

# Global models with D3 and D7 branes at singularities and moduli stabilisation

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- \* arxiv:1206.5237
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# Introduction

Two longstanding problems of string compactifications:

- 1) **Moduli stabilisation**;
- 2) **Derivation of GUT- or MSSM-like constructions**.

Md stab studied in many corners of Landscape. We chose to work in **type IIB**:


- **Fluxes stabilise** complex structure moduli and axiodilaton.
- Fluxes have **mild backreaction** to geometry (GKP).
- Viable **mechanisms to fix Kähler moduli**: KKLТ, LVS, D-terms.

[Balasubramanian, Berglund, Blumenhagen, Burgess, Conlon, Dasgupta, Denef, Douglas, Dudas, Giddings, Grimm, Haack, Hebecker, Kachru, Kallosh, Klemm, Linde, Louis, Lüст, Jockers, Maharana, Mayr, Plauschinn, Polchinski, Quevedo, Raby, Sethi, Stieberger, Taylor, Theisen, Trivedi, Weigand, Westphal, Zagermann....]

In the last years increasing of model building in type IIB with D7-branes

- In type IIB model building, one can use complex geometry techniques.
- **F-theory**: 7-brane/geometric moduli and 3-form/gauge fluxes unified.
- **Local models** with magnetized branes and branes at singularities, and recently **global models** (both perturb type IIB and F-theory).

[Aldazabal, Beasley, Berglund, Blumenhagen, Braun, Collinucci, Conlon, Donagi, Dudas, Grimm, Heckman, Ibanez, Kreuzer, Krippendorf, Marchesano, Marsano, Mayrhofer, Palti, Quevedo, Saulina, Schafer-Nameki, Shiu, Uranga, Vafa, Verlinde, Watari, Weigand, Wijnholt....]

Usually, 1) and 2) studied independently. → It's time to combine them! 

Three crucial issues in combining (global) model build with md stabilisation:

- \* There is tension between Kähler md stab by NP effects and chirality.

[Blumenhagen, Moster, Plauschinn]

⇒ Constraint on vis-sect flux: **no chirality at inters with NP cycle.**

Best place to put NP effect is **del Pezzo surface.** [Cicoli, Kreuzer, Mayrhofer]

(Also solution for  $h_{-}^{1,1}(X) > 0$  [Grimm, Kerstan, Palti, Weigand])

- \* Freed-Witten anomaly generically prevents more than one NP effect.

[Blumenhagen, Braun, Grimm, Weigand; Collinucci, Kreuzer, Mayrhofer, Walliser]

$\mathcal{F} = F - B = 0$  on NP-cycle →  **$B$  fixed. But generically  $\mathcal{F} \neq 0$  elsewhere.**

⇒ Kähler moduli stabilisation by only **one NP effect.**

(In specific examples one can have more cycles contributing.)

- \* Flux-generated D-terms can induce shrinking of 4-cycles (supporting visible sector), leading to the boundary of Kähler cone.

[Blumenhagen, Braun, Grimm, Weigand; Collinucci, Kreuzer, Mayrhofer, Walliser; Cicoli, Kreuzer, Mayrhofer]

⇒ If we don't want D3 at sing, **avoid visible sector on dP.**

( Moreover, control over EFT and stabilise Kähler moduli inside Kähler cone. )

Type IIB CY orientifolds, with D3/D7-branes and O3/O7-planes.

- ▶ **Phenomenological requirements** translate to **geometric properties** of the compact manifold.
  - \* Set of geometric **constraints** consistent with phenom viable model.
  - \* **Search** for glob defined compact manifold satisfying such constr's.
- ▶ We take CY 3-folds from reduced lists of **hypersurfaces in toric varieties** → allow to be very **explicit** on CY topology and systematic in the search.
- ▶ After choosing a proper O7-involution, take a phenomenologically interesting **brane setup with intersecting and (fluxed) D7-branes** or with **D3-branes at  $dP_n$  singularities**.
- ▶ Check **consistency conditions** (like D7/D5/D3-tadpole cancellation, FW anomaly cancellation,...).
- ▶ Assuming c.s. fixed by 3-form fluxes ( $W_0, g_s$  parameters), study **Kähler mod stab** in a way that **addresses previous issues** [Cicoli, Mayrhofer, R.V.].
- ▶ Fixed values of moduli set  $M_S, M_{SUSY}, \dots$  and the vacuum energy. In two explicit examples, we found a **dS vacuum** (D-terms [Cicoli, Krippendorff, Mayrhofer, Quevedo, R.V.] and F-terms [Louis, Rummel, R.V., Westphal] uplift).

