Basics of Event Generators II

Leif Lönnblad

Department of Theoretical Physics
Lund University

Terascale Monte Carlo School
DESY 08.04.21

Outline of Lectures

- Lecture I: Basics of Monte Carlo, the event generator strategy, matrix elements, LO/NLO, . . .
- Lecture II: Parton showers, Sudakov formfactors, initial/final state, angular ordering, $k_\perp$-factorization, . . .
- Lecture III: Underlying events, multiple interactions, minimum bias, pile-up, hadronization, decays, . . .

Outline of Lecture II

The purpose of parton showers is to generate real exclusive events on parton level down to a very low (almost non-perutbative) jet resolution scale μ.

Starting from an initial hard scattering eg. $\mathrm{e}^+\mathrm{e}^- \to \mathrm{q}\bar{\mathrm{q}}$ or $\mathrm{q}\bar{\mathrm{q}} \to Z^0$, we basically need

$$
\begin{aligned}
\sigma_{+0} &= \sigma_0(1 + C_{01}\alpha_s + C_{02}\alpha_s^2 + C_{03}\alpha_s^3 + \ldots) \\
\sigma_{+1} &= \sigma_0(C_{11}\alpha_s + C_{12}\alpha_s^2 + C_{13}\alpha_s^3 + \ldots) \\
\sigma_{+2} &= \sigma_0(C_{22}\alpha_s^2 + C_{23}\alpha_s^3 + C_{24}\alpha_s^4 + \ldots) \\
&\vdots
\end{aligned}
$$

Tree-level generators only gives us inclusive events.

NLO generators only gives us one extra parton.

The purpose of parton showers is to generate real exclusive events on parton level down to a very low (almost non-perutbative) jet resolution scale μ.

Starting from an initial hard scattering eg. $\mathrm{e^+e^-} \to \mathrm{q\bar{q}}$ or $\mathrm{q\bar{q}} \to Z^0$, we basically need

$$\begin{aligned}
\sigma_{+0} &= \sigma_0(1 + C_{01}\alpha_s + C_{02}\alpha_s^2 + C_{03}\alpha_s^3 + \ldots) \\
\sigma_{+1} &= \sigma_0(C_{11}\alpha_s + C_{12}\alpha_s^2 + C_{13}\alpha_s^3 + \ldots) \\
\sigma_{+2} &= \sigma_0(C_{22}\alpha_s^2 + C_{23}\alpha_s^3 + C_{24}\alpha_s^4 + \ldots) \\
&\vdots
\end{aligned}$$

Tree-level generators only gives us inclusive events.

NLO generators only gives us one extra parton.

The purpose of parton showers is to generate real exclusive events on parton level down to a very low (almost non-perutbative) jet resolution scale μ.

Starting from an initial hard scattering eg. $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{q}\bar{\mathrm{q}}$ or $\mathrm{q}\bar{\mathrm{q}} \rightarrow Z^0$, we basically need

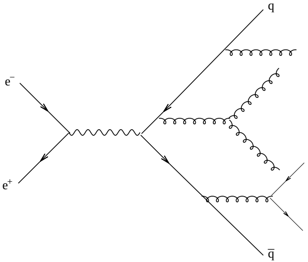
$$\begin{aligned}
\sigma_{+0} &= \sigma_0(1 + C_{01}\alpha_\mathrm{s} + C_{02}\alpha_\mathrm{s}^2 + C_{03}\alpha_\mathrm{s}^3 + \ldots) \\
\sigma_{+1} &= \sigma_0(C_{11}\alpha_\mathrm{s} + C_{12}\alpha_\mathrm{s}^2 + C_{13}\alpha_\mathrm{s}^3 + \ldots) \\
\sigma_{+2} &= \sigma_0(C_{22}\alpha_\mathrm{s}^2 + C_{23}\alpha_\mathrm{s}^3 + C_{24}\alpha_\mathrm{s}^4 + \ldots) \\
&\vdots
\end{aligned}$$

Tree-level generators only gives us inclusive events.

NLO generators only gives us one extra parton.

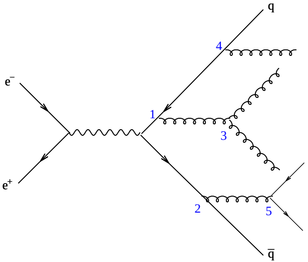
Final-State Showers

The tree-level matrix element for an *n*-parton state can be approximated by a product of splitting functions corresponding to a sequence of one-parton emissions from the zeroth order state.



