Generalized Orbifolds of Landau-Ginzburg Models

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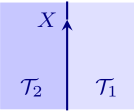
based on work in progress with I. Brunner and N. Carqueville

Motivation

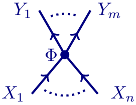
- describe ordinary orbifolds via **defects** $\rightsquigarrow$ **generalized orbifolds**
- many standard results on orbifolds can be derived in a simpler & more conceptual way in the defect approach, e.g. **Cardy condition**
  $\rightsquigarrow$ carries over to the generalized setting
- compute **arbitrary topological correlators**:
  - bulk/boundary correlators ($\rightsquigarrow$ **eff. superpotentials, D-brane charges**)
  - defect action on bulk fields
  - ...

  using the matrix factorization description of defects in LG models
- LG models important because of **CY/LG** and **CFT/LG** correspondences

Defects in 2D field theories



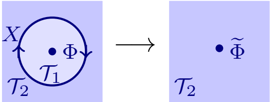
- A **defect** X is a 1D interface between two theories $\mathcal{T}_1$ and $\mathcal{T}_2$ together with a gluing condition on fields
- defects can form **junctions** $\rightsquigarrow$ junction fields:



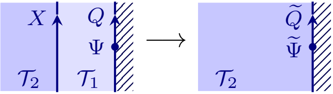
- usually want to preserve some symmetry (e.g. conformal, SUSY, . . .)
- X is a **topological defect** if all correlators inv. under deformations of X.

  use this to map objects of $\mathcal{T}_1$ to objects of $\mathcal{T}_2$ (and vice versa):

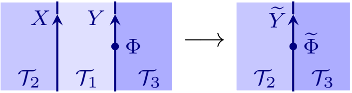
  bulk fields:



  boundary (fields):



  defect (junction fields):



Defect description of orbifolds

- ordinary orbifolds: theory $\mathcal{T}$ with finite symmetry group G
  $\rightsquigarrow$ orbifold theory $\mathcal{T}/G$:

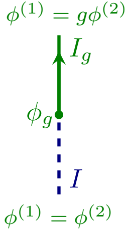
  twisted fields

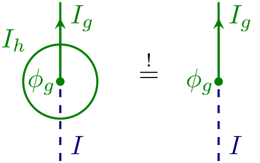
  $$\phi_g(e^{2\pi i}z) = g\phi_g(z)$$

  orbifold projection

  $$P_{\mathsf{orb}}|\phi_g\rangle \equiv \frac{1}{G}\sum_{h\in G} h|\phi_g\rangle \stackrel{!}{=} |\phi_g\rangle$$

- **defect perspective**:



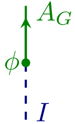


$\forall h \in G$

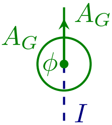
Generalized orbifolds

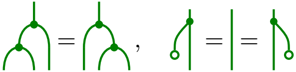
- assemble I_g to $A_G = \bigoplus_{g \in G} I_g$

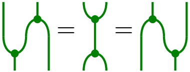
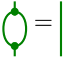
  twisted fields – elements in $\mathrm{Hom}(I, A_G)$

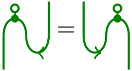
  

  orbifold projector

  $P_{\text{orb}}(\phi) \; =$ 

- **generalized orbifolds** – allow any "nice" A – (symm.) sep. Frob. algebra
  - algebra: $\exists$  $: A \otimes A \to A\,,$  $: I \to A,$ with 

    Frobenius:  separable: 

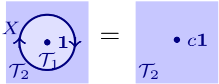
    $\rightsquigarrow$ associative OPE, P_{orb}, bulk pairings, ...
  - symmetric: 

    $\rightsquigarrow$ spectral flow operator $\rightsquigarrow$ iso RR $\leftrightarrow$ (c,c), open/closed TFT

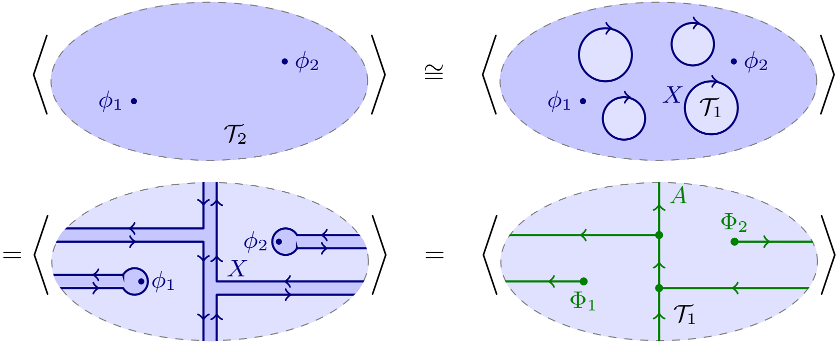
- **important class** $A = X^\dagger \otimes X$