- 1 D-branes at singularities, local and global models
- 2 Explicit example with global embedding of local models and moduli stabilisation
- 3 Transitions among quivers
- 4 Conclusions and outlook.

# D-branes at singularities

and

## local/global models

# D-branes at singularities and quiver theories

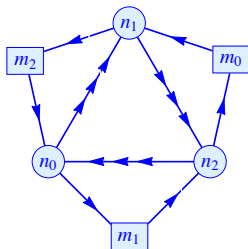
Consider models where the CY  $X$  develops point-like singularities. One can then obtain chiral matter by putting D3 branes on top of sing.

- D3-branes split into **fractional branes**. [Douglas,Moore; Douglas,Diaconescu,Gomis] Associated **quiver theory**: for each fractional brane we draw a node, for each massless open string from brane  $i$  to brane  $j$  we draw an arrow.
- So far great attention on phenomenologically interesting **local** models, with MSSM-like gauge group and spectrum. [Aldazabal,Ibanez,Quevedo,Uranga; Berenstein,Jejjala,Leigh; Verlinde,Wijnholt; Dolan,Krippendorf,Quevedo...]
- We want globally defined compact models. Need to **embed local quiver model into** an orientifold of a **compact singular  $CY_3$** .  
( Non-trivial: compact manifold very specific, in order to realise the embedding and to have moduli stabilisation; on top of it, charge cancellation. )
- See [Diaconescu, Florea, Kachru, Svrcek; Buican, Malyshev, Morrison, H.Verlinde, Wijnholt] for first global embeddings of  $dP_n$  singularities, and [Balasubramanian, Berglund, Braun, García-Etxebarria] for systematic construction of toric singularities in compact CYs and embedding of quivers (with flavour D7-branes).

# $dP_0$ singularity and associated quiver theory

We consider pt-like sing obtained by shrinking one  $dP_0 = \mathbb{C}P^2$  in the  $CY_3$ .

- ▷ It is the orbifold sing  $\mathbb{C}^3/\mathbb{Z}_3$ : There are 3 fractional branes, in one-to-one corresp with irreducible rep of orbif group.
- ▷ Associated quiver theory  $SU(n_0) \times SU(n_1) \times SU(n_2)$ , with flav branes:



Anomaly cancellation constrains multiplicities of flav branes:

$$m_0 = m + 3(n_1 - n_0) \quad m_1 = m \quad m_2 = m + 3(n_1 - n_2) \quad (m \in \mathbb{Z})$$

To globally embed the local model, need to express fractional and flavour branes in terms of the geometry of the global model.



# D-brane charges

D-brane charges encoded into the Mukai charge vector:

$$\Gamma_{\mathcal{E}} = [D] \cdot \text{ch}(\mathcal{E}) \cdot \sqrt{\frac{\text{Td}(TD)}{\text{Td}(ND)}} \quad \hookrightarrow \begin{cases} 2\text{-form} \leftrightarrow \text{D7-charge} \\ 4\text{-form} \leftrightarrow \text{D5-charge} \\ 6\text{-form} \leftrightarrow \text{D3-charge} \end{cases}$$

- \* **Fractional branes** described by well-chosen collection of sheaves, supported on shrinking cycle (7-branes). For  $\mathbb{C}^3/\mathbb{Z}_3$  fractional branes:

$$\text{ch}(\mathcal{E}_0) = -1 + H - \frac{1}{2} H \wedge H, \quad \text{ch}(\mathcal{E}_1) = 2 - H - \frac{1}{2} H \wedge H, \quad \text{ch}(\mathcal{E}_2) = -1$$

and  $D = \mathcal{D}_{dP_0}$  ( $H$  is hyperplane class of  $dP_0 = \mathbb{P}^2$ ).

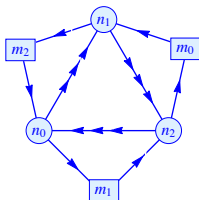
- \* **Flavour branes** are D7-branes wrapping **holomorphic 4-cycles**  $\mathcal{D}_{\text{flav}}$  that **pass through the sing** (i.e.  $\mathcal{D}_{\text{flav}} \cdot \mathcal{D}_{dP_0} \neq 0$ ):

$$\Gamma_{D7_{\text{flav}}} = \mathcal{D}_{\text{flav}} + \mathcal{D}_{\text{flav}} \wedge \mathcal{F}_{\text{flav}} + \mathcal{D}_{\text{flav}} \wedge \left[ \frac{1}{2} \mathcal{F}_{\text{flav}}^2 + \frac{1}{24} \mathcal{C}_2(\mathcal{D}_{\text{flav}}) \right]$$

Local models only say what are the **local charges** of  $D7_{\text{flav}}$ , i.e. restriction of  $\Gamma_{D7_{\text{flav}}}$  on  $\mathcal{D}_{dP_0}$ .

