The μ term and neutrino masses

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Motivation

The MSSM

- 👍 solves hierarchy problem, unification, . . .
- 👎 allows proton decay, . . . ; Solved by matter parity, but does not forbid

$$\mathscr{W} \supset \mu \, H_u \, H_d \ .$$

Why is the μ term of the order of the electroweak scale?

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Why is the μ term of the order of the electroweak scale?

Giudice–Masiero (GM) mechanism can explain this ...

. . . but μ term has to be absent in the first place.

Check for discrete symmetries that forbid μ term and allow for GM mechanism.

Outline

1. Motivation ✓
2. Giudice–Masiero mechanism
3. Anomalies
4. Symmetries with Majorana or Dirac neutrinos
5. Conclusions

Giudice–Masiero mechanism

- Giudice–Masiero:

$$K \supset k_{H_u H_d} \frac{X^\dagger}{M_\mathrm{P}} H_u H_d + \text{h.c.} ,$$

with spurion $X = \theta\,\theta\, F_X$.

Giudice and Masiero [1988]

- For $\langle F_X \rangle \sim m_{3/2} M_\mathrm{P}$

Effective superpotential term

$$\mathscr{W}_\mathrm{eff} \sim \frac{F_X}{M_\mathrm{P}} H_u H_d =: \mu_\mathrm{eff} H_u H_d ,$$

μ_eff of the order $m_{3/2}$.

μ term has to be absent at tree–level

Find symmetries that forbid μ term.

- □ Strong arguments against global symmetries.

cf. Banks and Seiberg [2011]

- □ Require unification
  ➥ symmetry should commute with SU(5).
- □ Anomaly freedom
  ➥ allow for Green–Schwarz mechanism.

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- □ Anomaly freedom
  ➥ allow for Green–Schwarz mechanism.

Check constraints for discrete $\mathbb{Z}_M^R$ symmetries.

Anomaly constraints

- Anomaly coefficients of the MSSM

$$A_3^R := A_{\mathrm{SU}(3)_\mathrm{C}-\mathrm{SU}(3)_\mathrm{C}-\mathbb{Z}_M^R} ,$$
$$A_2^R := A_{\mathrm{SU}(2)_\mathrm{L}-\mathrm{SU}(2)_\mathrm{L}-\mathbb{Z}_M^R} ,$$
$$A_1^R := A_{\mathrm{U}(1)_Y-\mathrm{U}(1)_Y-\mathbb{Z}_M^R} .$$

Unification requires universality

$$A_3^R = A_2^R = A_1^R = \rho \mod \eta \quad \text{with} \quad \eta := \begin{cases} M/2 , & \text{if } M \text{ even} , \\ M , & \text{if } M \text{ odd} . \end{cases}$$

- ρ indicates Green–Schwarz (GS) mechanism
  - ➥ $\rho = 0$ conventional anomaly freedom.
  - ➥ $\rho \neq 0$ GS axion shift.

Green and Schwarz [1984]

't Hooft anomaly matching

- Only one anomaly coefficient at SU(5) level:

$$A_{\mathsf{SU}(5)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathsf{SU}(5)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathsf{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta\ .$$

't Hooft et al. [1980] , Csáki and Murayama [1998]

- After GUT breaking (leaving $\mathbb{Z}_M^R$ unbroken)
  ➥ mismatch of gaugino contributions to the anomalies.

$$A^{\mathsf{SU}(5)\ \text{broken}}_{\mathsf{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} - A^{\mathsf{SU}(5)\ \text{broken}}_{\mathsf{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} \stackrel{!}{=} 0$$

$$\Leftrightarrow A^{\mathsf{SU}(5)}_{\mathsf{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} - 2q_\theta - \left(A^{\mathsf{SU}(5)}_{\mathsf{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} - 3q_\theta\right) \stackrel{!}{=} 0\ .$$

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$$\Leftrightarrow A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)_\mathrm{C}^2-\mathbb{Z}_M^R} - \mathbf{2q_\theta} - \left(A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)_\mathrm{L}^2-\mathbb{Z}_M^R} - \mathbf{3q_\theta}\right) \stackrel{!}{=} 0 \ .$$

Require split multiplets below the GUT scale.

$\mathbb{Z}_M^R$ symmetries

- Higgs fields are split multiplets, i.e. cancel mismatch:

$$\frac{1}{2}\left(q_{H_u} + q_{H_d} - 2q_\theta\right) = q_\theta \mod \eta \Leftrightarrow q_{H_u} + q_{H_d} = 2q_{\mathscr{W}} \mod 2\eta$$

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- Only R symmetries, i.e. $q_{\mathscr{W}} \neq 0$, can forbid μ term.
  Hall et al. [2002] , Lee et al. [2011b]
- No continuous R symmetries in MSSM.
  Chamseddine and Dreiner [1996]

Discrete Abelian R symmetries ☞ $\mathbb{Z}_M^R$ symmetries.