Final-State Showers

The tree-level matrix element for an n-parton state can be approximated by a product of splitting functions corresponding to a sequence of one-parton emissions from the zeroth order state.



We can then order the emissions acording to some resolution scale, ρ, so that $\rho_1 \gg \rho_2 \gg \rho_3 \gg \ldots$

We have the standard DGLAP splitting kernels

$$
\begin{aligned}
\mathcal{P}_{\mathrm{q}\to\mathrm{qg}}(\rho,z)d\rho dz &= \frac{\alpha_{\mathrm{s}}}{2\pi}dz\frac{d\rho}{\rho}C_F\frac{1+z^2}{1-z} \\
\mathcal{P}_{\mathrm{g}\to\mathrm{gg}}(\rho,z)d\rho dz &= \frac{\alpha_{\mathrm{s}}}{2\pi}dz\frac{d\rho}{\rho}N_C\frac{(1-z(1-z))^2}{z(1-z)} \\
\mathcal{P}_{\mathrm{g}\to\mathrm{q\bar{q}}}(\rho,z)d\rho dz &= \frac{\alpha_{\mathrm{s}}}{2\pi}dz\frac{d\rho}{\rho}T_R\,(z^2+(1-z)^2)
\end{aligned}
$$

where ρ is the squared invariant mass or transverse momentum, and z is the energy (light-cone momenta) fraction taken by one of the daughters.

We now to make the events exclusive. This is done by saying that the first emission at some ρ_1 is given by the splitting kernel multiplied by the probability that there has been no emission above that scale.

In a given interval $d\rho$ we have the no-emission probability

$$\left(1 - \sum_{bc} \int dz\, \mathcal{P}_{a\to bc}(z,\rho)\right) d\rho$$

Integrating from ρ_1 up to some maximum scale, ρ_0 we get

$$\Delta(\rho_0,\rho_1) = \exp\left(-\sum_{bc} \int_{\rho_1}^{\rho^0} d\rho \int dz\, \mathcal{P}_{a\to bc}(z,\rho)\right)$$

In the same way we get the probability to have the *n*th emission at some scale ρ_n

$$P(\rho_n) = \sum_{abc} \int dz\, \mathcal{P}_{a\to bc}(\rho_n, z) \times \exp\left(- \sum_{abc} \int_{\rho_n}^{\rho_{n-1}} d\rho' \int dz'\, \mathcal{P}_{a\to bc}(z', \rho') \right)$$

Integrating we get schematically

$$
\begin{aligned}
\sigma_{+0} &= \sigma_0 \Delta_{S0} = \sigma_0(1 + C^{PS}_{01}\alpha_s + C^{PS}_{02}\alpha_s^2 + \ldots) \\
\sigma_{+1} &= \sigma_0 C^{PS}_{11}\alpha_s \Delta_{S1} = \sigma_0(C^{PS}_{11}\alpha_s + C^{PS}_{12}\alpha_s^2 + C^{PS}_{13}\alpha_s^3 + \ldots) \\
\sigma_{+2} &= \sigma_0 C^{PS}_{22}\alpha_s^2 \Delta_{S2} = \sigma_0(C^{PS}_{22}\alpha_s^2 + C^{PS}_{23}\alpha_s^3 + C^{PS}_{24}\alpha_s^4 + \ldots) \\
&\vdots
\end{aligned}
$$

We still need a cutoff, ρ_{cut}, and the coefficients C^{PS}_{nn} diverges as $\log^{2n} \rho_{\mathrm{max}}/\rho_{\mathrm{cut}}$

but the Sudakovs corresponds to the an approximate resummation of all virtual terms and makes things finite, and we can use $\rho_{\mathrm{cut}} \sim 1$ GeV.

Integrating we get schematically

$$
\begin{aligned}
\sigma_{+0} &= \sigma_0 \Delta_{S0} = \sigma_0(1 + C_{01}^{\mathrm{PS}}\alpha_s + C_{02}^{\mathrm{PS}}\alpha_s^2 + \ldots) \\
\sigma_{+1} &= \sigma_0 C_{11}^{\mathrm{PS}}\alpha_s \Delta_{S1} = \sigma_0(C_{11}^{\mathrm{PS}}\alpha_s + C_{12}^{\mathrm{PS}}\alpha_s^2 + C_{13}^{\mathrm{PS}}\alpha_s^3 + \ldots) \\
\sigma_{+2} &= \sigma_0 C_{22}^{\mathrm{PS}}\alpha_s^2 \Delta_{S2} = \sigma_0(C_{22}^{\mathrm{PS}}\alpha_s^2 + C_{23}^{\mathrm{PS}}\alpha_s^3 + C_{24}^{\mathrm{PS}}\alpha_s^4 + \ldots) \\
&\vdots
\end{aligned}
$$

We still need a cutoff, ρ_{cut}, and the coefficients C_{nn}^{PS} diverges as $\log^{2n} \rho_{\mathrm{max}}/\rho_{\mathrm{cut}}$

but the Sudakovs corresponds to the an approximate resummation of all virtual terms and makes things finite, and we can use $\rho_{\mathrm{cut}} \sim 1$ GeV.

