 1-loop Chern-Simons terms

- 6d gravitational anomalies

[Alvarez-Gaume,Witten] [Sagnotti]

$$H - V = 273 - 29T$$

hypermultiplets, vector multiplets, tensor multiplets

$$a^\alpha \Omega_{\alpha\beta} a^\beta = 9 - T$$

Green-Schwarz mechanism: $a^\alpha \Omega_{\alpha\beta} B_2^\beta \wedge \mathrm{Tr}(\mathcal{R} \wedge \mathcal{R})$

- 6d anomalies captured by 5d 1-loop Chern-Simons terms

	spin-1/2 fermion ψ	self-dual tensor $B_{\mu\nu}$	spin-3/2 fermion ψ_μ
$k_{AFF} \int A \wedge F \wedge F$	$-\frac{1}{48\pi^2}\, q^3 \cdot c_{1/2}$	$-\frac{1}{48\pi^2}\, q^3 \cdot (-4\, c_B)$	$-\frac{1}{48\pi^2}\, q^3 \cdot (5\, c_{3/2})$
$k_{ARR} \int A \wedge \mathrm{tr}\,(R \wedge R)$	$-\frac{1}{384\pi^2}\, q \cdot c_{1/2}$	$-\frac{1}{384\pi^2}\, q \cdot (8\, c_B)$	$-\frac{1}{384\pi^2}\, q \cdot (-19\, c_{3/2})$

q is the charge of the field under the U(1)-field A, $c_{1/2}, c_B, c_{3/2}$ is 6d chirality

$\Rightarrow$ summation over all Kaluza-Klein modes, zeta-function regularization

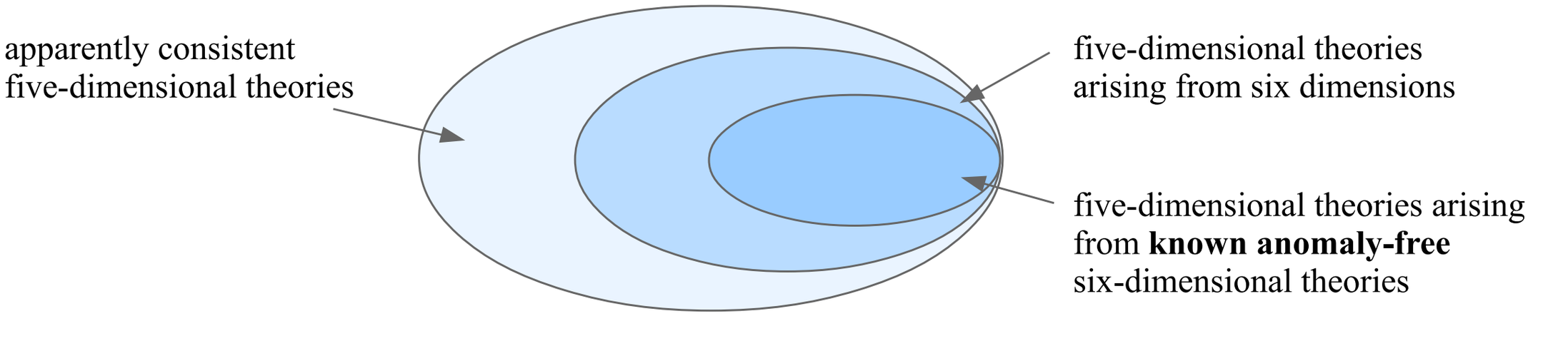
[Bonetti,TG,Hohenegger 1302] → F. Bonetti's talk

6d origins of 5d theories

[Bonetti,TG,Hohenegger 1303]

- can one pose the reversed question:

Given a 5d effective theory, can one determine whether this theory arises from a 6d theory that is **anomaly free?**



- analyzing 5d Chern-Simons terms: split $S^{\rm CS}_{\rm class} + S^{\rm CS}_{1-{\rm loop}}$

  check if correlations to potentially lift to 6d anomaly free theory

- same strategy allows to check if 5d, N=4 theory can arise from **anomaly-free** Abelian (2,0) theory

What about non-Abelian (2,0) theory?

