Terascale Accelerator School

10-14 March 2008, DESY Hamburg

Novel Accelerator Concepts

S. Khan, University of Hamburg

1. Introduction
- Acceleration and its limitations

2. Plasma-wakefield acceleration
- Short-pulse lasers
- laser-driven plasma acceleration
- bunch-driven plasma acceleration

3. Two-beam (wake field) acceleration
- Wake fields and impedance
- Acceleration concept
- CLIC

4. Inverse free-electron lasers
- Synchrotron radiation
- Free-electron lasers
- Acceleration concept
- Examples

5. Other ideas

1. Introduction

1.1 Acceleration and its limitations

Two tasks in particle accelerators

(i) acceleration: electric field
(ii) focussing: magnetic field (usually)

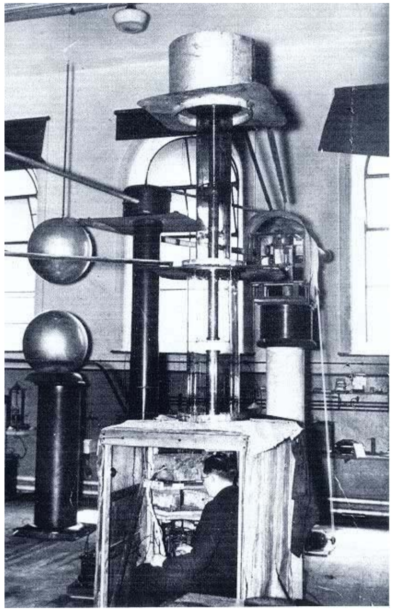
$$\vec{F} = e \cdot \vec{E} + e \cdot \left(\vec{v} \times \vec{B} \right)$$

typ. 20 MV/m (electric field $\vec{E}$) **typ. 1 T** (magnetic field $\vec{B}$)

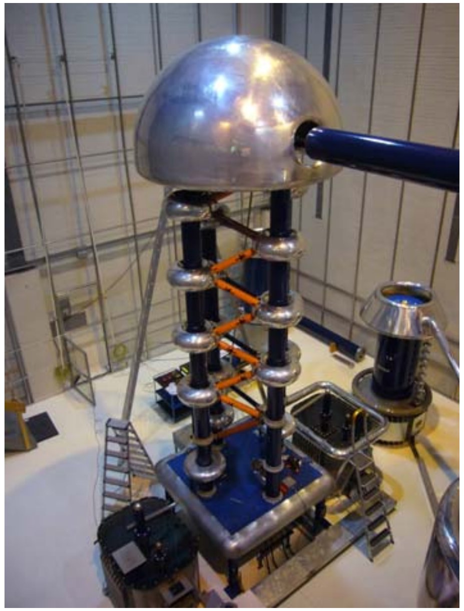
How to create electric fields?

- static or time-varying voltage (displaced charges)
- induction (time-varying magnetic field)
- electromagnetic waves (rf technology)
- new concepts (laser, wake fields)

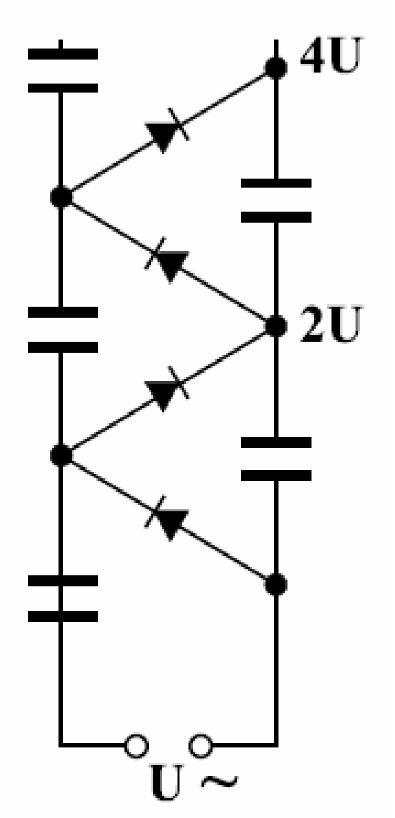
Static voltage, electrostatic field (displaced charges)
- Cockroft-Walton generator



Cavendish Laboratory
Cambridge, Massachusetts
800 keV in 1932



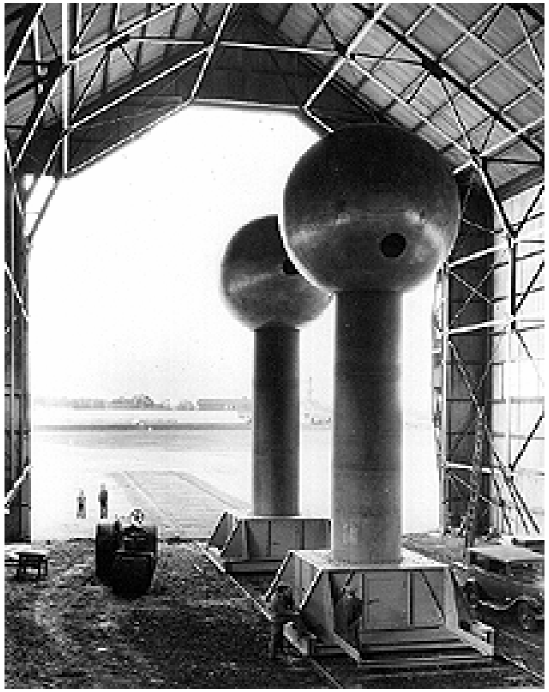
Paul-Scherrer-Institute (CH)



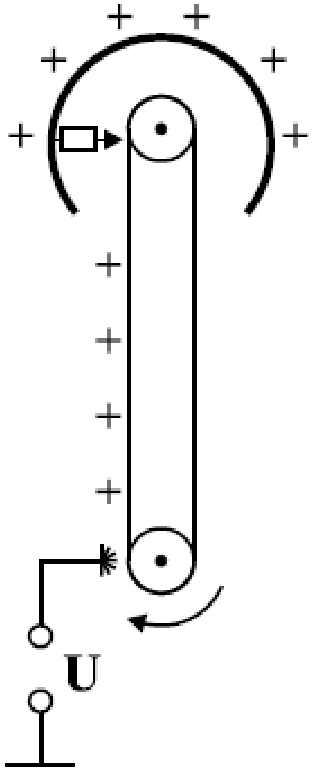
J. Cockroft,
E. Rutherford
and E.T.S. Walton

Static voltage, electrostatic field (displaced charges)
- Van-de-Graaf generator

Tanden Van de Graaf accelerator
(Western Michigan University)

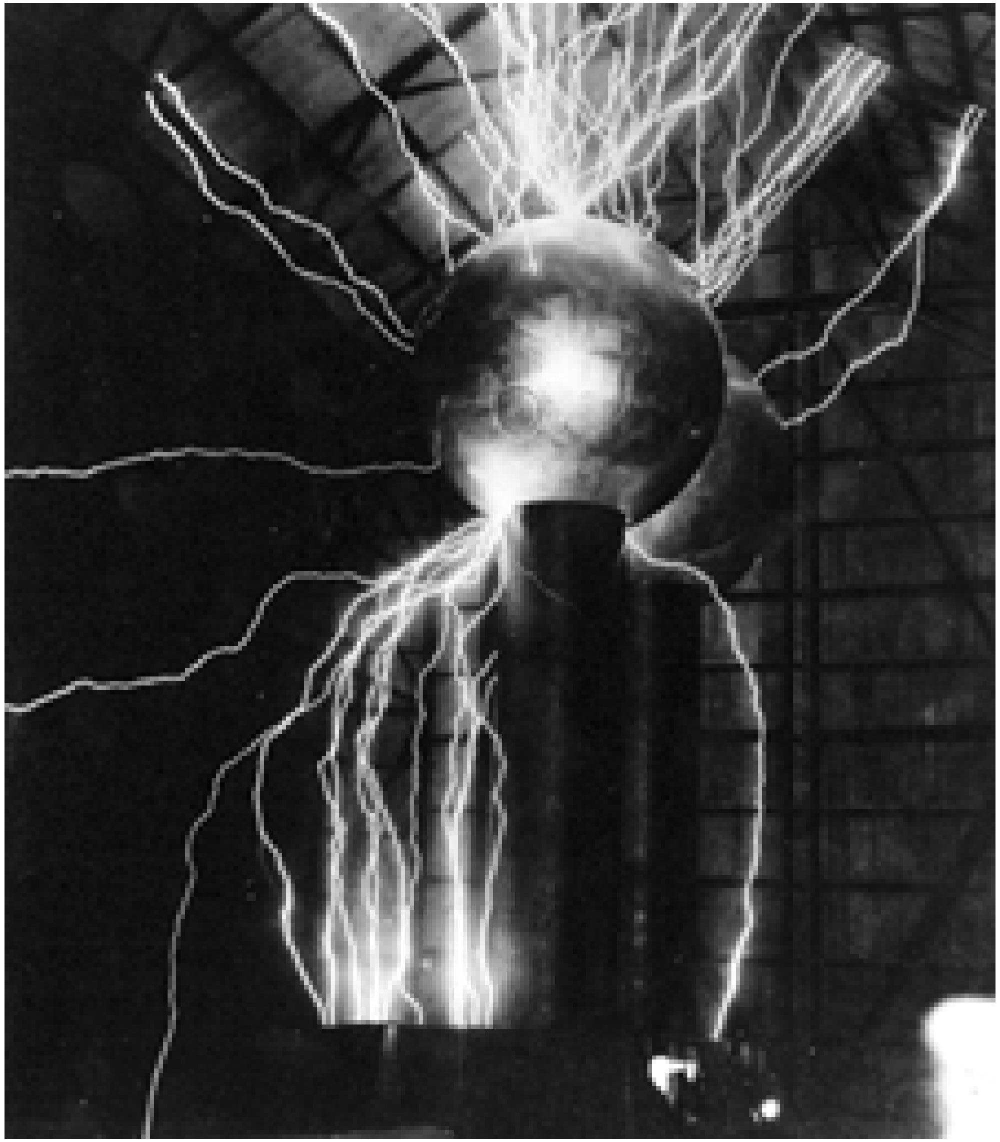


MIT 1932



R. Van de Graaf

Limitation of electrostatic accelerators



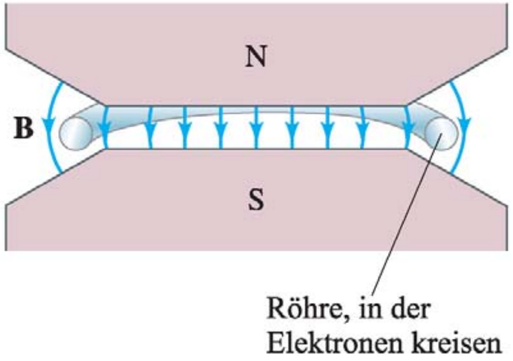
breakdown

air 3 MV/m

SF_6 8 MV/m

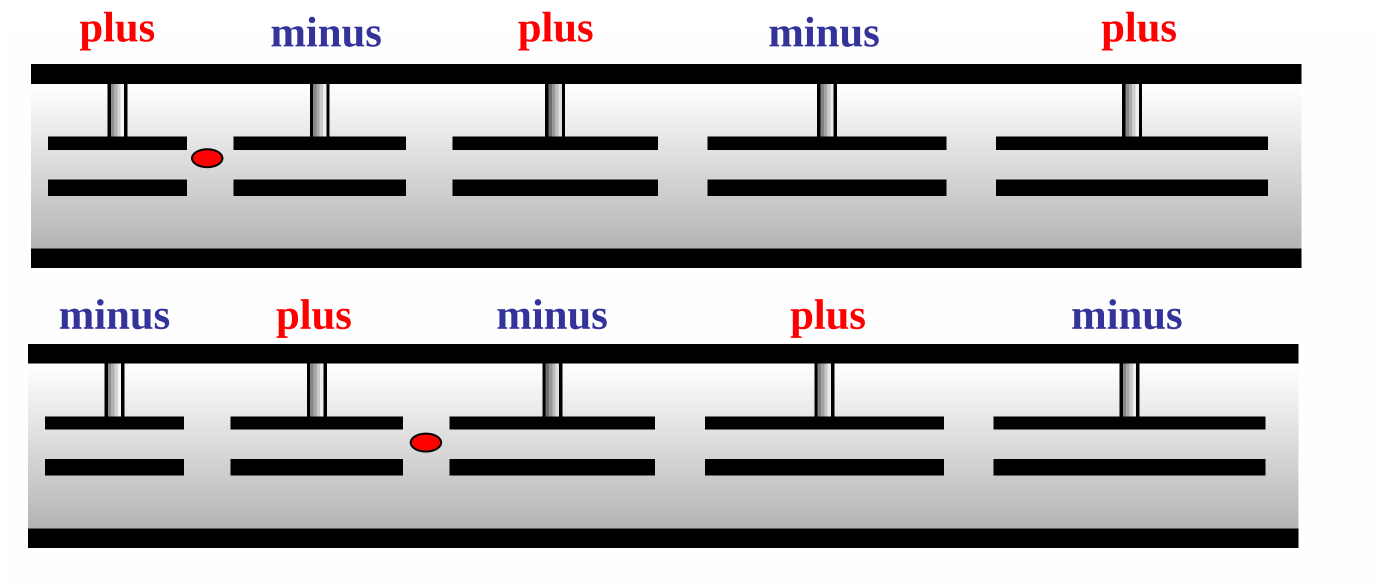
Induction (time-varying magnetic field)
- betatron

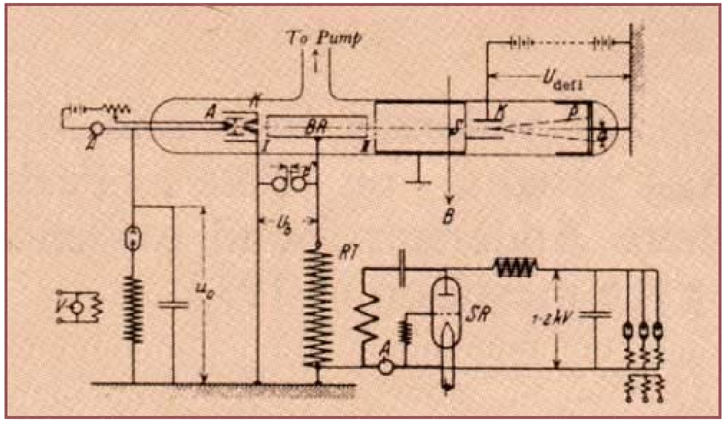
D. Kerst 1940



$$B_R = \frac{1}{2}\langle B \rangle + B_{\text{hom}} \qquad \text{(Wideröe condition)}$$

Time-varying voltage (~kHz to MHz)



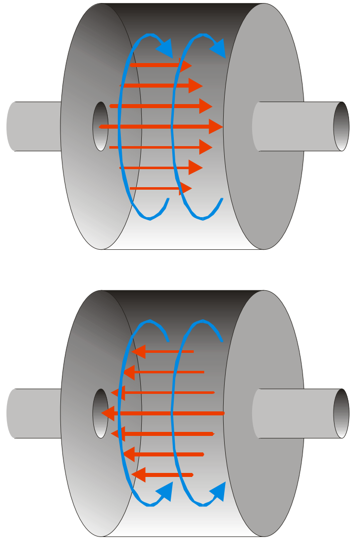


first linear accelerator, Aachen 1928

Rolf Wideröe

Radiofrequency voltage (~GHz)

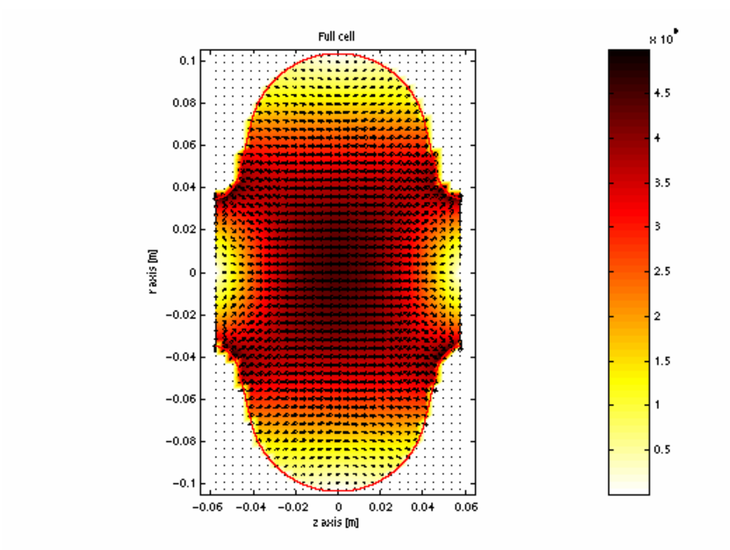
TTF 9-cell superconducting cavity

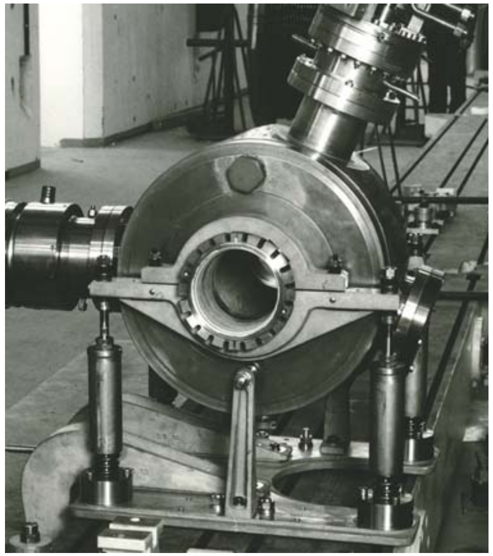


$t = 0$

E **B**

$t = \frac{T_{\mathrm{rf}}}{2}$

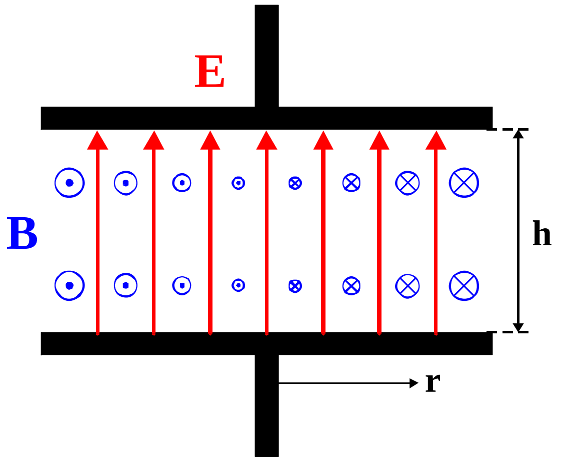




DORIS 1-cell „pillbox“ cavity

Pillbox cavity and J_0

Feynman, Leighton, Sands
***Lecture Notes on Physics* Vol. II Ch. 23**



$$E_1 = E_0 e^{i\omega t}$$

$$\oint_S \vec{B}_1 d\vec{s} = \frac{1}{c^2}\frac{\partial}{\partial t}\int_A \vec{E}_1 d\vec{a} \quad \rightarrow \quad B_1 = i\frac{\omega r}{2c^2}E_0 e^{i\omega t}$$

$$\oint_S \vec{E}_2 d\vec{s} = -\frac{\partial}{\partial t}\int_A \vec{B}_1 d\vec{a} \quad \rightarrow \quad E_2 = -\frac{\omega^2 r^2}{4c^2}E_0 e^{i\omega t}$$

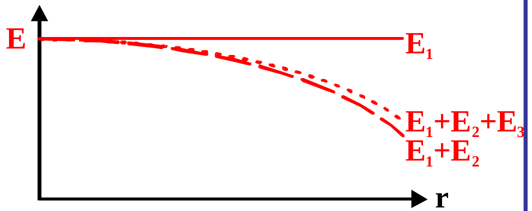
$$\oint_S \vec{B}_2 d\vec{s} = \frac{1}{c^2}\frac{\partial}{\partial t}\int_A \vec{E}_2 d\vec{a} \quad \rightarrow \quad B_2 = -i\frac{\omega^3 r^3}{16c^4}E_0 e^{i\omega t}$$

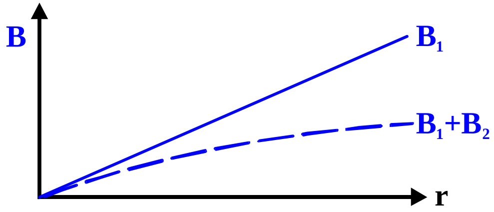
$$\oint_S \vec{E}_3 d\vec{s} = -\frac{\partial}{\partial t}\int_A \vec{B}_2 d\vec{a} \quad \rightarrow \quad E_3 = +\frac{\omega^4 r^4}{64c^4}E_0 e^{i\omega t}$$

$\ldots$

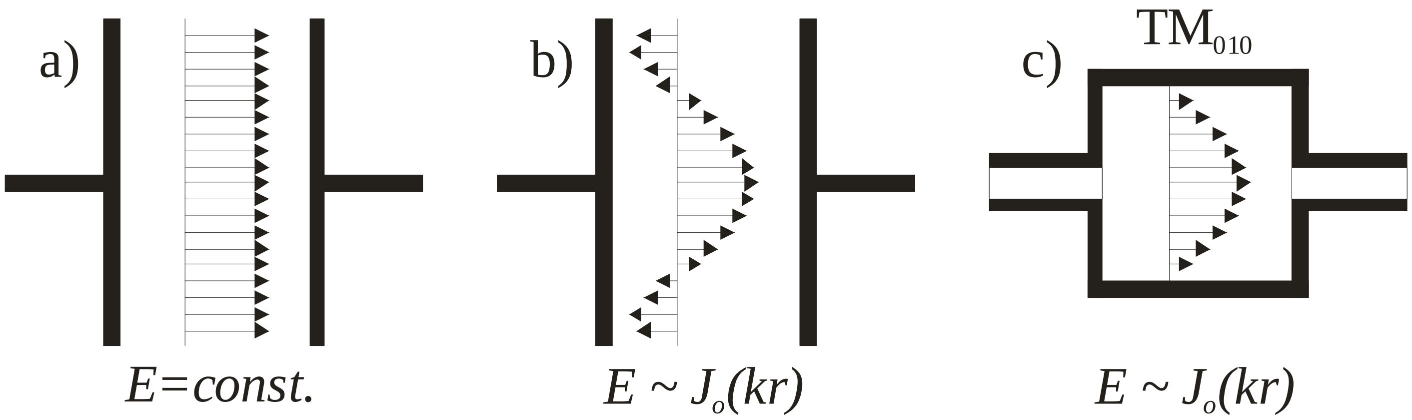
$$E = E_0 e^{i\omega t}\left\{1 - \frac{1}{(1!)^2}\left(\frac{\omega r}{2c}\right)^2 + \frac{1}{(2!)^2}\left(\frac{\omega r}{2c}\right)^4 - \ldots\right\}$$

$$B = \frac{i}{c}E_0 e^{i\omega t}\left\{\left(\frac{\omega r}{2c}\right) - \frac{1}{1!2!}\left(\frac{\omega r}{2c}\right)^3 + \ldots\right\}$$





Plattenkondensator mit Wechselspannung



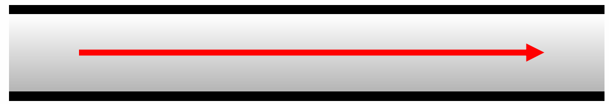
$$E = E_0 e^{i\omega t}\left\{1 - \frac{1}{(1!)^2}\left(\frac{\omega r}{2c}\right)^2 + \frac{1}{(2!)^2}\left(\frac{\omega r}{2c}\right)^4 - \ldots\right\} = E_0 e^{i\omega t} J_0\left(\frac{\omega r}{c}\right)$$

$$B = \frac{i}{c} E_0 e^{i\omega t}\left\{\left(\frac{\omega r}{2c}\right) - \frac{1}{1!2!}\left(\frac{\omega r}{2c}\right)^3 + \ldots\right\} = \frac{i}{c} E_0 e^{i\omega t} J_0'\left(\frac{\omega r}{c}\right)$$

$$J_0(2.405) = 0 \rightarrow \omega = 2.405\frac{c}{R}$$

Radiofrequency voltage (~GHz)

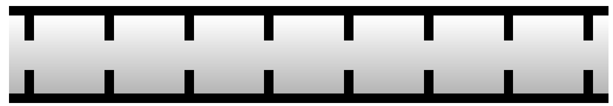
a)



$$v_{\text{phase}} > c$$

disk-loaded waveguide e.g. SLAC

b)

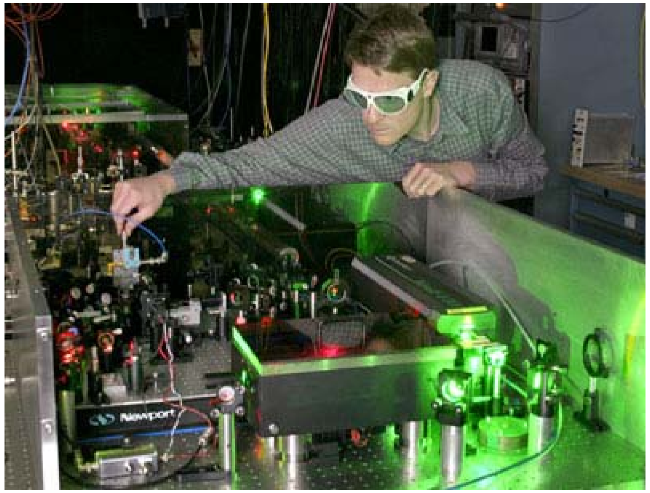


$$v_{\text{phase}} \;\square\; c$$

SLAC two-mile accelerator

European X-ray FEL (XFEL)

Short laser pulses

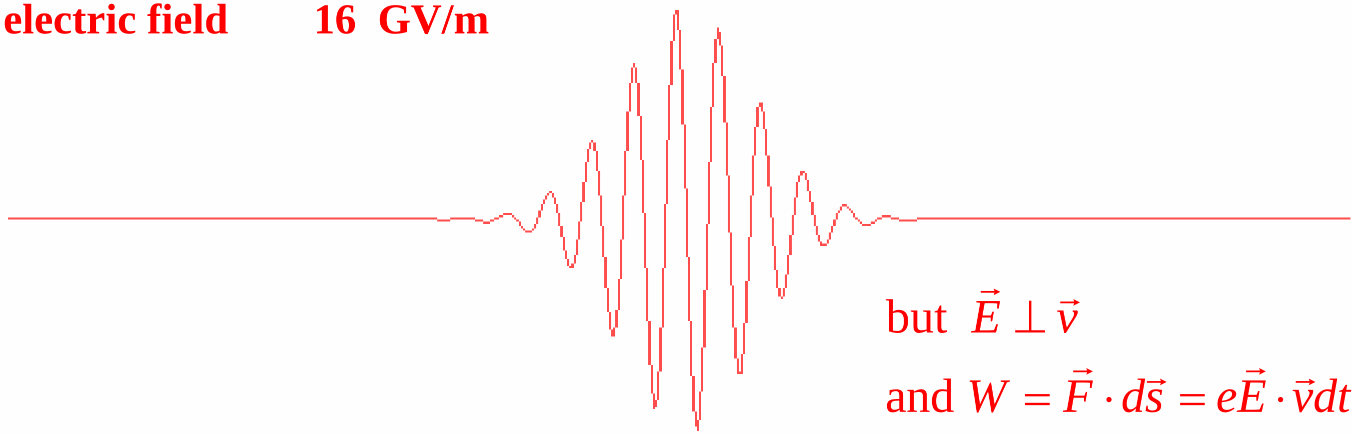


typically Ti:sapphire laser systems,

example (moderate system ~300 k$)

wavelength	**800 nm**
pulse length	**30 fs or 9 μm (fwhm)**
pulse energy	**3 mJ @ 1 kHz rep.rate**
spot size	**500 x 500 μm (fwhm)**

energy density	**1.1 GJ/m^3**
power	**0.1 TW**
electric field	**16 GV/m**



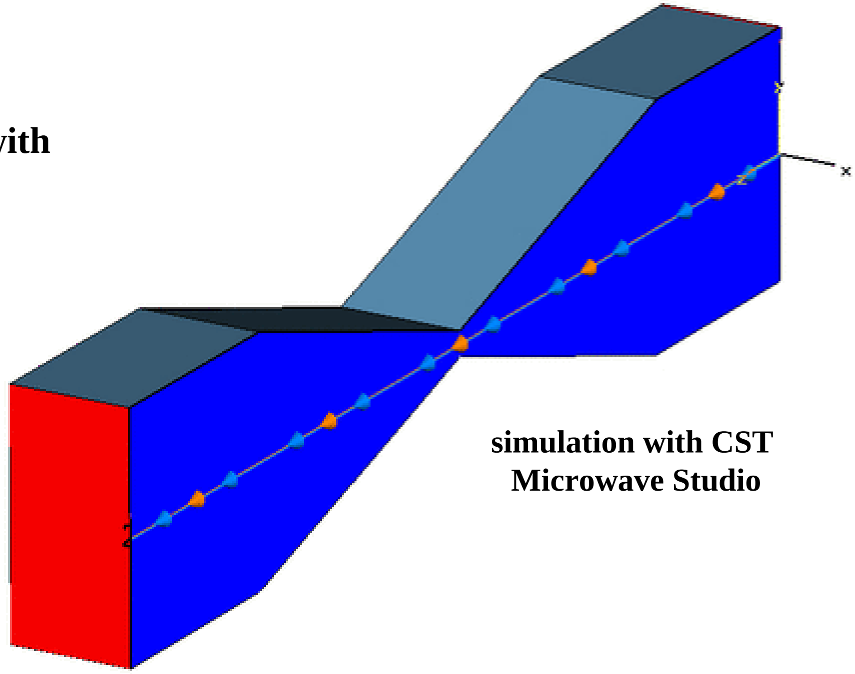
but $\vec{E} \perp \vec{v}$

and $W = \vec{F} \cdot d\vec{s} = e\vec{E} \cdot \vec{v} dt$

Wake fields

charged-particle beam in vacuum pipe with
- non-uniform geometry
- finite conductivity

