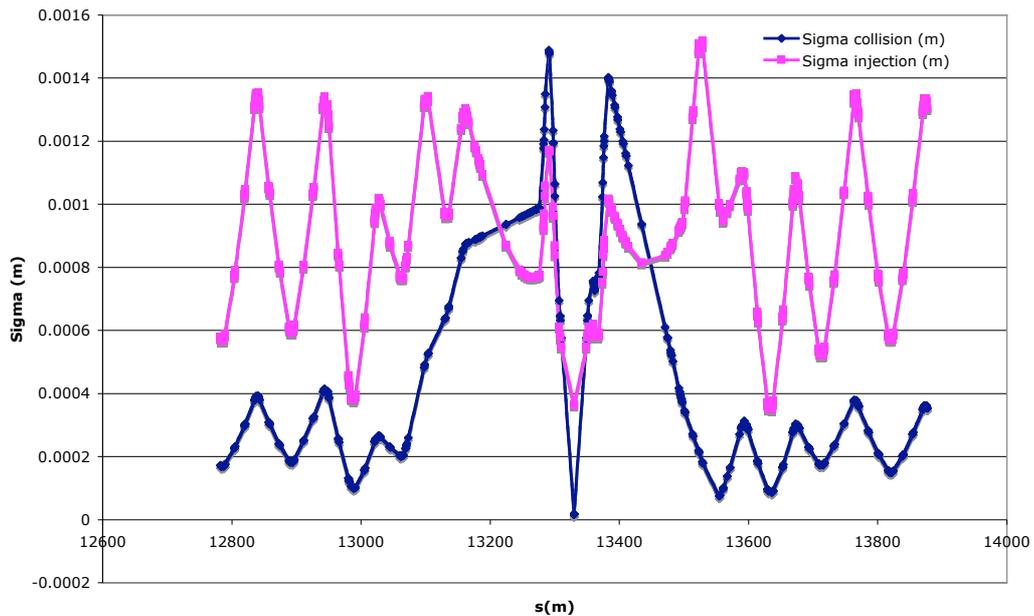
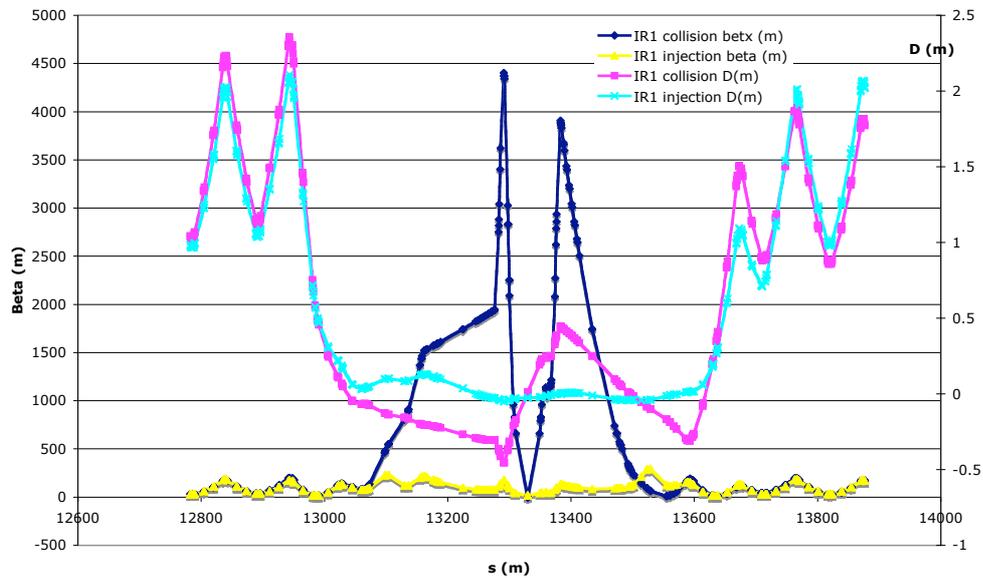