Part II: Gauged 6D supergravities and symmetry-breaking vacua

Generalize 6d F-theory reductions

- consider M-theory reduction with:
  - background flux $G_4 = \langle dC_3 \rangle = \theta_\Lambda \tilde{\omega}^\Lambda$ on compact geometry
  - geometric flux: SU(3) structure manifold $\hat{Z}_6$ (non-Calabi-Yau space)

$$dJ = e_{K\Lambda} v^\Lambda \beta^K, \qquad J = v^\Lambda \omega_\Lambda$$
$$d\Omega = Z^K e_{K\Lambda} \tilde{\omega}^\Lambda \ , \qquad \Omega = Z^K \alpha_K - F_K \beta^K$$

$J,\ \Omega$ no-where vanishing real 2-form, complex 3-form

[Gurierri,Louis,Micu,Waldram] [Grana,Louis Waldram]

- 5d supergravity has gauged hypermultiplets (gauged shift symmetries)
  - M-theory three-form: $C_3 = \mathcal{C}_3 + \xi^K \alpha_K - \tilde{\xi}_K \beta^K + A^\Lambda \wedge \omega_\Lambda$
  - gauged hypers:

$$Dq^u = \begin{cases} d\Phi + 2A^\Lambda \theta_\Lambda \ , & \text{if } q^u = \Phi \ , \\ d\tilde{\xi}_K + A^\Lambda e_{K\Lambda} \ , & \text{if } q^u = \tilde{\xi}_K \ , \\ dq^u \ , & \text{if } q^u \neq \Phi, \tilde{\xi}_K \ . \end{cases}$$

universal hyper

5d to 6d gauged supergravity

[TG,Pugh]

- Scalar potential determined by 'gauge potentials' $\mathcal{P}^i_\Lambda$ (transforming in SU(2))

$$v^\Lambda \mathcal{P}^3_\Lambda = \frac{i}{8\mathcal{V}} \int_{\hat{Z}_6} J \wedge G_4 \qquad v^\Lambda \mathcal{P}^1_\Lambda + i v^\Lambda \mathcal{P}^2_\Lambda = \frac{i}{8\sqrt{\mathcal{V}}} e^{\frac{1}{2}K_c} \int_{\hat{Z}_6} \Omega \wedge dJ$$

- Take M-theory to F-theory limit: 6d (1,0) gauged supergravity
  $\Rightarrow$ lift 5d hypersector from to 6d hypersector
  - background G-flux: $\hat{V}^{(6)}_{\text{flux}} = \frac{1}{32\Omega_{\alpha\beta}\hat{j}^\alpha b^\beta \hat{\mathcal{V}}^2} C^{-1ij}\theta_i\theta_j$

    $\Rightarrow$ potential can arise from 7-brane flux in F-theory

  - geometric flux: $\hat{V}^{(6)}_{U(1)} = \frac{1}{32\Omega_{\alpha\beta}\hat{j}^\alpha b^\beta} C^{-1ij}\left(\frac{1}{\mathcal{V}^2} e_{\kappa i} e_{\lambda j} \xi^\kappa \xi^\lambda + \frac{e^{K_c}}{\mathcal{V}} e_{\kappa i} e_{\lambda j} z^\kappa \bar{z}^\lambda\right)$

    $\Rightarrow$ potential for geometrically massive U(1)'s in F-theory

    [TG,Weigand] [TG,Kerstan,Palti,Weigand]

