Non-geometric fluxes, non-geometry, and non-commutativity

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arXiv:1106.4015 by D. A., M. Larfors, D. Lüst, P. Patalong
arXiv:1202.3060 by D. A., O. Hohm, M. Larfors, D. Lüst, P. Patalong
arXiv:1204.1979 by D. A., O. Hohm, M. Larfors, D. Lüst, P. Patalong
arXiv:1211.6437 by D. A., M. Larfors, D. Lüst, P. Patalong
arXiv:1303.0251 by D. A., and work in progress...

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Non-geometry, mostly in 4d and 10d SUGRA

Restrict to NSNS sector: $g_{mn},\ b_{mn},\ \phi$.

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hep-th/0508133 by J. Shelton, W. Taylor, B. Wecht

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  ⇒ away from standard geometry: non-geometry
  10d fields look ill-defined: not single-valued, global issues

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$\hookrightarrow$ global issues $\Rightarrow$ integration?

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  - field redefinition on NSNS fields $\Rightarrow$ Q, R in 10d Lag.

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- Reveals more (geometrical) structure:
  - field redefinition in double field theory (DFT)
    $\hookrightarrow$ manifestly covariant action w.r.t diffeomorphisms
    R-flux: tensor, Q-flux: connection $\Rightarrow$ geom. role

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    R-flux: tensor, Q-flux: connection $\Rightarrow$ geom. role
  - relations between non-geometry, the new fields, and non-commutativity of the string coordinates

Field redefinition

Presentation

Key object: β: antisymmetric bivector β^{mn}.

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Motivations from Generalized Complex Geometry/SUGRA
Arguments in GCG: β related to non-geometry / to ${Q_k}^{mn}$, R^{kmn}

hep-th/0609084, arXiv:0708.2392 by P. Grange, S. Schäfer-Nameki

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β appears via a reparametrization of the gen. metric $\mathcal{H}$:

$$\mathcal{H} = \begin{pmatrix} g - bg^{-1}b & bg^{-1} \\ -g^{-1}b & g^{-1} \end{pmatrix} = \begin{pmatrix} \tilde{g} & -\tilde{g}\beta \\ \beta\tilde{g} & \tilde{g}^{-1} - \beta\tilde{g}\beta \end{pmatrix}, \ \tilde{g}: \text{ new metric}$$

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$$\Leftrightarrow \quad \begin{matrix} g = (\tilde{g}^{-1}+\beta)^{-1}\tilde{g}^{-1}(\tilde{g}^{-1}-\beta)^{-1} \\ b = -(\tilde{g}^{-1}+\beta)^{-1}\beta(\tilde{g}^{-1}-\beta)^{-1} \end{matrix} \quad \Leftrightarrow \quad \begin{matrix} (g+b) = (\tilde{g}^{-1}+\beta)^{-1} \\ \text{(useful for DFT)} \end{matrix}$$

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Field redefinition: $(g, b, \phi) \leftrightarrow (\tilde{g}, \beta, \tilde{\phi})$, β favored for non-geom.

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Field redefinition: $(g, b, \phi) \leftrightarrow (\tilde{g}, \beta, \tilde{\phi})$, β favored for non-geom.

Apply it on NSNS Lagrangian?
β could be related to non-geo. fluxes $\Rightarrow$ would they appear?

Rewriting of the NSNS Lagrangian

$$\mathcal{L} = e^{-2\phi}\sqrt{|g|}\left(\mathcal{R}(g) + 4(\partial\phi)^2 - \frac{1}{2.3!}H_{kmn}H_{pqr}\ g^{kp}g^{mq}g^{nr}\right)$$

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(assumption: $\beta^{km}\partial_m \cdot = 0$)