The divergencies comes from the soft and collinear poles in the splitting kernels, eg.

$$\int_{\rho_c}^{\rho_0} d\rho \int dz\, \mathcal{P}_{q\to qg}(\rho, z) \sim \int_{\rho_c}^{\rho_0} \frac{\alpha_s d\rho}{\rho} \ln(\rho_0/\rho) \sim \alpha_s \ln^2(\rho_0/\rho_c)$$

Parton showers systematically resums all orders of $\alpha_s^n \ln^{2n}(\rho_0/\rho_c)$ which is the main part of the higher order corrections.

The divergencies comes from the soft and collinear poles in the splitting kernels, eg.

$$\int_{\rho_c}^{\rho_0} d\rho \int dz\, \mathcal{P}_{q\to qg}(\rho, z) \sim \int_{\rho_c}^{\rho_0} \frac{\alpha_s d\rho}{\rho} \ln(\rho_0/\rho) \sim \alpha_s \ln^2(\rho_0/\rho_c)$$

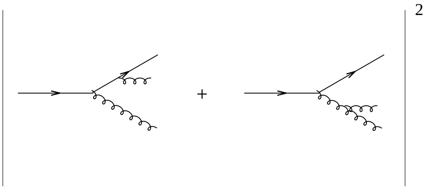
Parton showers systematically resums all orders of $\alpha_s^n \ln^{2n}(\rho_0/\rho_c)$ which is the main part of the higher order corrections.

However if there is no strong ordering, $\rho_1 \gg \rho_2 \gg \rho_3 \gg \ldots$, the PS approximation breaks down

Parton showers cannot model several hard jets very well. Especially the correlations between hard jets are poorly described.

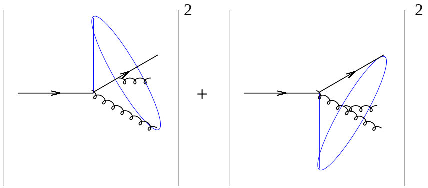
Angular Ordering

The splitting probabilities means that coherence effects are not taken into account



Angular Ordering

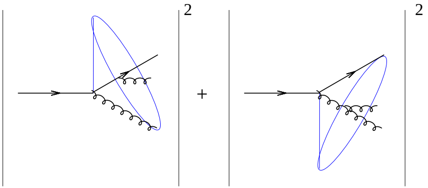
The splitting probabilities means that coherence effects are not taken into account



Most coherence effects can be taken into account by angular ordering.

Angular Ordering

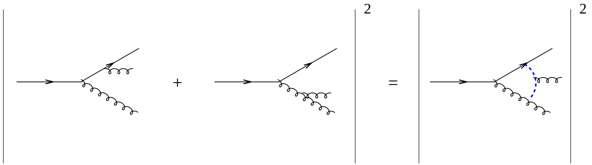
The splitting probabilities means that coherence effects are not taken into account



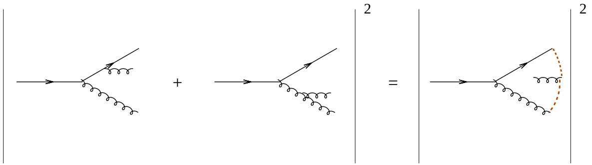
Most coherence effects can be taken into account by angular ordering.

Some angular correlations can also be taken into account by adjusting the azimuthal angles after a shower is generated. (cf. lecture on Wednesday)

Coherence effects can be included directly, by considering gluon radiation from colour dipoles between colour-connected partons.



Coherence effects can be included directly, by considering gluon radiation from colour dipoles between colour-connected partons.



Rather than iterating $1 \to 2$ parton splitting we iterate $2 \to 3$ splittings. Each emission from a dipole will create two new dipoles which can continue radiating.

This was first implemented in the ARIADNE generator. Recently similar schemes have been implemented in PYTHIA, SHERPA and VINCIA.

