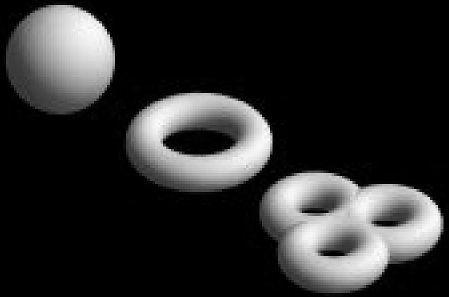

Bad Honnef
18/03/2013

Worldsheet Realization of The Refined Topological String (II)

Based on work with I. Antoniadis, I. Florakis, S. Hohenegger and K.S. Narain (hep-th/1302.6993)

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Plan

- Introduction & Recapitulation
- The Refined Couplings in Heterotic
    - Setup
    - Heterotic computation
- Nekrasov's Partition Functions
    - 4D-field theory limit
    - Radius deformations
- Conclusion And Outlook

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- In the field theory limit, they reduce to Nekrasov's partition function for $\varepsilon_+=0$

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- Topological string partition function computed by gravitational couplings F_g
- In the field theory limit, they reduce to Nekrasov's partition function for $\varepsilon_+=0$
- Satisfy holomorphic anomaly equations

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The Refined Couplings in Heterotic

Antoniadis, Florakis, Hohenegger, Narain, AZA (‘13)

1. Setup and insertions
2. Computation of the couplings
3. Structure of the deformation

Ansatz

- Compute gravitational couplings involving two anti-self-dual gravitons, anti-self-dual graviphotons and self-dual vector partners of the $\bar{U}$-modulus

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- Identify the vertex operators

- Solve for the sigma-model deformation

Vertex Operators in Heterotic

- Anti-self-dual graviton (Riemann tensor)

$$\begin{cases} V_R\left(p_1\right) &= \left(\partial Z^2 - i p_1 \chi^1 \chi^2\right) \bar{\partial} Z^2 e^{i p_1 Z^1} \\ V_R\left(\bar{p}_1\right) &= \left(\partial \bar{Z}^2 - i \bar{p}_1 \bar{\chi}^1 \bar{\chi}^2\right) \bar{\partial} \bar{Z}^1 e^{i \bar{p}_1 \bar{Z}^1} \end{cases}$$

- Anti-self-dual graviphoton

$$\begin{cases} V_G\left(p_1\right) &= \left(\partial X - i p_1 \chi^1 \Psi\right) \bar{\partial} Z^2 e^{i p_1 Z^1} \\ V_G\left(\bar{p}_1\right) &= \left(\partial X - i \bar{p}_1 \bar{\chi}^1 \Psi\right) \bar{\partial} \bar{Z}^2 e^{i \bar{p}_1 \bar{Z}^1} \end{cases}$$

- Self-dual Ū-vectors

$$\begin{cases} V_{\bar{U}}\left(p_1\right) &= \left(\partial \bar{Z}^2 - i p_1 \chi^1 \bar{\chi}^2\right) \bar{\partial} X e^{i p_1 Z^1} \\ V_{\bar{U}}\left(\bar{p}_1\right) &= \left(\partial Z^2 - i \bar{p}_1 \bar{\chi}^1 \chi^2\right) \bar{\partial} X e^{i \bar{p}_1 \bar{Z}^1} \end{cases}$$

Setup and contractions

- In the Heterotic String on $K3 \times T^2$

$$\left\langle (V_-^R)^2 \; (V_-^G)^{2g-2} \; (V_+^{\bar{U}})^{2n} \right\rangle$$

Setup and contractions

- In the Heterotic String on K3 x T²

$$\left\langle (V_-^R)^2\ (V_-^G)^{2g-2}\ (V_+^{\bar{U}})^{2n} \right\rangle$$

- Making all possible contractions, the correlators factorizes

$$\langle \text{Bosonic directions} \rangle \times \langle \text{Fermionic directions} \rangle$$

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- The T² bosonic coordinates only contribute zero modes

Bosonic part

- A typical correlator is of the form

$$\langle (Z^1\bar{\partial}Z^2)^{g+1}(\bar{Z}^1\bar{\partial}\bar{Z}^2)^{g+1}(Z^1\partial\bar{Z}^2)^{n-m}(\bar{Z}^1\partial Z^2)^{n-m}(\partial X)^{2g}(\bar{\partial}X)^{2n}\rangle$$