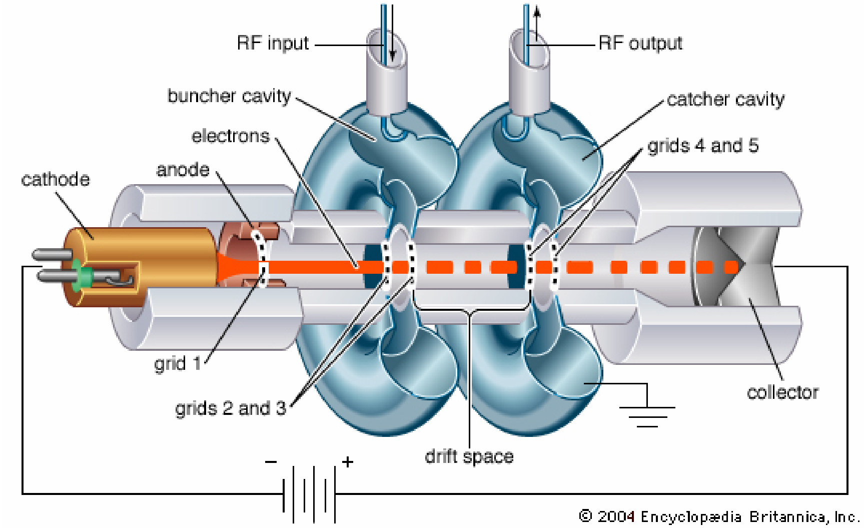
wake turbulence from heavy aircraft



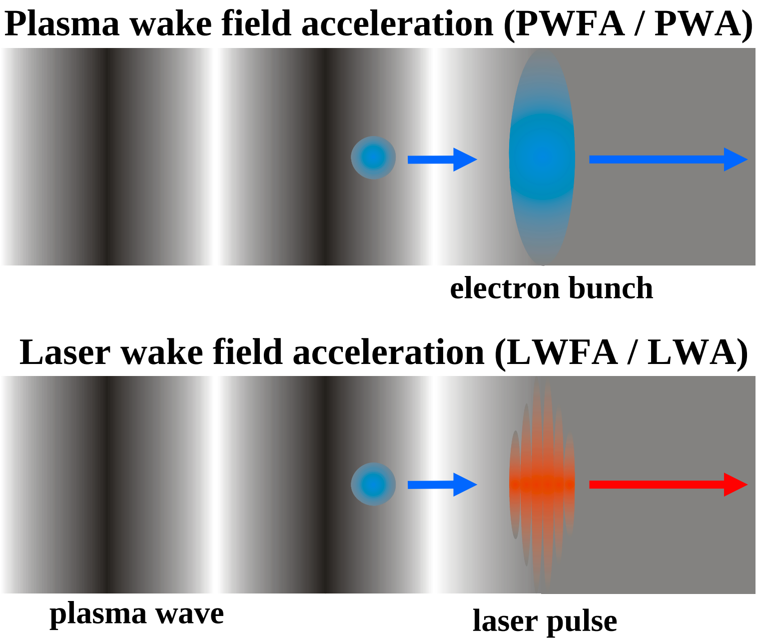
simulation with CST Microwave Studio

Example: the klystron amplifier

Russel and Sigurd Varian, 1951



2 Plasma-wakefield acceleration



field $E = \frac{m_e \omega_p c}{e}$ with plasma frequency $\omega_p = \sqrt{\frac{n_p e^2}{\varepsilon_0 m_e}}$

e.g. $n_p = 10^{18}\ \mathrm{cm}^{-3} \rightarrow \omega_p = 5.6 \cdot 10^{-13}\ \mathrm{s}^{-1} \rightarrow E = 1.0\ \mathrm{GeV/cm}$

field $E = \dfrac{m_e \omega_p c}{e}$ with plasma frequency $\omega_p = \sqrt{\dfrac{n_p e^2}{\varepsilon_0 m_e}}$

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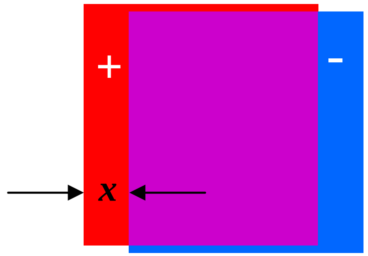


$$a = -a_0 \sin \omega_p t$$

$$x = x_0 \sin \omega_p t = \frac{a_0}{\omega_p^2} \sin \omega_p t$$

$$x_0 = \frac{a_0}{\omega_p^2} = \frac{F}{m_e \omega_p^2} = \frac{e \cdot E}{m_e \omega_p^2}$$

$$k_p \cdot x_0 = \omega_p c \cdot x_0 \approx 1$$

$$\rightarrow \qquad E \approx \frac{m_e \omega_p c}{e}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{n_p e x}{\varepsilon_0}$$

$$\ddot{x} + \frac{F}{m_e} = \ddot{x} + \underbrace{\frac{n_p e^2}{\varepsilon_0 m_e}}_{\omega_p^2} x = 0$$

$$\omega_p = \sqrt{\frac{n_p e^2}{\varepsilon_0 m_e}}$$

Suggested reading

VOLUME 43, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 1979

Laser Electron Accelerator

T. Tajima and J. M. Dawson
Department of Physics, University of California, Los Angeles, California 90024
(Received 9 March 1979)

An intense electromagnetic pulse can create a weak of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. Existing glass lasers of power density 10^{18}W/cm^2 shone on plasmas of densities 10^{18} cm^{-3} can yield gigaelectronvolts of electron energy per centimeter of acceleration distance. This acceleration mechanism is demonstrated through computer simulation. Applications to accelerators and pulsers are examined.

theory („wave breaking“, „bubble“)
A. Pukhov and J. Meyer-ter-Vehn, Appl. Phys. B (2002), 355

breakthrough („monoenergetic electrons“) in 2004:
S.P.D. Mangles et al., Nature 431 (2004), 535
C.G.R. Geddes et al., Nature 431 (2004), 538
J. Faure et al., Nature 431 (2004), 541

most recent energy records:
W.P. Leemans et al., Nature Physics 21 (2006), 696 (LWA with 1 GeV electrons over 3.3 cm at LBNL)
H. Schwoerer et al., Nature 439 (2006), 445 (LWA with 1.2 MeV protons at Jena/Germany)
I. Blumenfeld et al., Nature 445 (2007), 741 (PWA energy doubling of 42 GeV electrons at SLAC)

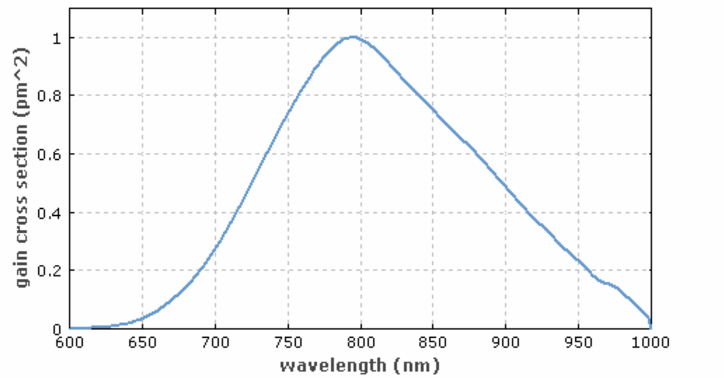
overview:
C. Joshi, Particle Accelerator Conference 2007, Albuquerque, 3845 [www.jacow.org]
C. Joshi, Scientific American (Feb 2006) or Spektrum der Wissenschaft (Aug 2006), 56
11th Advanced Acceletaror Concepts Workshop, Stony Brook (New York), AIP Proceedings 737
12th Advanced Accelerator Concepts Workshop, Lake Geneva (Wiskonsin), AIP Proceedings 887
[DESY library, electronic books]

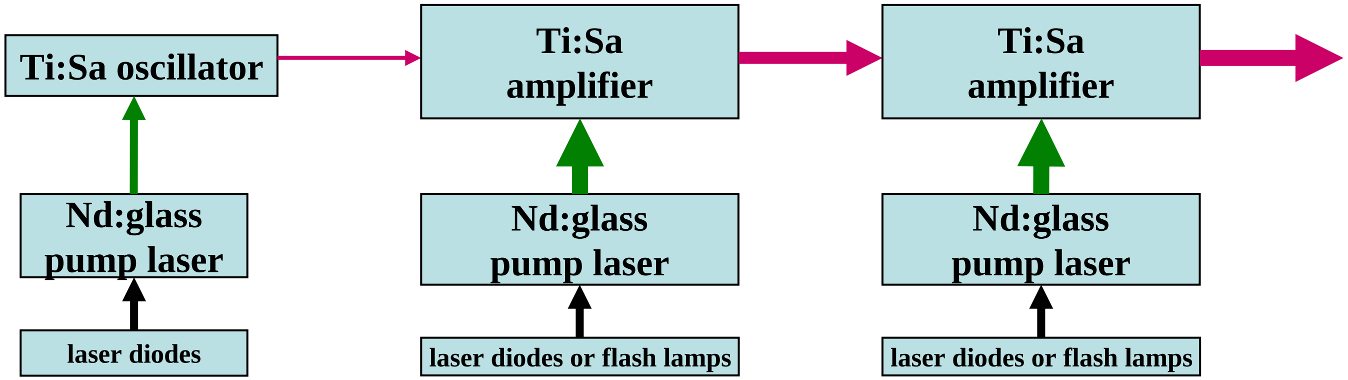
2.1 Short-pulse lasers

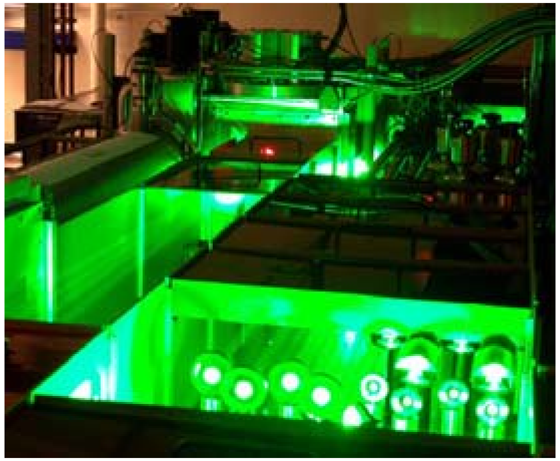
usually Ti^{3+}:sapphire

time-bandwidth product

$$\tau_{rms} \cdot \omega_{rms} \approx 0.5$$

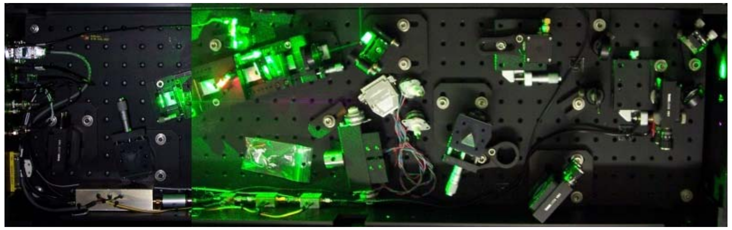


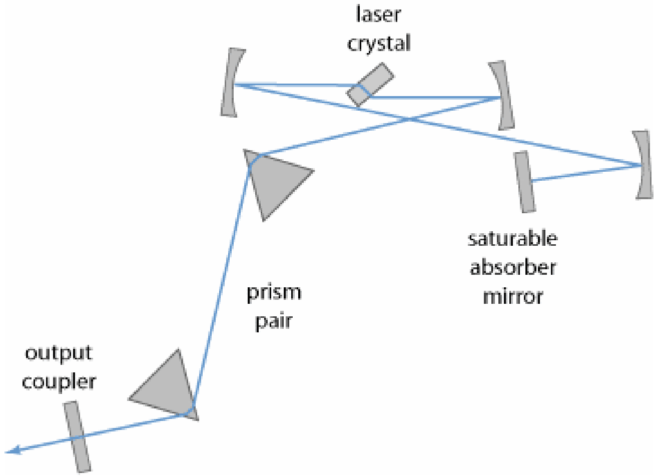


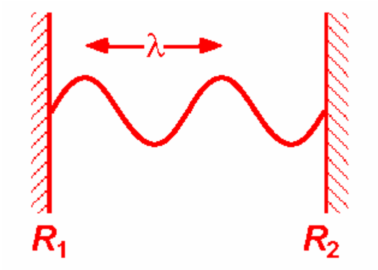


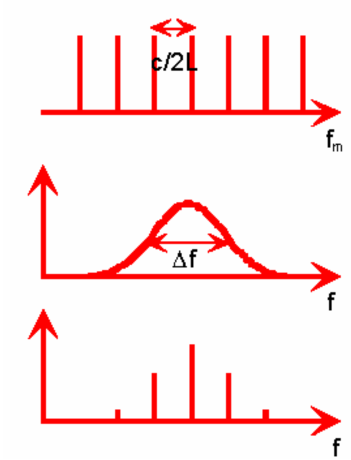
buzz word #1: mode locking

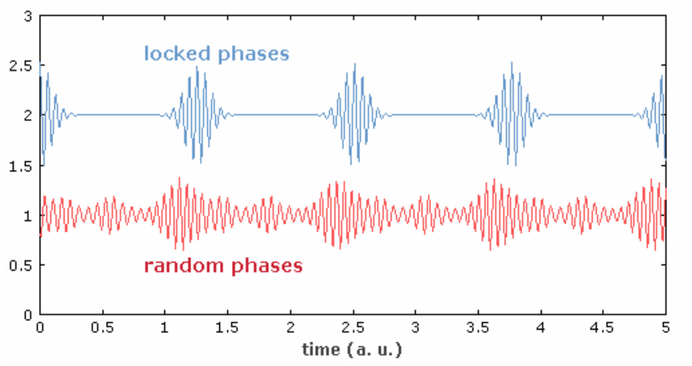
Ti:sapphire oscillator





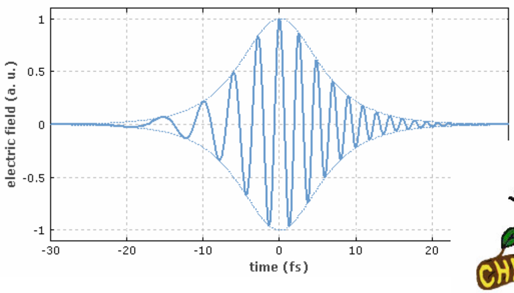






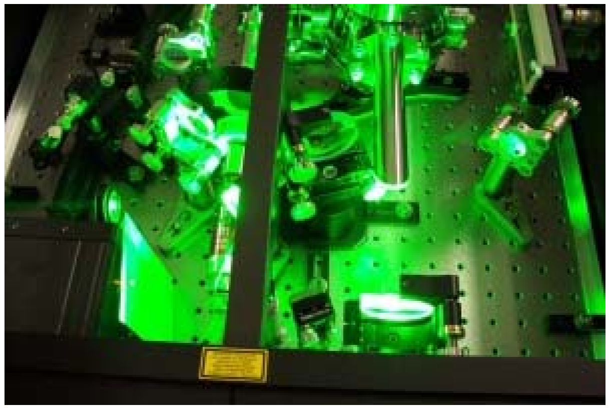
buzz word #2: chirped-pulse amplification (CPA)

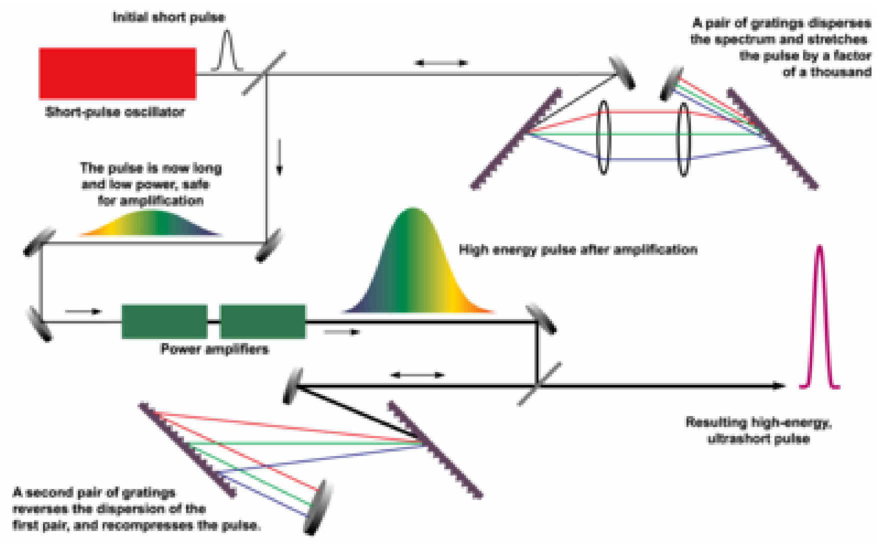
D. Strickland and G. Mourou, Opt. Commun 56 (1985), 219

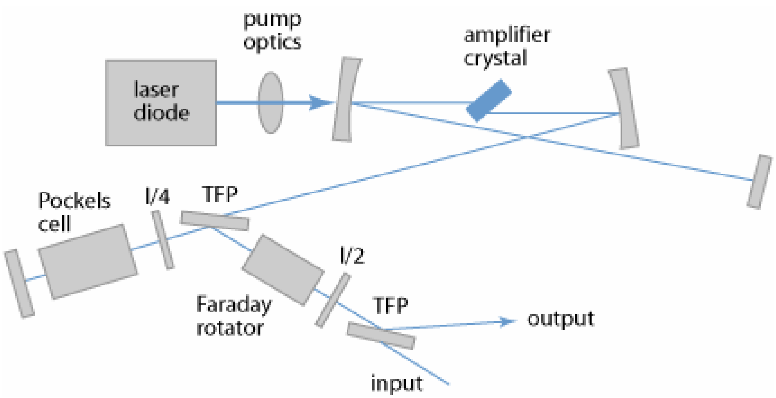


chirp

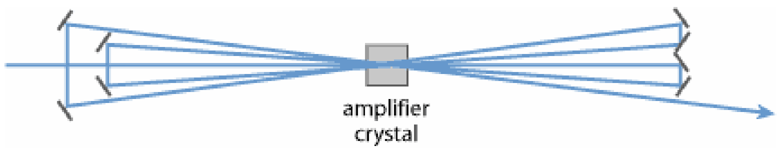
$\nu(t)$



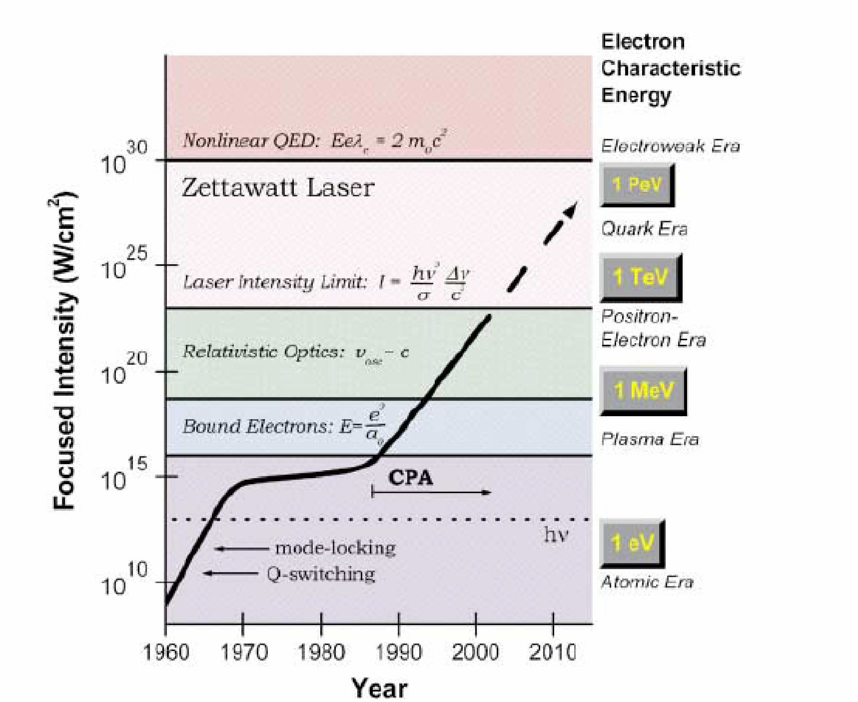




regenerative and multipass amplifiers



Laser intensity frontier

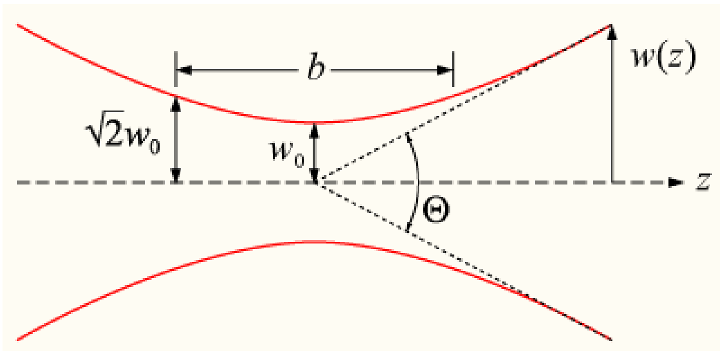


T. Tajima, G. Mourou,
Phys. Rev. Special Topics – Accel. Beams 5 (2003), 031301

Gaussian laser pulses

$$w(z) = 2\sigma(z) = w_0\sqrt{1+\left(M^2 z / z_R\right)^2}$$

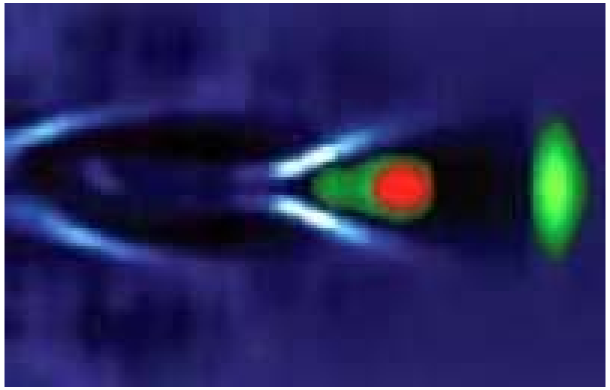
with Rayleigh length $z_R = \dfrac{\pi w_0^2}{\lambda}$



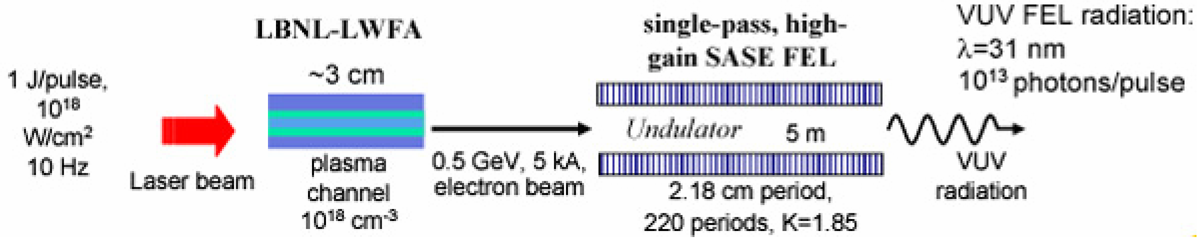
small beam waist = large divergence

2.2 Laser-driven plasma wake field acceleration (LWA)

LOASIS/LBNL Berkeley (USA)
PBPL/UCLA Los Angeles (USA)
University of Michigan (USA)
LOA Palaiseau (France)
RAL Chilton (UK)
University of Strathclyde (UK)
University of Tokyo (Japan)
Lund Laser Centre (Sweden)
MPQ Garching (Germany)
University of Jena (Germany) ?
FZ Dresden/Rossendorf (Germany) ?
. . .

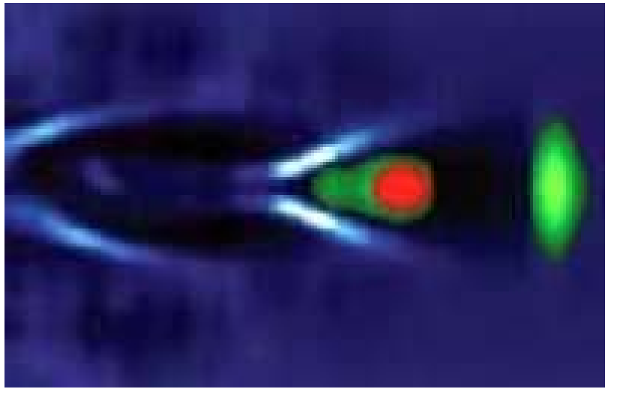


goals:
- „table-top“ free-electron lasers
- pushing for higher energy (10 GeV and more)



How to obtain „monochromatic“ beams?