Evolution Variables

How do we choose the evolution variable, ρ?

The most natural choice is to choose a variable which isolates both the soft and collinear poles in the splitting kernel. This is the case for $\rho = p_{\perp}^2$ as used in eg. ARIADNE.

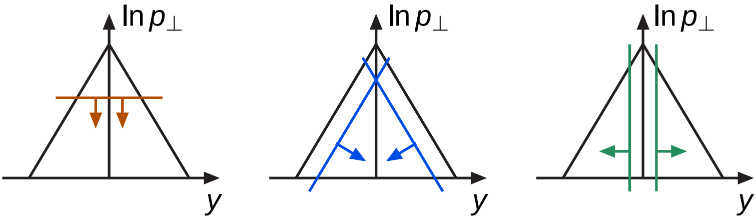
In old versions of PYTHIA and SHERPA the evolution variable is the virtuality Q^2 which in principle is fine except that $\alpha_s(p_{\perp}^2)$ may diverge for any given Q^2. Also angular ordering needs to be imposed in separately.

In HERWIG the ordering is in angle, which ensures angular ordering, but does not isolate the soft pole, and an additional cutoff is needed.

Transverse momentum $\rho = p_\perp^2$

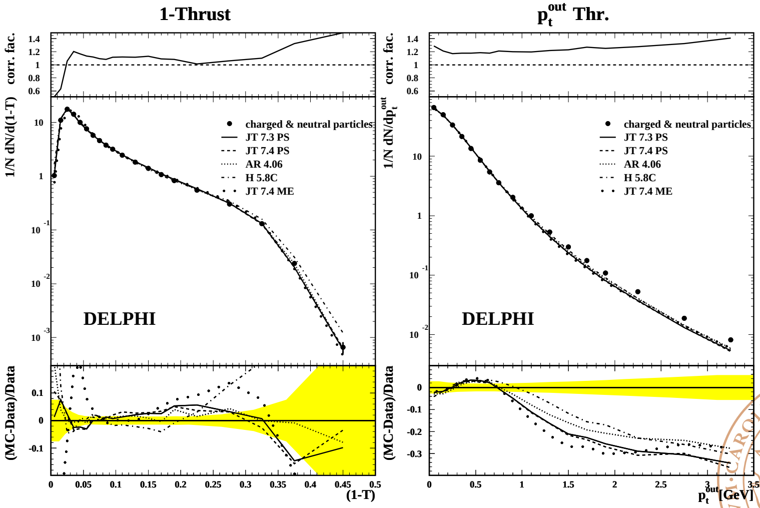
Virtuality $\rho = Q^2 \sim \frac{p_\perp^2}{z(1-z)}$

Angle $\rho \sim E^2\theta^2 \sim \frac{p_\perp^2}{z^2(1-z)^2}$



The Sixth Commandment of Event Generation

Final-state parton showers did really well at LEP



How do we generate a parton shower emission?

$$P(t) = f(t) \exp\left(-\int_t^{t_{\max}} \mathcal{P}(t')\,dt'\right)$$

We can do the standard transformation method

$$\int_t^{t_{\max}} dtP(t) = \exp\left(-\int_t^{t_{\max}} \mathcal{P}(t')\,dt'\right) = \int_r^1 p_R(t)dt = 1 - r$$

So if $\mathcal{P}$ has a simple primitive function F we get

$$t = F^{-1}(\ln r)$$

but $\mathcal{P}$ is never simple...

How do we generate a parton shower emission?

$$P(t) = f(t) \exp\left(-\int_t^{t_{\max}} \mathcal{P}(t')\,dt'\right)$$

We can do the standard transformation method

$$\int_t^{t_{\max}} dtP(t) = \exp\left(-\int_t^{t_{\max}} \mathcal{P}(t')\,dt'\right) = \int_r^1 p_R(t)dt = 1 - r$$

So if $\mathcal{P}$ has a simple primitive function F we get

$$t = F^{-1}(\ln r)$$

but $\mathcal{P}$ is never simple...

Assume g is a simple function with a simple primitive function G such that $g(t) \geq \mathcal{P}(t)$, $\forall t$. Then we can use the following algorithm

- start with $t_0 = t_{\max}$;
- select $t_i = G^{-1}(G(t_{i-1}) - \ln R)$, i.e. according to $g(t)$, but with the constraint that $t_i < t_{i-1}$,
- compare a (new) R with the ratio $\mathcal{P}(t_i)/g(t_i)$; if $\mathcal{P}(t_i)/g(t_i) \leq R$, then return to point 2 for a new try;
- otherwise t_i is retained as final answer.