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- Computed by the generating function

$$G^{\mathrm{b}}(\epsilon_-,\epsilon_+) = \left\langle \exp\left[-\epsilon_- \int d^2z\ \partial X(Z^1\bar{\partial}Z^2 + \bar{Z}^2\bar{\partial}\bar{Z}^1) - \right.\right.$$
$$\left.\left. -\epsilon_+ \int d^2z\ (Z^1\partial\bar{Z}^2 + Z^2\partial\bar{Z}^1)\bar{\partial}X \right] \right\rangle$$

Fermionic part

- A generic fermionic correlator is of the form

$$\langle \chi^1\chi^2(x)\bar{\chi}^1\bar{\chi}^2(y)\prod_{i=1}^{m}\chi^1\bar{\chi}^2(u_i)\bar{\chi}^1\chi^2(u_i')\rangle_s$$

Fermionic part

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- Summing over the spin structures, this can be rewritten as a correlator in the odd spin structure

$$\langle \chi^1\bar{\chi}^1(x)\,\chi^2\bar{\chi}^2(y)\rangle\left\langle \prod_{i=1}^{m}\chi^4\chi^5(u_i)\,\bar{\chi}^4\bar{\chi}^5(u_i')\right\rangle_{h,g}$$

Fermionic part

$$\langle\chi^1\bar{\chi}^1(x)\,\chi^2\bar{\chi}^2(y)\rangle\left\langle\prod_{i=1}^{m}\chi^4\chi^5(u_i)\,\bar{\chi}^4\bar{\chi}^5(u_i')\right\rangle_{h,g}$$

- Computed by the generating function

$$G^{\mathrm{f}}\begin{bmatrix}h\\g\end{bmatrix}(\epsilon_+)=\left\langle e^{-\epsilon_+\int(\chi^4\chi^5-\bar{\chi}^4\bar{\chi}^5)\bar{\partial}X}\right\rangle_{h,g}$$

Structure of the deformation

$$S_{\text{def}}^{\text{bos}} = \tilde{\epsilon}_- \int d^2z \, (\, Z^1 \bar{\partial} Z^2 + \bar{Z}^2 \bar{\partial} \bar{Z}^1 \,) + \check{\epsilon}_+ \int d^2z \, (\, Z^1 \partial \bar{Z}^2 + Z^2 \partial \bar{Z}^1 \,)$$

Structure of the deformation

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$$\epsilon_- \, \langle \partial X \rangle \qquad\qquad \epsilon_+ \, \langle \bar{\partial} X \rangle$$

Structure of the deformation

$$S_{\mathrm{def}}^{\mathrm{bos}} = \tilde{\epsilon}_{-} \int d^2 z \, \Big(\underbrace{Z^1 \bar{\partial} Z^2 + \bar{Z}^2 \bar{\partial} \bar{Z}^1}_{\mathrm{SU(2)}_L \text{ current}} \Big) + \check{\epsilon}_{+} \int d^2 z \, \Big(\underbrace{Z^1 \partial \bar{Z}^2 + Z^2 \partial \bar{Z}^1}_{\mathrm{SU(2)}_R \text{ current}} \Big)$$

Structure of the deformation

$$S^{\text{bos}}_{\text{def}} = \tilde{\epsilon}_- \int d^2z\, \big(\underbrace{Z^1\bar{\partial}Z^2 + \bar{Z}^2\bar{\partial}\bar{Z}^1}_{\text{SU(2)}_{\text{L}}\text{ current}}\big) + \check{\epsilon}_+ \int d^2z\, \big(\underbrace{Z^1\partial\bar{Z}^2 + Z^2\partial\bar{Z}^1}_{\text{SU(2)}_{\text{R}}\text{ current}}\big)$$

- Effective SU(2) R-symmetry current

$$S^{\text{def}}_{\text{ferm}} = \check{\epsilon} \int d^2z (\chi^4\chi^5 + \text{c.c.})$$

Computation of the couplings

- The generating functions can be computed exactly since they are quadratic in the fields

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- For the fermionic directions

$$G^{\mathrm{f}}\left[{h \atop g}\right](\check{\epsilon}_+) = \frac{\theta\left[{1+h \atop 1+g}\right](\check{\epsilon}_+;\tau)\theta\left[{1-h \atop 1-g}\right](\check{\epsilon}_+;\tau)}{\eta^2}\; e^{\frac{\pi}{\tau_2}\check{\epsilon}_+^2}$$

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- For the bosonic directions, the result is more involved