- „bubble“ regime
- wave breaking



2001/2002: simulations

until 2004: electron beams < 200 MeV with 100% energy spread

in 2004: up to 170 MeV with few % energy spread (RAL, LOA, LBNL)
- **wave breaking**
- **increase interaction length by (i) wider beam = larger Rayleigh length**
  (ii) channeling with precursor laser pulses

but: precursor laser requires high density and dephasing length

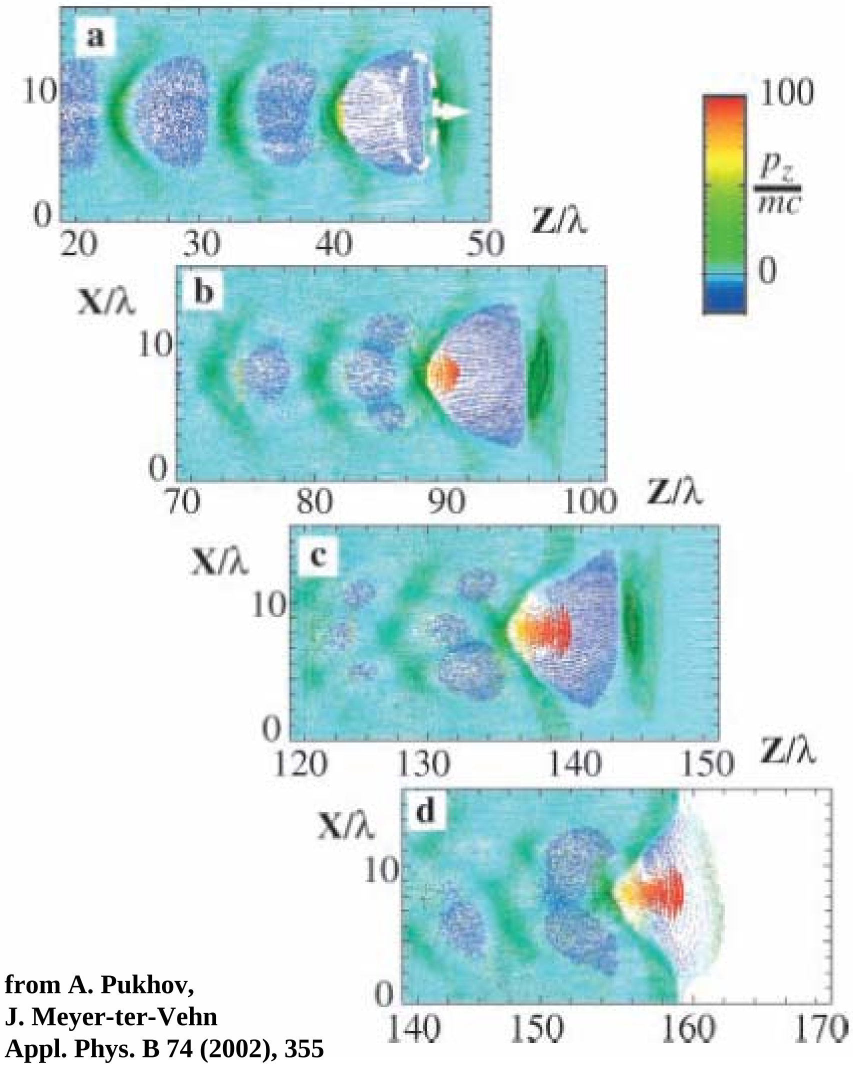
in 2006: 1 GeV electrons using gas capillary as a waveguide with electric discharge (LBNL)

$$L_d \propto n_p^{-3/2}$$

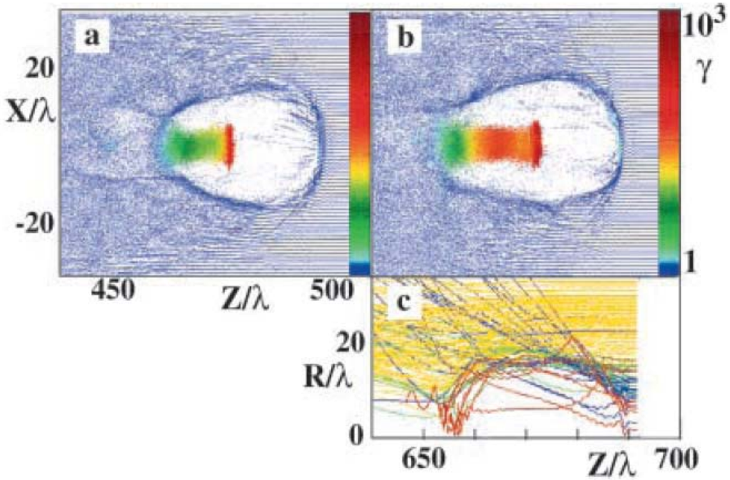
in 2007: undulator synchrotron radiation from laser-plasma accelerator (Jena)

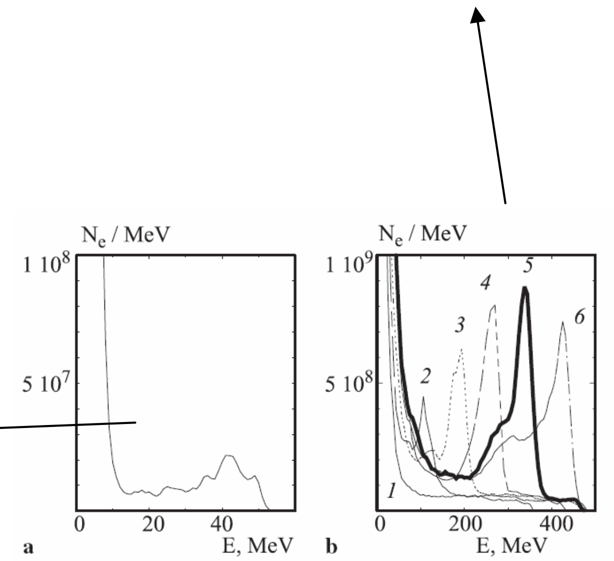
Simulations

at the wave-breaking threshold (20 mJ, 6.6 fs)



above the threshold (12 J, 33 fs)

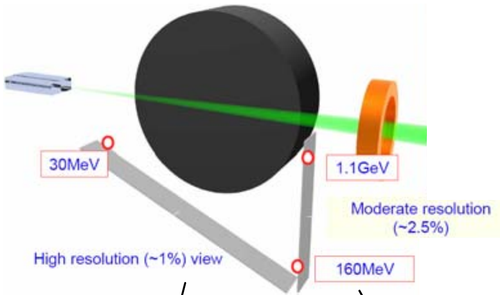


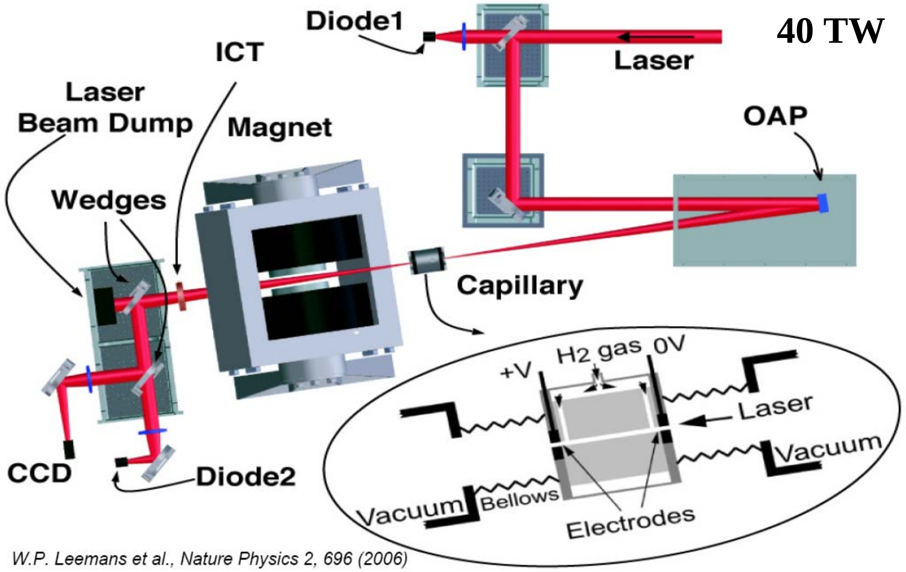


from A. Pukhov,
J. Meyer-ter-Vehn
Appl. Phys. B 74 (2002), 355

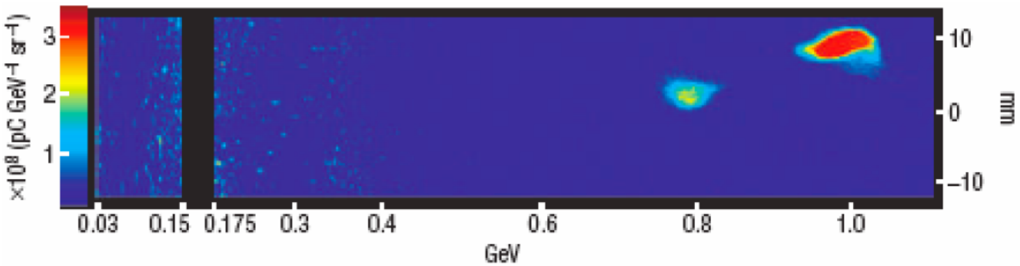
Experiment at LOASIS/Berkeley

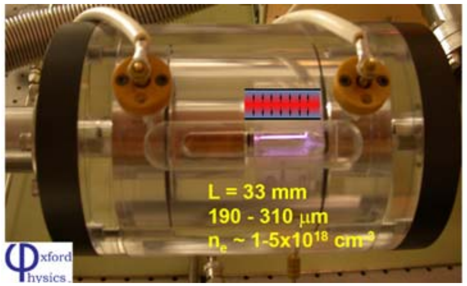
Nature Physics 2 (2006), 696





W.P. Leemans et al., Nature Physics 2, 696 (2006)





results:		
	energy approx.	1 GeV
	charge	>30 pC
	energy spread	2.5% rms
	divergence	2.0 mrad

gas capillary with discharge ~150 ns prior to laser pulse

Alternatives

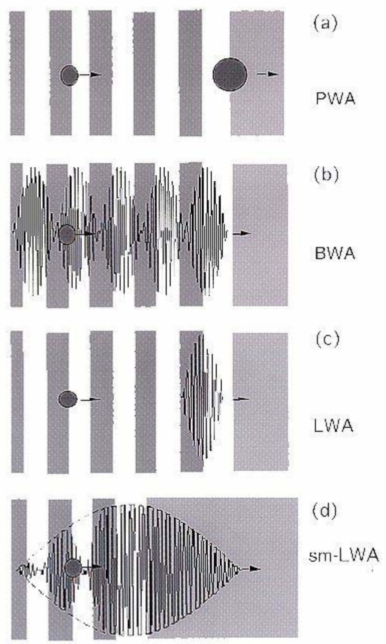
LWA **laser pulse length ideally $\lambda_p/2$**

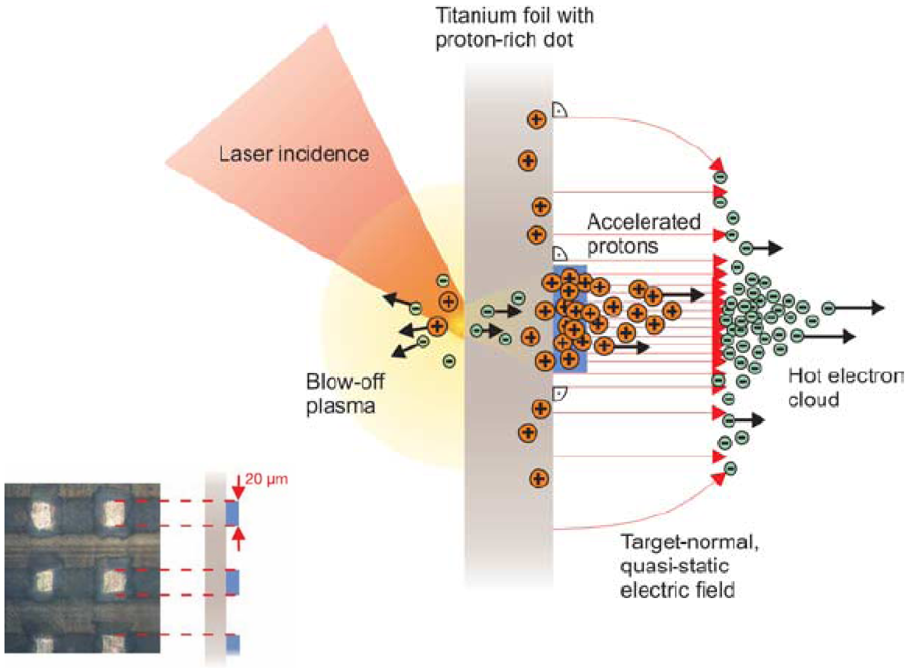
self-modulated LWA **laser pulse length > λ_p**

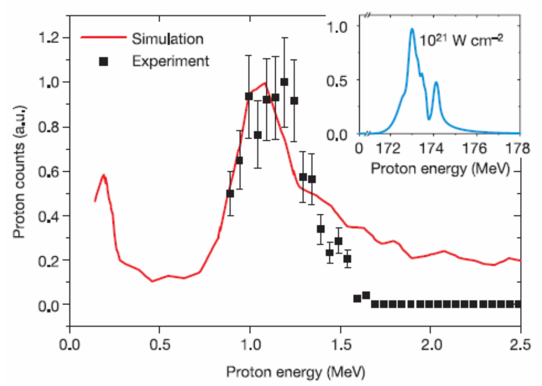
beatwave plasma acceleration

direct laser acceleration $\Delta E = e\vec{E}\cdot\vec{v} = eEv_{\perp}$

laser-plasma acceleration of protons, using mircostructured targets (Jena)







H. Schwoerer et al., Nature 439 (2006), 445

Towards 10 GeV and higher?

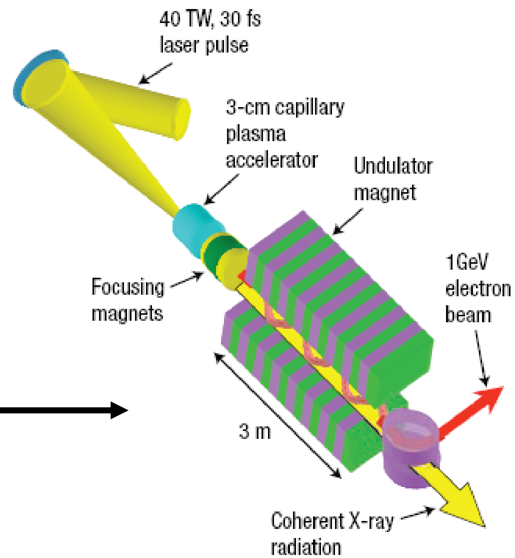
- goal: 1 nC, 10 GeV = 10 J per bunch

- higher laser pulse energy and power, e.g. 100 J / 100 fs = 1 PW (PetaWatt)

- longer gas capillary (several 10 cm), lower density, longer dephasing length

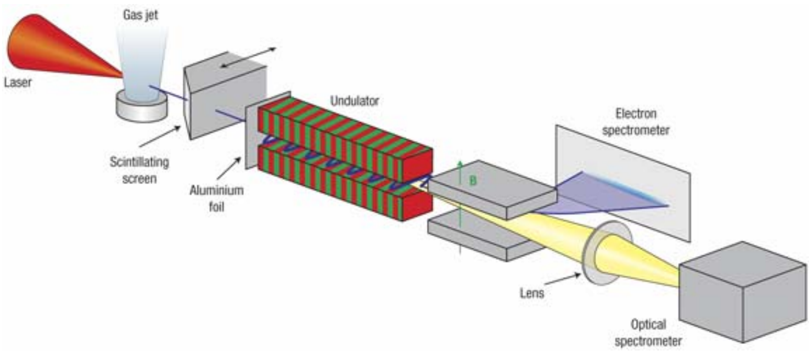
- staging: inject electron bunch into plasma bubble

?

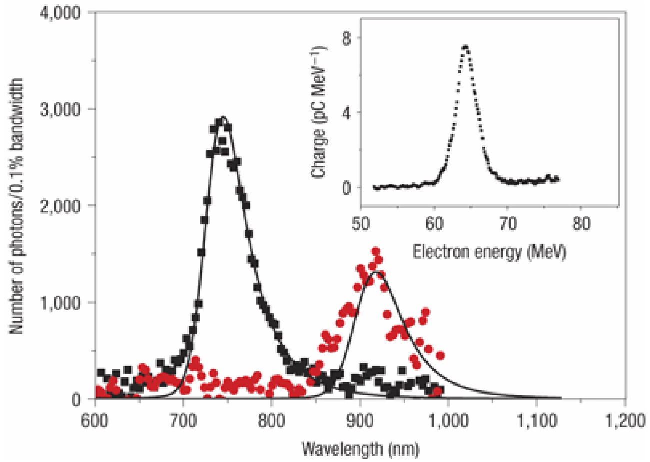


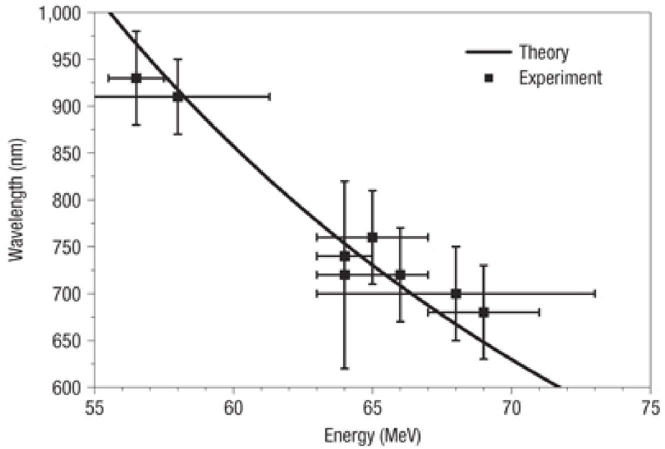
Undulator radiation from laser plasma acceleration

Univ. Jena
(Nature Physics 4 (2008), 130



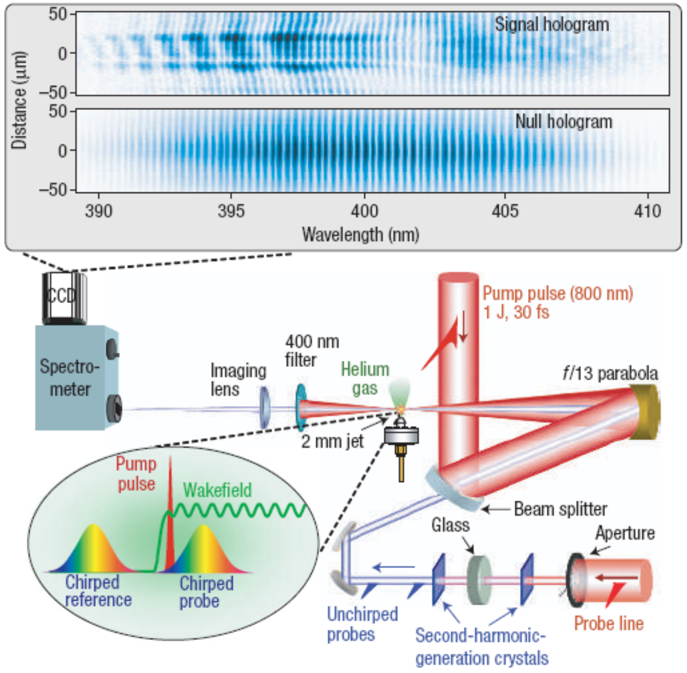
$$\lambda = \frac{\lambda_U}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

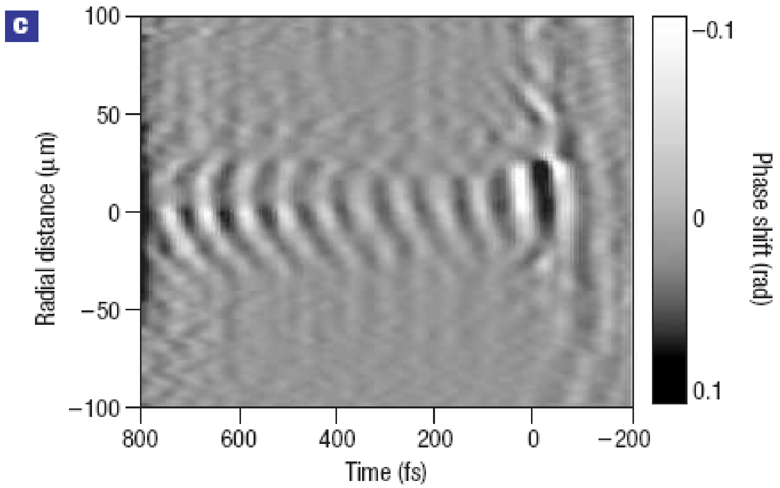


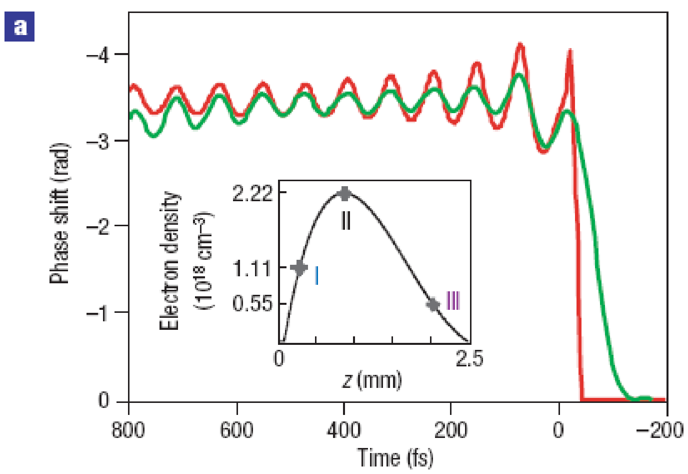


Snapshots of laser wake fields

N. H. Matlis et al.,
Nature Physics 2 (2006), 749



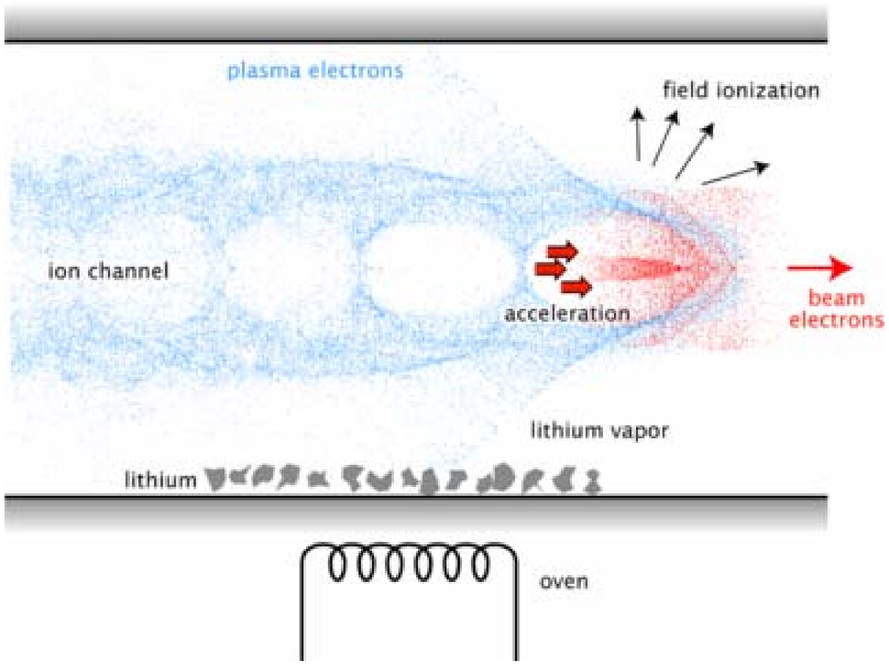




2.3 Bunch-driven plasma wake field acceleration (PWA)

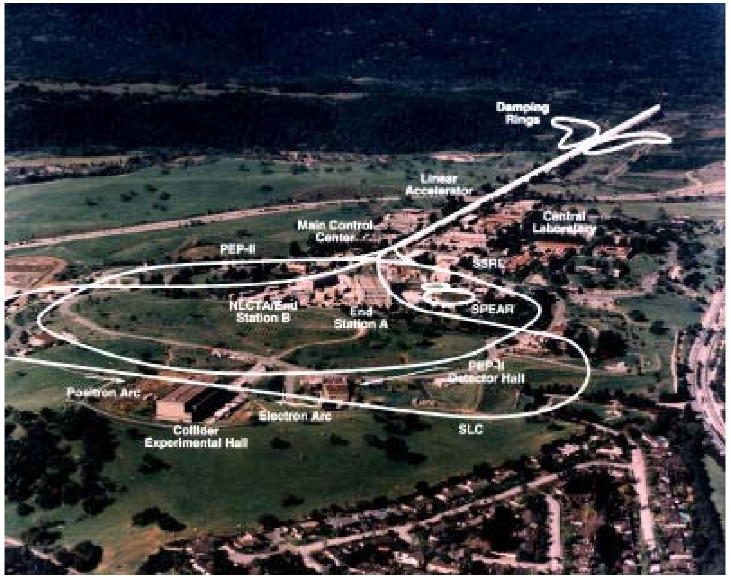
Argonne National Lab (USA) – late 1980s
Fermilab Batavia (USA) – 1990s
PBPL/UCLA Los Angeles (USA)
SLAC/Stanford Menlo Park (USA)
...

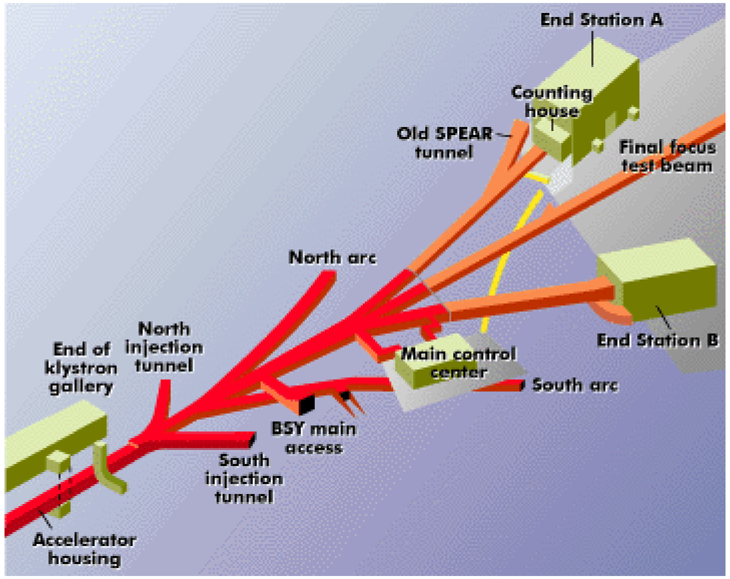
goal:
- double the energy of a linear accelerator („afterburner“)

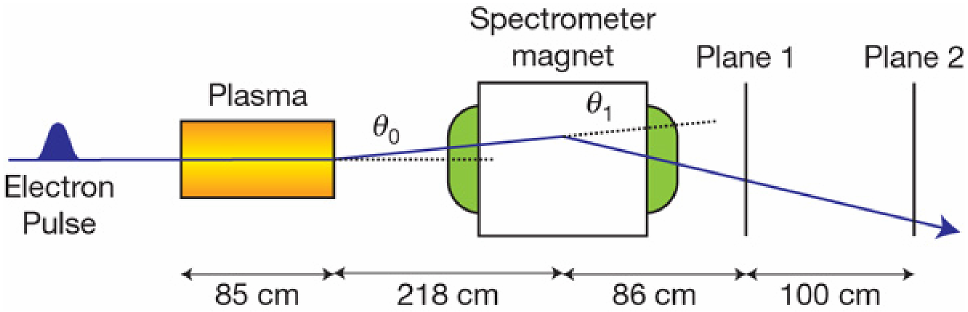


Experiment at the Final Focus Test Beam facility at SLAC

Nature 445 (2007), 741

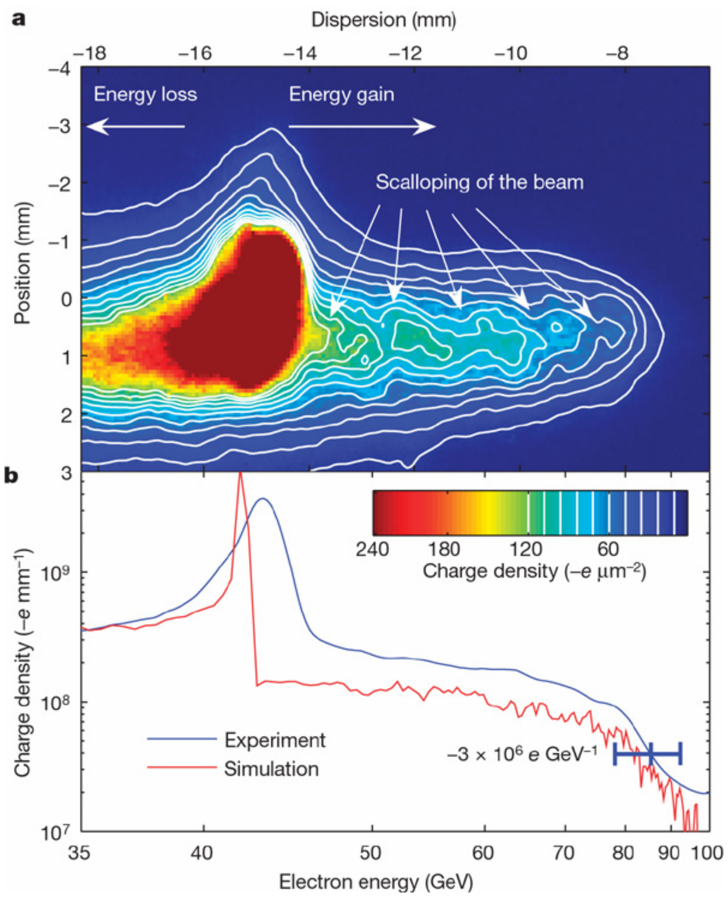






Experimental Results

Nature 445 (2007), 741



electron bunches

beam energy 42 GeV
bunch length 50 fs
bunch charge 2.9 nC

Li vapour

density 2.7e17 cm^{-3}

max. energy gain

43 GeV (85 cm column) = 52 GV/m
29 GeV (113 cm column)

Simulations

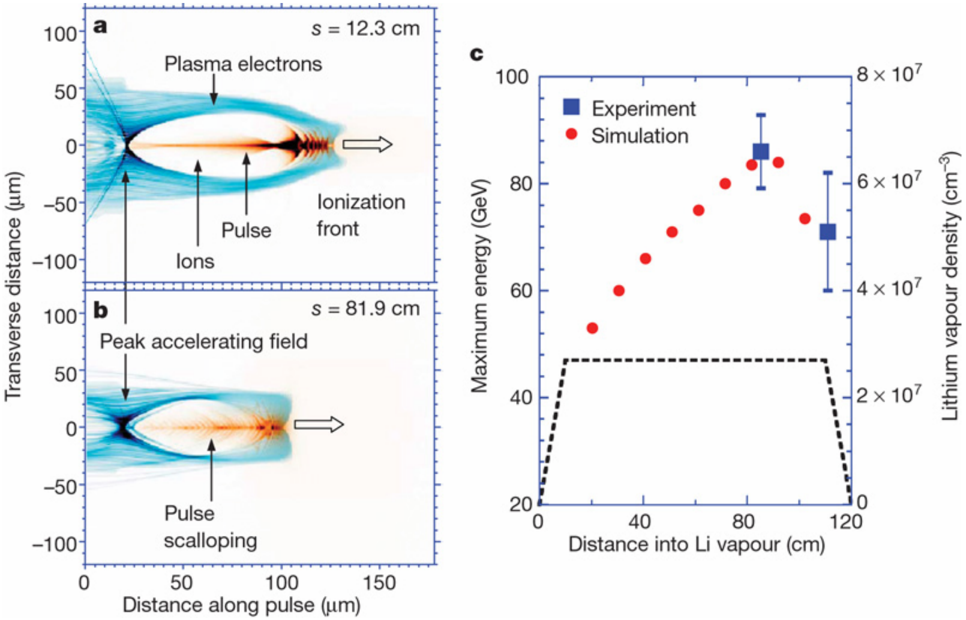
Nature 445 (2007), 741

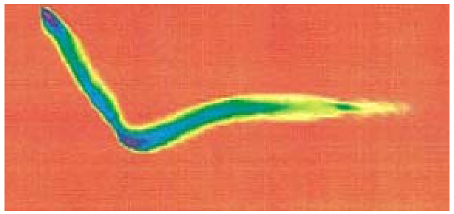
detrimental effects

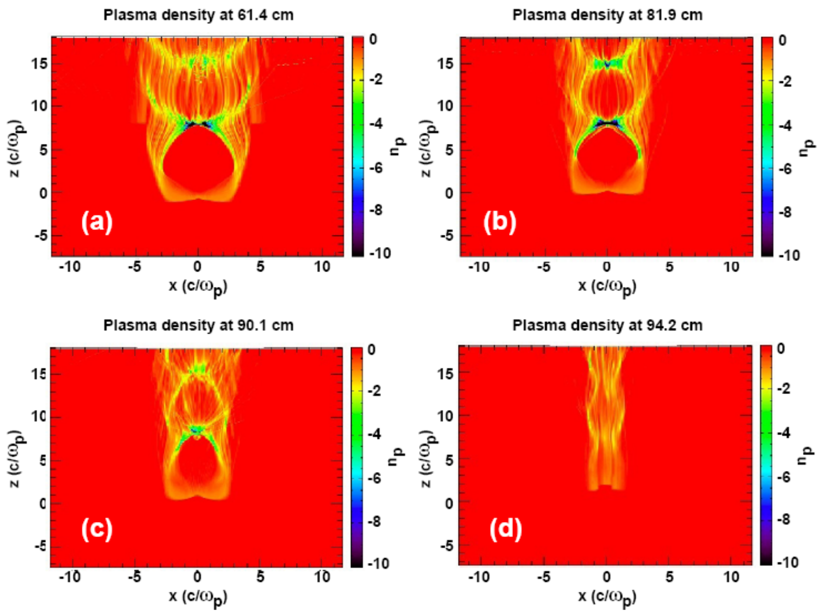
- energy depletion
  (by wake production)