    → T. Weigand's talk

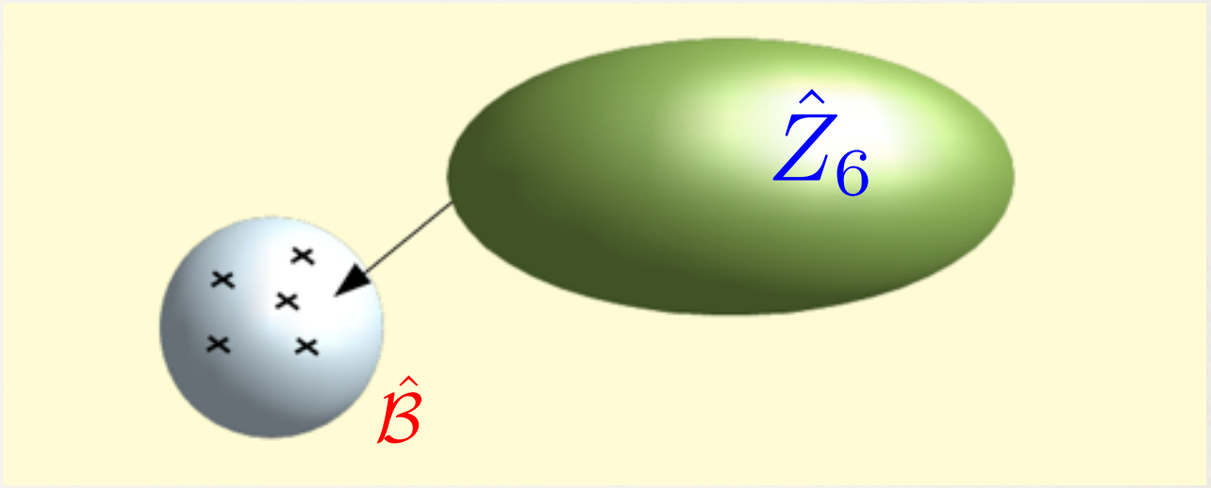
Vacua of the 6d gauged theory

- gauged 6d (1,0) theories admit no Minkowski vacua

- new Ansatz: $\mathbb{R}^{3,1} \times \hat{\mathcal{B}}$ $\quad ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \Omega(z,\bar{z}) dz d\bar{z}$

Solution:

- volume of $\hat{Z}_6$ develops profile on $\hat{\mathcal{B}}$ due to G-flux
  $\Rightarrow$ sources needed
- complex structure moduli develop profile on $\hat{\mathcal{B}}$
  $\Rightarrow$ cosmic string solutions: new 7-branes wrapping base B_2 of $\hat{Z}_6$
- universal hypermultiplet scalar Φ develops profile on $\hat{\mathcal{B}}$
- new U(1)-fluxes needed on $\hat{\mathcal{B}}$ (complete G-flux, s.t. it is self-dual)



- Solution matches with N=1, 4d fourfold reduction on $\hat{Z}_8$ with base $\hat{\mathcal{B}} \times B_2$

compare with warped solution [Becker,Becker]

Supersymmetry breaking and chirality

- study vanishing of fermion variations on this background

  $\Rightarrow$ Killing spinor equation implies that only **4 supercharges** can be preserved in 4d

- supersymmetry breaking corresponds to N=2 → N=1 in 4d
  however, is accompanied by 6d → 4d reduction

  recently studied [Louis,Smyth,Triendl]

- derived N=1, 4d effective action obtained in $\hat{\mathcal{B}}$ - reduction

  $\Rightarrow$ matches F-theory reduction on with G-flux and geom. massive U(1)'s

  compare [TG] [TG,Kerstan,Palti,Weigand]

- 4d theory can admit a chiral spectrum:

  - chiral spectrum can be 'measured' by 3d Chern-Simons terms: (generated at 1-loop in 3d Coulomb branch)

    $$\Theta_{ij} = \int_{\hat{Z}_8} G_4 \wedge \omega_i \wedge \omega_j$$

  - 3d Chern-Simons terms for solution: $\Theta_{ij} = \mathcal{V}_{ijk}\theta^k$

    1-loop Chern-Simons terms in 5d

Conclusions

- M-theory to F-theory limit allows to study 6d theories with self-dual tensors
  - 6d pseudo-action via 5d Kaluza-Klein actions
    → proposed non-Abelian generalization of 5d tensor action
  - 5d Chern-Simons terms capture quantum information about 6d theory
    → non-Abelian generalization?
  - α' - corrections → also exist in 4d, N=1 compactifications

- background G-flux and geometric fluxes induce 6d gauged supergravity
  - no 6d Minkowski vacua: G-flux ensures spontaneous compactificaiton
  - compactifying solutions with 4d Minkowski and 2d compact space
    → vacua admit all key features of 4d F-theory vacua but are simpler
  - spontaneous supersymmetry breaking from 8 to 4 supercharges
    → since 6d → 4d reduction: can still have 4d chiral spectrum

The End.
Thank you!