# Restrictions on $m_i$

From number of chiral intersections between flav and fract branes  $\Rightarrow$  local flav brane charges



$$\Gamma_{D7_0}^{\text{loc}} = (m + 3(n_1 - n_0))H \left[1 + \frac{1}{2}H\right]$$

$$\Gamma_{D7_1}^{\text{loc}} = -2mH[1 + H]$$

$$\Gamma_{D7_2}^{\text{loc}} = (m + 3(n_1 - n_2))H \left[1 + \frac{3}{2}H\right]$$

$$\text{with } \Gamma_{D7}^{\text{loc}} \equiv \Gamma_{D7_{\text{flav}}} |_{dP_0} = aH + bH^2$$

- \* In particular,  $\mathcal{D}_{\text{flav}} |_{dP_0} = aH$  is 2-form P.D. to the intersection  $\mathcal{D}_{\text{flav}} \cap \mathcal{D}_{dP_0}$ .
- \*  $\mathcal{D}_{\text{flav}}$  is **connected** and  $\mathcal{D}_{\text{flav}} \cap \mathcal{D}_{dP_0}$  is effective curve  $\Rightarrow a > 0$ .

The locally induced D7-charge of a flav brane must be **positive** multiple of  $H$ .

$$\Rightarrow \quad 0 \leq -m \leq 3(n_1 - \max\{n_0, n_2\})$$

$\hookrightarrow$  **Not all local models are realised globally.** E.g., if  $n_0 = n_1 = n_2$  then  $m_i = 0$ .

Explicit global embedding  
of a local model  
with moduli stabilisation

# Explicit global quiver model: geometry

We take  $CY_3 X$ , that is a hypersurface in a 4d toric ambient variety:

| $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ | $z_6$ | $z_7$ | $z_8$ | $D_{eqX}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-----------|
| 1     | 1     | 1     | 0     | 3     | 3     | 0     | 0     | 9         |
| 0     | 0     | 0     | 1     | 0     | 1     | 0     | 0     | 2         |
| 0     | 0     | 0     | 0     | 1     | 1     | 0     | 1     | 3         |
| 0     | 0     | 0     | 0     | 1     | 0     | 1     | 0     | 2         |

$$SR = \{z_4 z_6, z_4 z_7, z_5 z_7, z_5 z_8, z_6 z_8, z_1 z_2 z_3\}$$

CY data obtained from PALP output [Kreuzer, Skarke].

- Hodge numbers:  $h^{1,1}(X) = 4$ ,  $h^{1,2}(X) = 112$ .
- Basis of  $H_4(X)$ :

$$\mathcal{D}_b = \mathcal{D}_4 + \mathcal{D}_5 = \mathcal{D}_6 + \mathcal{D}_7, \quad \mathcal{D}_{q_1} = \mathcal{D}_4, \quad \mathcal{D}_{q_2} = \mathcal{D}_7, \quad \mathcal{D}_s = \mathcal{D}_8$$

- Intersection form  $l_3 = 27\mathcal{D}_b^3 + 9\mathcal{D}_{q_1}^3 + 9\mathcal{D}_{q_2}^3 + 9\mathcal{D}_s^3$ .
- There are three  $dP_0$  at  $z_4 = 0$ ,  $z_7 = 0$  and  $z_8 = 0$ .
- We have  $D_1|_{\mathcal{D}_{dP_0}} = H$  (where  $\mathcal{D}_{dP_0} = \mathcal{D}_{q_1}, \mathcal{D}_{q_2}, \mathcal{D}_a$ ).

