R-charge Conservation and More in Factorizable and Non-Factorizable Orbifolds

Based on: N. G. Cabo Bizet, T. Kobayashi, D. K. Mayorga Peña, S. L. Parameswaran, M.S., I. Zavala: arXiv:1301.2322[hep-th]

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Introduction & Motivation

- Heterotic orbifolds provide fertile framework for realistic string model building

  Blaszczyk, Buchmüller, Groot Nibbelink, Hamaguchi, Kim, Kobayashi, Kyae, Lebedev, Nilles, Quevedo, Raby, Ramos-Sanchez, Ratz, Rühle, Trapletti, Vaudrevange, Wingerter, ...

- Need to know couplings to understand dynamics of LEEFT
- Worldsheet CFT of orbifold compactifications is free
- Selection rules determine which couplings are forced to vanish
- **R-Symmetries**
  - solve numerous problems of supersymmetric SM extensions

    e.g. Chen, Fallbacher, Ratz, Vaudrevange
  - expected to originate from geometric symmetries in the UV

Outline

- Heterotic orbifolds & symmetries
- R-charge conservation from correlation functions
- Conclusions

Heterotic orbifolds & symmetries

- Toroidal heterotic $P = \mathbb{Z}_N$ orbifold compactification:

$$M_{10} = M_{3,1} \times \frac{T^6}{\mathbb{Z}^N} = M_{3,1} \times \frac{\mathbb{C}^3}{\mathbb{Z}_N \ltimes \Gamma_6} = M_{3,1} \times \frac{\mathbb{C}^3}{S}$$

  $\Gamma_6 = \Gamma_2 \times \Gamma_2' \times \Gamma_2''$ and $\mathbb{Z}^N \ni \theta = (\theta_1, \theta_2, \theta_3) \leftrightarrow$ orbifold is **factorizable**

- Orbifold boundary conditions:

$$Z^j(\sigma + \pi, \tau) = (\theta^k Z)^j(\sigma, \tau) + \lambda^j \qquad (\theta^k, \lambda) \in S \quad j = 1, 2, 3$$

- Action of P is not free $\rightarrow$ twisted strings located at **fixed points**:

$$\theta^k z_\mathrm{f} = z_\mathrm{f} + \lambda\,,$$

  which fall in **conjugacy classes**: $z_\mathrm{f} \sim z_\mathrm{f}' \Leftrightarrow z_\mathrm{f}' = h z_\mathrm{f}$.

Heterotic orbifolds & symmetries

- Symmetries of the Orbifold geometry

$$\mathrm{O}(6) \supset \mathrm{Aut}(\Gamma_6) = G \ltimes F \ltimes E \ltimes D$$

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  $\rightarrow$ candidates for symmetries leading to R-symmetries in IR

Heterotic orbifolds & symmetries

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	Lattice	Twist	Orbifold Automorphisms
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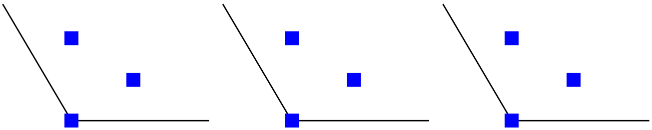

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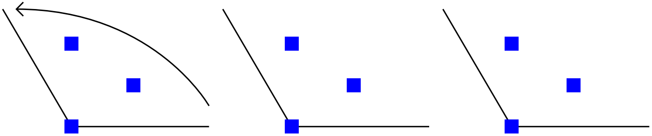

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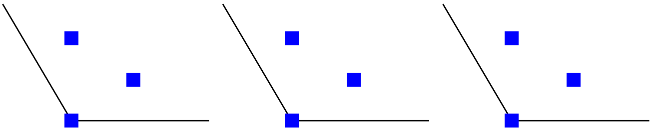

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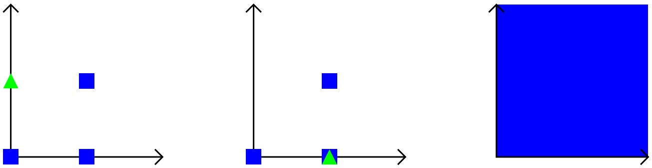

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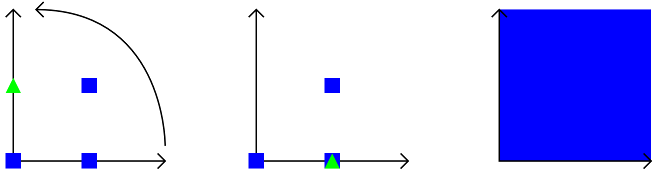