- beam breakup
  (instability)

- head erosion
  (not focussed by the ion column)

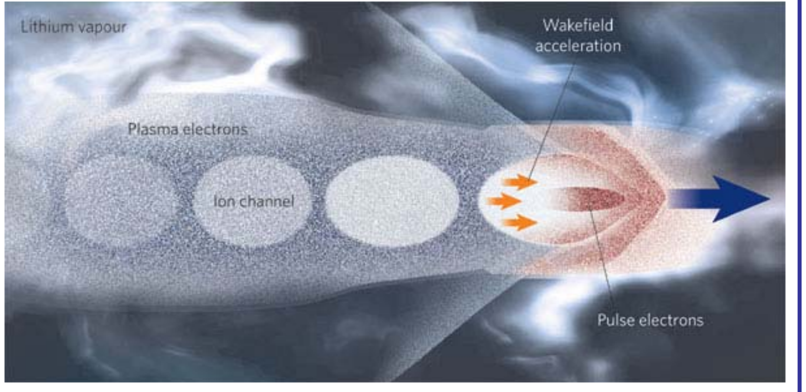






PWA – Status

- no „monoenergetic“ beams yet
- two-bunch experiments (only preliminary)



Example: proposed SLC afterburner

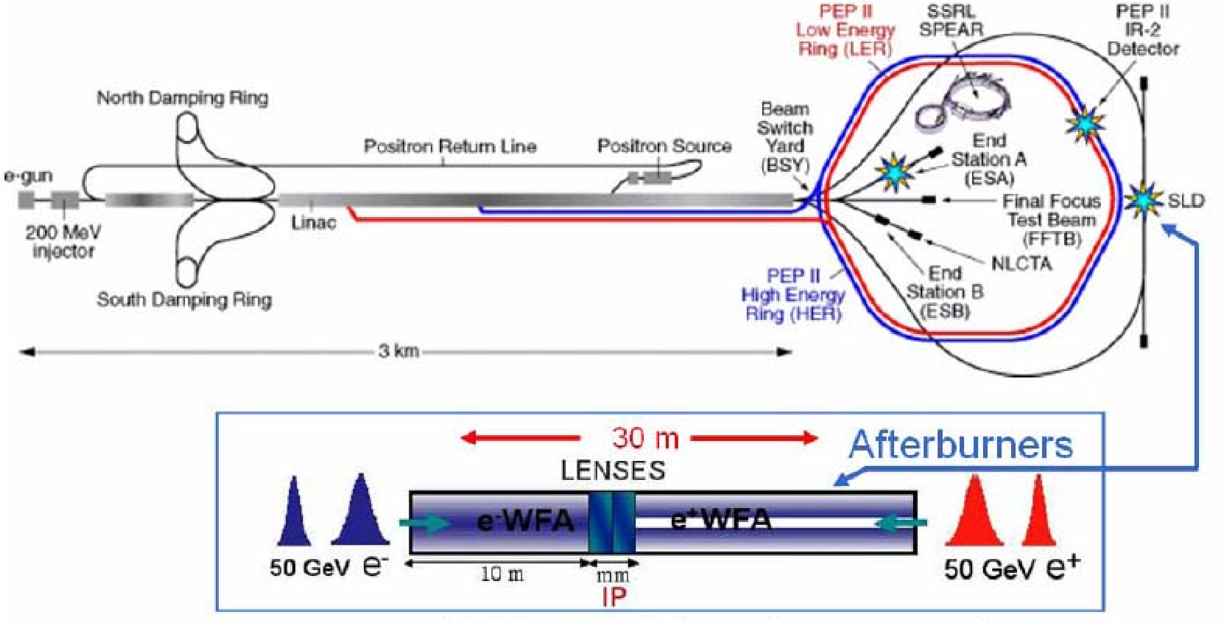


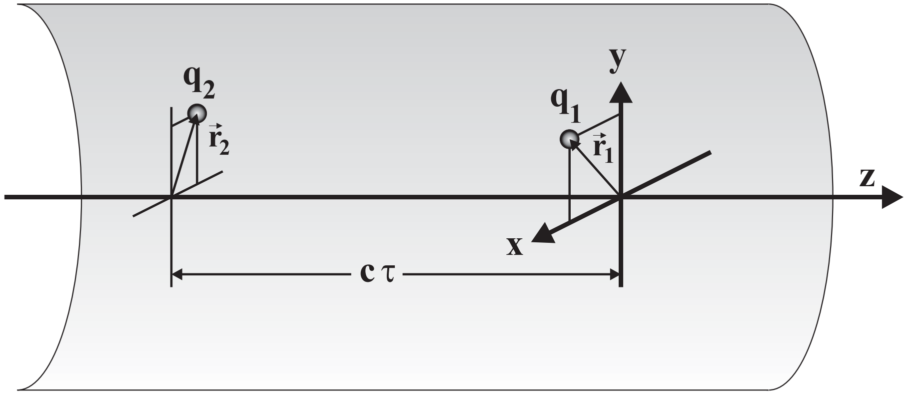
FIGURE 1. Schematic of the SLC Afterburner (from Tom Katsouleas).

T. Raubenheimer, Eleventh Advanced Accelerator Concepts Workshop, Stony Brook 2004

INTERMISSION

3. Two-beam (wake field) acceleration

3.1 Wake fields and impedance

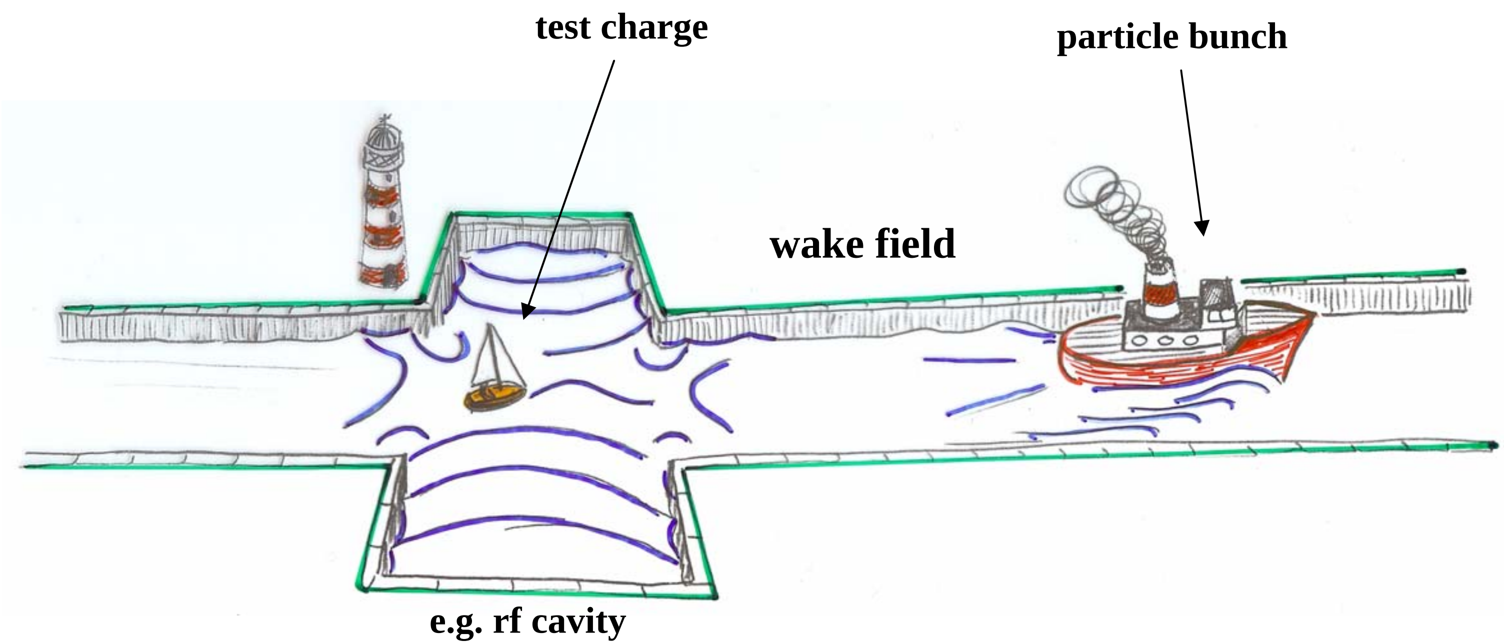


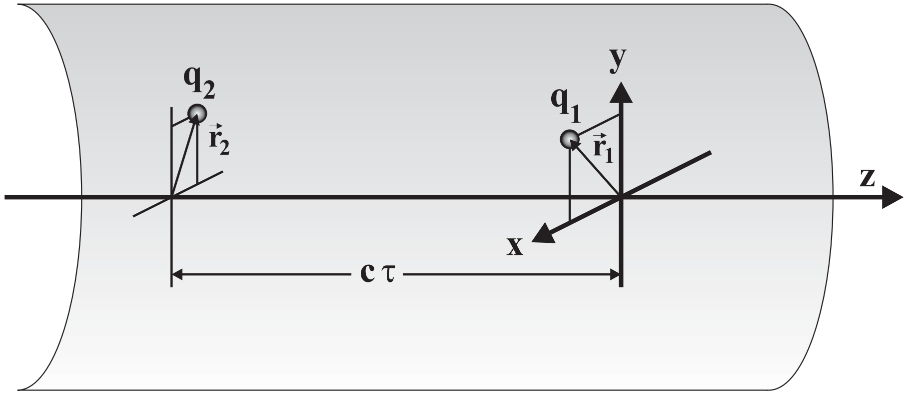
Force on a trailing charge 2 due to the presence of charge 1

$$F(r_1,s_1,r_2,s_2,t) = q_2\left\{E(r_1,s_1,r_2,s_2,t) + v \times B(r_1,s_1,r_2,s_2,t)\right\}$$

Wake function (general): time-integrated force per unit charges

$$W(r_1,r_2,\tau) = -\frac{c}{q_1}\int dt\left\{E(r_1,r_2,\tau,t) + v \times B(r_1,r_2,\tau,t)\right\}$$


test charge
particle bunch
wake field
e.g. rf cavity



Force on a trailing charge 2 due to the presence of charge 1

$$F\left(r_1,s_1,r_2,s_2,t\right)=q_2\left\{E\left(r_1,s_1,r_2,s_2,t\right)+v\times B\left(r_1,s_1,r_2,s_2,t\right)\right\}$$

Wake function (general): time-integrated force per unit charges

$$W\left(r_1,r_2,\tau\right)=-\frac{c}{q_1}\int dt\left\{E\left(r_1,r_2,\tau,t\right)+v\times B\left(r_1,r_2,\tau,t\right)\right\}$$

Longitudinal wake function

$$W_{\parallel}\left(r_1,r_2,\tau\right)=-\frac{\Delta U}{q_1q_2}=-\frac{c}{q_1}\int dtE_z\left(r_1,r_2,\tau,t\right)$$

Transverse wake function

$$W_{\perp}\left(r_1,r_2,\tau\right)=-\frac{c}{q_1}\int dt\left\{E_{\perp}\left(r_1,r_2,\tau,t\right)+\left[v\times B\left(r_1,r_2,\tau,t\right)\right]_{\perp}\right\}$$

Wake function

Time integrated force caused by a pointlike unit charge, acting on a trailing pointlike unit charge

Wake potential

Time integrated force caused by an extended unit charge, acting on a trailing pontlike unit charge

Expample: longitudinal wake potential (superposition principle)

$$V_{\parallel}(\tau) = \int_{-\infty}^{\infty} dt \; W_{\parallel}(t) \, j(\tau - t)$$

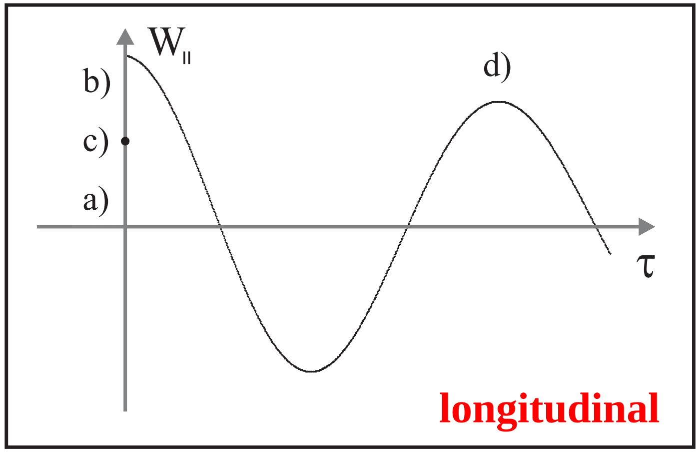
Impedance

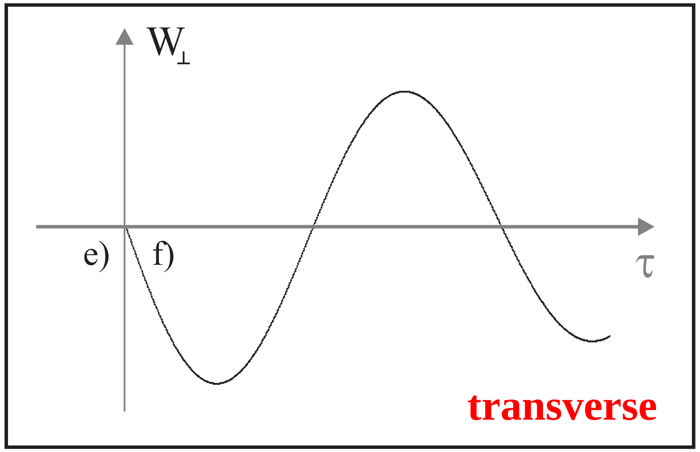
Fourier transform of the wake function

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} d\tau W_{\parallel}(\tau) \exp(-i\omega\tau)$$

$$Z_{\perp}(\omega) = i \int_{-\infty}^{\infty} d\tau W_{\perp}(\tau) \exp(-i\omega\tau)$$

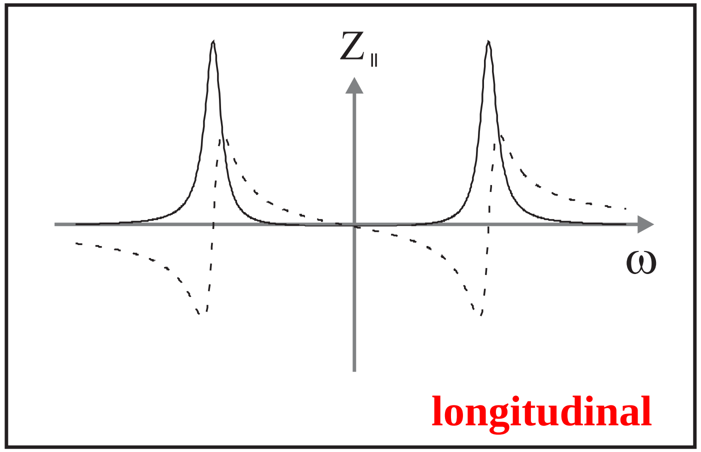
Properties of wake functions

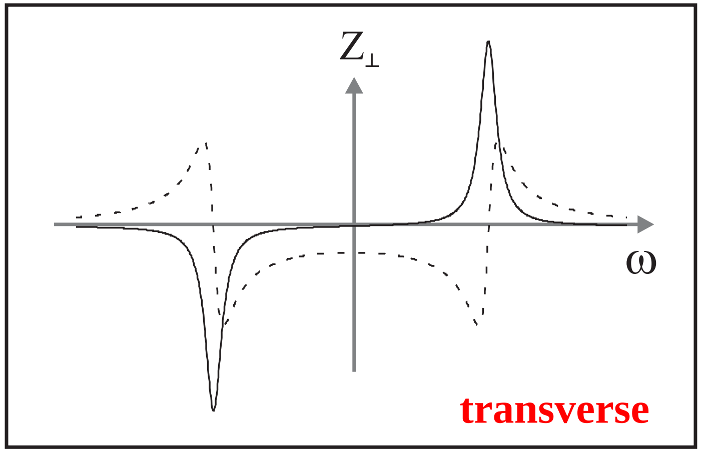




- a) „causality“: zero at $\tau<0$
- b) Long. wake function at $\tau=0+$ positiv (energy loss)
- c) Long. wake function at $\tau=0$ is 1/2 of that at $\tau=0+$ „fundamental theorem of beam loading“
- d) Long. wake function never larger than at $\tau=0+$
- e) Trans. wake function is zero at $\tau=0$
- f) Trans. wake function negativ for small τ
- g) Trans. wake function has maximum at zeros of the long. wake function (trans.=sin-like; long.=cos-like)

Properties of impedance

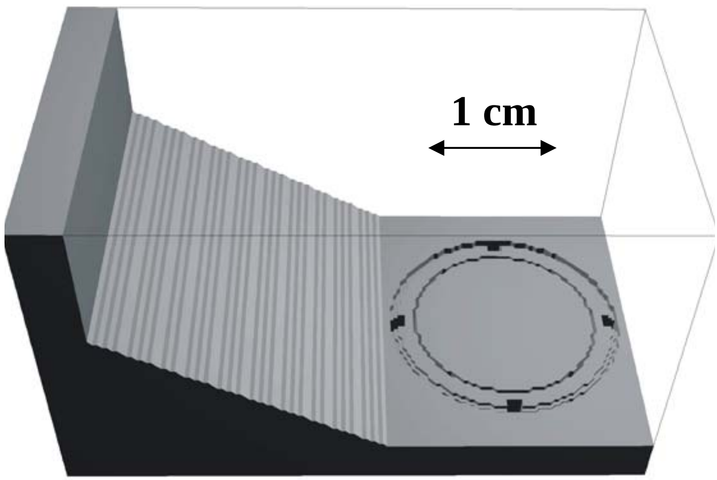


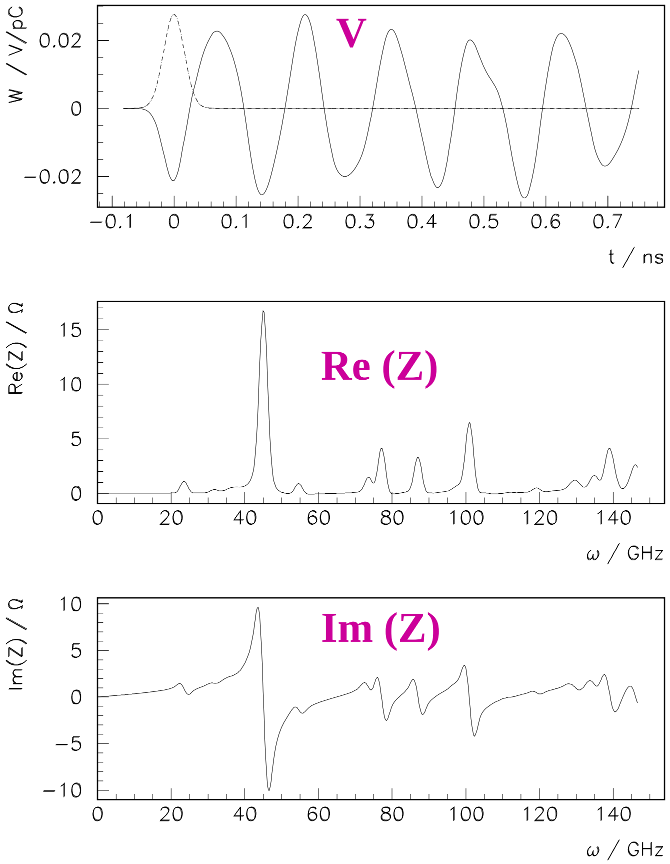


a) Long. impedance symmetric about 0, trans. impedance anti-symmetric
b) Long. impedance positiv, trans. impedance positiv for $\omega>0$, negativ for $\omega<0$
c) real und imaginary part are not independent
(maximum of real part = zero of imaginary part)
d) Wake function can be calculated from the real or imaginary part alone

Example: beam position monitor

calculated in small time steps





$$V_{\parallel}(\tau) = \int_{-\infty}^{\infty} dt\ W_{\parallel}(t)\, j(\tau - t)$$

$$Z_{\parallel}(\omega) = \frac{\tilde{V}(\omega)}{J(\omega)} = \frac{\int_{-\infty}^{\infty} d\tau V_{\parallel}(\tau)\exp(-i\omega\tau)}{J(\omega)}$$

Impedance (Wake) and beam instability

Particle bunches perform oscillations
- transverse: betatron oscillation
- longitudinal: synchrotron oscillation

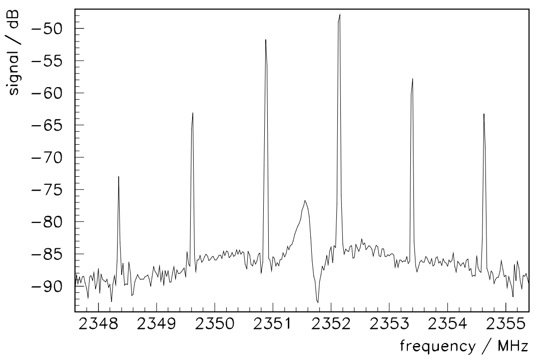
$$\propto \exp\left(i\{\omega + \Delta\omega\}t\right)$$

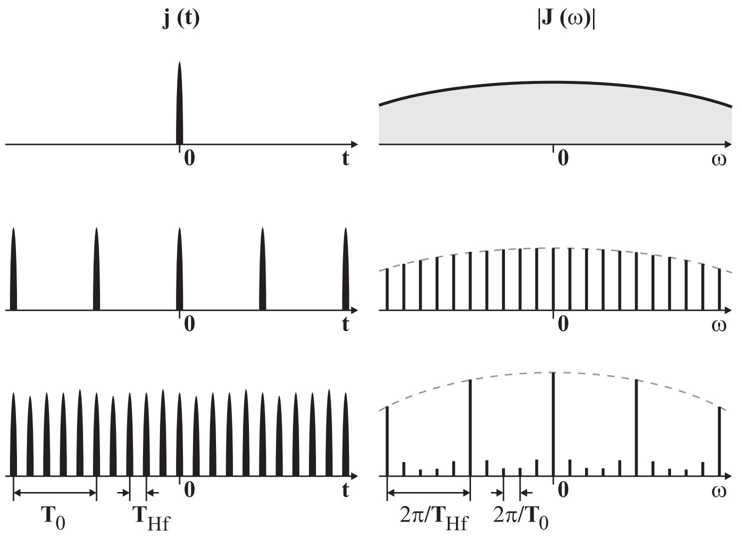
Real part of impedance causes*)
- imaginary frequency shift
- increases (or damps) oscillation

Imaginary part of impedance causes*)
- real frequency shift
- changes the frequency

***) similar concepts of impedance in electricity, mechanics, acoustics**

Impedance is potentially harmful, when beam spectrum and impedance overlap





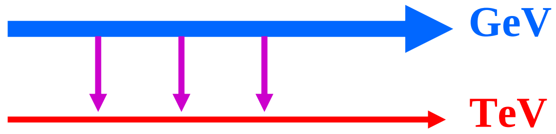
wake fields can also be used for acceleration ...

2.2 Acceleration concept

power = voltage (beam energy) x current

(i) drive beam: low energy, high current

(ii) main beam: high energy, low current



How to transfer the power from the drive beam to the main beam?

basic concepts (W. Schnell, E. Sessler, ... 1980s)

- accelerate drive beam with induction linac, rf generation by FEL radiation
- accelerate drive beam by s.c. cavities, rf generation by (n.c.) cavities

s.c. cavities limited to ~50 MV/m

design goal 100-150 MV/m

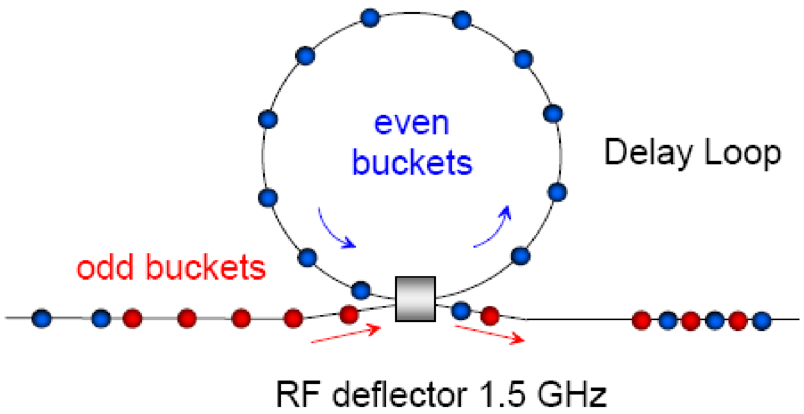
Drive beam: high beam current

- high bunch charge (prone to instabilities)

- many bunches and high rf frequency (at CLIC: initially 30 GHz, now 12 GHz)

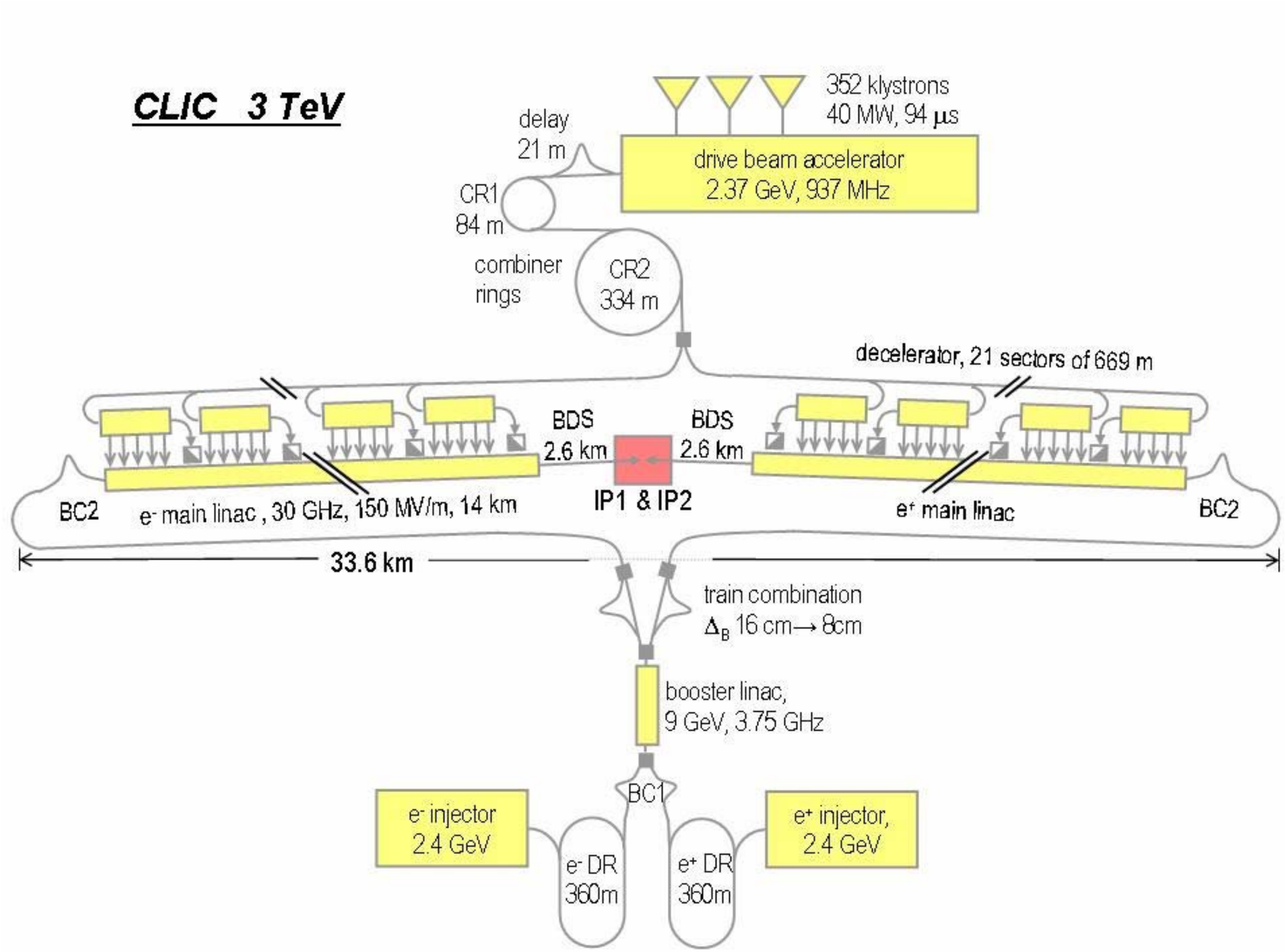
Linac with low (feasible) rf frequency + combiner ring