- Giudice–Masiero: $H_u\, H_d$ has to be neutral
  $\hookrightarrow$ $q_{H_u} + q_{H_d} = 0 \mod M$
  $\hookrightarrow$ $2q_{\mathscr{W}} = 4q_\theta = 0 \mod M$
  $\hookrightarrow$ **$M = 4\times$ integer and $q_\theta = M/4$.**

Classification

Check for **anomaly–free** discrete symmetries that

1. are flavor–universal and Abelian, i.e. $\mathbb{Z}_M^R$,
2. commute with SU(**5**),
3. forbid μ **term**,
4. allow usual Yukawa couplings,
5. are compatible with the **Giudice–Masiero mechanism**.

Differentiate between cases of Majorana and Dirac neutrinos.

Majorana neutrinos

For Majorana neutrinos

⑥ allow Weinberg neutrino mass operator

➥ $2q_{\overline{\mathbf{5}}} + 2q_{H_u} = 2q_\theta \mod M \quad \curvearrowright \quad q_{\overline{\mathbf{5}}} = q_\theta - q_{H_u} \mod M/2.$

From up– and down–type Yukawa couplings, using ⑥,

- $q_{\mathbf{10}} = q_\theta + q_{H_u} - q_{H_d} \mod M/2,$
- $q_{H_u} = q_{H_d} = 0 \mod M/2.$

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From up– and down–type Yukawa couplings, using ❻,

- $q_{\mathbf{10}} \;=\; q_\theta + q_{H_u} - q_{H_d} \mod M/2,$
- $q_{H_u} \;=\; q_{H_d} \;=\; 0 \mod M/2.$

All together: $\boldsymbol{q_{10} \;=\; q_{\overline{5}} \;=\; q_\theta \mod M}$

Symmetry commutes with $\mathrm{SO}(10) \implies$ unique $\mathbb{Z}_4^R$.

Babu et al. [2003] , Lee et al. [2011a]

Dangerous dimension–four and –five proton decay operators also forbidden.

Dirac neutrino Yukawa couplings

- Effective Dirac neutrino couplings

$$K \supset k_{LH_u\bar{\nu}} \frac{X^\dagger}{M_\mathrm{P}^2} L\, H_u\, \bar{\nu} + \text{h.c.}\,,$$

with $\bar{\nu}$ right–handed neutrino. Recall $\langle F_X \rangle \sim m_{3/2}\, M_\mathrm{P}$.

Connection size of μ term and small neutrino masses

$$Y_\nu \sim \frac{m_{3/2}}{M_\mathrm{P}} \sim \frac{\mu}{M_\mathrm{P}}\,.$$

Arkani-Hamed et al. [2001]

- $L\, H_u\, \bar{\nu}$ has to be absent at tree–level $\Longrightarrow$ adjust $q_{\bar{\nu}}$.

Dirac neutrinos

For Dirac neutrinos

- ❻ forbid Weinberg neutrino mass operator,
- ❼ $L\,H_u\,\bar{\nu}$ has to be neutral, i.e.

$$q_{\bar{\nu}} = -q_{H_u} - q_L \mod M \ .$$

Dirac neutrinos

For Dirac neutrinos

❻ forbid Weinberg neutrino mass operator,

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Check for symmetries with ❶ – ❼ up to order $M = 36$:

15 $\mathbb{Z}_M^R$ symmetries

Inequivalence tested by comparing monomials of the Hilbert basis.

Kappl et al. [2011]

Dirac $\mathbb{Z}_M^R$ symmetries up to $M = 36$

M	q_{10}	$q_{\overline{5}}$	q_{H_u}	q_{H_d}	q_θ	ρ	$q_{\bar{\nu}}$
4	0	0	2	2	1	1	2
4	2	2	2	2	1	1	0
8	1	5	2	6	2	2	1
12	1	9	4	8	3	3	11
12	2	6	2	10	3	3	4
12	4	0	10	2	3	3	2
16	1	13	6	10	4	4	13
24	1	21	10	14	6	6	17
28	1	25	12	16	7	7	19
28	2	22	10	18	7	7	24
28	4	16	6	22	7	7	6
32	1	29	14	18	8	8	21
36	1	33	16	20	9	9	23
36	2	30	14	22	9	9	28
36	4	24	10	26	9	9	2

$\mathbb{Z}_8^R$ example

Hilbert basis:

- Superpotential terms (R charge $= 2q_\theta$)

$$\mathscr{M}^{(i)} = \mathscr{M}_{\text{in}}^{(i)} \prod_{j=1} \left(\mathscr{M}_{\text{hom}}^{(j)}\right)^{\eta_j} \qquad \text{with} \qquad \eta_j \in \mathbb{N}\,.$$

- Inhomogeneous monomials (R charge $= 2q_\theta$)

$$\bar{\nu}^4 \;;\; L\,L\,\overline{E}\,\bar{\nu} \;;\; L\,H_d\,\overline{E} \;;\; \left(L\,L\,\overline{E}\right)^4 \;;\; \left(L\,L\,\overline{E}\right)^2 (L\,H_u)^2 \;;\; (L\,H_u)^4 \;,$$

- Homogeneous monomials (R charge $= 0$)

$$\bar{\nu}^8 \;;\; L\,H_u\,\bar{\nu} \;;\; (L\,H_u)^8 \;;\; \left(L\,L\,\overline{E}\right)^5 \bar{\nu} \;;\; \left(L\,L\,\overline{E}\right)^4 \left(L\,H_d\,\overline{E}\right) \;;$$
$$H_u\,H_d \;;\; \left(L\,L\,\overline{E}\right) \bar{\nu}^5 \;;\; \left(L\,H_d\,\overline{E}\right) \bar{\nu}^4 \;;\; \left(L\,L\,\overline{E}\right)^2 \left(L\,H_d\,\overline{E}\right) (L\,H_u)^2 \;;$$
$$\left(L\,L\,\overline{E}\right)^8 \;;\; \left(L\,H_d\,\overline{E}\right)^2 \;;\; \left(L\,L\,\overline{E}\right) \left(L\,H_d\,\overline{E}\right) \bar{\nu} \;;\; \left(L\,L\,\overline{E}\right)^2 \bar{\nu}^2 \;;$$
$$\left(L\,L\,\overline{E}\right)^3 (L\,H_u) \;;\; \left(L\,H_d\,\overline{E}\right) (L\,H_u)^4 \;;\; \left(L\,L\,\overline{E}\right) (L\,H_u)^3 \;.$$

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$$\boldsymbol{H_u\,H_d} \;;\; \left(L\,L\,\overline{E}\right) \bar{\nu}^5 \;;\; \left(L\,H_d\,\overline{E}\right) \bar{\nu}^4 \;;\; \left(L\,L\,\overline{E}\right)^2 \left(L\,H_d\,\overline{E}\right) (L\,H_u)^2 \;;$$
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$$H_u\,H_d \;;\; \left(L\,L\,\overline{E}\right) \bar{\nu}^5 \;;\; \left(L\,H_d\,\overline{E}\right) \bar{\nu}^4 \;;\; \left(L\,L\,\overline{E}\right)^2 \left(L\,H_d\,\overline{E}\right) (L\,H_u)^2 \;;$$
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Conclusions

- Giudice–Masiero (GM) mechanism creates effective μ term:

$$K \supset k_{H_u H_d} \frac{X^\dagger}{M_\mathrm{P}} H_u H_d + \text{h.c.}$$

$$\Rightarrow \mathscr{W}_\mathrm{eff} \sim \frac{F_X}{M_\mathrm{P}} H_u H_d =: \mu_\mathrm{eff} H_u H_d \ .$$

- μ has to be absent before, only R–symmetries can do that.
- Check for anomaly–free, discrete R–symmetries compatible with SU(5) GUTs and GM mechanism.
  - For Majorana neutrinos unique $\mathbb{Z}_4^R$ symmetry.
  - Class of possible $\mathbb{Z}_M^R$ symmetries for Dirac neutrinos.
- Size of Dirac neutrino Yukawa coupling related to μ term:

$$\mu \sim \langle \mathscr{W} \rangle / M_\mathrm{P}^2 \sim m_{3/2} \quad \text{and} \quad Y_\nu \sim \mu / M_\mathrm{P} \ .$$

Backup

MSSM anomaly coefficients

- In the MSSM the anomaly coefficients $A_3^R := A_{\mathrm{SU}(3)_\mathrm{C}-\mathrm{SU}(3)_\mathrm{C}-\mathbb{Z}_M^R}$, $A_2^R := A_{\mathrm{SU}(2)_\mathrm{L}-\mathrm{SU}(2)_\mathrm{L}-\mathbb{Z}_M^R}$ and $A_1^R := A_{\mathrm{U}(1)_Y-\mathrm{U}(1)_Y-\mathbb{Z}_M^R}$ read

$$
\begin{aligned}
A_3^R &= \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^g + q_{\mathbf{\overline{5}}}^g\right) - 3q_\theta \ , \\
A_2^R &= \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^g + q_{\mathbf{\overline{5}}}^g\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 5q_\theta \ , \\
A_1^R &= \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^g + q_{\mathbf{\overline{5}}}^g\right) + \frac{3}{5}\left[\frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 11q_\theta\right] \ .
\end{aligned}
$$

Anomaly universality

- Couplings between superfield $S|_{\theta=0} = s + \mathrm{i}\, a$ and the supersymmetric field strengths $W^{(i)}$,

$$\mathscr{L}_{\text{axion}} \supset \sum_i \int \mathrm{d}^2\theta \, \frac{c_i}{8} \, S \, W_\alpha^{(i)} W^{(i)\,\alpha} \, .$$

- Unequal c_i will spoil unification after S acquires VEV.