Generalized orbifolds

- [Fröhlich, Fuchs, Runkel, Schweigert '09], [Carqueville, Runkel '12]
- take $A = X^\dagger \otimes X$, where $X : \mathcal{T}_1 \to \mathcal{T}_2$ with invertible quantum dimension,

  i.e. 

  with $c \in \mathbb{C} \setminus \{0\}$
- $\mathcal{T}_1$ and $\mathcal{T}_2$ are related by a generalized orbifold:



- "twisted fields" in $\mathcal{T}_1 \leftrightarrow$ bulk fields in $\mathcal{T}_2$

Landau-Ginzburg models

- An $\mathcal{N} = (2,2)$ SUSY LG model is a 2D QFT with action:

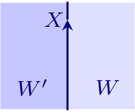
$$S = \int d^2z\, d^4\theta\ K(X_i, \bar{X}_i) + \frac{1}{2}\left(\int d^2z\ d^2\theta\ W(X_i)|_{\bar{\theta}^{\pm}=0} + c.c.\right)$$

  X_i - chiral superfields, $K(X_i, \bar{X}_i)$ - Kähler potential, $W(X_i)$ - **superpotential**

- LG models not conformal, but for W homogeneous $\rightsquigarrow$ flow to an IR fixed pt.
- The **CFT in IR** is characterized solely by W. One can extract information about the CFT from properties of W, e.g. chiral primary fields $\leftrightarrow \frac{\mathbb{C}[X_i]}{(\partial W)}$
- Many $\mathcal{N} = 2$ CFTs can be described as IR fixed pts. of LG orbifolds, in particular, **CY compactifications** in stringy regime of Kähler moduli space.
- from here on: topologically B-twisted LG models

Defects in LG models

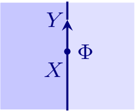
- Defects between LG models with superpotentials $W(x_1,..,x_n)$, $W'(y_1,..,y_m)$ described by **matrix factorizations** of $W' - W$. [Brunner, Roggenkamp '07] [Kapustin, Li '02]



- A matrix factorization of a polynomial f is a polynomial matrix d s.t.

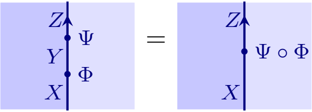
$$d^2 = f \cdot \mathbf{1}$$

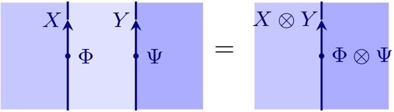
- Junction fields Φ from X to Y are given by polynomial matrices in the **cohomology** $\mathrm{Hom}(X,Y)$ of the operator



$$D_{XY}\Phi = d_Y\Phi - (-1)^{|\Phi|}\Phi\, d_X$$

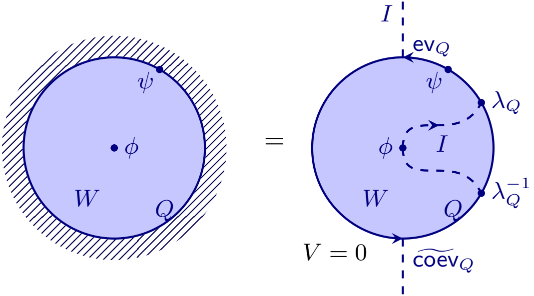
- Defect OPE given by matrix multiplication, defect fusion by $\otimes$ of m.f.





Defects in LG models

- one can compute **arbitrary topological correlators** using defects and m.f.
- first express everything in terms of defects, then evaluate using m.f. by composing horizontally ($\otimes$) and vertically ($\circ$)
- e.g. bulk/boundary correlator:



$$= \mathrm{Res}\left[\frac{\phi\ \mathsf{STr}\big[\partial_{x_1} d_Q \ldots \partial_{x_m} d_Q\ \psi\big]\underline{\mathrm{d}x}}{\partial_{x_1} W \ldots \partial_{x_m} W}\right]$$

- λ, ev, coev are canonical maps known explicitly for any defect X [Carqueville, Murfet '12]
- natural language for top. defects – bicategories (with adjoints)

LG orbifolds – bulk sector

- study twisted **RR** and **(c,c)-fields** [Intriligator, Vafa '90]
- unproj. RR fields in g-th sector