## Solutions for the LHC lectures

- 1) The bending radius for one magnet is equal to  $\rho(m)=3.3356 p \text{ (GeV)}/B(T) = 2803 \text{ m}$ . Then angular kick given by every magnet is  $\theta(\text{mrad})=L/\rho= 5.1 \text{ mrad}$ . Then  $N_{\text{magnets}}=2*\pi/5.1 \text{ mrad} = 1232$ .
- 2) The gravitational force give a  $\Delta y=0.5*g*t^2$ . Assuming  $g=10 \text{ m/s}^2$  to cover the 28 mm one would take about 74.8 ms. The revolution frequency of 11.245 kHz gives one turn every 88.9  $\mu\text{s}$ , so it makes about 841 turns.
- 3) The magnetic energy stored equals  $E_n=1/2 * L * I^2=6.93 \text{ MJ}$ .
- 4) The relevant plots are shown below:

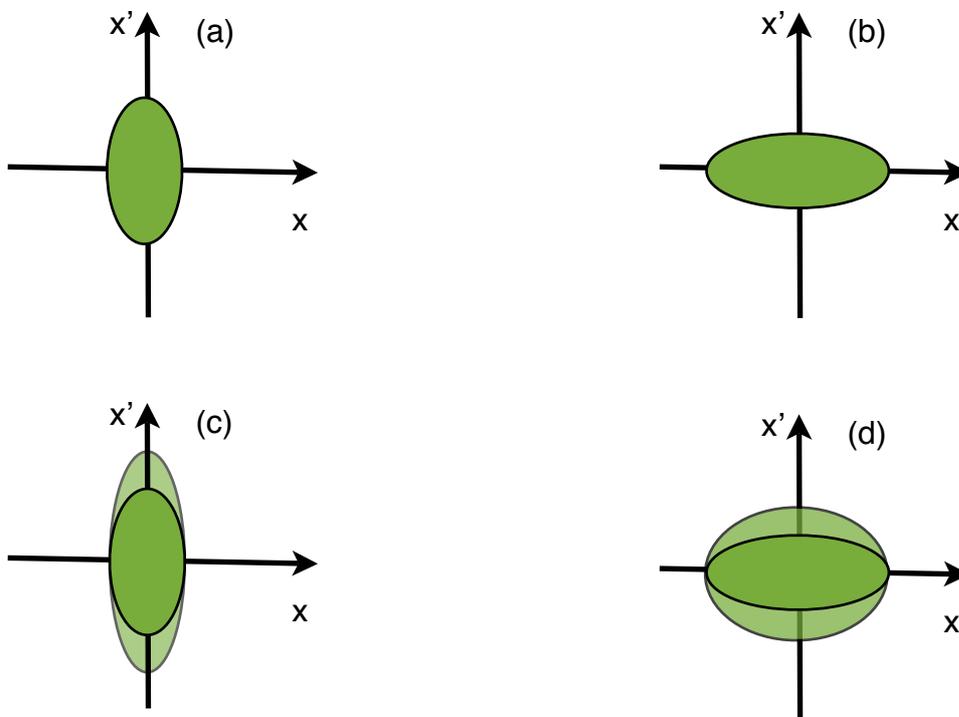


The luminosity can be computed by:

$$L = \frac{N^2 \cdot f \cdot n_b}{4\pi \cdot \sigma_x^* \cdot \sigma_y^*} \cdot F$$

which leads to  $L(7 \text{ TeV})= 1.1971 \cdot 10^{34}/\text{cm}^2/\text{s}$  while  $L(450 \text{ GeV})=2.4988 \cdot 10^{31}/\text{cm}^2/\text{s}$ , three orders of magnitude less.

5) The effect of the multiple scattering is to provide an angular kick, meaning adding divergence to the beam. In  $xx'$  and  $yy'$  planes, the beam emittance is defined roughly as the area of the ellipse occupied by the beam. When the optical beta is a the minimum, this



ellipse is upright, i.e. the beam is small and the angular divergence is large, like in figure (a). When the beta is large, the ellipse is lying on the x axis (b). Adding divergence to the beam means to stretch the ellipse, like in figure (c) and (d).

By taking the ratio of the ellipse surfaces before the bottle and after passing the bottle, one realises that in the case of (c) the area, meaning the emittance, is increased less than in the case of (d). The place to put the bottle is where the minimum of the beta occurs, i.e. in the LHC at the IPs, in the middle of one of the experiments.