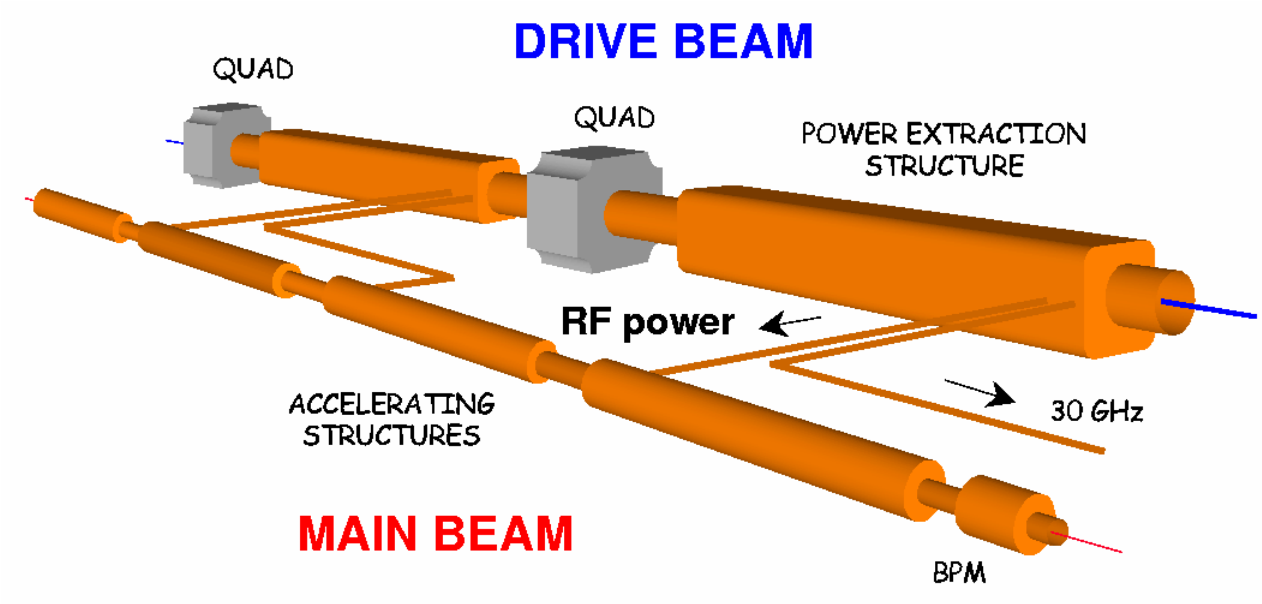
from long, low-current
to short, high-current bunch train



2.3 CLIC at CERN (Compact LInear Collider Study)

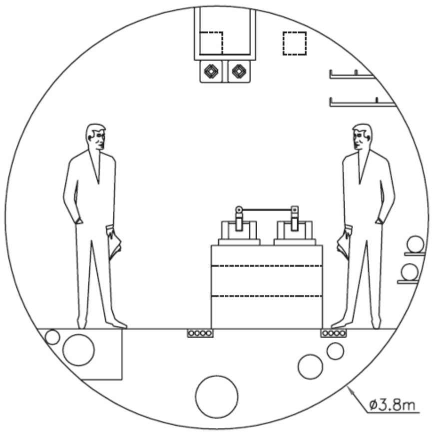
(now 42 km)





Energy stored in the drive beam

- no power-generating devices (klystrons, ...) in the tunnel



Recent developments

- two-beam acceleration demonstrated at CTF II (CLIC test facility) ~ 200 MV/m

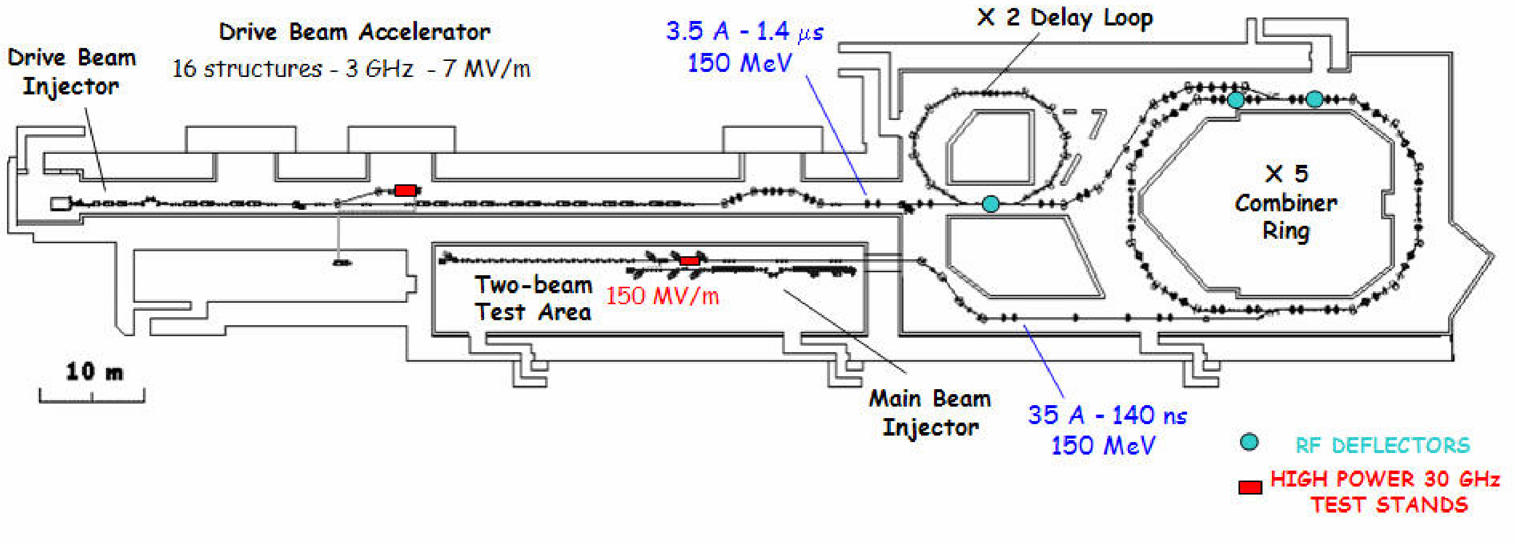
- construction of CTF3

- CLIC parameters reconsidered

Rf frequency from 30 to 12 GHz

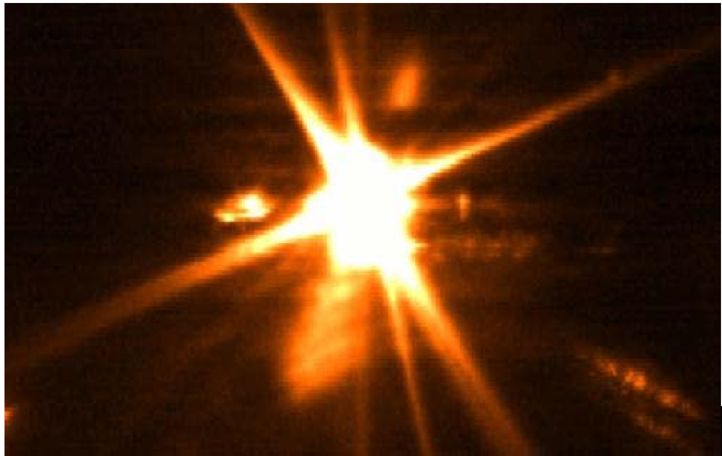
Gradient from 150 to 100 MV/m

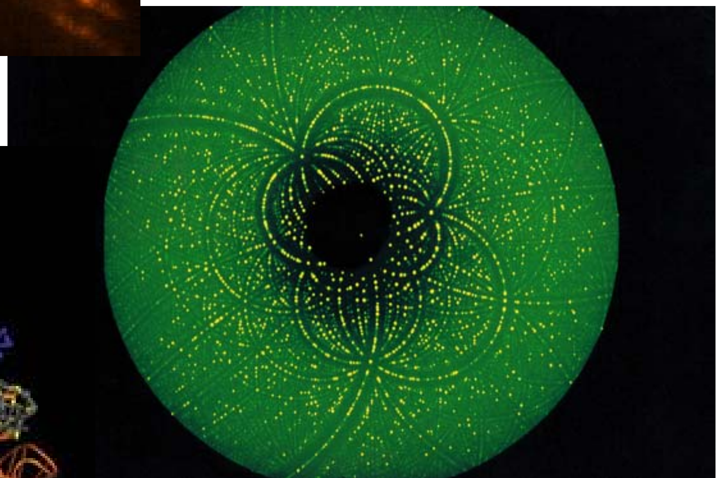
Linac length from 34 to 42 km

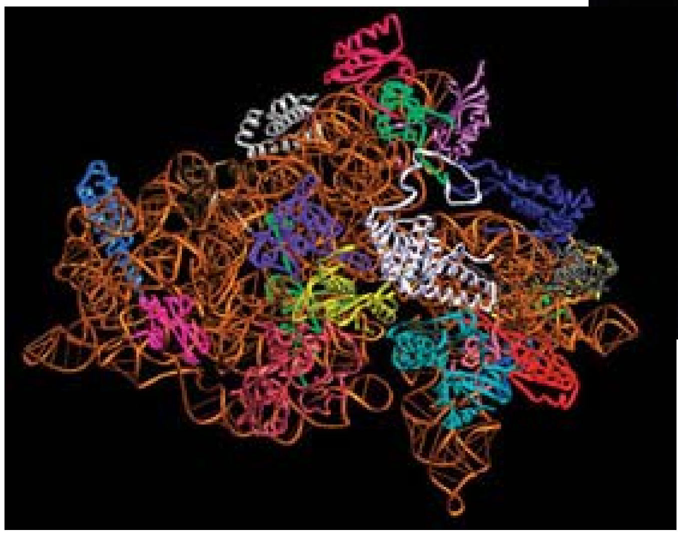


4 Inverse free-electron lasers

4.1 Synchrotron radiation

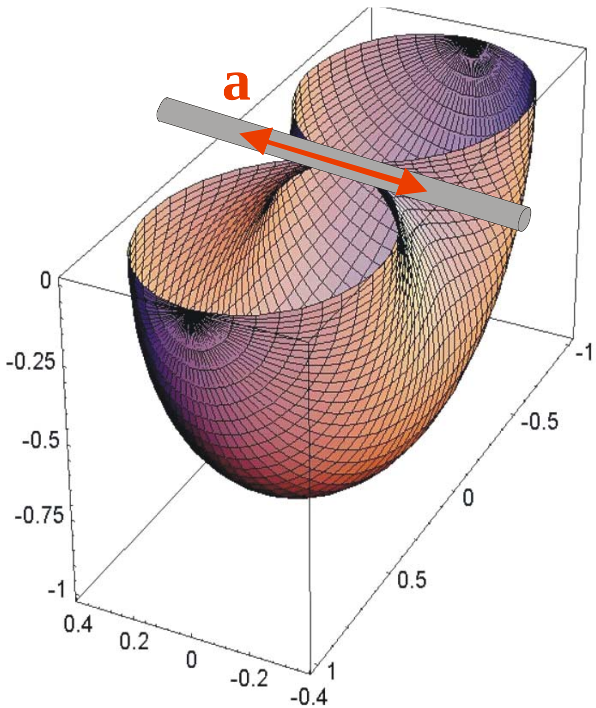




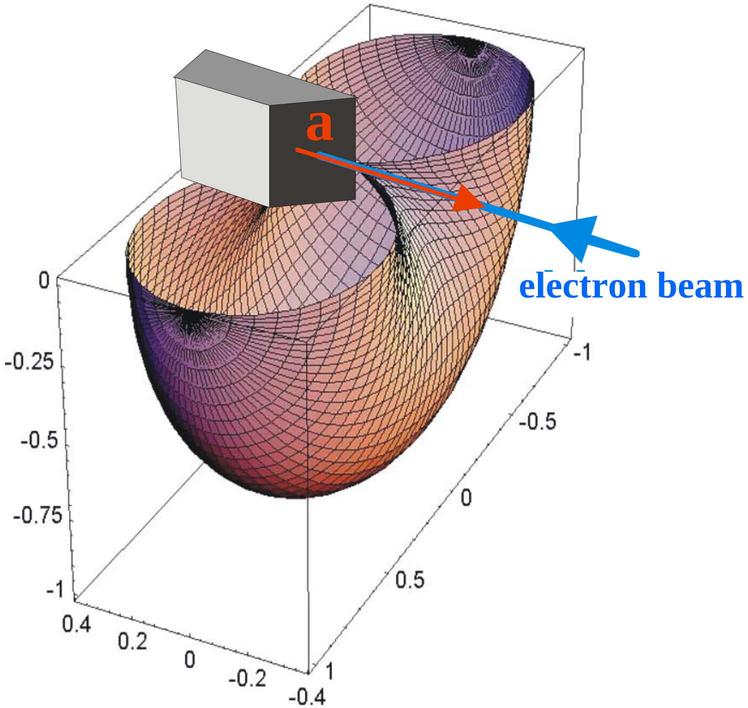


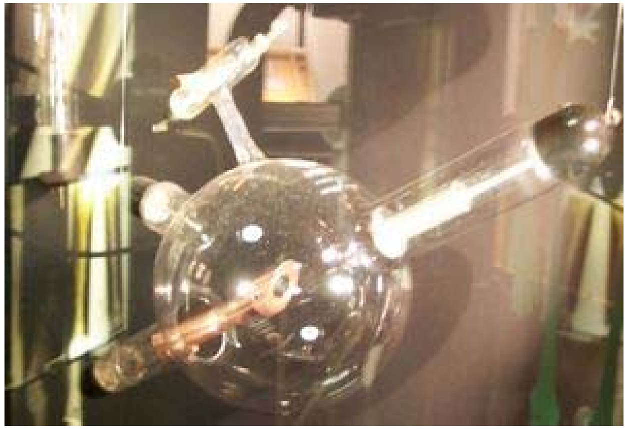
Radiation from accelerated electrons

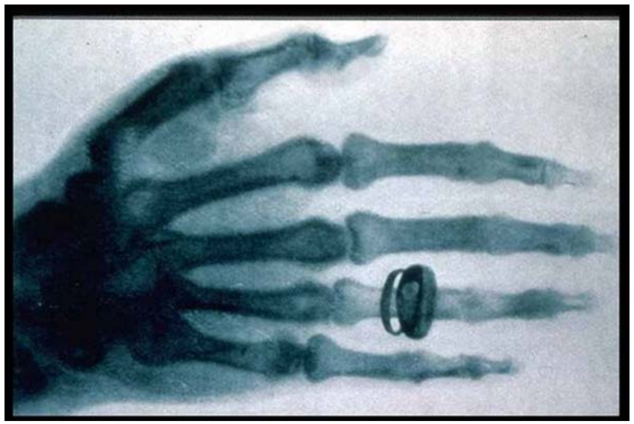
radio transmitter



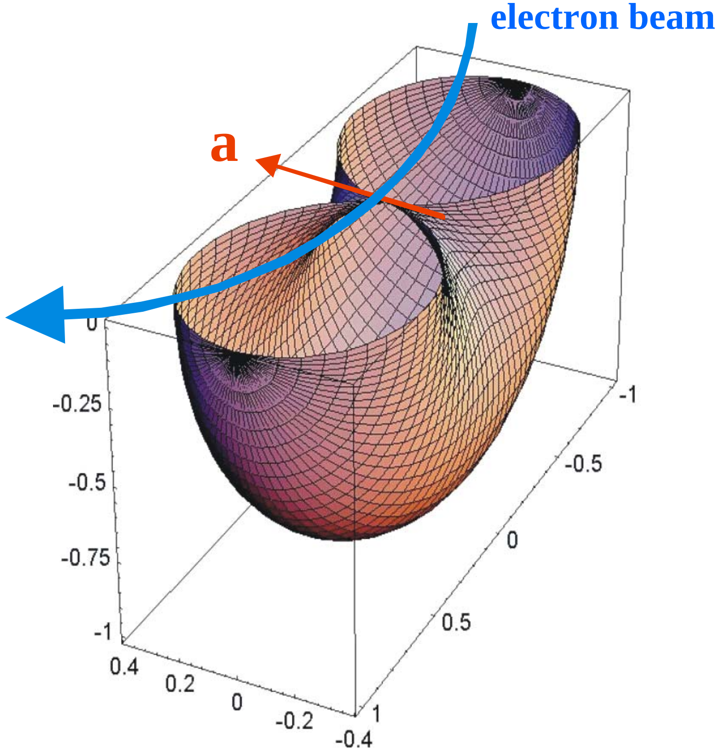
x-ray tube

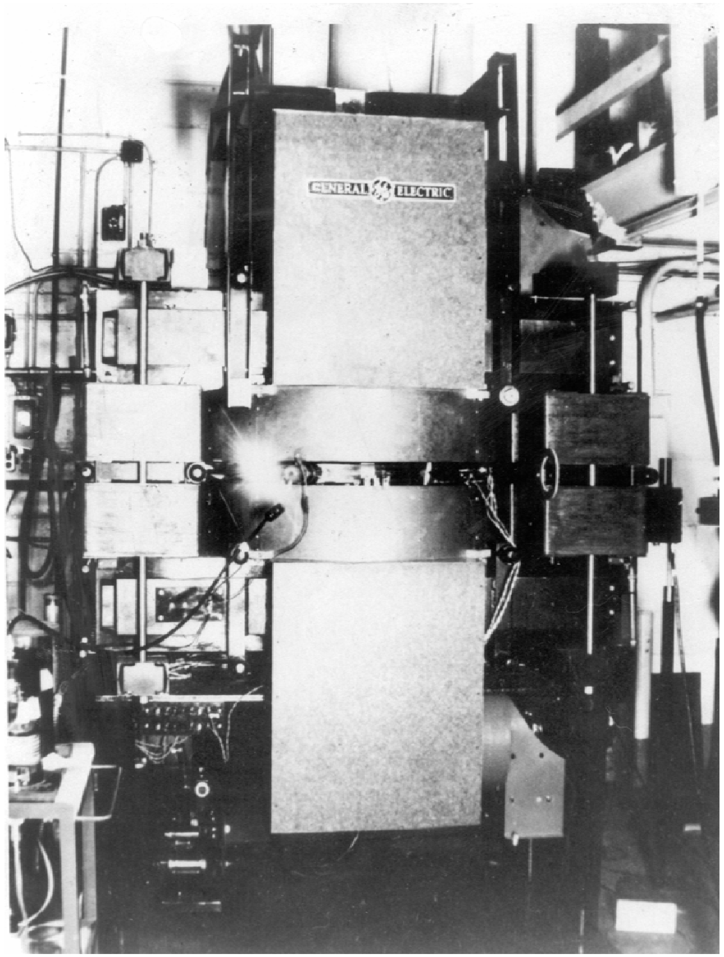






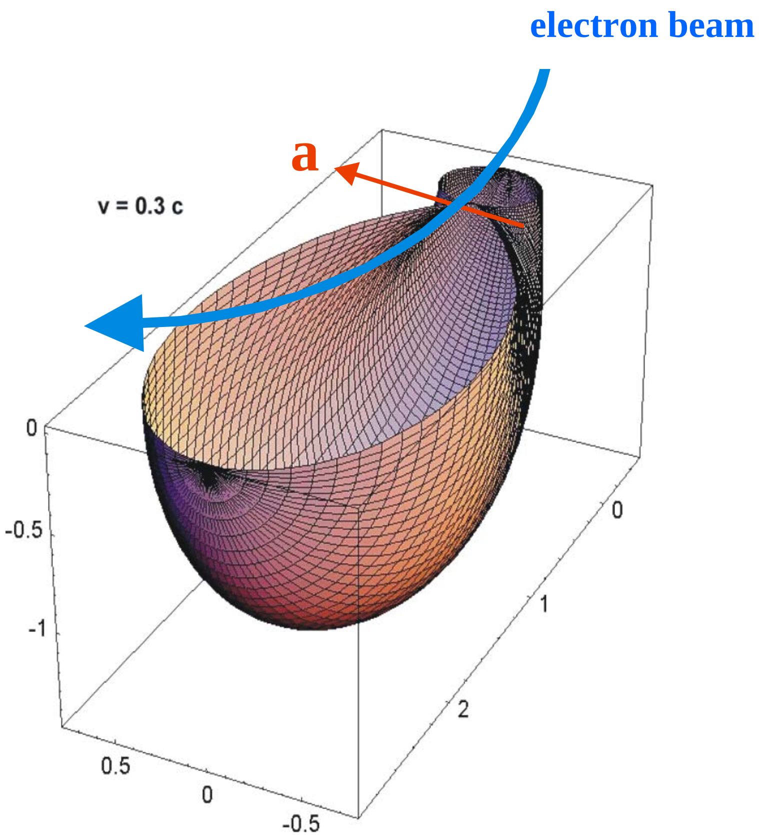
synchrotron radiation source



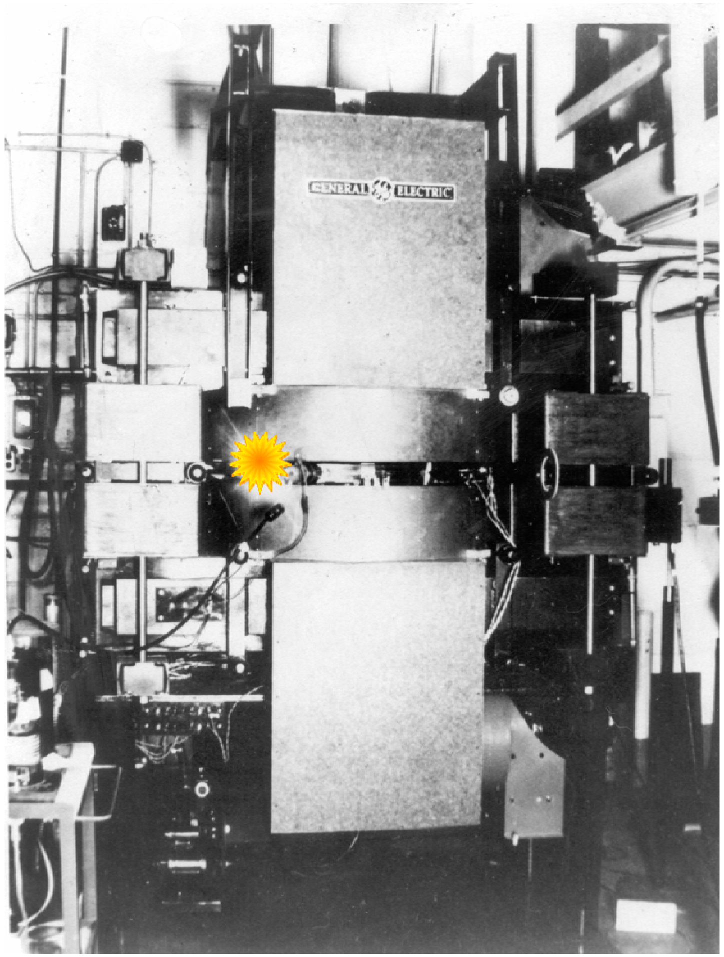


(co-moving system)

synchrotron radiation source



(lab system)



Theory of synchrotron radiation

Consider the magnetic vector potential *A* and electric scalar potential *φ*

Wave equations ... **... and their solution „ansatz“**

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \frac{1}{\varepsilon_0}\frac{\vec{v}\rho}{c} \quad \leftarrow \quad \vec{A}(t) = \frac{1}{4\pi c^2 \varepsilon_0}\int \left.\frac{\vec{v}\rho(x,y,z)}{R}\right|_{t_r} dxdydz$$

$$\nabla^2 \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{1}{\varepsilon_0}\rho \quad \leftarrow \quad \varphi(t) = \frac{1}{4\pi c^2 \varepsilon_0}\int \left.\frac{\rho(x,y,z)}{R}\right|_{t_r} dxdydz$$

„retarded“ time **distance charge-observer** **unit vector**

$$t_r = t - \frac{R(t_r)}{c} \qquad \vec{R} = (x_r - x, y_r - y, z_r - z) \qquad \vec{n} = \frac{\vec{R}}{R}$$

consequence $\int \left.\frac{\rho}{R}\right|_{t_r} dxdydz \neq \frac{q}{R}$ $\rightarrow$ $A(P,t) = \frac{1}{4\pi c^2 \varepsilon_0}\frac{q}{R}\left.\frac{\vec{\beta}}{1+\vec{n}\vec{\beta}}\right|_{t_r}$

leads to Liénard-Wiechert potentials
and after a few trivial manipulations ... $\rightarrow$ $\varphi(P,t) = \frac{1}{4\pi c^2 \varepsilon_0}\frac{q}{R}\left.\frac{1}{1+\vec{n}\vec{\beta}}\right|_{t_r}$

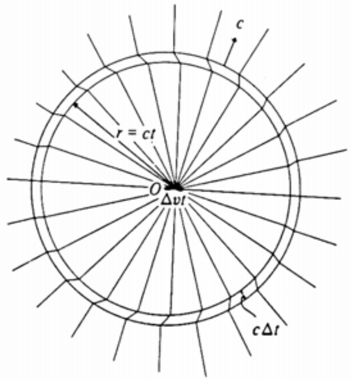
$$4\pi\varepsilon_0 \frac{\vec{E}(t)}{q} = \frac{1-\beta^2}{r^3}\left(\vec{R}+R\vec{\beta}\right)_r + \frac{1}{cr^3}\left.\left(\vec{R}\times\left[\left(\vec{R}+R\vec{\beta}\right)_r \times \frac{d\vec{\beta}}{dt_r}\right]\right)\right|_r$$

and likewise for the magnetic field

~1/R² Coulomb regime **~1/R radiation regime**

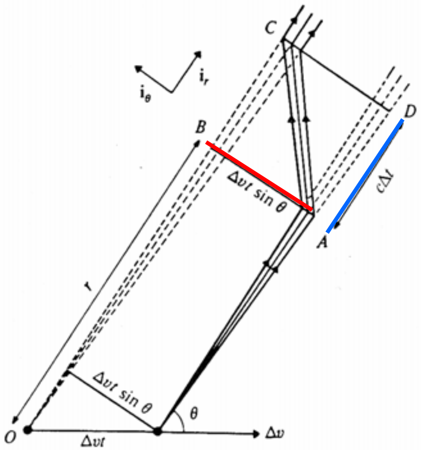
J. J. Thomson's argument

$$\frac{E_\theta}{E_r} = \frac{\Delta v \cdot t \cdot \sin\theta}{c \cdot \Delta t}$$



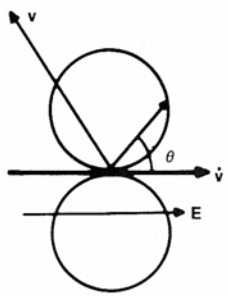
$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{q}{4\pi\varepsilon_0 \cdot r \cdot c \cdot t}$$ **Coulomb**

$$E_\theta = \frac{q}{4\pi\varepsilon_0 \cdot r \cdot c \cdot t} \cdot \frac{\Delta v \cdot t \cdot \sin\theta}{c \cdot \Delta t} = \frac{q \cdot \ddot{r} \cdot \sin\theta}{4\pi\varepsilon_0 \cdot c^2 \cdot r}$$



Energy flux per time into solid angle *dΩ*

$$\frac{dW}{dt} d\Omega = \varepsilon_0 \cdot c \cdot E^2 d\Omega = \frac{q^2 \cdot \ddot{r}^2 \cdot \sin^2\theta}{16\pi^2\varepsilon_0 \cdot c^3}$$



$$\frac{dW}{dt} = \frac{q^2 \cdot \ddot{r}^2}{6\pi \cdot \varepsilon_0 \cdot c^3}$$ **Larmor's formula**