  $|\phi_g\rangle = \prod_{\Theta_i^g \in \mathbb{Z}} (X_i)^{l_i} |0\rangle_{RR}^g$

  action of $h \in G$ on $|\phi_g\rangle$

  $hX_ih^{-1} \equiv h_i^j X_i = e^{2\pi i \Theta_i^h} X_i$

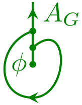
  $h|0\rangle_{RR}^g = \det(h)\; e^{2\pi i \sum_{\Theta_i^h \in \mathbb{Z}} \Theta_i^g} \; |0\rangle_{RR}^g$

  defect/m.f. perspective:

  compute cohomology Hom(I, I_g)

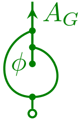
  $\phi_g = \prod_{\Theta_i^g \in \mathbb{Z}} (X_i)^{l_i} \prod_{\Theta_i^g \notin \mathbb{Z}} \omega_i^g$
  [Brunner, Roggenkamp '07]

  evaluate the diagram

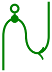
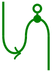
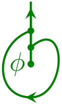
  $P_{\text{orb}}^{RR}(\phi) =$ 

  $\rightsquigarrow$ reproduces the spectrum and phases above

- (c,c)-fields similarly – unproj. spectrum iso to RR sector via spectral flow,

  but action $h|0\rangle_{(c,c)}^g$ different – reproduced by $P_{\text{orb}}^{(c,c)}(\phi) =$ 

LG orbifolds – bulk sector

- The **spectral flow** op. $U_{1/2,1/2}$ corresponds to untwisted RR vacuum $|0\rangle_{RR}$
- identify $|0\rangle_{RR} \leftrightarrow$ 
- one can show $P^{RR}_{\mathsf{orb}}($  $) \equiv$  $=$  $\Leftrightarrow A$ symmetric, i.e.  $=$ 
- further, for A **symmetric** $P^{RR}_{\mathsf{orb}}(\phi) =$ 

$=$ 

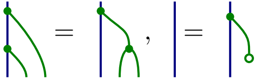
$= P^{(c,c)}_{\mathsf{orb}}(\phi)$

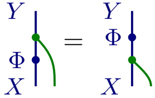
  $\implies$ iso projected (c,c) $\leftrightarrow$ RR
- this generalizes the analysis for ordinary orbifolds

Boundary/defect sector, correlators

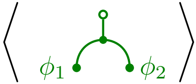
- in orbifold, boundaries/defects are A-**modules**, fields on them **module maps**

$: X \otimes A \longrightarrow X$ s.t.

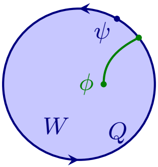




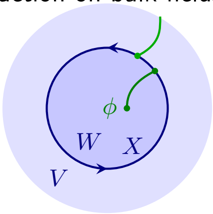
- for $A = A_G$ this reproduces G-equivariant matrix factorizations
- bulk pairing:



- disk correlators:

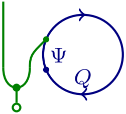


defect action on bulk fields:



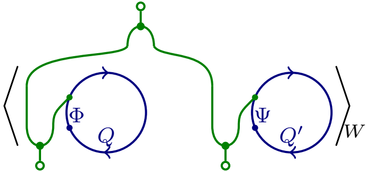
- can be computed explicitly for any A

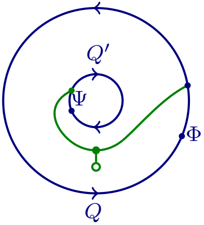
Cardy condition

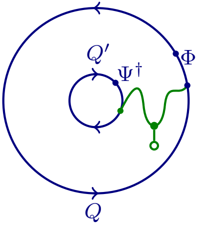
- **boundary-bulk map** $\beta^Q_{\text{orb}}(\Psi) :=$ 

- Theorem: The **Cardy condition** holds for (generalized) LG orbifolds, i.e. $\left\langle \beta^Q_{\text{orb}}(\Phi), \beta^{Q'}_{\text{orb}}(\Psi) \right\rangle_{(W,A)} = \text{str}({}_\Psi m_\Phi)$

Proof:

$\left\langle \beta^Q_{\text{orb}}(\Phi), \beta^{Q'}_{\text{orb}}(\Psi) \right\rangle_{(W,A)} =$  $=$

$=$ 

$=$ 

$= \text{str}({}_\Psi m_\Phi)$

using only the general (algebraic) properties of A and X

Summary & Outlook

- describe orbifolds via **defects** $\rightsquigarrow$ **generalized orbifolds** (quantum symmetry)
- standard results on orbifolds can be carried over to the generalized setting
- for LG models, compute arbitrary topological correlators in the orbifold theory using m.f. $\rightsquigarrow$ e.g. **RR-charges** of D-branes, **eff. superpotentials** on CY

Outlook:

- understand discrete torsion in the generalized setting
- find **new equivalences** between theories via a generalized orbifold construction (e.g. between different **CY** manifolds)