Anomaly matching II

- One anomaly coefficient

$$A_{\mathsf{SU}(5)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathsf{SU}(5)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathsf{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta \ ,$$

with

$$A^{\text{matter}}_{\mathsf{SU}(5)^2-\mathbb{Z}_M^R} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right) - 6q_\theta \ .$$

- At the GUT level:

$$A^{\mathsf{SU}(5)}_{\mathsf{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathsf{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathsf{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} + 3q_\theta + \frac{1}{2}\cdot 2\cdot 2\cdot q_\theta \ ,$$

$$A^{\mathsf{SU}(5)}_{\mathsf{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathsf{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathsf{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} + 2q_\theta + \frac{1}{2}\cdot 2\cdot 3\cdot q_\theta \ .$$

References I

Nima Arkani-Hamed, Lawrence J. Hall, Hitoshi Murayama, David Tucker-Smith, and Neal Weiner. Small neutrino masses from supersymmetry breaking. *Phys.Rev.*, D64: 115011, 2001. doi: 10.1103/PhysRevD.64.115011.

K. S. Babu, Ilia Gogoladze, and Kai Wang. Natural r-parity, mu-term, and fermion mass hierarchy from discrete gauge symmetries. *Nucl. Phys.*, B660:322–342, 2003.

Tom Banks and Nathan Seiberg. Symmetries and Strings in Field Theory and Gravity. *Phys.Rev.*, D83:084019, 2011. doi: 10.1103/PhysRevD.83.084019.

Ali H. Chamseddine and Herbert K. Dreiner. Anomaly free gauged R symmetry in local supersymmetry. *Nucl.Phys.*, B458:65–89, 1996. doi: 10.1016/0550-3213(95)00583-8.

Mu-Chun Chen, Michael Ratz, Christian Staudt, and Patrick K.S. Vaudrevange. The mu Term and Neutrino Masses. *Nucl.Phys.*, B866:157–176, 2013. doi: 10.1016/j.nuclphysb.2012.08.018.

Csaba Csáki and Hitoshi Murayama. Discrete anomaly matching. *Nucl. Phys.*, B515: 114–162, 1998.

G. F. Giudice and A. Masiero. A natural solution to the mu problem in supergravity theories. *Phys. Lett.*, B206:480–484, 1988.

References II

Michael B. Green and John H. Schwarz. Anomaly Cancellation in Supersymmetric $D = 10$ Gauge Theory and Superstring Theory. *Phys. Lett.*, B149:117–122, 1984.

Lawrence J. Hall, Yasunori Nomura, and Aaron Pierce. R symmetry and the mu problem. *Phys. Lett.*, B538:359–365, 2002. doi: 10.1016/S0370-2693(02)02043-9.

Rolf Kappl, Michael Ratz, and Christian Staudt. The Hilbert basis method for D-flat directions and the superpotential. *JHEP*, 1110:027, 2011. doi: 10.1007/JHEP10(2011)027.

Hyun Min Lee, Stuart Raby, Michael Ratz, Graham G. Ross, Roland Schieren, Kai Schmidt-Hoberg, and Patrick K.S. Vaudrevange. A unique Z_4^R symmetry for the MSSM. *Phys.Lett.*, B694:491–495, 2011a. doi: 10.1016/j.physletb.2010.10.038.

Hyun Min Lee, Stuart Raby, Michael Ratz, Graham G. Ross, Roland Schieren, Kai Schmidt-Hoberg, and Patrick K.S. Vaudrevange. Discrete R symmetries for the MSSM and its singlet extensions. *Nucl.Phys.*, B850:1–30, 2011b. doi: 10.1016/j.nuclphysb.2011.04.009.

(ed.) 't Hooft, Gerard, (ed.) Itzykson, C., (ed.) Jaffe, A., (ed.) Lehmann, H., (ed.) Mitter, P.K., et al. Recent Developments in Gauge Theories. Proceedings, Nato Advanced Study Institute, Cargese, France, August 26 - September 8, 1979. *NATO Adv.Study Inst.Ser.B Phys.*, 59:pp.1–438, 1980.