Synchrotron radiation from bending (dipole) magnets

typical half-opening angle

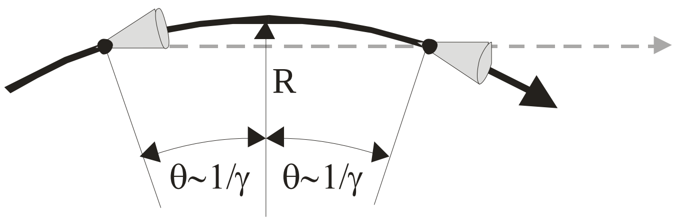
$$\theta \approx \frac{1}{\gamma}$$

total radiated power

$$P \propto \frac{1}{(mc^2)^4} \frac{E^4}{R^2}$$

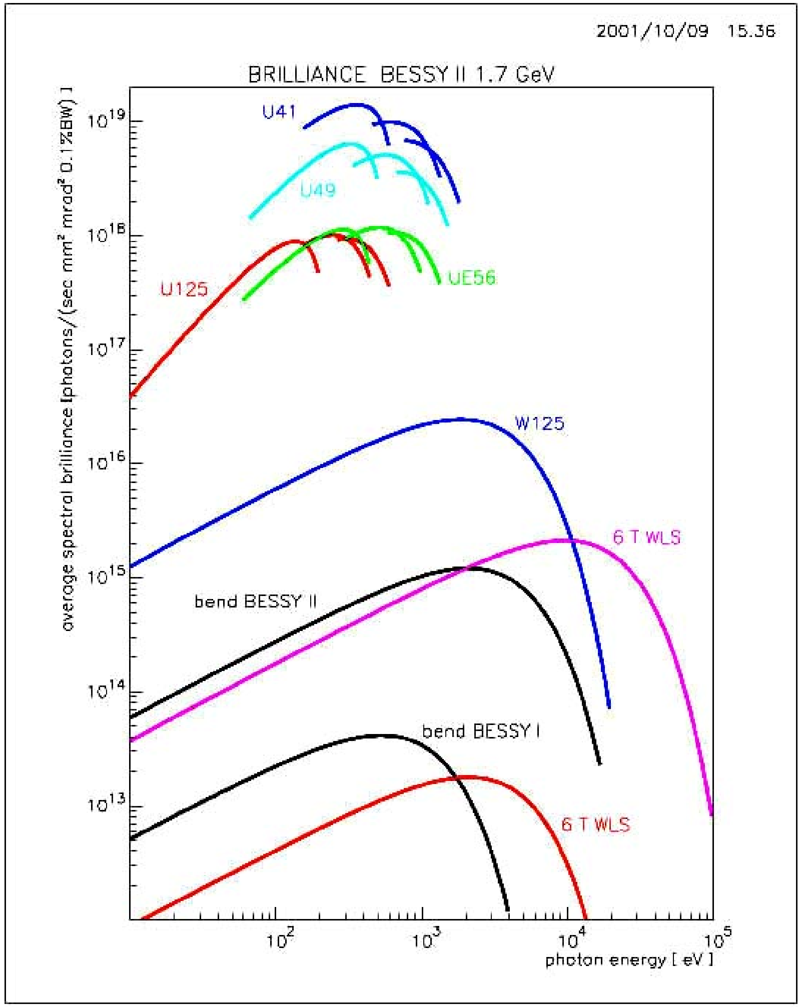
spectrum (angle integrated)

$$\frac{dP}{d\omega} \propto \frac{E^4}{R^2} \cdot S\left(\frac{E_{\text{photon}}}{E_{\text{photon}}^{\text{crit}}}\right)$$



qualitative estimate of the spectrum

$$E_{\text{typical}} \cdot \Delta t \approx h$$

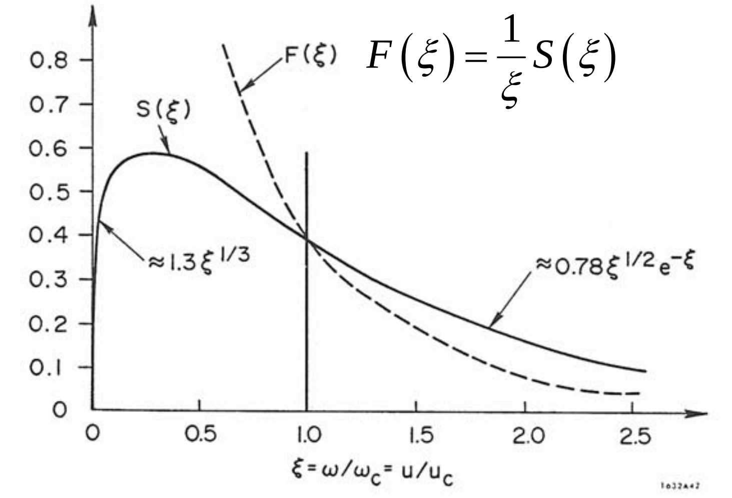


spectral brilliance (or brightness)

$$= \frac{\text{photons / second}}{\text{mm}^2 \ \text{mrad}^2 \ 0.1\% \text{ bandwidth}}$$

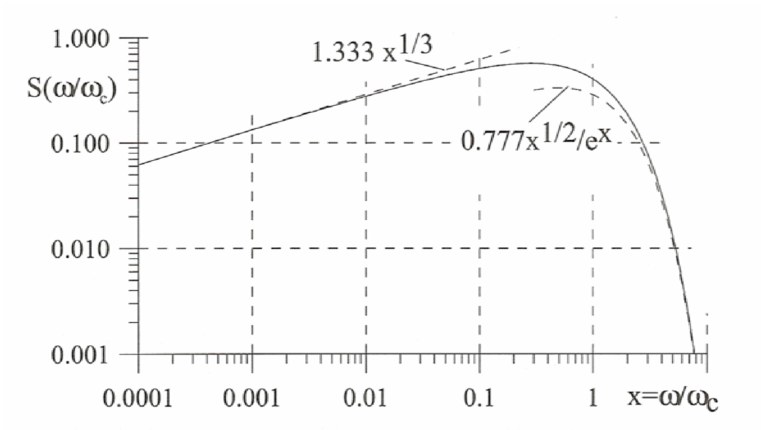
Spectrum from bending magnet (integrated over angles)

linear plot



D. Carey, The Optics of Charged Particle Beams

double-logarithmic plot



H. Wiedemann, Synchrotron Radiation

$$\frac{dP}{d\omega} \propto \frac{E^4}{R^2} S\left(\frac{\omega}{\omega_c}\right)$$

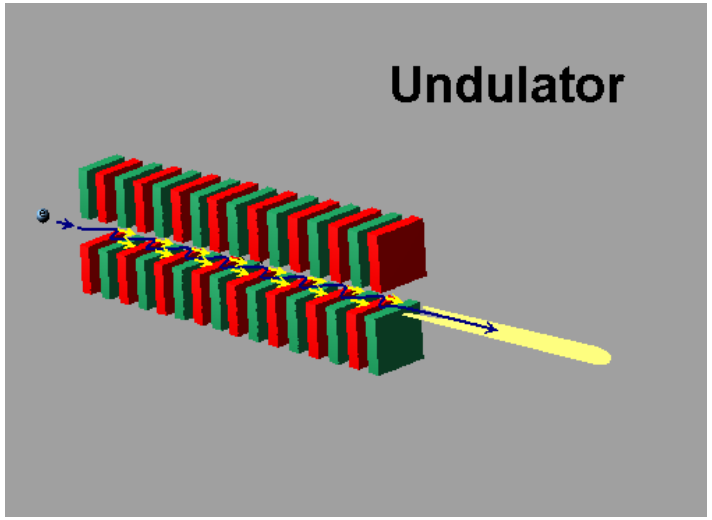
where

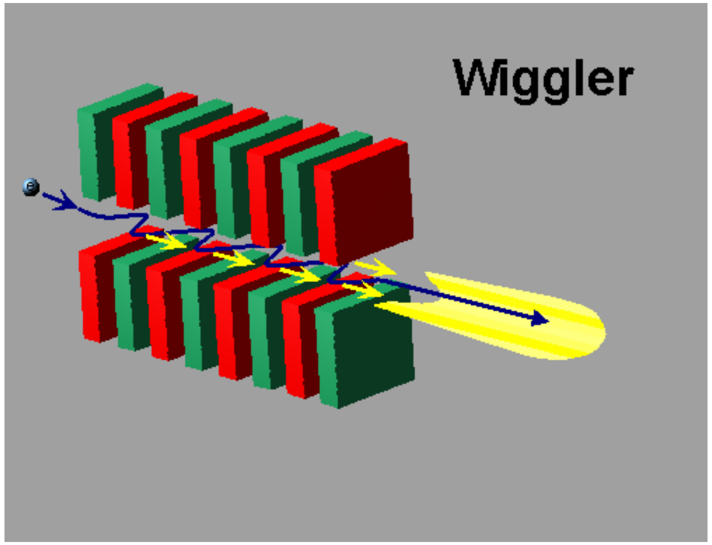
$$S\left(\frac{\omega}{\omega_c}\right) = \frac{9\sqrt{3}}{8\pi}\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(x)\,dx$$

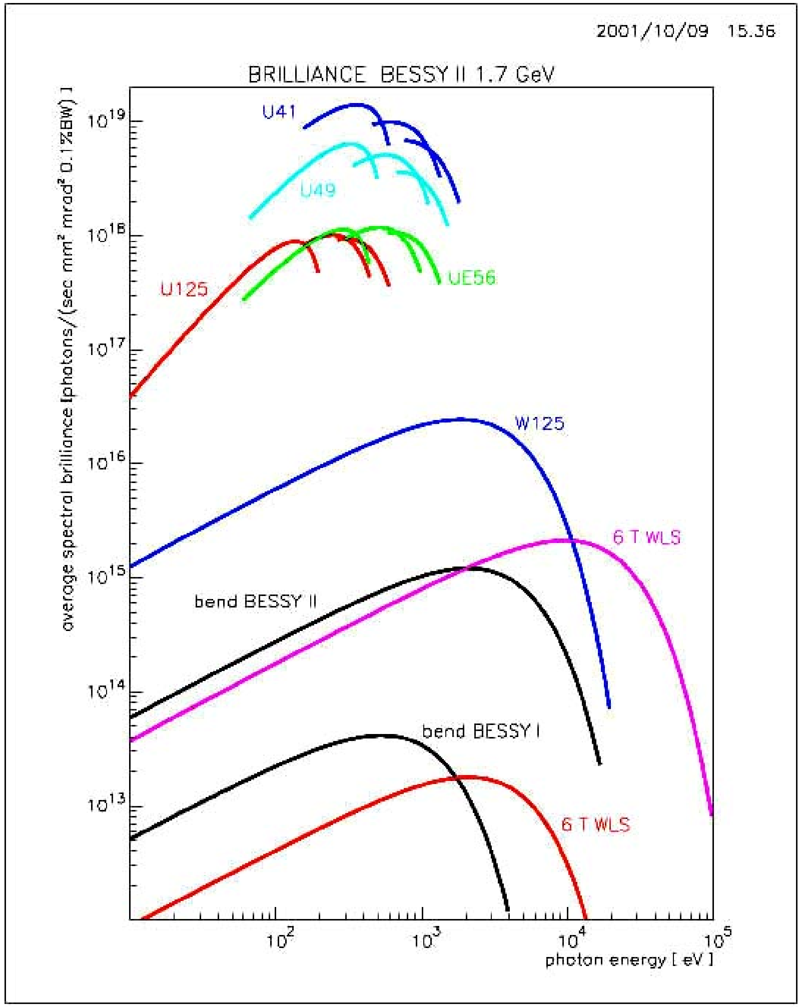
$$P_\sigma = \frac{7}{8}P_\gamma \qquad P_\pi = \frac{1}{8}P_\gamma$$

horizontal / vertical polarization

Wigglers and undulators





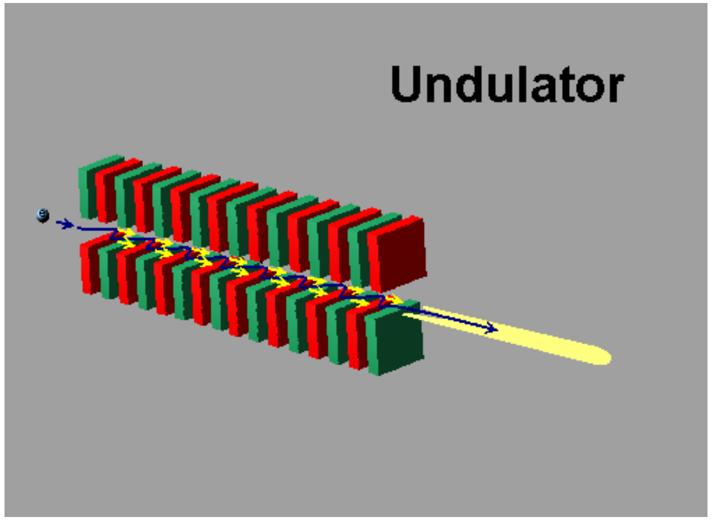


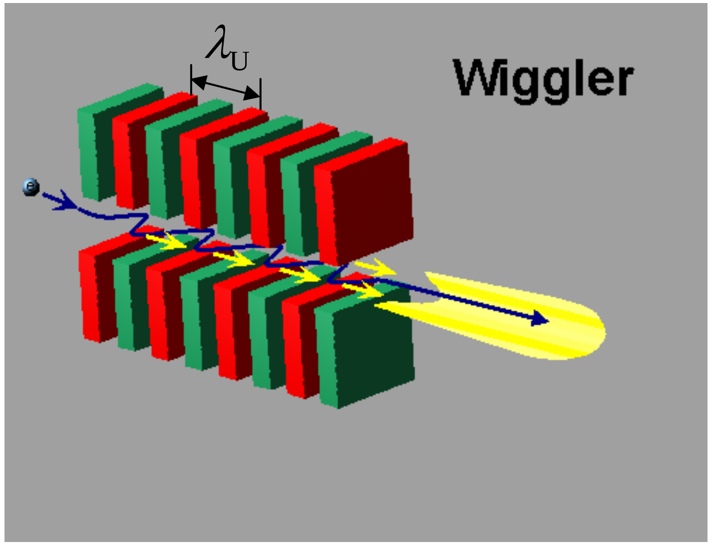
spectral brilliance (or brightness)

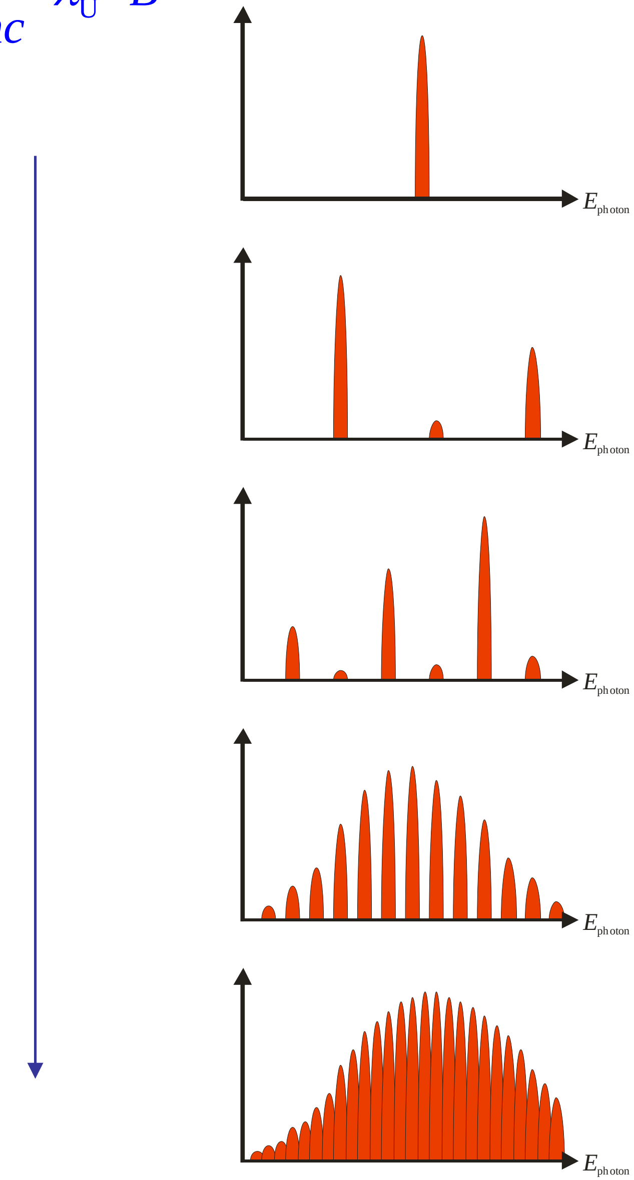
$$= \frac{\text{photons / second}}{\text{mm}^2\ \text{mrad}^2\ 0.1\%\ \text{bandwidth}}$$

strength parameter *K*

$$K \equiv \frac{e}{2\pi mc} \cdot \lambda_U \cdot B$$







Radiation from undulators

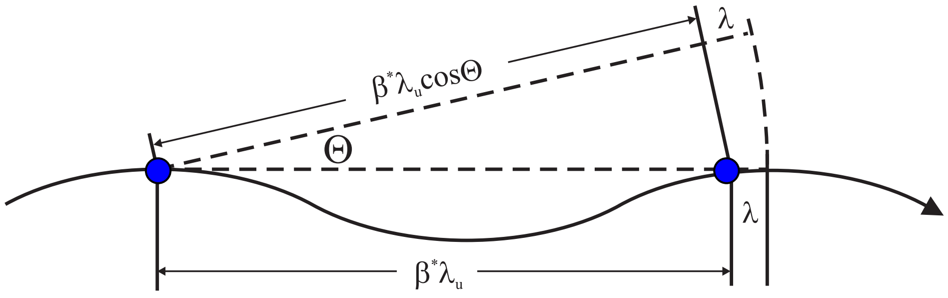
typical half-opening angle $$\theta \approx \frac{1}{\gamma\sqrt{N}}$$

undulator line width $$\frac{\Delta \mathrm{E}}{\mathrm{E}} \approx \frac{1}{N}$$

first-harmonic wavelength $$\lambda = \frac{\lambda_{\mathrm{U}}}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

$$K \equiv \frac{e}{2\pi mc} \cdot \lambda_{\mathrm{U}} \cdot B$$

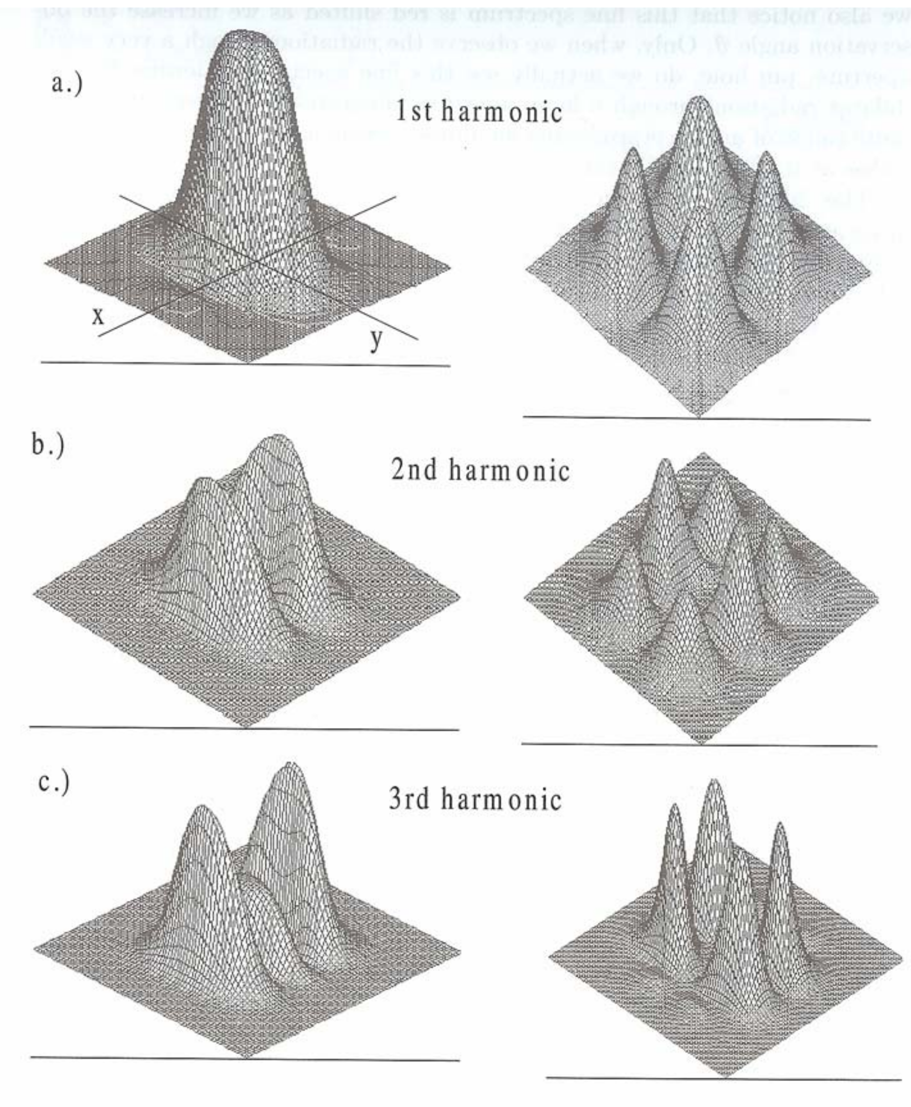
$$\text{max. angle} = \frac{\mathrm{K}}{\gamma}$$



electrons lag behind the radiation by one wavelength per undulator period („slippage“)

Angular distributions of undulator radiation

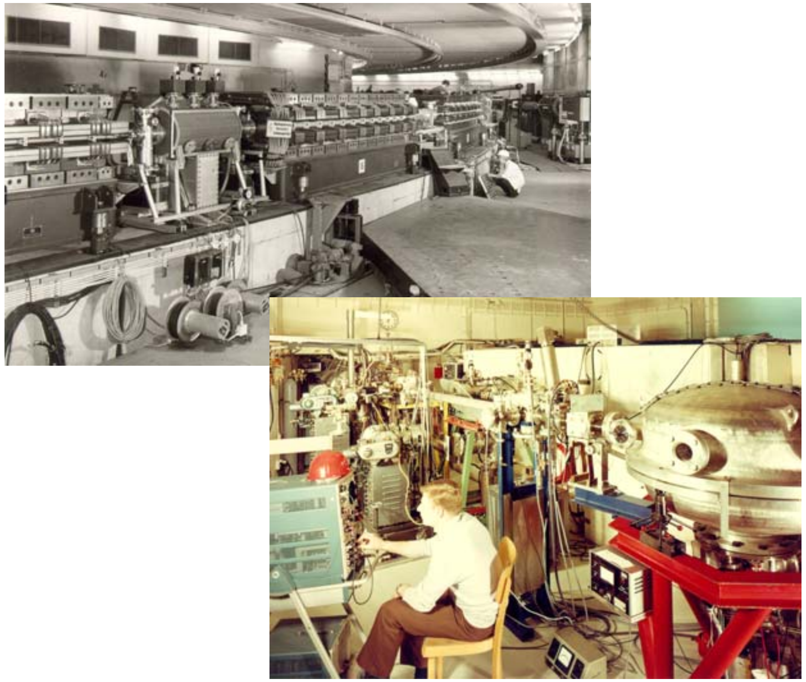
σ-Mode **π-Mode**

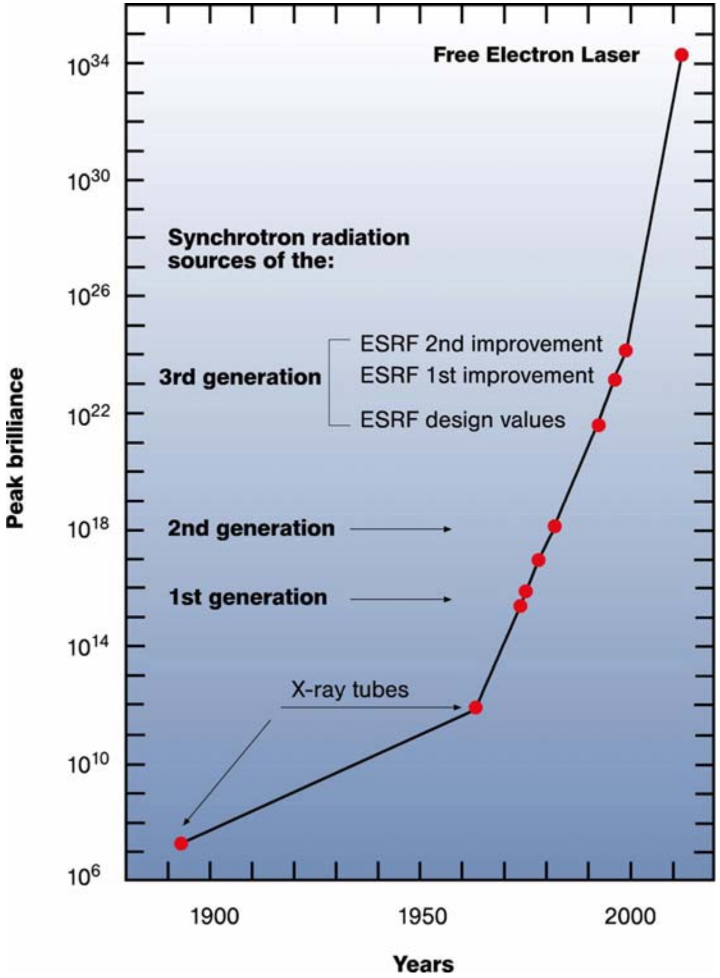


from H. Wiedemann, Synchrotron Radiation

History: three „generations“

1970s: parasitic use of e^+e^- colliders
1980s: dedicated electron storage rings
1990s: high-brilliance sources

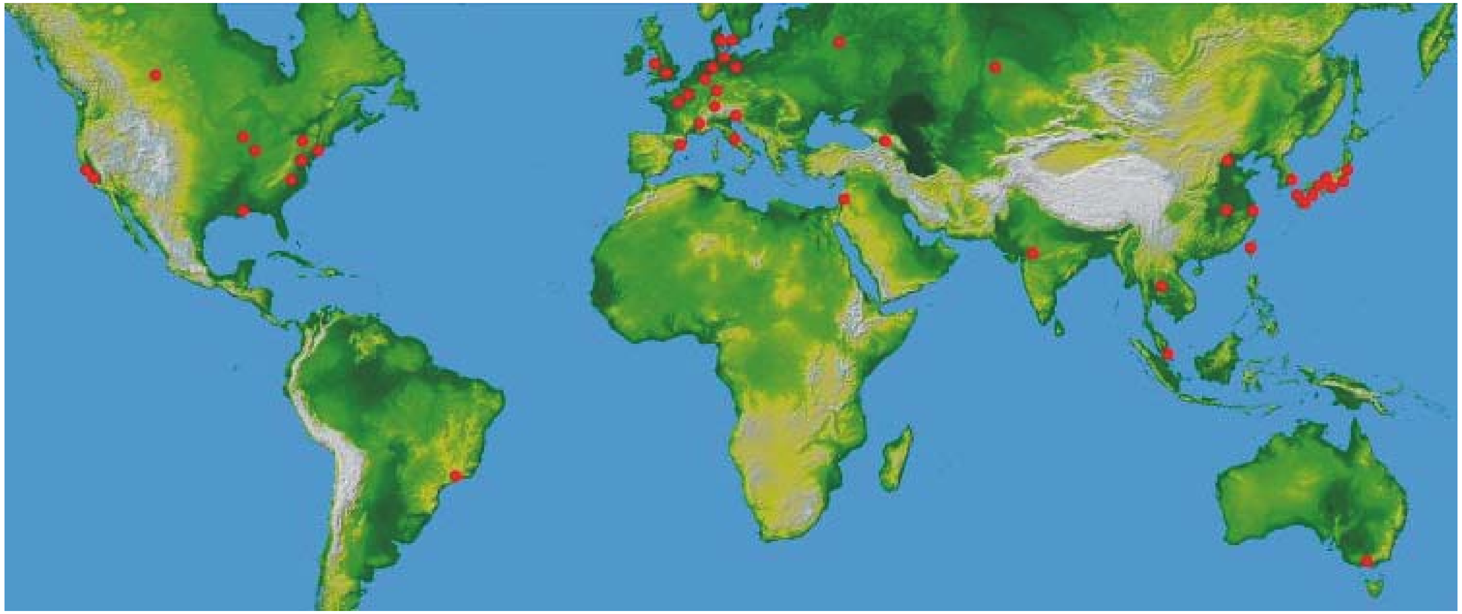




Two energy regimes

1-3 GeV electrons: VUV, „soft“ x-rays
6-8 GeV electrons: „hard“ x-rays

Synchrotron radiation sources worldwide



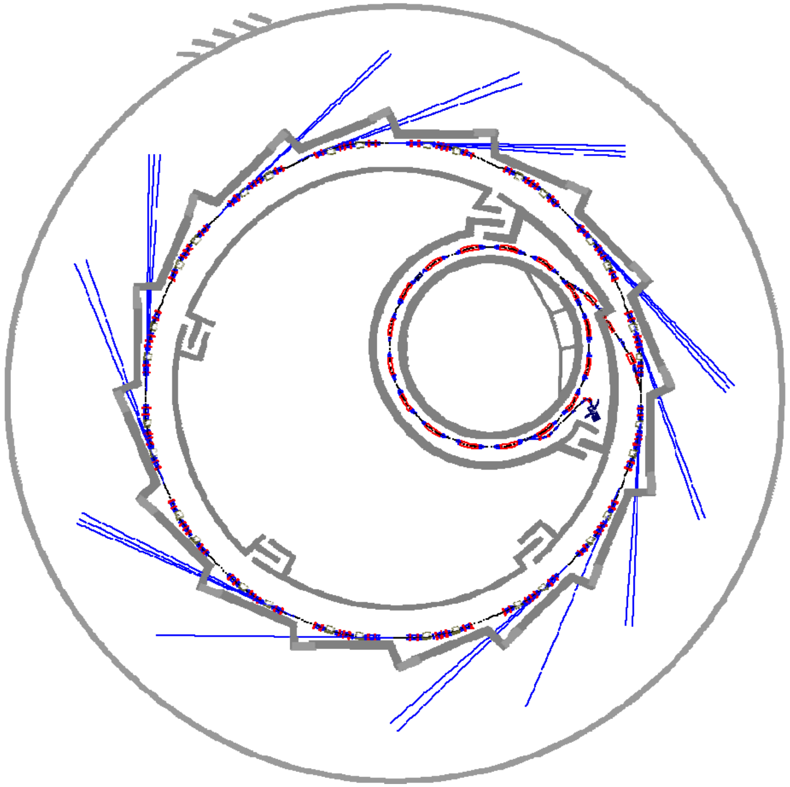
ALS (USA) 1.9 GeV

Diamond (UK) 3 GeV

SPring-8 (Japan) 8 GeV

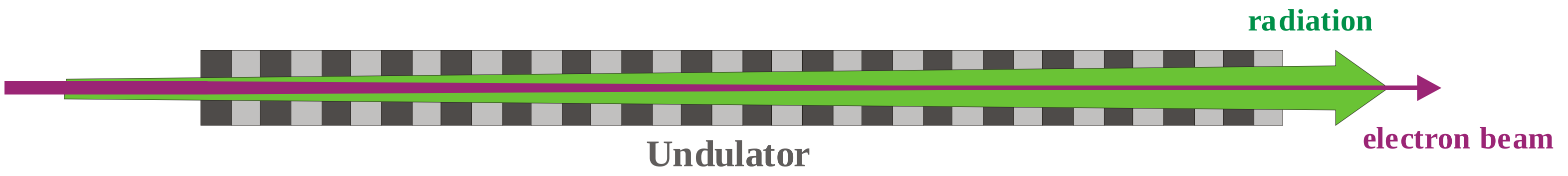
A typical synchrotron radiation source

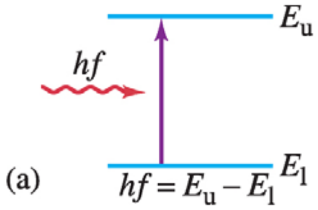
(and my favorite)



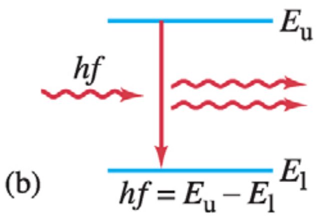
4.2 Free-electron lasers (FELs)

Is the free-electron laser
... a particle accelerator?
... just another electron tube?
... synchrotron light source?
... or is it a LASER?