$$\begin{aligned}
\mathcal{R}(g) = \mathcal{R}(\tilde{g}) &- \partial_k \tilde{g}_{su}\partial_m \tilde{g}_{pq}\left(2\tilde{g}^{km}\tilde{g}^{uq}\tilde{g}^{ps} + 2\tilde{g}^{pq}\tilde{g}^{ks}\tilde{g}^{mu} + \frac{1}{2}\tilde{g}^{uq}\tilde{g}^{sm}\tilde{g}^{kp}\right)\\
&- \tilde{g}_{pq}\partial_k\beta^{pk}\partial_m\beta^{qm} - \frac{1}{2}\tilde{g}_{pq}\partial_k\beta^{qm}\partial_m\beta^{pk}\\
&+ 2\tilde{g}^{km}\tilde{g}^{pq}\partial_k\partial_m\tilde{g}_{pq} + 2\tilde{g}^{km}(G^{-1})_{pq}\partial_k\partial_m G^{qp}\\
&+ \partial_m G^{vl}\Big(-2\tilde{g}^{mr}\tilde{g}^{ks}(G^{-1})_{lv}\partial_k\tilde{g}_{rs} - \tilde{g}^{rs}\tilde{g}^{km}(G^{-1})_{lv}\partial_k\tilde{g}_{rs}\\
&\quad + \tilde{g}^{ms}\tilde{g}^{ru}(G^{-1})_{lu}\partial_v\tilde{g}_{rs} - \tilde{g}^{km}\tilde{g}^{rs}(G^{-1})_{ls}\partial_k\tilde{g}_{vr}\Big)\\
&+ \partial_m G^{vl}\Big((G^{-1})_{lq}\partial_v G^{qm} + \frac{1}{2}g_{lq}\partial_v G^{mq}\Big)\\
&- \partial_m G^{vl}\partial_k G^{ps}\frac{1}{2}\tilde{g}^{km}\Big(2(G^{-1})_{lv}(G^{-1})_{sp} + 5(G^{-1})_{sv}(G^{-1})_{lp} + g_{sl}\tilde{g}_{pv}\Big)\\
&\text{where } G = \tilde{g}^{-1} + \beta\ .
\end{aligned}$$

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$$\mathcal{L} = e^{-2\phi}\sqrt{|g|}\left(\mathcal{R}(g) + 4(\partial\phi)^2 - \frac{1}{2.3!}H_{kmn}H_{pqr}\ g^{kp}g^{mq}g^{nr}\right)$$
$$= e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\left(\mathcal{R}(\tilde{g}) + 4(\partial\tilde{\phi})^2 - \frac{1}{2}|Q|^2\right) + \partial(\dots) = \tilde{\mathcal{L}} + \partial(\dots)$$

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where $G = \tilde{g}^{-1} + \beta$.

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$\Rightarrow$ also get $R^{mnp} = 3\ \beta^{k[m}\partial_k\beta^{np]}$, $|R|^2 = \frac{1}{3!}R^{kmn}R^{pqr}\ \tilde{g}_{kp}\tilde{g}_{mq}\tilde{g}_{nr}$

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NSNS Lagrangian $\mathcal{L}$ rewritten in terms of new fields
$\Rightarrow$ 10d $\tilde{\mathcal{L}}$, with Q-, R-fluxes appearing

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Relation to 4d Q-, R-fluxes/non-geo. terms?

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Relation to 4d Q-, R-fluxes/non-geo. terms?

$\Rightarrow$ dimensional reduction, 4d potential

Rewriting of the NSNS Lagrangian

$$\mathcal{L} = e^{-2\phi}\sqrt{|g|}\left(\mathcal{R}(g) + 4(\partial\phi)^2 - \frac{1}{2.3!}H_{kmn}H_{pqr}\ g^{kp}g^{mq}g^{nr}\right)$$

$$= e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\left(\mathcal{R}(\tilde{g}) + 4(\partial\tilde{\phi})^2 - \frac{1}{2}|Q|^2 + \dots - \frac{1}{2}|R|^2\right) + \partial(\dots)$$

where $Q_k{}^{mn} = \partial_k\beta^{mn}$, $|Q|^2 = \frac{1}{2!}Q_k{}^{mn}Q_p{}^{qr}\ \tilde{g}^{kp}\tilde{g}_{mq}\tilde{g}_{nr}$

(assumption: $\beta^{km}\partial_m\cdot = 0$)

Without the assumption

$\Rightarrow$ also get $R^{mnp} = 3\ \beta^{k[m}\partial_k\beta^{np]}$, $|R|^2 = \frac{1}{3!}R^{kmn}R^{pqr}\ \tilde{g}_{kp}\tilde{g}_{mq}\tilde{g}_{nr}$

NSNS Lagrangian $\mathcal{L}$ rewritten in terms of new fields
$\Rightarrow$ 10d $\tilde{\mathcal{L}}$, with Q-, R-fluxes appearing
Relation to 4d Q-, R-fluxes/non-geo. terms?
$\Rightarrow$ dimensional reduction, 4d potential

$\hookrightarrow$ action, integration $\Rightarrow$ global aspects

The dimensional reduction

The 4d scalar potential

Split 10d $\Rightarrow$ 4d max. sym. space-time $\times$ 6d compact $\mathcal{M}$
Compactification ansatz: $\mathrm{d}s_{10}^2 = \mathrm{d}s_4^2 + \mathrm{d}s_6^2$ (no warp factor), b, β purely internal