Consider the various ways in which one can select a specific scale t. The probability that the first try works, $t = t_1$, i.e. that no intermediate t values need be rejected, is given by

$$p_0(t) = e^{-\int_t^{t_{\max}} g(t')\,dt'} g(t) \frac{\mathcal{P}(t)}{g(t)} = \mathcal{P}(t) e^{-\int_t^{t_{\max}} g(t')\,dt'}$$

The probability that we have thrown away one intermediate value t_1

$$\begin{aligned} p_1(t) &= \int_t^{t_{\max}} dt_1 e^{-\int_{t_1}^{t_{\max}} g(t')\,dt'} g(t_1) \left[1 - \frac{\mathcal{P}(t_1)}{g(t_1)}\right] \\ & \quad e^{-\int_t^{t_1} g(t')\,dt'} g(t) \frac{\mathcal{P}(t)}{g(t)} \end{aligned}$$

$$p_1(t) = p_0(t) \int_t^{t_{\max}} dt_1 \left[g(t_1) - \mathcal{P}(t_1)\right]$$

Similarly we get

$$\begin{aligned} p_2(t) &= p_0(t) \int_t^{t_{\max}} dt_1 \left[g(t_1) - \mathcal{P}(t_1)\right] \int_t^{t_1} dt_2 \left[g(t_2) - \mathcal{P}(t_2)\right] \\ &= p_0(t) \frac{1}{2} \left(\int_t^{t_{\max}} \left[g(t') - \mathcal{P}(t')\right] dt' \right)^2 \end{aligned}$$

$$\begin{aligned} p_{tot}(t) &= \sum_{i=0}^{\infty} p_i(t) = p_0(t) \sum_{i=0}^{\infty} \frac{1}{i!} \left(\int_t^{t_{\max}} \left[g(t') - \mathcal{P}(t')\right] dt' \right)^i \\ &= \mathcal{P}(t) e^{-\int_t^{t_{\max}} g(t')\, dt'} e^{\int_t^{t_{\max}} \left[g(t') - \mathcal{P}(t')\right] dt'} \\ &= \mathcal{P}(t) e^{-\int_t^{t_{\max}} \mathcal{P}(t')\, dt'} \end{aligned}$$

Also if several things may happen, $\mathcal{P}_1(t)$, $\mathcal{P}_2(t)$,$\mathcal{P}_3(t)$, ... the probability of i happening first is

$$\mathcal{P}_i(t) \times \prod_j e^{-\int_t^{t_{\max}} \mathcal{P}_j(t')\,dt'}$$

Simply generate a scale for each i according to

$$\mathcal{P}_i(t) \times e^{-\int_t^{t_{\max}} \mathcal{P}_i(t')\,dt'}$$

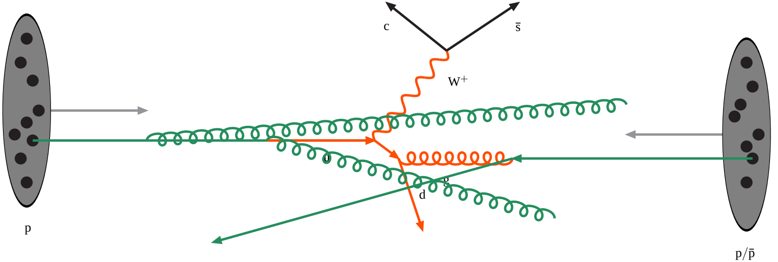
and pick the process with the largest scale.

Initial-State Showers

For incoming hadrons, we need to consider the evolution of the parton densities. Using collinear factorization and DGLAP evolution we have (with $t = \log k_\perp^2/\Lambda^2$)

$$\frac{df_b(x,t)}{dt} = \sum_a \int \frac{dx'}{x'} f_a(x',t) \frac{\alpha_s}{2\pi} P_{a\to b}\left(\frac{x}{x'}\right)$$

We can interpret this as during a small increase $\mathrm{d}t$ there is a probability for parton a with momentum fraction x' to become resolved into parton b at $x = zx'$ and another parton c at $x' - x = (1-z)x'$.