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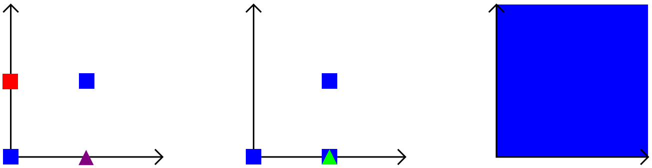

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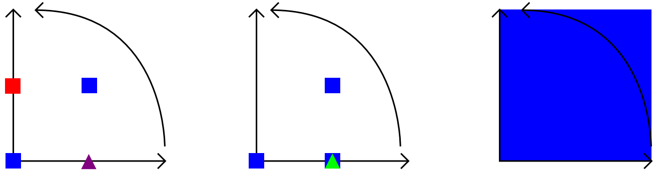

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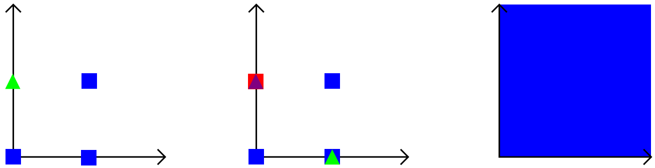

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- non-factorizable:

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$\mathbb{Z}_{6-\mathrm{II}}$	SU(6)×SU(2)	$\frac{1}{6}(1,2,-3)$	θ
$\mathbb{Z}_7$	SU(7)	$\frac{1}{7}(1,2,-3)$	θ
$\mathbb{Z}_{8-\mathrm{I}}$	SO(5)×SO(9)	$\frac{1}{8}(2,1,-3)$	$\theta,\ (\theta_1)^2$
$\mathbb{Z}_{8-\mathrm{II}}$	SO(8)×SO(4)	$\frac{1}{8}(1,3,-4)$	$\theta,\ \theta_3$
$\mathbb{Z}_{12-\mathrm{I}}$	SU(3)×F_4	$\frac{1}{12}(4,1,-5)$	$\theta,\ \theta_1$
$\mathbb{Z}_{12-\mathrm{II}}$	F_4×SO(4)	$\frac{1}{12}(1,5,-6)$	$\theta,\ \theta_3$

R-charge conservation from correlation functions

- Consider $\psi\psi\phi^{L-2}$ **tree-level couplings** $\langle V_\mathrm{F} V_\mathrm{F} V_\mathrm{B} \ldots V_\mathrm{B} \rangle$ to learn about $W \subset \Phi^L$
- **Vertex operators** for emission of k^{th} twisted sector string

$$V_\mathrm{B} = e^{-\phi} \prod_{i=1}^{3} (\partial X^i)^{\mathcal{N}_\mathrm{L}^i} \; (\partial \bar{X}^i)^{\bar{\mathcal{N}}_\mathrm{L}^i} \; e^{\mathrm{i} q_{sh}^m H^m} \; e^{\mathrm{i} p_{sh}^I X^I} \; \sigma^i_{(k,\psi)}$$

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- $\sigma_{(k,\psi)}$: **twist fields**

$$\boxed{\sigma_{(k,\psi)} = \sum_{r=0}^{l-1} e^{-2\pi\mathrm{i} r\gamma} \sigma_{(k,\theta^r f)}}$$

Lauer, Mas, Nilles '91; Erler, Jungnickel, Lauer, Mas '92

cf. $|\psi\rangle = |f\rangle + e^{-2\pi\mathrm{i}\gamma} |\theta f\rangle + \dots + e^{-2\pi\mathrm{i}(l-1)\gamma} |\theta^{l-1} f\rangle$
with l: smallest integer s.t. $\theta^l f = f + \lambda$.