Light
Amplification by
Stimulated
Emission of
Radiation



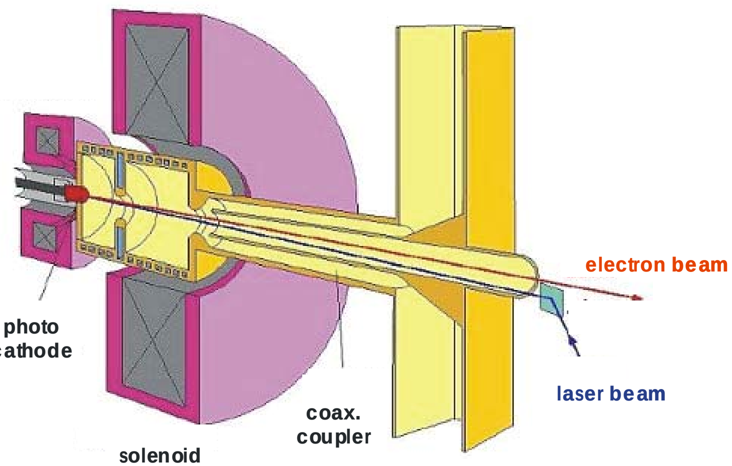
(i) a free-electron laser amplifies light

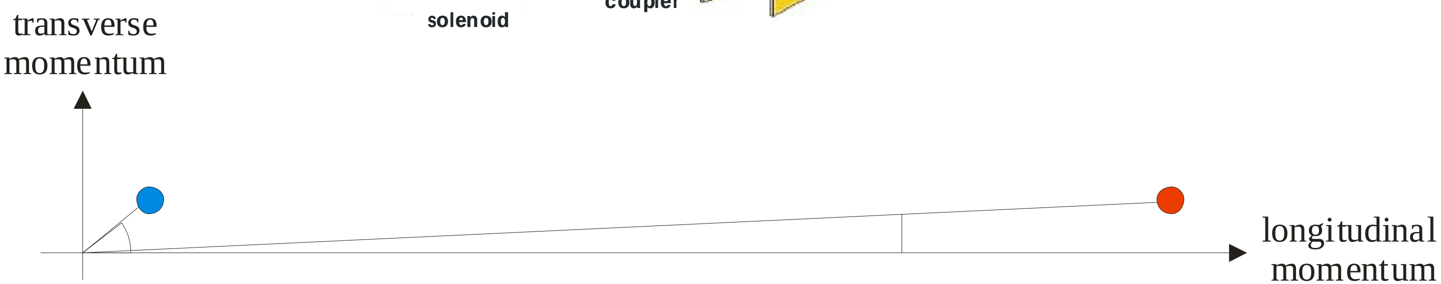
(ii) radiation accelerates electrons and accelerated electrons emit radiation

X-ray FELs require high charge density

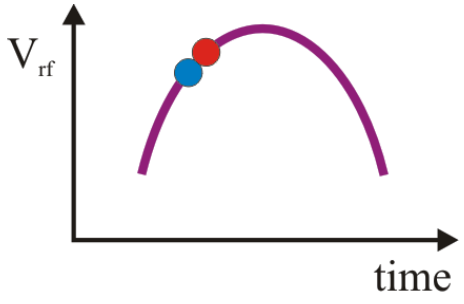
Low beam emittance
Short bunches

Low emittance: photocathode rf gun + „adiabatic" damping ~ 1/γ

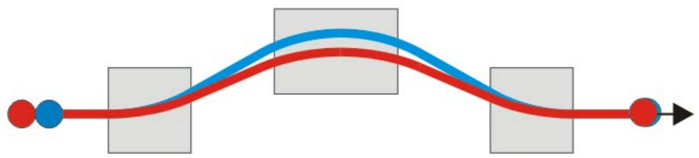




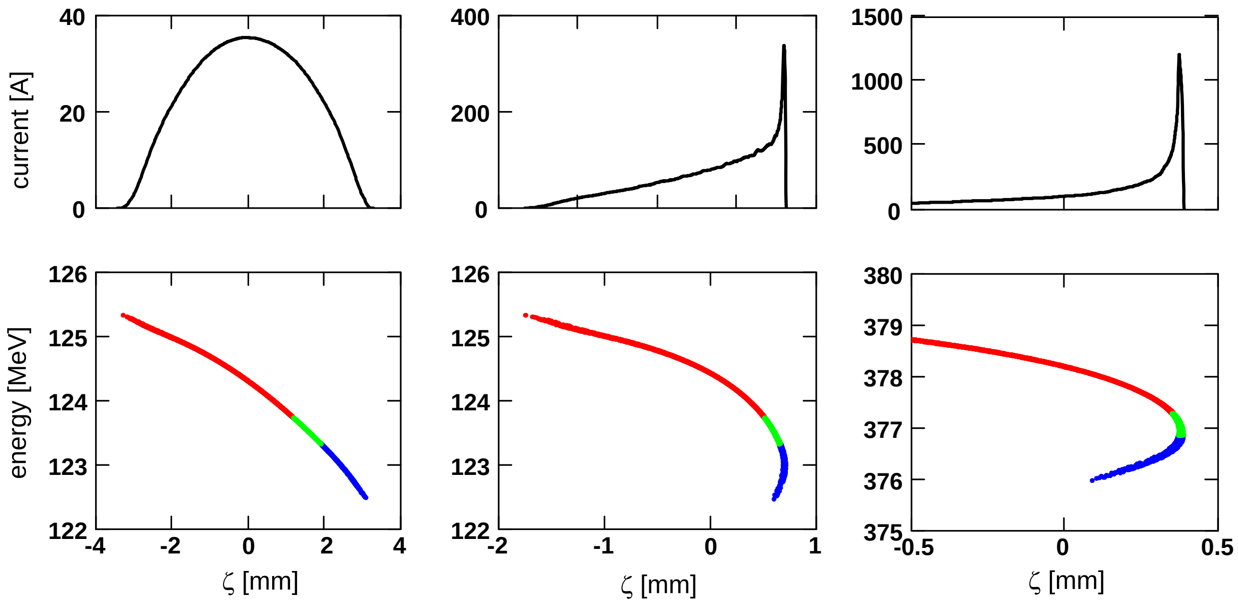
Short bunches: bunch compression (cf. laser compressor)



(i) energy chirp by „off-crest“ acceleration



(ii) energy-dependent path length in a magnetic chicane

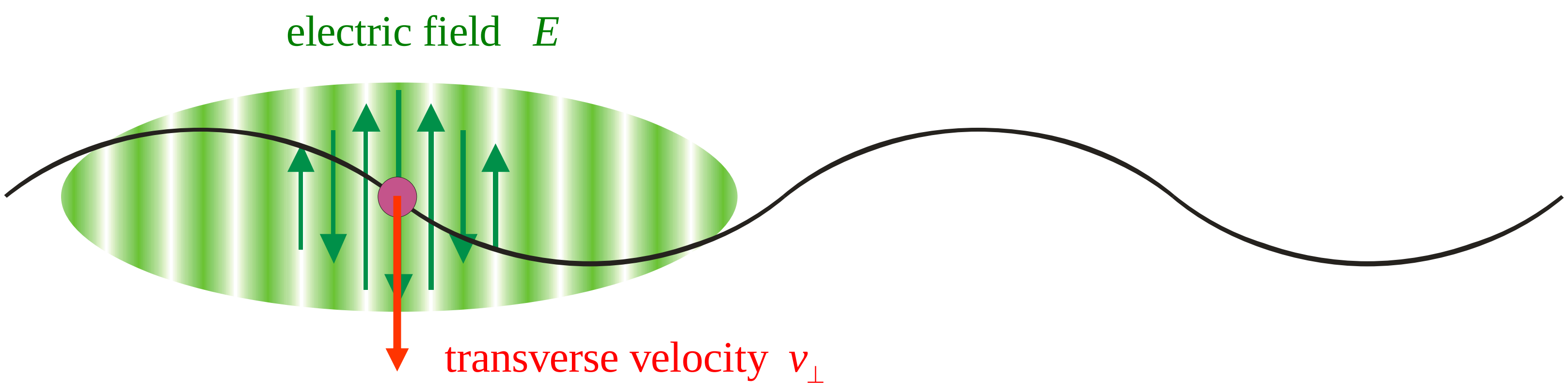


Energy exchange between radiation and electrons

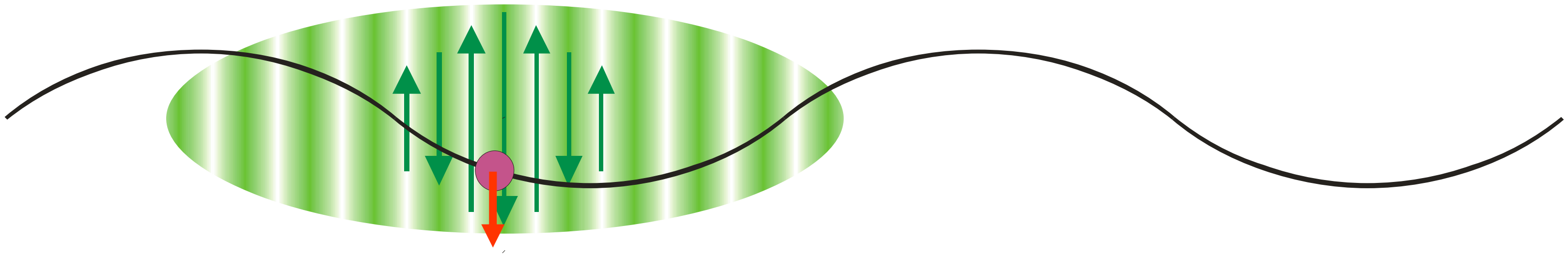
assume a given radiation field (whatever its origin) ***E***

and a relativistic electron moving in an undulator with velocity ***v***

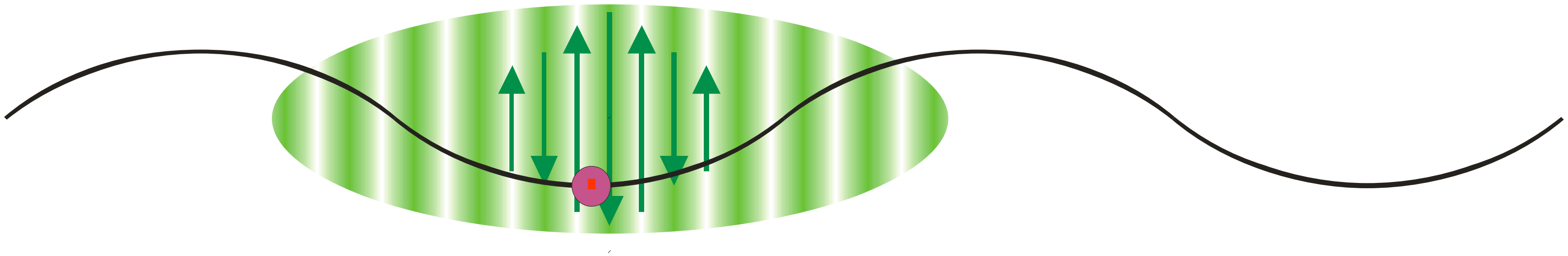
$$\Delta E = -e \cdot \vec{E} \cdot d\vec{s} = -e \cdot E \cdot v_{\perp} \cdot dt$$



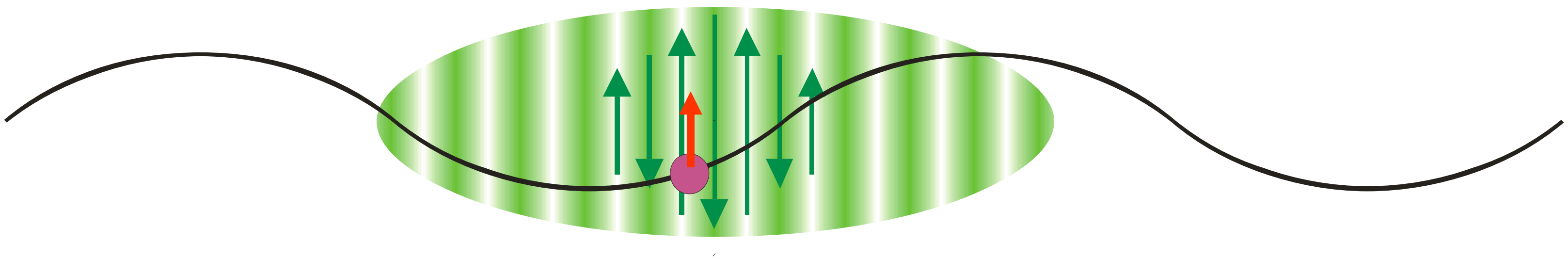
$$\Delta E = -e \cdot \vec{E} \cdot d\vec{s} = -e \cdot E \cdot v_{\perp} \cdot dt$$



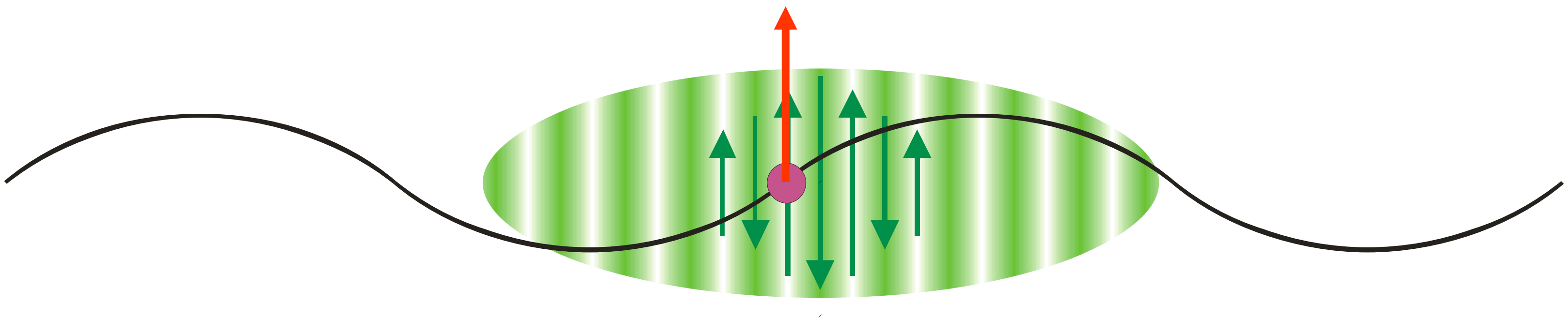
$$\Delta E = -e \cdot \vec{E} \cdot d\vec{s} = -e \cdot E \cdot v_{\perp} \cdot dt$$



$$\Delta E = -e \cdot \vec{E} \cdot d\vec{s} = -e \cdot E \cdot v_{\perp} \cdot dt$$



$$\Delta E = -e \cdot \vec{E} \cdot d\vec{s} = -e \cdot E \cdot v_{\perp} \cdot dt$$



$$\lambda_L = \frac{\lambda_U}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

Energy exchange – calculated

$$E = E_0 \cos(k_L z - \omega_L t) \qquad v_\perp = \frac{cK}{\gamma}\cos(k_U z) \qquad k_U = \frac{2\pi}{\lambda_U}$$

$$\Delta E = -e \cdot \vec{E} \cdot d\vec{s} = -e \cdot E \cdot v_\perp \cdot dt$$

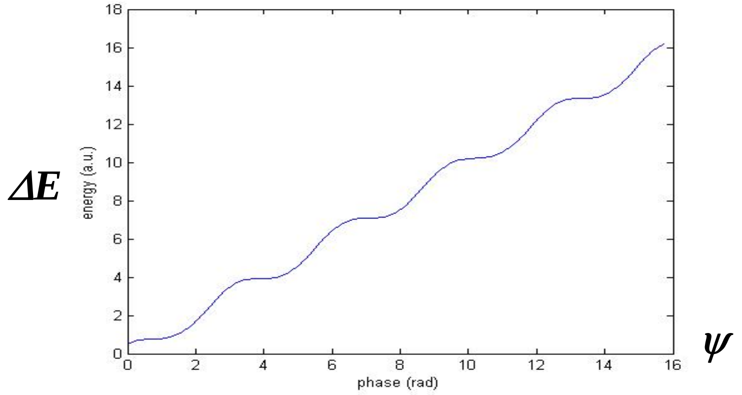
$$\Delta E = -\frac{ecKE_0}{\gamma}\cos(k_U z)\cos(k_L z - \omega_L t + \psi_0)$$

$$= -\frac{ecKE_0}{2\gamma}\left\{\cos\left(\left[k_L + k_U\right] - \omega_L t + \psi_0\right) + \cos\left(\left[k_L + k_U\right] - \omega_L t + \psi_0\right)\right\}$$

constant phase for

$$\lambda_L = \frac{\lambda_U}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

oscillates as $\cos(2k_U z)$



The pendulum equation

$$\lambda_\mathrm{L} = \frac{\lambda_\mathrm{U}}{2\gamma_\mathrm{r}^2}\left(1+\frac{K^2}{2}\right)$$

some cosmetic changes

$$\eta \equiv \frac{\gamma - \gamma_\mathrm{r}}{\gamma_\mathrm{r}} \qquad \varphi \equiv \left[k_\mathrm{L} + k_\mathrm{U}\right]\bar{\beta}ct - \omega_\mathrm{L} t + \psi_0 + \frac{\pi}{2} \quad \rightarrow \quad \boxed{\frac{d\eta}{dt} = \frac{eE_0 K}{2m_\mathrm{e} c\gamma_\mathrm{r}^2}\sin\varphi}$$

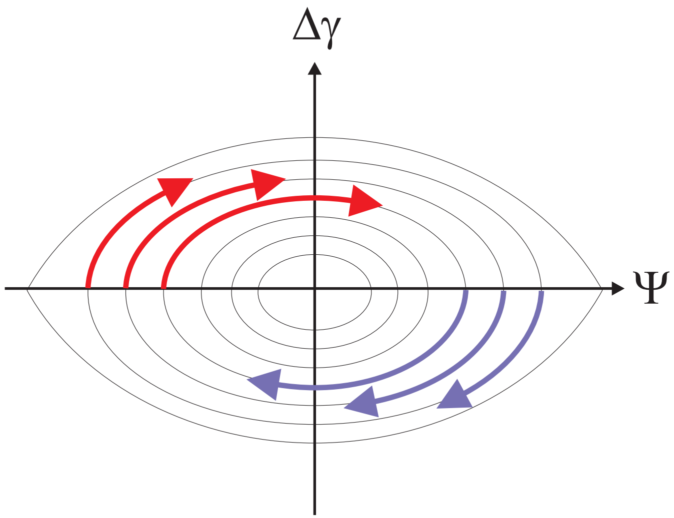
phase-dependent energy change

$$\frac{d\varphi}{dt} = \left[k_\mathrm{L} + k_\mathrm{U}\right]\bar{\beta}c - \omega_\mathrm{L} t \approx k_U c - \frac{k_L c\left(1+\frac{K^2}{2}\right)}{2\gamma^2} = \ldots \qquad \boxed{\frac{d\varphi}{dt} = 2k_U c\eta}$$

energy-dependent phase change

$$\bar{\beta} = \left\{1 - \frac{1}{2\gamma^2}\left(1+\frac{K^2}{2}\right)\right\}$$

Combining these two coupled differential equations leads to the pendulum equation



$$\frac{d\eta}{dt} = \frac{eE_0K}{2m_e c\gamma_r^2}\sin\varphi$$

phase-dependent energy change

$$\frac{d\varphi}{dt} = 2k_U c\eta$$

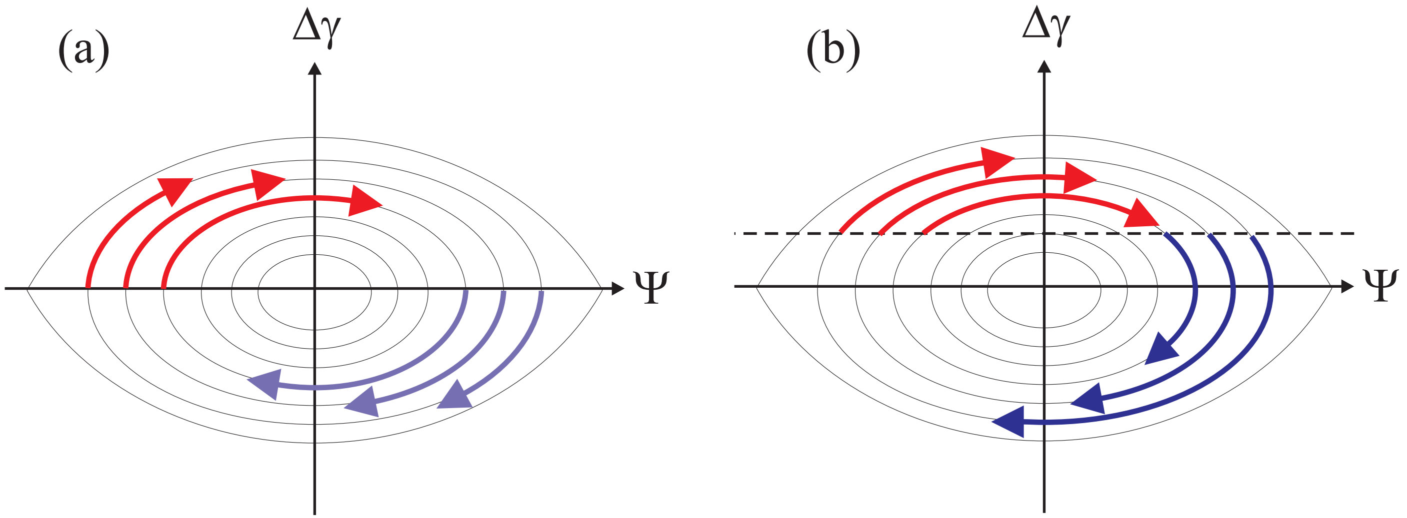
energy-dependent phase change

pendulum equation

$$\frac{d^2\varphi}{dt^2} + \Omega^2 \cdot \sin\varphi = 0$$

with $\Omega^2 = \frac{eE_0Kk_U}{m_e\gamma_r^2}$

Electron motion in phase space



(a) equal energy gain and loss (no net energy transfer)

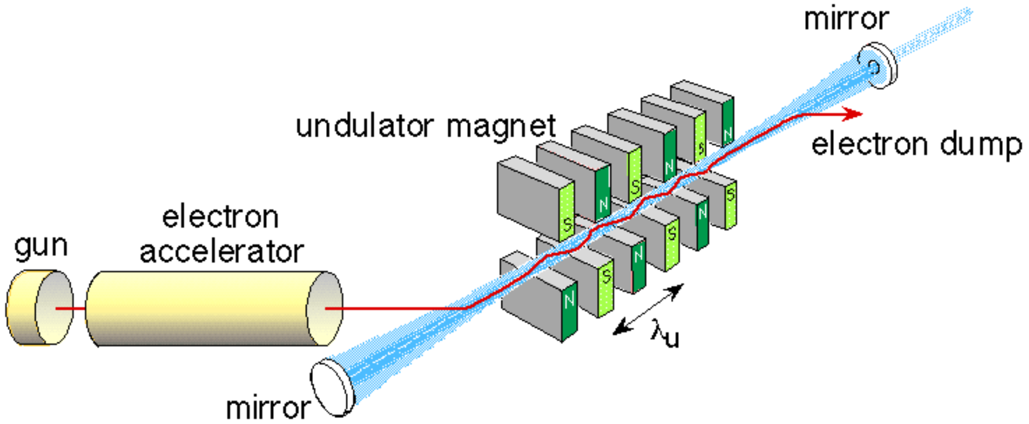
(b) more energy loss than gain (second-order effect)

pendulum equation

$$\frac{d^2\varphi}{dt^2} + \Omega^2 \cdot \sin\varphi = 0$$

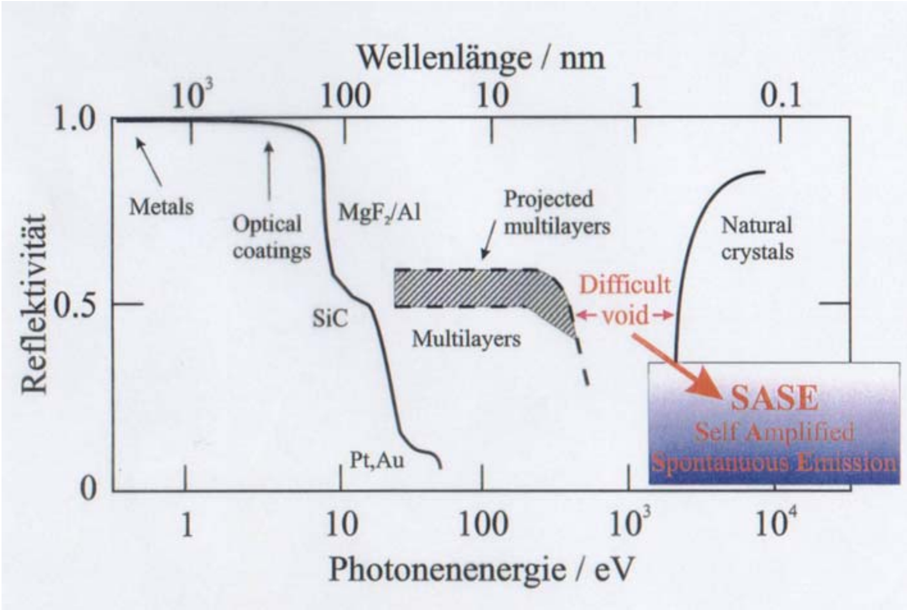
with $\Omega^2 = \frac{eE_0 K k_\mathrm{U}}{m_\mathrm{e}\gamma_\mathrm{r}^2}$

Low-gain FEL: undulator with mirrors (oscillator)



either linear accelerator or storage ring

High-gain FEL: no mirrors for small wavelengths



consequences
- single pass
- high gain
- *E*-field no longer constant

Electron motion in phase space - revisited

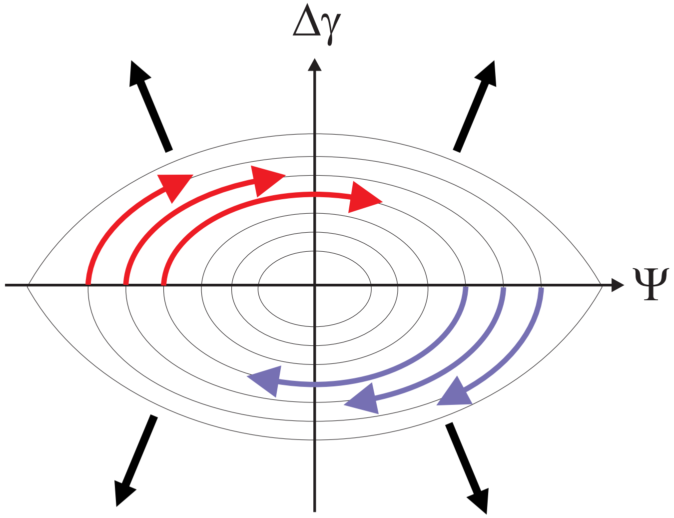
$$\frac{d\eta}{dt} = \frac{eE_0K}{2m_e c\gamma_r^2}\sin\varphi$$