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Only two 4d scalar fields: volume ρ and dilaton σ:

$$g_{6ij} = \rho \; g^{(0)}_{6ij} \ , \quad e^{-\phi} = e^{-\phi^{(0)}} \; \sigma \rho^{-\frac{3}{2}} \ , \ e^{\phi^{(0)}} = g_s$$

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$$S_E = M_4^2 \int \mathrm{d}^4x \ \sqrt{|g^E|} \left(\mathcal{R}_4^E + \mathrm{kin} - \frac{1}{M_4^2} V(\rho,\sigma) \right)$$

arXiv:0712.1196 by E. Silverstein

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where $V(\rho, \sigma) = \sigma^{-2} \left(\rho^{-3} \ V_H^0 + \rho^{-1} \ V_{\mathcal{R}}^0 \right)$

$$V_H^0 = \frac{M_4^2}{v_0} \int \mathrm{d}^6x \ \sqrt{|g_6^{(0)}|} \ \frac{1}{12} H_{ijk}^{(0)} H_{lmn}^{(0)} \ g_6^{il(0)} g_6^{jm(0)} g_6^{kn(0)}$$

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Non-geometric terms? Most general (NSNS) potential:

$$V(\rho, \sigma) = \sigma^{-2} \left(\rho^{-3} \ V^0_H + \rho^{-1} \ V^0_{\mathcal{R}} + \rho \ V^0_Q + \rho^3 \ V^0_R \right)$$

arXiv:0711.2512 by M. P. Hertzberg, S. Kachru, W. Taylor, M. Tegmark

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With $\tilde{\mathcal{L}}$ instead of $\mathcal{L}$, we get the ✓ 4d potential

We get $V(\rho,\sigma) = \sigma^{-2}\left(\rho^{-1}\ V^0_{\mathcal{R}} + \rho\ V^0_Q + \rho^3\ V^0_R\right)$ where

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$$V^0_R = \frac{M_4^2}{v_0}\int \mathrm{d}^6x\ \sqrt{|\tilde{g}^{(0)}_6|}\ \frac{1}{12} R^{ijk(0)} R^{lmn(0)}\ \tilde{g}^{(0)}_{6il}\tilde{g}^{(0)}_{6jm}\tilde{g}^{(0)}_{6kn}$$

$$\begin{aligned}V^0_Q = -\frac{M_4^2}{v_0}\int \mathrm{d}^6x\ \sqrt{|\tilde{g}^{(0)}_6|}\ \Bigg(&-\frac{1}{4}\tilde{g}_{ik}\tilde{g}_{jl}\tilde{g}^{rs}\ {Q_r}^{kl}\,{Q_s}^{ij} + \frac{1}{2}\tilde{g}_{pq}\,{Q_k}^{lp}\,{Q_l}^{kq} \\ &+ \tilde{g}_{jl}\tilde{g}_{pq}\beta^{jm}\left({Q_k}^{lp}\partial_m\tilde{g}^{kq} + \partial_k\tilde{g}^{lp}\ {Q_m}^{kq}\right) \\ &- \frac{1}{4}\tilde{g}_{ik}\tilde{g}_{jl}\tilde{g}_{pq}\left(\beta^{pr}\beta^{qs}\partial_r\tilde{g}^{kl}\partial_s\tilde{g}^{ij} - 2\beta^{ir}\beta^{js}\partial_r\tilde{g}^{lp}\partial_s\tilde{g}^{kq}\right) \\ &+ \frac{1}{2\sqrt{|\tilde{g}|}}\ \tilde{g}^{pq}\partial_k\tilde{g}_{pq}\ \partial_m\left(\sqrt{|\tilde{g}|}\tilde{g}_{ij}\beta^{ik}\beta^{jm}\right)\Bigg)^{(0)}_6\end{aligned}$$

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$\tilde{\mathcal{L}}$ and 10d Q, R give the ✓ 4d potential (give a 10d origin)

We get $V(\rho,\sigma) = \sigma^{-2}\left(\rho^{-1}\ V_{\mathcal{R}}^0 + \rho\ V_Q^0 + \rho^3\ V_R^0\right)$ where

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4d Q is not clearly identified... Work in progress...
↪ DFT brings an interpretation for the Q-terms, the role of Q

Global aspects

Dim. red.: integrate over a concrete background $\Rightarrow$ possible?