# Orientifold projection and Kähler moduli

We take an orientifold involution that exchanges two (shrinking)  $dP_0$ s:

$$\sigma : \quad z_4 \leftrightarrow z_7 \quad \text{and} \quad z_5 \leftrightarrow z_6 \quad (h_{-}^{1,1}(X)=1 \text{ and } h_{+}^{1,1}(X)=3)$$

- The two  $dP_0$ s  $\mathcal{D}_{q_1} = D_4$  and  $\mathcal{D}_{q_2} = D_7$  are exchanged.
- There are **no O3-planes** and **two O7-planes**:  $O7_1$  at  $z_4 z_5 - z_6 z_7 = 0$  and  $O7_2$  at  $z_8 = 0 \rightarrow [O7_1] = \mathcal{D}_b$  and  $[O7_2] = \mathcal{D}_s$ .
- O7-planes **do not intersect the (shrinking)  $dP_0$ s** and do not intersect each others.
- Symmetric **Kähler form**:  $J = t_b \mathcal{D}_b + t_s \mathcal{D}_s + t_{\text{shr}}(\mathcal{D}_{q_1} + \mathcal{D}_{q_2})$ :

$$\text{vol}(\mathcal{D}_{q_1}) = \text{vol}(\mathcal{D}_{q_2}) = \frac{9}{2} t_{\text{shr}}^2, \quad \text{vol}(\mathcal{D}_s) = \frac{9}{2} t_s^2, \quad \text{vol}(X) = 3\left(\frac{3}{2} t_b^3 + t_{\text{shr}}^3 + \frac{1}{2} t_s^3\right)$$

- **Kähler cone**:

$$t_b + t_s > 0 \quad -t_s > 0 \quad t_b + t_{\text{shr}} > 0 \quad -t_{\text{shr}} > 0$$

Singular CY at  $t_{\text{shr}} \rightarrow 0$ .

# Model without flavour branes

Visible sector from  $N = 3$  D3-branes on top of each (of the two) sing.  
 $n_0 = n_1 = n_2 = 3$

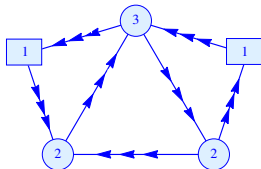
- $dP_0$  quiver theory (trification model -  $SU(3)_c \times SU(3)_L \times SU(3)_R$  with chiral spectrum  $3 \left[ \left( 3, \bar{3}, 1 \right) + \left( 1, 3, \bar{3} \right) + \left( \bar{3}, 1, 3 \right) \right]$ ).

To cancel D7-charge of O7-plane:  
put 4 D7 (plus images) on top of each O7-plane.

- Hidden group  $SO(8) \times SO(8)$ .
- FW flux  $F_s = -\frac{D_s}{2}$  cancelled by choosing  $B = -\frac{D_s}{2}$ .  
↳  $\mathcal{F}_s = F_s - B = 0 \rightarrow$  pure  $SO(8)$  SYM on rigid  $\mathcal{D}_s$  (gaugino condens).
- FW flux  $F_b = -\frac{D_b}{2}$  also cancelled if  $B = -\frac{D_b}{2} - \frac{D_s}{2}$ .  
↳ There are adjoint scalars; if they are lifted by appropriate flux [Biachi, Collinucci, Martucci], we can have non-pert effects also from  $\mathcal{D}_b$  (KKLT).

# Model *with* flavour branes

We want to realise the following quiver theory at the sing:



Left-Right symmetric model

$$SU(3)_c \times SU(2)_L \times SU(2)_R$$

with  $n_0 = n_2 = 2$ ,  $n_1 = 3$  and  $m = 0$   
( $m_0 = m_2 = 3$  and  $m_1 = 0$ )

Local  $D7_{\text{flav}}$  charges:  $\Gamma_{D7_0}^{\text{loc}} = 3H(1 + \frac{1}{2}H)$  and  $\Gamma_{D7_2}^{\text{loc}} = 3H(1 + \frac{3}{2}H)$

- **D7-charge:** for both flav branes  $\mathcal{D}_{\text{flav}}|_{\mathcal{D}_{q_1}} = 3H$ . Hence,

$$\mathcal{D}_{\text{flav}} = 3\mathcal{D}_1 + \alpha^b \mathcal{D}_b + \alpha^s \mathcal{D}_s + \alpha^{q_2} \mathcal{D}_{q_2}$$

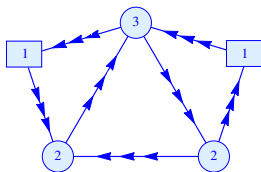
Require they do not intersect  $\mathcal{D}_s$  and  $\mathcal{D}_{q_2}$ :

$$\mathcal{D}_{\text{flav}}^{(0)} = 3\mathcal{D}_1 + \mathcal{D}_s + \mathcal{D}_{q_2} + \alpha_0^b \mathcal{D}_b = (1 + \alpha_0^b) \mathcal{D}_b - \mathcal{D}_{q_1}$$

$$\mathcal{D}_{\text{flav}}^{(2)} = 3\mathcal{D}_1 + \mathcal{D}_s + \mathcal{D}_{q_2} + \alpha_2^b \mathcal{D}_b = (1 + \alpha_2^b) \mathcal{D}_b - \mathcal{D}_{q_1}$$