← **depends on *E*-field**

phase-dependent energy change

$$\frac{d\varphi}{dt} = 2k_U c\eta$$

energy-dependent phase change



low gain:
change of *E*-field ignored

high-gain:
change of *E*-field significant

Electron motion in phase space - revisited

$$\frac{d\eta}{dt} = \frac{eE_0 K}{2m_e c \gamma_r^2} \sin\varphi$$

phase-dependent energy change

$$\frac{d\varphi}{dt} = 2k_U c\eta$$

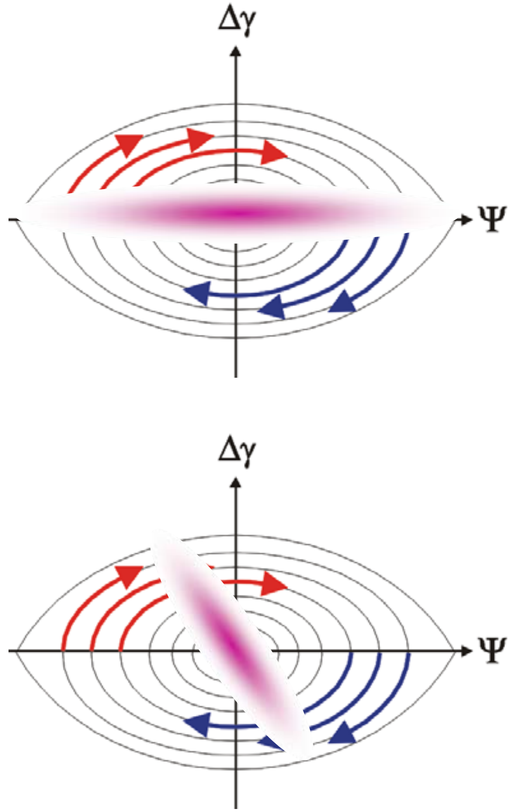
energy-dependent phase change

$$\tilde{j}_1 = -n_e ec \frac{2}{N} \sum_{n=1}^{N} \exp(-i\varphi_n)$$

modulation of the current density

$$\frac{d\tilde{E}_x}{dz} = -\frac{\mu_0 cK}{4\gamma} \tilde{j}_1$$

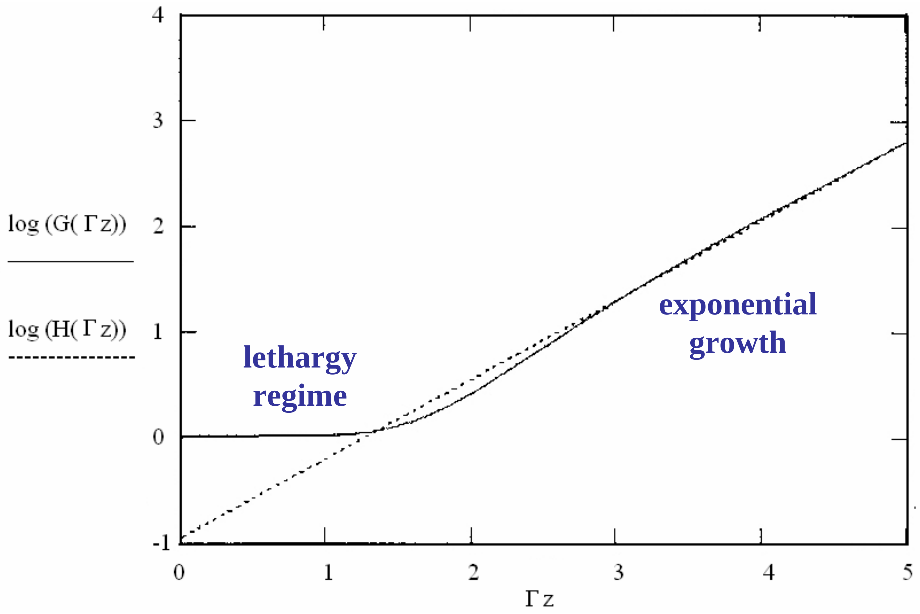
current-dependent field change

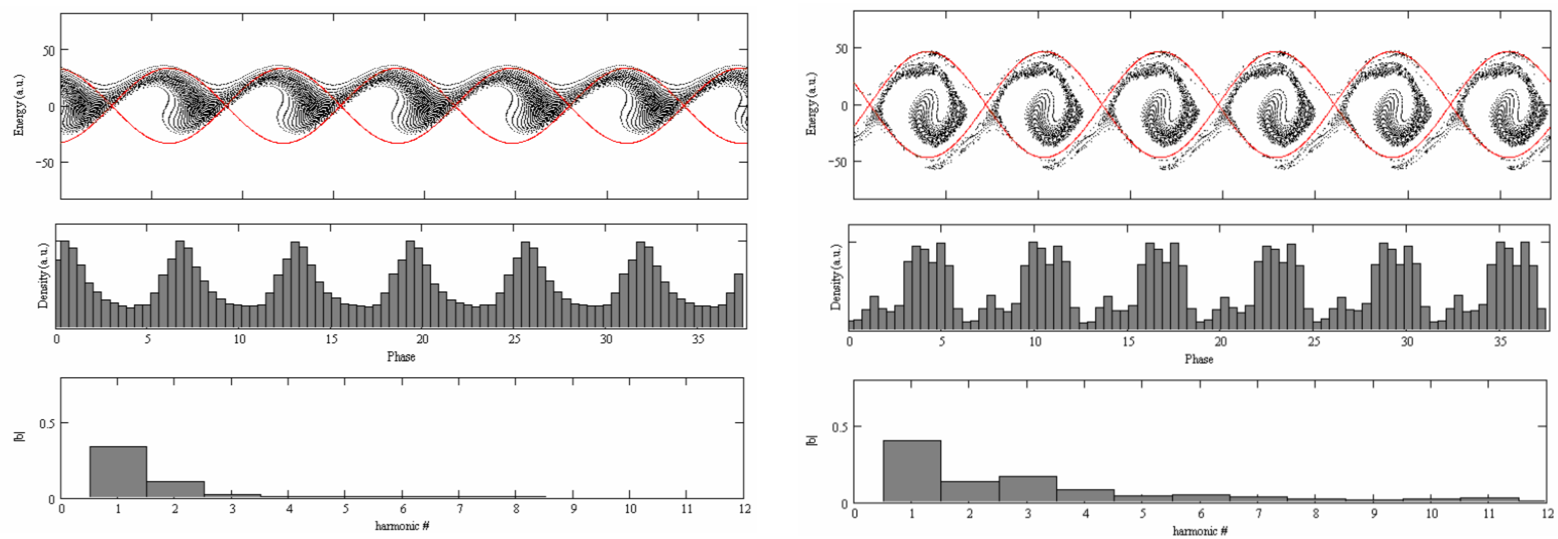


From here

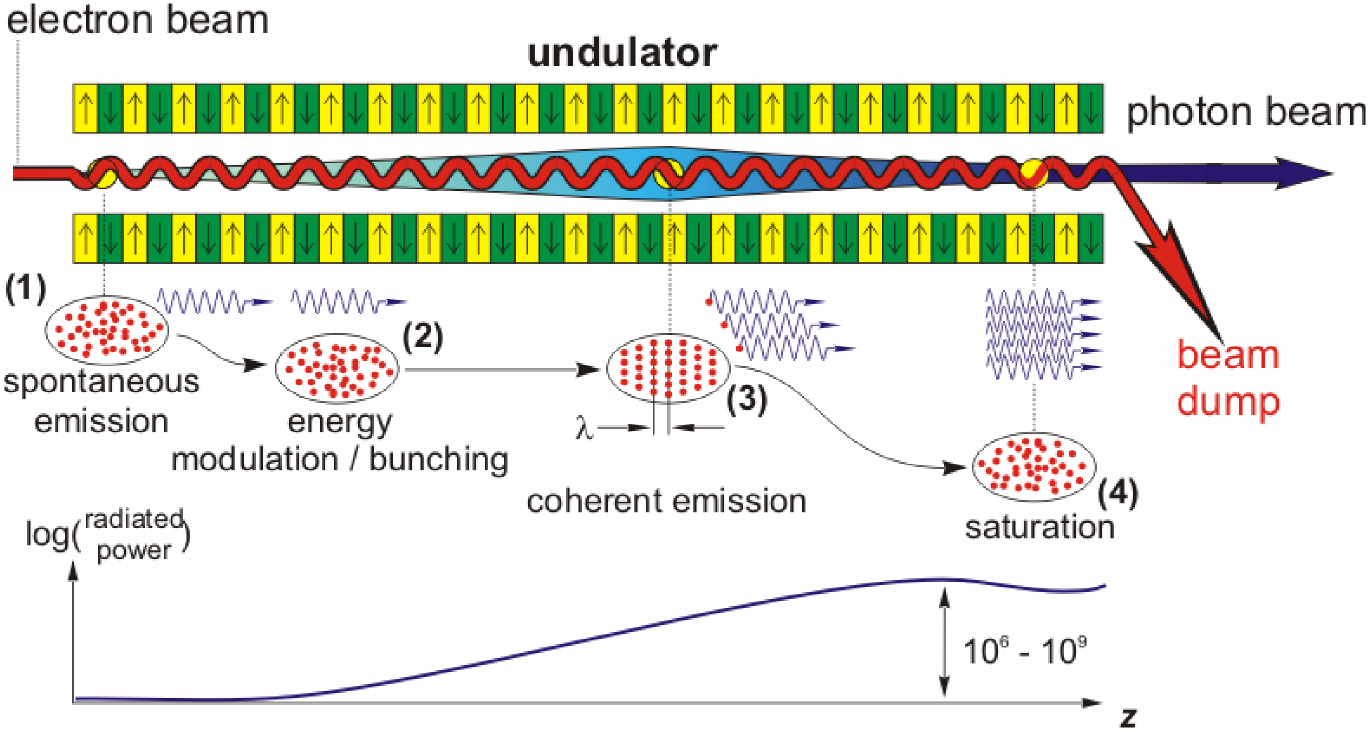
... analytical treatment:
predicts exponential growth

... numerical simulation to
study the dynamics in detail



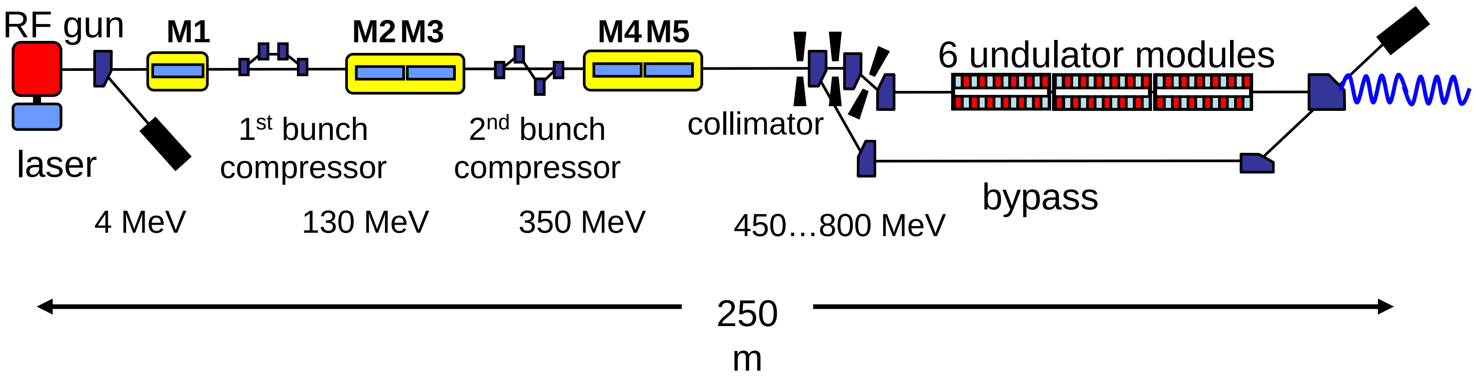


SASE (self-amplified spontaneous emission)



... starting from spontaneous undulator radiation

Example: FLASH



superconducting L-band (1.3 GHz) linac

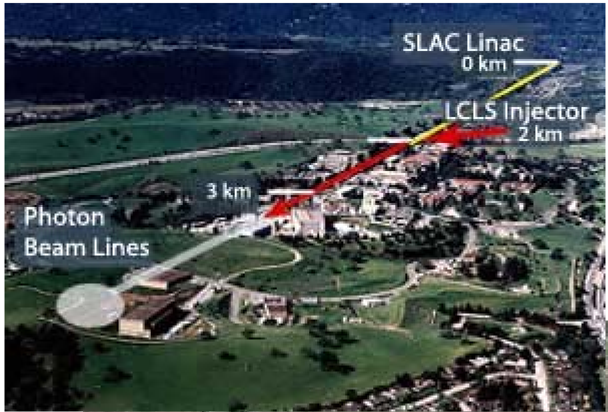
wavelength 6 nm with 1 GeV (γ ~ 2000)
pulse energy up to 40 μJ (~ 10^{12} photons)
repetition rate 10 Hz
pulse duration ~10 fs

$$\lambda = \frac{\lambda_U}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

with $\lambda_U = 27.3$ mm

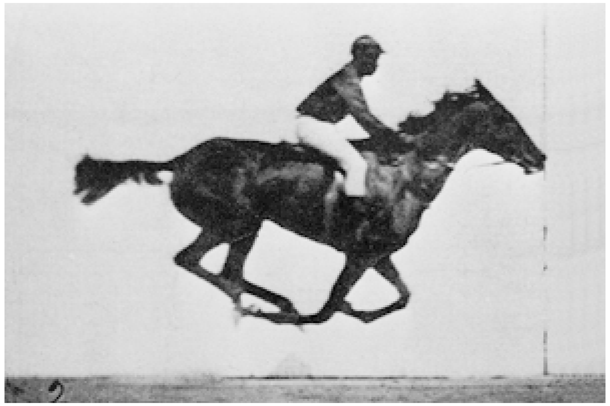
and $K = 1.23$

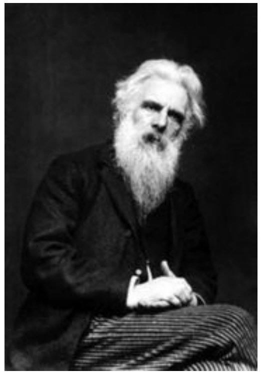
Example: LCLS (Linear Coherent Light Source, Stanford)



normal-conducting S-band (2.9 GHz) SLAC linac
commissioning in 2009

beam energy 15 GeV
wavelength 0.15 nm

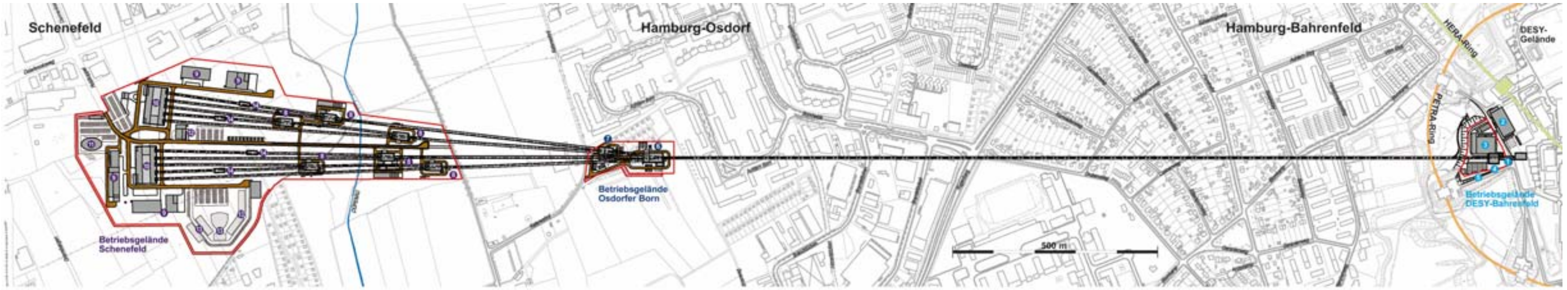




Eadweard Muybridge (1830-1904)

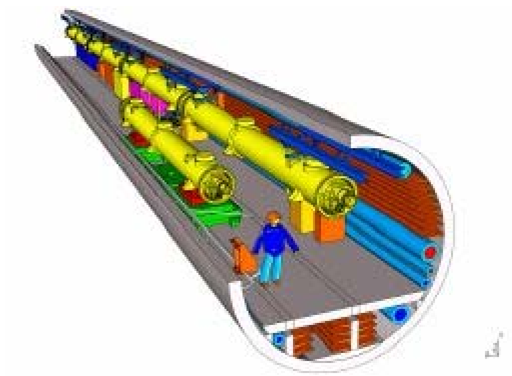
Amasa Leland Stanford (1824-1893)

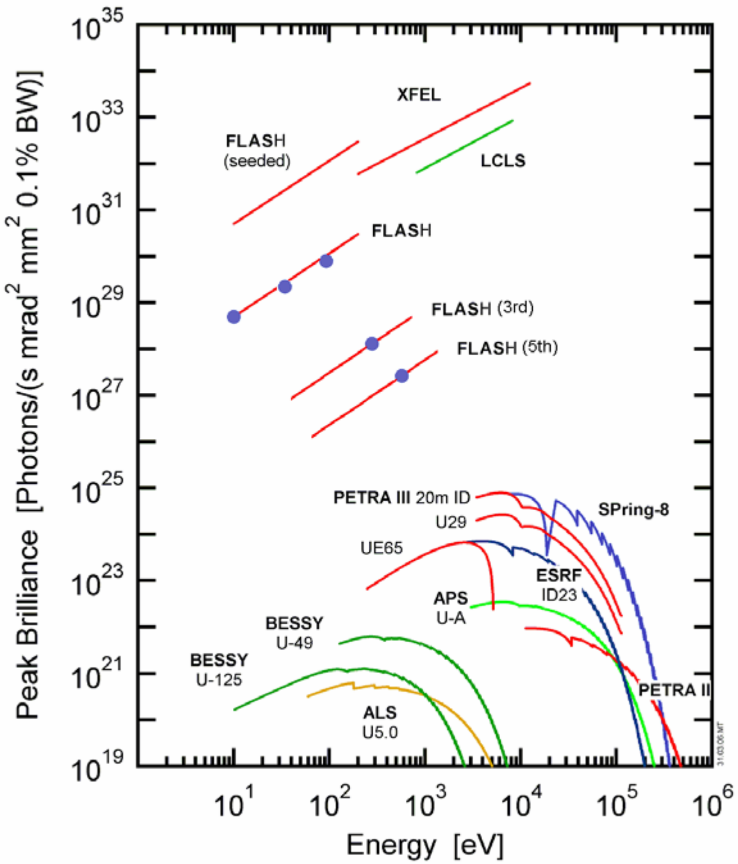
Example: European XFEL (DESY, commissioning 2013)



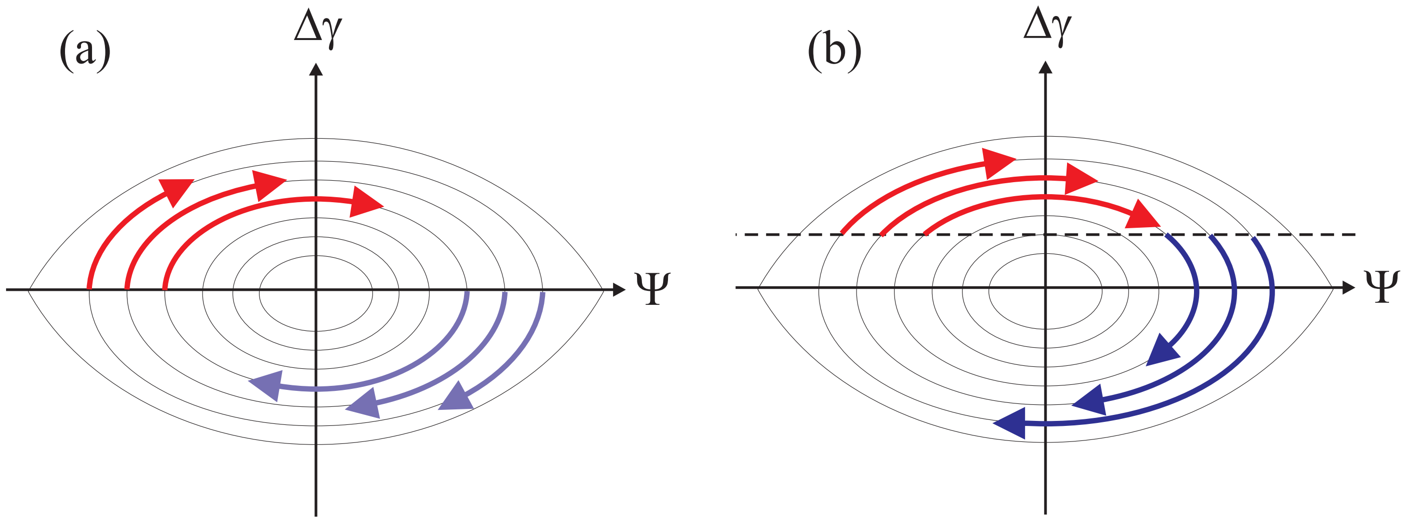
beam energy 20 GeV with superconducting L-band linac

wavelength 0.09 nm





4.3 Concept of inverse Free-electron lasers (FELs)



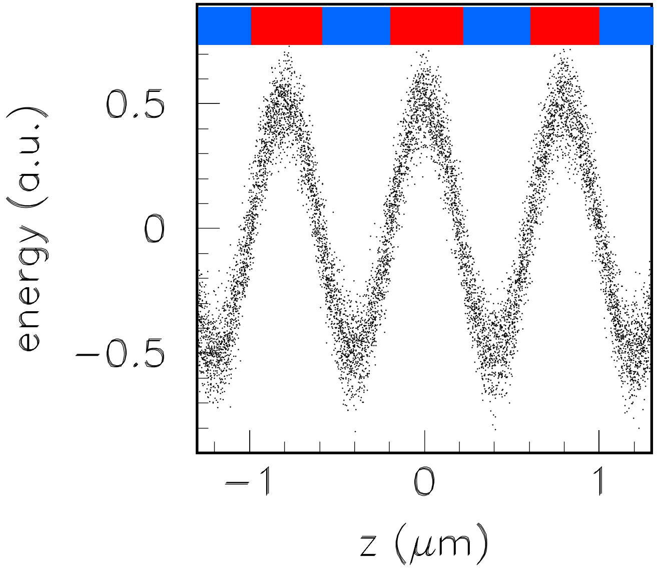
(a) equal energy gain and loss (no net energy transfer)

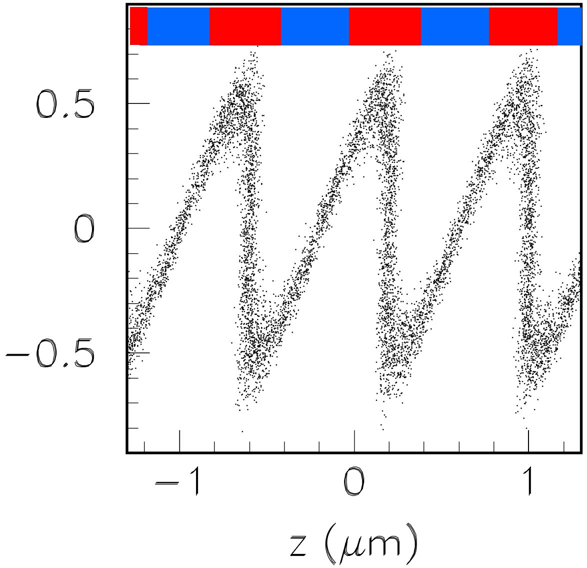
(b) more energy loss than gain (second-order effect)

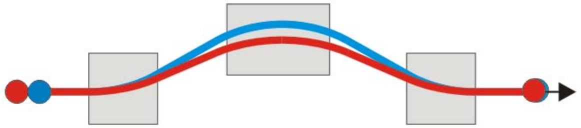
(c) more energy gain than loss, if starting from $\Delta\gamma < 0$

4.3 Concept of inverse Free-electron lasers (FELs)

two steps: **1. moderate energy modulation** **2. laser interaction after microbunching**





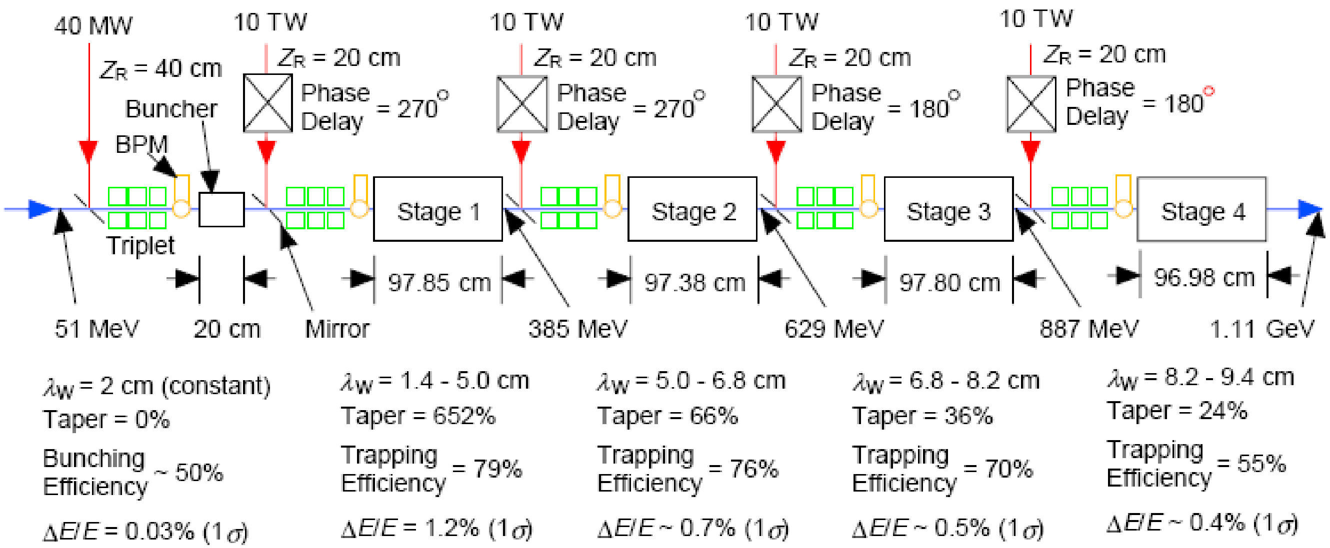


microbunching chicane

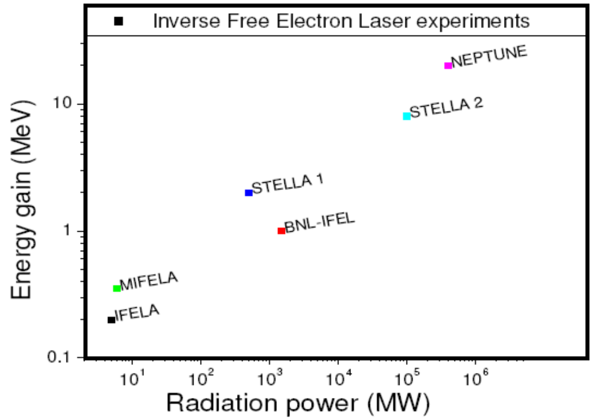
4.4 Examples

Conceptual Design for a 1-GeV IFEL Accelerator

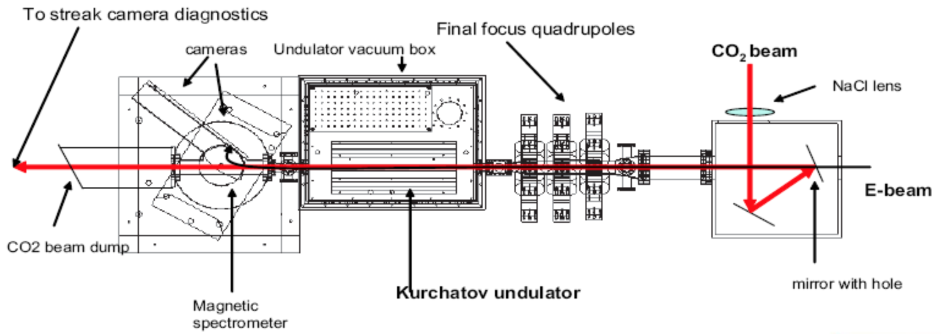
W. D. Kimura et al., Eleventh Advanced Accelerator Concepts Workshop, Stony Brook 2004



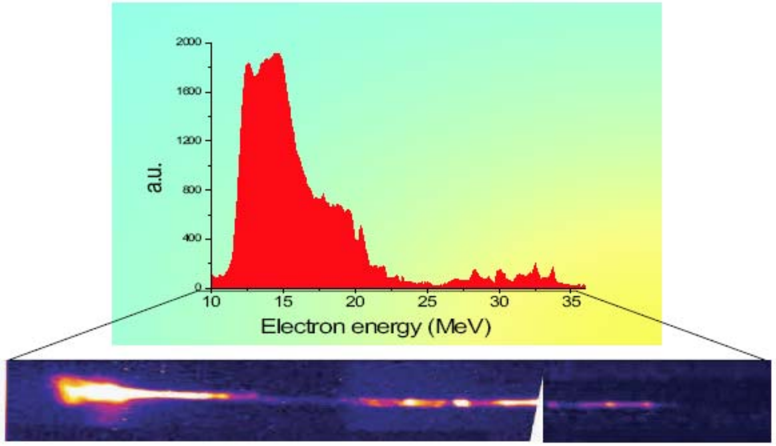
4.4 Examples



NEPTUNE (UCLA)



14.5 MeV beam
400 GW laser



P. Musumeci et al.,
Eleventh Advanced Accelerator Concepts Workshop,
Stony Brook 2004

5. Other ideas

only two general remarks:

(i) the Lawson-Woodward Theorem:
Direct acceleration by laser fields is only possible in the proximity to material boundary conditions (e.g. apertures, dielectric material etc.)

$$\Delta W = e \int_{-\infty}^{\infty} \vec{E} \cdot \vec{v} dt$$

(ii) the „Acceleration Theorem“:
acceleration by an external field requires the existence of spontaneous radiation

fields $E_{laser} + E_{spont}$ total field energy $W \propto |E|^2$

$$W = W_{laser} + W_{spont} + 2\sqrt{A_{laser} A_{spont}} \cos\phi$$

e.g. inverse free-electron laser

but consider also: transition radiation, Smith-Purcell radiation, Cherenkov radiation etc.