Bosonic correlator

- Decomposition of the bosonic correlator

$$G^{\mathrm{bos}}(\epsilon_-,\epsilon_+) = G_{\mathrm{hol}}(\epsilon_-,\epsilon_+) \times G_{\text{non-hol}}(\epsilon_-,\epsilon_+)$$

Bosonic correlator

- Decomposition of the bosonic correlator

$$G^{\text{bos}}(\epsilon_-,\epsilon_+) = G_{\text{hol}}(\epsilon_-,\epsilon_+) \times G_{\text{non-hol}}(\epsilon_-,\epsilon_+)$$

- The almost (anti-)holomorphic piece is standard

$$G_{\text{hol}}(\epsilon_-,\epsilon_+) = \frac{(2\pi)^2(\epsilon_-^2 - \epsilon_+^2)\,\bar{\eta}(\bar{\tau})^6}{\bar{\theta}_1(\tilde{\epsilon}_- - \tilde{\epsilon}_+;\bar{\tau})\,\bar{\theta}_1(\tilde{\epsilon}_- + \tilde{\epsilon}_+;\bar{\tau})}\; e^{-\frac{\pi}{\tau_2}(\tilde{\epsilon}_-^2+\tilde{\epsilon}_+^2)}$$

Bosonic correlator

- The second factor is completely non-holomorphic

$$\log[G_{\text{non-hol}}(\epsilon_-,\epsilon_+)] =$$

$$\sum_{k=1}^{\infty}\frac{1}{k}\sum_{\ell=0}^{k}\binom{k}{\ell}(-)^{\ell}\,\tau_2^{\ell-k}\sum_{\substack{r=0\\k+r\in 2\mathbb{Z}}}^{\infty}\binom{k+r-1}{r}\tilde{\epsilon}_+^{\ell}\,\check{\epsilon}_+^{k-\ell}\Big[(\tilde{\epsilon}_- - \tilde{\epsilon}_+)^r + (-\tilde{\epsilon}_- - \tilde{\epsilon}_+)^r\Big]\,\Phi^*_{k-\ell,r+k}$$

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Selberg-Poincaré series

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Selberg-Poincaré series

$$\Phi_{\alpha,\beta}(\tau,\bar{\tau}) = \sum_{N>0}\sum_{(c,d)=1}\frac{\tau_2^{\alpha}}{N^{\beta}|c\tau+d|^{2\alpha}(c\tau+d)^{\beta-2\alpha}}\,e^{-2\pi i\frac{\kappa}{N}\frac{a\bar{\tau}+b}{c\bar{\tau}+d}}$$

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- Modular invariant regularization

Bosonic correlator

- Fourier expansion from the Selberg-Poincaré series

$$\tau_2^{-\alpha}\,\Phi_{\alpha,\beta}(\tau,\bar{\tau}) = 2\zeta(\beta) + 2\tau_2^{1-\beta}\left\{C_0^{\alpha,\beta} + \sum_{n>0}\left[C_n^{\alpha,\beta}(\tau_2)\,q^n + I_n^{\alpha,\beta}(\tau_2)\,\bar{q}^n\right]\right\}$$

Bosonic correlator

- Fourier expansion from the Selberg-Poincaré series

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$$\left\{\begin{aligned} C_n^{\alpha,\beta}(\tau_2) &= \frac{(2\pi)^\beta(-i)^{\beta-2\alpha}}{\Gamma(\beta-\alpha)}\,(n\tau_2)^{\beta-1}\,\sigma_{1-\beta}(n)\,(4\pi n\tau_2)^{-\frac{\beta}{2}}\,e^{2\pi n\tau_2}\,W_{\frac{\beta}{2}-\alpha,\frac{\beta-1}{2}}(4\pi n\tau_2)\\ I_n^{\alpha,\beta}(\tau_2) &= \frac{(2\pi)^\beta(-i)^{\beta-2\alpha}}{\Gamma(\alpha)}\,(n\tau_2)^{\beta-1}\,\sigma_{1-\beta}(n)\,(4\pi n\tau_2)^{-\frac{\beta}{2}}\,e^{2\pi n\tau_2}\,W_{\alpha-\frac{\beta}{2},\frac{\beta-1}{2}}(4\pi n\tau_2)\\ C_0^{\alpha,\beta} &= 2^{2-\beta}\pi(-i)^{\beta-2\alpha}\frac{\Gamma(\beta-1)\zeta(\beta-1)}{\Gamma(\alpha)\Gamma(\beta-\alpha)} \end{aligned}\right.$$