# Model *with* flavour branes

We want to realise the following quiver theory at the sing:



Left-Right symmetric model

$$SU(3)_c \times SU(2)_L \times SU(2)_R$$

with  $n_0 = n_2 = 2$ ,  $n_1 = 3$  and  $m = 0$   
( $m_0 = m_2 = 3$  and  $m_1 = 0$ )

Local  $D7_{\text{flav}}$  charges:  $\Gamma_{D7_0}^{\text{loc}} = 3H(1 + \frac{1}{2}H)$  and  $\Gamma_{D7_2}^{\text{loc}} = 3H(1 + \frac{3}{2}H)$

- **D5-charge:** fluxes are different on two flav branes:  $\mathcal{F}_0|_{\mathcal{D}_{q_1}} = \frac{1}{2}H$  and  $\mathcal{F}_2|_{\mathcal{D}_{q_1}} = \frac{3}{2}H$ . Moreover, pull-back of  $\mathcal{D}_s$  and  $\mathcal{D}_{q_2}$  on divisor  $3\mathcal{D}_1 + \mathcal{D}_s + \mathcal{D}_{q_2} + \alpha^b \mathcal{D}_b$  are trivial. Hence,

$$\mathcal{F}_{\text{flav}}^{(0)} = \frac{1}{2}\mathcal{D}_1 + \beta_0^b \mathcal{D}_b$$

$$\mathcal{F}_{\text{flav}}^{(2)} = \frac{3}{2}\mathcal{D}_1 + \beta_2^b \mathcal{D}_b$$

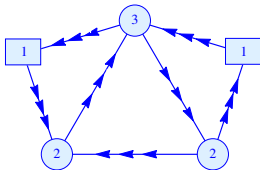
- **D3-charge:**

$$\mathcal{D}_{\text{flav}} \cdot \left[ \frac{1}{2} \mathcal{F}_{\text{flav}}^2 + \frac{\mathcal{C}_2(\mathcal{D}_{\text{flav}})}{24} \right]$$



# Model *with* flavour branes

We want to realise the following quiver theory at the sing:



Left-Right symmetric model

$$SU(3)_c \times SU(2)_L \times SU(2)_R$$

with  $n_0 = n_2 = 2$ ,  $n_1 = 3$  and  $m = 0$   
 ( $m_0 = m_2 = 3$  and  $m_1 = 0$ )

The charges from the quiver system (including fractional branes) are

$$\Gamma_{\text{fractionalD3}} = 2\Gamma_{\varepsilon_0} + 3\Gamma_{\varepsilon_1} + 2\Gamma_{\varepsilon_2} = 2\mathcal{D}_{q_1} + 2\mathcal{D}_{q_1} \wedge D_1 - \frac{3}{2}dVol_X^0$$

$$\Gamma_{D7_0^{\text{flav}}} = (\mathcal{D}_b - \mathcal{D}_{q_1}) + (\mathcal{D}_b - \mathcal{D}_{q_1}) \wedge \left( \frac{1}{2}D_1 + \beta_0^b \mathcal{D}_b \right) + \left\{ 5 + \frac{3}{2}\beta_0^b(1 + 9\beta_0^b) \right\} dVol_X^0$$

$$\Gamma_{D7_2^{\text{flav}}} = (\mathcal{D}_b - \mathcal{D}_{q_1}) + (\mathcal{D}_b - \mathcal{D}_{q_1}) \wedge \left( \frac{3}{2}D_1 + \beta_2^b \mathcal{D}_b \right) + \left\{ 7 + \frac{27}{2}\beta_2^b(1 + \beta_2^b) \right\} dVol_X^0$$

where  $\int_X dVol_X^0 = 1$  and we chose  $\alpha_0^b = \alpha_2^b = 0$ .

$$\text{Summing 3 vect's, with } \beta_0^b = \beta_0^b = 0: \Gamma_{\text{quiver}}^{z_4=0} = 2\mathcal{D}_b + 2\mathcal{D}_b \wedge D_1 + \left( \frac{27}{2} - 3 \right) dVol_X^0$$