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Correlation function factors into several parts:

$$\mathcal{F}_{3pt} = \left\langle \mathrm{e}^{\mathrm{i}\sum_{\alpha=1}^{3} p^{I}_{\mathrm{sh},\alpha}\cdot X^{I}(z_\alpha)} \right\rangle \times \left\langle \mathrm{e}^{\mathrm{i}\sum_{\alpha=1}^{3} q^{m}_{\mathrm{sh},\alpha}\cdot H^{m}(z_\alpha)} \right\rangle$$

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- Space group selection rule: *b.c. allow twisted strings to join*
- Rule 5: Depending on $\{k_\alpha\}$
  - only anti-holomorphic instantons: $\mathcal{N}^i_{\mathrm{L}} \leq \bar{\mathcal{N}}^i_{\mathrm{L}}$
  - only holomorphic instantons: $\mathcal{N}^i_{L} \geq \bar{\mathcal{N}}^i_{\mathrm{L}}$
  - no instantons: $\mathcal{N}^i_{L} = \bar{\mathcal{N}}^i_{\mathrm{L}}$

Kobayashi, Parameswaran, Ramos-Sánchez, Zavala '11

R-charge conservation from correlation functions

- Use **rules** & perform **split** $\partial X = \partial X_{\mathrm{cl}} + \partial X_{\mathrm{qu}}$ with $\bar{\partial}\partial X_{\mathrm{cl}} = 0$:

$$\mathcal{F} = \sum_{r_1=0}^{l_1} \cdots \sum_{r_3=0}^{l_3} e^{-2\pi \mathrm{i} \sum_{\alpha=1}^{3} r_\alpha \gamma_\alpha} \prod_{i=1}^{3} \mathcal{F}^i_{\mathrm{aux}}$$

with

$$\mathcal{F}^i_{\mathrm{aux}} = \sum_{X^i_{\mathrm{cl}}} \mathrm{e}^{-S^i_{\mathrm{cl}}} (\partial X^i_{\mathrm{cl}})^{\mathcal{N}^i_{\mathrm{L}} - \bar{\mathcal{N}}^i_L - \bar{\mathcal{N}}^i_R} f(|\partial X^i_{\mathrm{cl}}|^2, \partial X^i_{\mathrm{cl}} \partial \bar{X}^i_{\mathrm{cl}}) \\ \times \left\langle \sigma^i_{(k_1, \theta^{r_1} f_1)} \cdots \sigma^i_{(k_{\mathrm{L}}, \theta^{r_{\mathrm{L}}} f_{\mathrm{L}})} \right\rangle_{\mathrm{qu}}$$

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$$\mathcal{F}^i_{\rm aux} = {\color{red}\sum_{X^i_{\rm cl}} \mathrm{e}^{-S^i_{\rm cl}} (\partial X^i_{\rm cl})^{\mathcal{N}^i_{\rm L} - \bar{\mathcal{N}}^i_L - \bar{\mathcal{N}}^i_R} f(|\partial X^i_{\rm cl}|^2, \partial X^i_{\rm cl} \partial \bar{X}^i_{\rm cl})} \times {\color{blue}\left\langle \sigma^i_{(k_1, \theta^{r_1} f_1)} \cdots \sigma^i_{(k_{\rm L}, \theta^{r_{\rm L}} f_{\rm L})} \right\rangle_{\rm qu}}$$

- Observe:
  - quantum part **independent** of fixed point position

R-charge conservation from correlation functions

- Use **rules** & perform **split** $\partial X = \partial X_{\mathrm{cl}} + \partial X_{\mathrm{qu}}$ with $\bar{\partial}\partial X_{\mathrm{cl}} = 0$:

$$\mathcal{F} = \sum_{r_1=0}^{l_1} \cdots \sum_{r_3=0}^{l_3} e^{-2\pi\mathrm{i}\sum_{\alpha=1}^{3} r_\alpha \gamma_\alpha} \prod_{i=1}^{3} \mathcal{F}_{\mathrm{aux}}^{i}$$

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$$\times \left\langle \sigma^{i}_{(k_1, \theta^{r_1} f_1)} \cdots \sigma^{i}_{(k_{\mathrm{L}}, \theta^{r_{\mathrm{L}}} f_{\mathrm{L}})} \right\rangle_{\mathrm{qu}}$$

- Observe:
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  - ∂X_{cl} **enjoy symmetries** from D

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- Observe:
  - quantum part **independent** of fixed point position
  - ∂X_{cl} **enjoy symmetries** from D
  - prime planes yield **common factor**

R-charge conservation from correlation functions

Use rot. **symmetries** of $\partial X^i_{\rm cl}$:

- prime planes:

$$\mathcal{F}^j \sim (1)^{(\mathcal{N}^j_{\rm L} - \bar{\mathcal{N}}^j_{\rm L} - \bar{\mathcal{N}}^j_{\rm R})} + (\theta_j)^{(\mathcal{N}^j_{\rm L} - \bar{\mathcal{N}}^j_{\rm L} - \bar{\mathcal{N}}^j_{\rm R})} + \cdots + (\theta_i^{(N^j-1)})^{(\mathcal{N}^j_{\rm L} - \bar{\mathcal{N}}^j_L - \bar{\mathcal{N}}^j_R)}$$

$$\Rightarrow \boxed{\sum_\alpha \left(q^j_{\rm sh} - \mathcal{N}^j_{\rm L} + \bar{\mathcal{N}}^j_{\rm L} \right)_\alpha = 1 \mod \mathrm{N}^j}$$

R-charge conservation from correlation functions

Use rot. **symmetries** of $\partial X^i_{\rm cl}$:

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$$\Rightarrow \boxed{\sum_\alpha \left(q^j_{\rm sh} - \mathcal{N}^j_{\rm L} + \bar{\mathcal{N}}^j_{\rm L}\right)_\alpha = 1 \mod \mathrm{N}^j}$$

- non-prime planes:

$$\mathcal{F} \sim \prod_{i\neq j} \sum_{|X^i_{cl}|} \sum_{n=0}^{N-1} e^{-S^i_{cl}} (|\partial X^i_{\rm cl}|\theta_i^n)^{(\mathcal{N}^i_{\rm L}-\bar{\mathcal{N}}^i_{\rm L}-\bar{\mathcal{N}}^i_{\rm R})}\, e^{-2\pi \mathrm{i}\, n \sum_{\alpha=1}^L \gamma_\alpha}$$

$$\Rightarrow \boxed{\sum_\alpha \left(\sum_{i\neq j} v^i \left(q^i_{\rm sh} - \mathcal{N}^i_{\rm L} + \bar{\mathcal{N}}^i_{\rm L}\right)_\alpha + \gamma_\alpha\right) = \left(\sum_{i\neq j} v^i\right) \mod 1}$$

Conclusions

To build realistic models we must **understand couplings** in LEEFT

- **Selection rules** for superpotential couplings can be identified via L-point correlators
- R-charge conservation can be understood from **symmetries among instanton solutions**
- Traditional R-charge conservation rule only applies for **prime planes** in factorizable Orbifolds
- In general it gets a contribution from the γ-**phases** and is **summed** over non-prime planes
  $\Rightarrow$ Redefinition of R-charges of the fields!
  $\Rightarrow$ Generically more couplings allowed!
- In special cases there are further symmetries $\rightarrow$ 'Rule 6'

Conclusions

To build realistic models we must **understand couplings** in LEEFT

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- Traditional R-charge conservation rule only applies for **prime planes** in factorizable Orbifolds
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Thank you!

Rule 6

Consider **example**: $\mathrm{T}^2/\mathbb{Z}_6$ on G_2 lattice:

- θ-**action**: $\theta e_1 = -e_1 - e_2$, $\theta e_2 = 3e_1 + 2e_2$
- θ^2 sector **fixed points**: $z_\mathrm{f} = 0\,,\ \ e_2/2\,,\ \ 2e_2/3$
- $\theta^2\theta^2\theta^2$ coupling has two contributions:

$$\mathcal{F} = \quad e^{-2\pi\mathrm{i}\gamma_3}\sum_{X_\mathrm{cl}} e^{-S_\mathrm{cl}}(\partial X_\mathrm{cl})^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}}\left\langle\sigma_{(\theta^2,0)}\sigma_{(\theta^2,e_1/3)}\sigma_{(\theta^2,\theta e_1/3)}\right\rangle$$
$$+e^{-2\pi\mathrm{i}\gamma_2}\sum_{X_{cl}} e^{-S_{cl}}(\partial X_{cl})^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}}\left\langle\sigma_{(\theta^2,0)}\sigma_{(\theta^2,\theta e_1/3)}\sigma_{(\theta^2,e_1/3)}\right\rangle\,,$$

- **overall factor**

$$\mathcal{F} \sim \quad e^{-2\pi\mathrm{i}\gamma_3}\left((1)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}} + (\theta^2)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}} + (\theta^4)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}}\right)$$
$$+e^{-2\pi\mathrm{i}\gamma_2}\left((\theta)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}} + (\theta^3)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}} + (\theta^5)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}}\right)$$

- Selection rule:

$$\boxed{\sum_{\alpha=1}^{3}\mathcal{N}_{\mathrm{L}\,\alpha} - \bar{\mathcal{N}}_{\mathrm{L}\,\alpha} = 0 \mod 3}$$