6) The length of the beam orbit in a synchrotron is fixed by the constant RF frequency. If the circumference of the machine is changed, for example distorted by the Moon and Sun tides, the beam will pass no more through the center of the quadrupoles. The results is a different bending strength which will leads to an energy change.

The momentum compaction factor relates the relative variation of the machine circumference with the variation of machine beam energy, such that:

$$\frac{\Delta C}{C} = -\alpha_c \frac{\Delta E}{E}$$

This means that a  $\Delta C=1.23 \cdot 10^{-5}$  m variation causes a 10 MeV energy change, and with this conventions, an expanding ring induces a reduction of the energy.

For the experiment this effect should not be a problem. In an hadron collider, not like in a lepton collider, the energy of the colliding particles is not known precisely, due to the fact that are the partons (quarks and gluons) which interact, and not the protons.

1) The angular kick given by a dipole, as in the 1st exercise, is given by:

$$x' = 0.3 * \frac{L(m) * B(T)}{p(GeV)}$$

negative in this case to have a negative kick and let indicate it as y because is vertical. So  $y'=1.28 \cdot 10^{-4}$  rad. By applying the matrix formalism for a drift, a particle which is centered at the entrance of the magnet, it exits with an angle  $y'$ , and to reach 4 mm needs  $s=4 \text{ mm}/y=31.2$  m. After 31.2 m a second magnet has to be installed with the same bending strength with opposite polarity to compensate for the angle  $y'$  and put the beam again parallel. Then a second set of those dipoles, mirrored with respect to the IP has to be installed.

2) The synchrotron power radiated by one particle is

$$P = \frac{2c \times E^4 \times r_0}{3\rho^2 (m_0 \times c^2)^3}$$

where  $r_0$  is the classical radius of the given particle,  $E$  is the energy,  $m$  is the particle rest mass. By substituting the relevant values in the formulas, one gets about  $8.6 \cdot 10^6$  W for  $10^{12}$  electrons. For  $10^{14}$  protons one gets  $1.81 \cdot 10^3$  W. The energy per turn is obtained by multiplying the power irradiated by one particle times the inverse of the revolution frequency. The energy emitted by a 100 GeV electron per turn is about  $4.7 \cdot 10^9$  eV while for a proton at 7 TeV is  $1.01 \cdot 10^4$  eV, so few GeV for electrons and about 10 keV for protons. An electron at 7 TeV would irradiate  $1.14 \cdot 10^{17}$  eV.

3)The tune variation due to a quadrupolar error is given by:

$$\Delta Q = \frac{1}{4\pi} \cdot \oint \beta(s) \cdot \Delta K(s) ds$$

where  $\Delta K$  is the gradient variation given by the error, which can be a real quadrupole pulsing or the results of having the beam off axis with respect to a sextupole.

So, in this case:

$$\Delta K(s) = 2 \cdot \frac{D_x}{R_{ref}^2 \cdot \rho} \cdot b_3 \cdot 10^{-4} \cdot \frac{\Delta p}{p_0}.$$

because the dispersion  $D_x(s)$  causes a displacement  $dx$  at a given  $s$  proportional to the momentum deviation.

Then the tune variation can be computed from the first expression using the average

$$\begin{aligned} \Delta Q &= \frac{1}{2\pi} \cdot L \cdot \frac{\langle \beta \rangle \cdot \langle D_x \rangle}{R_{ref}^2 \cdot \rho} \cdot b_3 \cdot 10^{-4} \cdot \frac{\Delta p}{p_0} \\ &= 485.32 \cdot b_3 \cdot 10^{-4} \cdot \frac{\Delta p}{p_0} \end{aligned}$$

quantities mentioned in the text for one dipole. Then there are 1232 dipoles, so

$$\Delta Q \approx 59.8 \cdot b_3 \cdot \frac{\Delta p}{p_0}$$

which gives also the chromaticity in function of the  $b_3$ ,  $\xi = 59.8 \cdot b_3/Q$ . For 11 units of  $b_3$  one gets  $\xi = 657.8/Q$ .