Two O7-planes, one at  $\mathcal{D}_s$  and one at  $\mathcal{D}_b$ . We need to saturate their tadpole:

- Four D7-branes (plus images) wrapping  $\mathcal{D}_s$ . B-field cancel FW-flux.  $SO(8)$  gauge group.
- To cancel tadpole of  $[O7_1] = \mathcal{D}_b$ , we need two more D7-branes (plus images) wrapping  $\mathcal{D}_b$ .  $SO(4)$  gauge group  
(unless there is flux  $\mathcal{F}_b = (\frac{1}{2} + f)D_1$ :  $U(2)$  gauge group).
- D7- and D5-charges cancel globally. Net D3-charge:

$$\begin{aligned} Q_{D3}^{\text{tot}} &= Q_{D3}^{\text{quiver}; z_4=0} + Q_{D3}^{\text{quiver}; z_7=0} + Q_{D3}^{U(2)} + Q_{D3}^{SO(8)} \\ &= -63 - 3[\beta_0(1 + 9\beta_0) + 9\beta_2(1 + \beta_2) + 2f(1 + f)] \end{aligned}$$

Fluxes on the flavour branes and on the  $U(2)$ -stack can generate

- ▷ Chiral modes at their intersections (and bulk of  $U(2)$ ).
- ▷ Non-zero FI-terms.

'Step by step' stabilisation:

- Complex structure moduli and D7-deformations stabilised by fluxes.
- **D-terms** on the visible sector stabilise  $t_{\text{shr}} \rightarrow 0$ :

$$V_D = \frac{1}{\text{Re}(f_1)} \left( \sum_i q_{1i} K_i C_i - \xi_1 \right)^2 + \frac{1}{\text{Re}(f_2)} \left( \sum_i q_{2i} K_i C_i - \xi_2 \right)^2$$

For vanishing vev of matter fields  $C_i$ , min at  $\xi_1 = \xi_2 = 0$ , where

$$\xi_1 = -4q_1 \frac{\tau_+}{\mathcal{V}} \quad (\tau_+ \propto t_{\text{shr}}^2) \quad \xi_2 = -4q_2 \frac{b}{\mathcal{V}}$$

- **Gaugino condensation** on rigid  $\mathcal{D}_s$  (a  $dP_0$ ),  $W_0 \sim \mathcal{O}(1)$  and  $\alpha'$  corr:  
 $\hookrightarrow$  F-term potential stabilises  $\tau_s$  small and  $\mathcal{V}$  **LARGE**.

We can realize a **dS vacuum** in the LVS case:

- Switch on gauge flux on non-rigid  $SO(4)$  stack: it generates bulk chiral matter and FI-term.  $\Rightarrow$  **D-term uplift**:  $V_{\text{uplift}} \sim \frac{W_0^2}{\mathcal{V}^{8/3}}$ . We obtain a 'tiny' dS.

# Transitions

among

quivers

# Transitions in the example ?

Total charge of quiver system ( $SU(3) \times SU(2)^2$  plus two flav branes)

$$\Gamma_{\text{quiver}}^{Z_4=0} = 2\mathcal{D}_b + 2\mathcal{D}_b \wedge D_1 + \left(\frac{27}{2} - 3\right) d\text{Vol}_X^0$$

This may be realised by another configuration:

- Three D3-branes at the singularity, realising an  $SU(3)^3$
- Two D7-branes wrapping the divisor  $\mathcal{D}_b$  (i.e. *not passing through the singularity*) and with fluxes  $\mathcal{F} = \frac{1}{2}D_1, \frac{3}{2}D_1$ :

$$\Gamma_{\text{quiver}}^{SU(3)^3} = -3 d\text{Vol}_X^0$$

$$\Gamma_{D7_0} = \mathcal{D}_b + \mathcal{D}_b \wedge \frac{1}{2}D_1 + \frac{21}{4} d\text{Vol}_X^0$$

$$\Gamma_{D7_2} = \mathcal{D}_b + \mathcal{D}_b \wedge \frac{3}{2}D_1 + \frac{33}{4} d\text{Vol}_X^0$$

This suggests a **possible transitions** between the two susy configurations with the same charges.

# From $SU(3)^3$ to $SU(3)^2 \times SU(2)$

Let us study first an analogous transition

Initial configuration:

- $SU(3)^3$  quiver (without flav br) with total charge  $\Gamma_{\text{quiver}}^{SU(3)^3} = -3 dVol_X^0$ .
- Bulk brane with charge  $\Gamma_{D7_0} = \mathcal{D}_b + \mathcal{D}_b \wedge \frac{1}{2} D_1 + \frac{21}{4} dVol_X^0$ .