Bosonic correlator

- Fourier expansion from the Selberg-Poincaré series

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Divisor function

$$\begin{cases} C_n^{\alpha,\beta}(\tau_2) = \dfrac{(2\pi)^\beta(-i)^{\beta-2\alpha}}{\Gamma(\beta-\alpha)}\,(n\tau_2)^{\beta-1}\,\sigma_{1-\beta}(n)\,(4\pi n\tau_2)^{-\frac{\beta}{2}}\,e^{2\pi n\tau_2}\,W_{\frac{\beta}{2}-\alpha,\frac{\beta-1}{2}}(4\pi n\tau_2) \\ I_n^{\alpha,\beta}(\tau_2) = \dfrac{(2\pi)^\beta(-i)^{\beta-2\alpha}}{\Gamma(\alpha)}\,(n\tau_2)^{\beta-1}\,\sigma_{1-\beta}(n)\,(4\pi n\tau_2)^{-\frac{\beta}{2}}\,e^{2\pi n\tau_2}\,W_{\alpha-\frac{\beta}{2},\frac{\beta-1}{2}}(4\pi n\tau_2) \\ C_0^{\alpha,\beta} = 2^{2-\beta}\pi(-i)^{\beta-2\alpha}\dfrac{\Gamma(\beta-1)\zeta(\beta-1)}{\Gamma(\alpha)\Gamma(\beta-\alpha)} \end{cases}$$

Whittaker W-function

Bosonic correlator

- Asymptotic behavior of the non-holomorphic piece

$$G_{\text{non-hol}}(\epsilon_-, \epsilon_+) \quad \xrightarrow{\tau_2 \to \infty} \quad 1$$

- at a point where $P_L = P_R$

Computation of the couplings

- The generating function for the refined couplings can thus be expressed as

$$F(\epsilon_-,\epsilon_+) = \sum_{g,n} \frac{\epsilon_-^{2g}}{(g!)^2}\frac{\epsilon_+^{2n}}{(n!)^2} F_{g,n}$$

$$= \int_{\mathcal{F}} d\mu\, G^{\mathrm{b}}(\epsilon_-,\epsilon_+)\frac{1}{\eta^4\bar{\eta}^{24}}\frac{1}{2}\sum_{h,g=0}^{1} G^{\mathrm{f}}[^h_g](\check{\epsilon}_+)Z[^h_g]\ \Gamma_{(2,2+8)}(T,U,Y)$$

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$$
\begin{aligned}
F(\epsilon_-,\epsilon_+) &= \sum_{g,n} \frac{\epsilon_-^{2g}}{(g!)^2}\frac{\epsilon_+^{2n}}{(n!)^2} F_{g,n} \\
&= \int_{\mathcal{F}} d\mu\, G^{\mathrm{b}}(\epsilon_-,\epsilon_+)\frac{1}{\eta^4\bar{\eta}^{24}}\frac{1}{2}\sum_{h,g=0}^{1} G^{\mathrm{f}}\left[{}^{h}_{g}\right](\check{\epsilon}_+)\overbrace{Z\left[{}^{h}_{g}\right]}^{\text{K3 and } E_7 \text{ x SU(2) lattices}}\underbrace{\Gamma_{(2,2+8)}}_{T^2 \text{ and } E_8 \text{ lattices}}(T,U,Y)
\end{aligned}
$$

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  Heterotic computation

- Nekrasov’s Partition Functions

  4D-field theory limit
  Radius deformations

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Nekrasov’s partition functions

1. 4D field theory limit and N=2 gauge theory
2. Radius deformations and 5D-partition function

Field theory limit of the couplings

- Expand the amplitude around a Wilson-line enhancement point

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- The enhancement point is characterized by

$$P_L = P_R \equiv P = \frac{a_2 - U a_1}{\sqrt{(T - \bar{T})(U - \bar{U}) - \frac{1}{2}(\vec{Y} - \bar{\vec{Y}})^2}} \longrightarrow 0$$

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Complexified Wilson line of T^2

Field theory limit of the couplings

- The bosonic correlator

$$G^{\mathrm{bos}}(\epsilon_-,\epsilon_+) \quad \xrightarrow{\tau_2\to\infty} \quad \frac{\pi^2(\tilde{\epsilon}_-^2-\tilde{\epsilon}_+^2)}{\sin\pi(\tilde{\epsilon}_--\tilde{\epsilon}_+)\sin\pi(\tilde{\epsilon}_-+\tilde{\epsilon}_+)}$$

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- Only the untwisted sector (h=0) contributes and $Z\left[{0 \atop g}\right]\to 1$