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We have shown

$$\mathcal{L}(g, b, \phi) = \tilde{\mathcal{L}}(\tilde{g}, \beta, \tilde{\phi}) + \partial(\dots)$$

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Relation between 10d/4d non-geometry:
for a 10d non-geometric configuration,
field redefinition + dimensional reduction
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Possibility of describing “new” physics:
global properties of a solution get “improved” by
$(g,b,\phi) \rightarrow (\tilde{g},\beta,\tilde{\phi})$

More geometrical structure

Double field theory

arXiv:0904.4664, arXiv:0908.1792 by C. Hull, B. Zwiebach

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- $O(d,d)$ ($\sim$ T-d.) inv. theory: for $h = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(d,d)$
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Simple to perform field redefinition in DFT,
using $\mathcal{E} = (g+b) = (\tilde{g}^{-1}+\beta)^{-1}$, and $O(d,d)$ invariance

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  $\mathcal{L}_{\rm DFT}(\mathcal{E},\phi) \overset{\tilde{\partial}^i=0}{=\!=\!=\!=} \mathcal{L}_{\rm NSNS}(g,b,\phi) + \partial(\dots)$

Simple to perform field redefinition in DFT,
using $\mathcal{E} = (g+b) = (\tilde{g}^{-1}+\beta)^{-1}$, and $O(d,d)$ invariance
$\hookrightarrow \mathcal{L}_{\rm DFT}(\tilde{g},\beta,\tilde{\phi}) \overset{\tilde{\partial}^i=0}{=\!=\!=\!=} \tilde{\mathcal{L}}(\tilde{g},\beta,\tilde{\phi}) + \partial(\dots)$

Diffeomorphism covariance

Gauge sym. of DFT: diffeo.: $x^i \to x^i - \xi^i(x,\tilde{x})$, $\tilde{x}_i \to \tilde{x}_i - \tilde{\xi}_i(x,\tilde{x})$

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David
ANDRIOT

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$\tilde{\nabla}^i V^j \ = \ \tilde{D}^i V^j - \check{\Gamma}_k{}^{ij} V^k$ where

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Q-, R-fluxes help to make diffeomorphism cov. manifest in DFT
+ they get a geometrical role: R is a tensor
Q is not a tensor, rather a connection

Relations to non-commutativity

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- Open string: D-brane with a constant b-field B, leads to $[x^i(\tau), x^j(\tau)] = \mathrm{i}\theta^{ij}$, θ non-com. parameter

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hep-th/9908142 by N. Seiberg, E. Witten

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arXiv:1010.1361 by D. Lüst

arXiv:1202.6366 by C. Condeescu, I. Florakis, D. Lüst

arXiv:1211.6437 by D. A., M. Larfors, D. Lüst, P. Patalong

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- Non-associativity: R-flux is the parameter...
  $[[\mathcal{X}^i(\tau,\sigma), \mathcal{X}^j(\tau,\sigma)], \mathcal{X}^k(\tau,\sigma)] + \text{perm.} \sim R^{ijk}$

arXiv:1010.1263, arXiv:1106.0316, arXiv:1112.4611 by R. Blumenhagen, et al.

arXiv:1207.0926 by D. Mylonas, P. Schupp, R. Szabo

Conclusion and outlook

- GCG $\Rightarrow$ field redefinition $(g, b, \phi) \leftrightarrow (\tilde{g}, \beta, \tilde{\phi})$
  Rewriting NSNS Lag.: $\mathcal{L}(g, b, \phi) = \tilde{\mathcal{L}}(\tilde{g}, \beta, \tilde{\phi}) + \partial(\dots)$
  10d ${Q_k}^{mn} = \partial_k \beta^{mn}$ (for $\beta^{km}\partial_m \cdot = 0$), $R^{mnp} = 3\ \beta^{k[m}\partial_k \beta^{np]}$
- Dim. reduction of $\tilde{\mathcal{L}} \Rightarrow$ 4d non-geometric potential ✓
  Possible for 10d NSNS non-geom. $\Rightarrow \tilde{\mathcal{L}}(\tilde{g}, \beta, \tilde{\phi})$ globally ✓
  $\hookrightarrow$ Relation between 4d/10d non-geometry
  $\tilde{\mathcal{L}}(\tilde{g}, \beta, \tilde{\phi})$ could capture new physics ?
- Extend to RR sector (S-duality), or heterotic,
  and D-brane/O-plane (new objects, as exotic branes?)
  $\hookrightarrow$ new interesting backgrounds, 10d de Sitter solutions...
- Manifestly covariant DFT action w.r.t. diffeomorphisms
  Geometrical role of non-geometric fluxes:
  R: tensor, Q: connection
- Non-commutativity, non-geometry and new fields
  Get concrete closed string examples, study relations with open string...