Final configuration:

- $SU(3)^2 \times SU(2)$  quiver and one fractional brane with total charge

$$\Gamma_{\text{quiver}}^{SU(3)^2 \times SU(2)} = \mathcal{D}_b + \mathcal{D}_b \wedge \frac{1}{2} D_1 + \frac{9}{4} dVol_X^0.$$

- \* Charges are conserved.
- \* Before, a D7-brane that does not pass through the singularity and  $SU(3)^3$  quiver at the singularity.
- \* Afterwards, a D7(flavour)-brane passes through the sing and quiver gauge group changes!

Transition seems to occur when bulk D7 touches sing.

Consider the following space

|       |       |       |     |
|-------|-------|-------|-----|
| $x_0$ | $x_1$ | $x_2$ | $y$ |
| 1     | 1     | 1     | 3   |

- $\mathbb{C}^3/\mathbb{Z}_3$  singularity at  $x_0 = x_1 = x_2 = 0$ . Put 3 D3-branes there.
- Take D7-brane wrapping  $P_3(x_i) + \alpha y = 0$ .
- Let  $\alpha \rightarrow 0$ : D7 touches sing and transition on quiver should occur.

Let us see what happens in the resolved picture:

|       |       |       |     |     |
|-------|-------|-------|-----|-----|
| $x_0$ | $x_1$ | $x_2$ | $y$ | $z$ |
| 1     | 1     | 1     | 3   | 0   |
| 0     | 0     | 0     | 1   | 1   |

- $z = 0$  is the blown-up  $dP_0$ .
- Bulk D7 is  $z \cdot P_3(x_i) + \alpha y = 0$ .
- At  $\alpha \rightarrow 0$  D7 splits into flavour brane and a brane wrapping  $dP_0$ .

# Transitions in the example

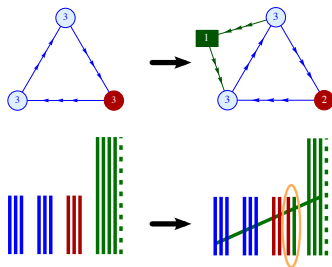
For an interpretation, go to large volume (blow-up). Effective description:

Take D7 on  $\mathcal{D}_b$  and split it into a brane on  $\mathcal{D}_b - \mathcal{D}_{q_1}$  and a brane on  $\mathcal{D}_{q_1}$  with

$$\Gamma_{D7_0^{\text{flav}}} = (\mathcal{D}_b - \mathcal{D}_{q_1}) + (\mathcal{D}_b - \mathcal{D}_{q_1}) \wedge \frac{1}{2} D_1 + 5 d\text{Vol}_X^0$$

$$\Gamma_{\mathcal{D}_{q_1}} = \mathcal{D}_{dP_0} + \mathcal{D}_{dP_0} \wedge \frac{1}{2} D_1 + \frac{1}{4} d\text{Vol}_X^0 = -\Gamma_{\varepsilon_0}$$

Splitting generates the flavour brane  $D7_0^{\text{flav}}$  and an ‘anti-fractional’ brane that annihilates with one fractional brane at the 0-node of the quiver.



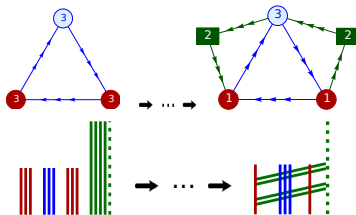
Repeating this, we can go to the L-R symmetric model and further to SM.



# Web of D-brane models at singularities

Claim: connect all the quiver models that can be embedded into CY three-fold

Bound given by D-brane charge conserv. E.g.



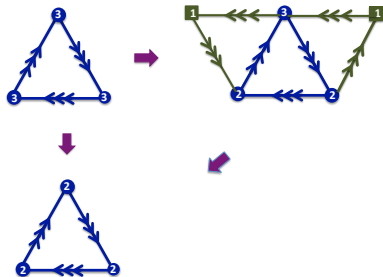
This phenomenon (described in one example) is valid for any global embed of  $dP_0$  quiver models, and should work analogously also for other  $dP_n$ .