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- Only the untwisted sector (h=0) contributes and $Z\left[{0 \atop g}\right] \to 1$

- The fermionic correlator

$$\frac{1}{2}\sum_{g=0,1}\theta\left[{1 \atop 1+g}\right](\check{\epsilon}_+;\tau)\ \theta\left[{1 \atop 1-g}\right](\check{\epsilon}_+;\tau) = -2\cos(2\pi\check{\epsilon}_+)q^{1/4}+\mathcal{O}(q^{5/4})$$

Field theory limit of the couplings

- Joining all contributions

$$F\left(\epsilon_-,\epsilon_+\right)\ \sim\ \left(\epsilon_-^2-\epsilon_+^2\right)\int_0^\infty\frac{dt}{t}\ \frac{-2\cos\left(2\epsilon_+t\right)}{\sin\left(\epsilon_--\epsilon_+\right)t\ \ \sin\left(\epsilon_-+\epsilon_+\right)t}\ e^{-\mu t}$$

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- The BPS-state that becomes massless has mass $\mu\sim\sqrt{(T-\bar{T})(U-\bar{U})}\bar{P}=a_2-\bar{U}a_1$

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- The BPS-state that becomes massless has mass $\mu \sim \sqrt{(T-\bar{T})(U-\bar{U})}\bar{P} = a_2 - \bar{U} a_1$

- As expected, the leading singularity is given by $\mu^{2-2g-2n}$

Field theory limit of the couplings

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- This is in perfect agreement with the gauge theory result of Nekrasov!

Nekrasov ('04)

Nekrasov, Okounkov ('03)

Field theory limit of the couplings

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- Not symmetric under $\varepsilon_{-} \leftrightarrow \varepsilon_{+}$

Radius deformations : D=6

- Decouple the winding modes by taking the large T_2 limit keeping U fixed

Radius deformations : D=6

- Decouple the winding modes by taking the large T_2 limit keeping U fixed
- The full Kaluza-Klein tower of states contributes in the α' ⟶ 0 limit yielding

$$\frac{F}{\epsilon_1\epsilon_2} \sim \frac{1}{2}\sum_{m_i}{}' \frac{U_2}{|m_1+Um_2|^2}\frac{e^{2\pi i(a_1m_1+a_2m_2)}}{\left(e^{i\pi\epsilon_1(m_1+Um_2)/U_2}-1\right)\left(e^{i\pi\epsilon_2(m_1+Um_2)/U_2}-1\right)}$$

$$+(\epsilon_i \rightarrow -\epsilon_i)$$

Radius deformations : D=5

- Taking one radius of the compactification torus to zero, one recovers the usual β-deformed partition function in 5D

$$\gamma_{\epsilon_1,\epsilon_2}(x|\beta) = \sum_{n=1}^{\infty} \frac{1}{n} \frac{e^{-\beta x}}{(e^{\beta n \epsilon_1} - 1)(e^{\beta n \epsilon_2} - 1)}$$

- With $\beta = 2\pi R_1$ and $x = -i\frac{a_1}{R_1}$

Nekrasov ('04)

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Conclusion

- The insertion of self-dual Ū-vectors in Heterotic provides a promising candidate for the refined topological string
  - Exactly solvable sigma-model
  - Reduces to the usual topological string upon setting $\varepsilon_+ = 0$

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  - Nekrasov's partition function in 4D

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Conclusion

- We reproduce known gauge theory results
    - Nekrasov's partition function in 4D
    - β-deformed version in 5D
- Universality of the ansatz (Type I)

Outlook

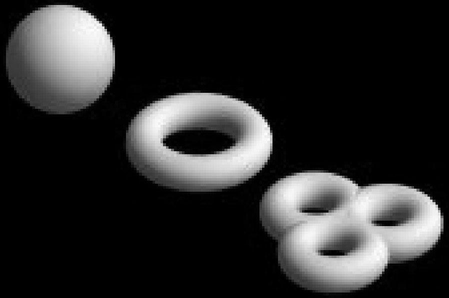
- Holomorphicity properties of the refined couplings

Outlook

- Holomorphicity properties of the refined couplings
    - Generalized recursion relations

Outlook

- Holomorphicity properties of the refined couplings
  - Generalized recursion relations
- Non-perturbative corrections
  - Type II dual



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18/03/2013

Worldsheet Realization of The Refined Topological String

Thank you!

Based on work with I. Antoniadis, I. Florakis, S. Hohenegger and K.S. Narain (hep-th/1303.6993)

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