# From $SU(3)^3$ to $SU(2)^3$

Transition from  $SU(3)^3$  to  $SU(2)^3$ :

Either take one D3-brane away from the singularity, or

- ↪ Take two bulk D7s and undergo transitions to  $SU(3) \times SU(2)^2$ .
- ↪ Recombine two flav branes and fractional brane at node  $n_1 = 3$ . This give a bulk D7 with non-abelian flux.
- ↪ D7 expels one D3-brane and split into the initial bulk D7s.



# Conclusions

- ✧ We studied how to **embed  $dP_0$  quiver** models **with and without flavour branes**, into a **compact** manifold.
- ✧ We found **restrictions on local models** that can be embedded in this way.
- ✧ We have described **explicit global embedding** of a  $dP_0$  quiver model.
- ✧ Geometric data described by **toric geometry**. This allowed us to make specific choice of brane setup and fluxes.
- ✧ We were able to **combine various mechanisms** to stabilise Kähler moduli, without violating global **consistency conditions** and overcoming problems found so far. We found  **$dS$  vacua**.
- ✧ We considered the possibility of **transitions** between quiver models.

- There is a long list of  $CY_3$  in PALP output: try to **automatise the search for consistent and phenomenological viable models**.
- Study **phenomenology of L-R symmetric (global) model**.
- Study **complex structure moduli stabilisation**: see if one can stabilise all of them and what 3-form fluxes one can switch on.
- Uplift to **F-theory** (more control over complex structure and open string moduli; flux quantisation).
- Understand the dynamics of transitions among quivers.

# Moduli stabilisation

# Moduli Stabilisation in Type IIB

Take Type IIB compactified on  $CY_3$   $X$  with orientifold invol  $(-1)^{F_L} \Omega_{p\sigma}$ .

- **Moduli:**  $h_-^{1,2}$  c.s.,  $h_+^{1,1}$   $\mathbb{C}$ -fied Kähler,  $h_-^{1,1}$  ( $B, C_2$ ) and  $S = e^{-\phi} + iC_0$ .
- The **tree-level 4D Kähler potential** takes the form [Grimm,Louis]:

$$K_{\text{tree}} = -2 \ln \mathcal{V} - \ln (S + \bar{S}) - \ln \left( -i \int_X \Omega \wedge \bar{\Omega} \right)$$

depends on c.s. md via  $\Omega$ , while on Kähler md via the CY vol  $\mathcal{V} = \frac{1}{6} \int_X J \wedge J \wedge J = \frac{1}{6} k_{ijk} t^i t^j t^k$ , where  $J = t^i \hat{D}_i$ .

- A tree-level **superpotential** is generated by turning on **bkgr fluxes**  $G_3 = F_3 + iSH_3$  ( $F_3 = dC_2$  and  $H_3 = dB_2$ ) [Gukov,Vafa,Witten]:

$$W_{\text{tree}} = \int_X G_3 \wedge \Omega$$

- **F-term potential:**

$$V_F^{\text{tree}} = e^K \left( |D_i W|^2 - 3|W|^2 \right) = e^K |D_i W|^2 \quad i \text{ over } S \text{ and c.s. md}$$

$\leftrightarrow$  Tree-level potential has no-scale structure; at min, **Kähler md are flat directions**, while c.s. md and  $S$  are fixed ( at  $D_i W = 0$  ).

# Kähler moduli stabilisation

Sources for Kähler moduli stabilisation → other terms in the potential

$$V = V_F^{\text{tree}} + V_D + \delta V_F^{\text{pert}} + \delta V_F^{\text{np}}$$

- $V_D$  : D-term potential (generated by fluxes on D7's) [Jockers,Louis].
  - $\delta V_F^{\text{pert}}$  : perturbative  $\alpha'$  [Becker,Becker,Haack,Louis] and  $g_s$  [Becker,Haack, Kors, Pajer] corrections to the Kähler potential  $K$ .
  - $\delta V_F^{\text{np}}$  : non-perturbative corrections to the superpotential  $W$  (E3-instantons or gaugino condensation on a D7-stack) [Witten; Kachru,Kalosh,Linde,Trivedi].
- At leading order in  $1/\mathcal{V}$ , **min at  $V_F^{\text{tree}} = 0$  and  $V_D = 0$ .**  
↔ dilaton and c.s. moduli fixed at their flux-stabilised values.
  - At subleading order → minimize  $\delta V_F$   
( $g_s$  and  $W_0 = \langle W_{\text{tree}} \rangle$  flux-dependent constants.  $K = -2 \ln \mathcal{V}$ , with c.s. and dilaton parts of  $K$  entering as an overall factor. )