c
s̄
W+
p
p/p̄
u
d

In a backward evolution scenario we start out with the hard sub-process at some scale $t_{\max}$

$$\sigma_0 \propto \hat{\sigma}_{ab\to X} f_a(x_a, t_{\max}) f_b(x_b, t_{\max})$$

and we get the relative probability for the parton a to be *unresolved* into parton c during a decrease in scale dt

$$d\mathcal{P}_a = \frac{df_a(x_a,t)}{f_a(x_a,t)} = |dt| \sum_c \int \frac{dx'}{x'} \frac{f_c(x',t)}{f_a(x_a,t)} \frac{\alpha_s}{2\pi} P_{c\to a}\left(\frac{x_a}{x'}\right)$$

Summing up the cumulative effect of many small changes dt, the probability for no radiation exponentiates and we get a Sudakov

$$\Delta_{S_a}(x_a, t_{\max}, t) = \exp\left\{ -\int_t^{t_{\max}} dt' \sum_c \int \frac{dx'}{x'} \frac{f_c(x',t')}{f_a(x_a,t')} \frac{\alpha_s(t')}{2\pi} P_{c\to a}\left(\frac{x_a}{x'}\right)\right\}$$

This now gives us the probability for the first backwards initial-state splitting

$$d\mathcal{P}_{ca} = \frac{\alpha_{\mathrm{s}}}{2\pi} P_{ac}(z) \frac{f_c(x_a/z, t)}{f_a(x_a, t)} dt \frac{dz}{z} \times \Delta_{S_a}(x_a, t_{\max}, t)$$

In a hadronic collision we first generate the hard scattering, then evolve the incoming partons backward to lower scales, and then alow for a final-state shower from all partons from the hard scattering and the initial-state shower.

This is like undoing the evolution of the PDFs

The small-x problem

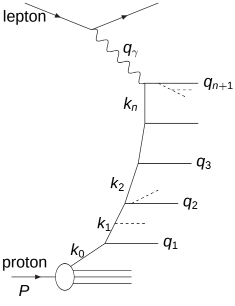
DGLAP evolution is not applicable if the hard scale is much smaller than the total energy and the virtuality of the incoming partons are not much smaller than the hard scale. (small x)

Collinear factorization $\Longrightarrow$ $k_\perp$-factorization

$$\int dx_a dx_b \hat{\sigma}_{ab\to X} f_a(x_a, Q^2) f_b(x_b, Q^2) \Longrightarrow$$

$$\int dx_a dx_b dk_{\perp a} dk_{\perp b} \hat{\sigma}^{\star}_{ab\to X} \mathcal{F}_a(x_a, k_{\perp a}, Q^2) \mathcal{F}_b(x_b, k_{\perp b}, Q^2)$$

$\mathcal{F}$ an unintegrated parton density.
$\hat{\sigma}^{\star}$ is the off-shell matrix element

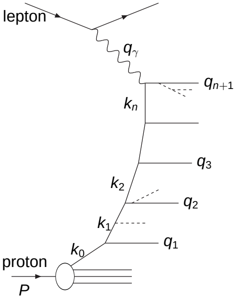


In DIS, the cross section is dominated by events with small $Q^2 = -q_\gamma^2$ and small x.

The available phase space for emitting partons is not limited by Q^2, but rather by the total hadronic energy, $W^2 \approx Q^2/x$.

The $1/z$ pole in the gluon splitting function makes it possible to emit many initial-state gluons even for small Q^2.

We need to take into account unordered evolution.



In DIS, the cross section is dominated by events with small $Q^2 = -q_\gamma^2$ and small x.

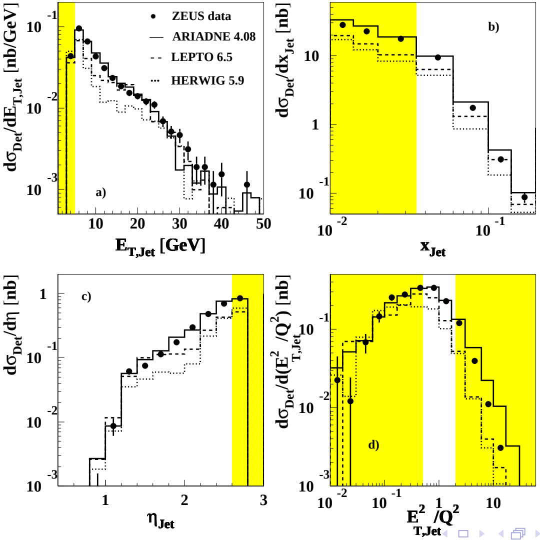
The available phase space for emitting partons is not limited by Q^2, but rather by the total hadronic energy, $W^2 \approx Q^2/x$.

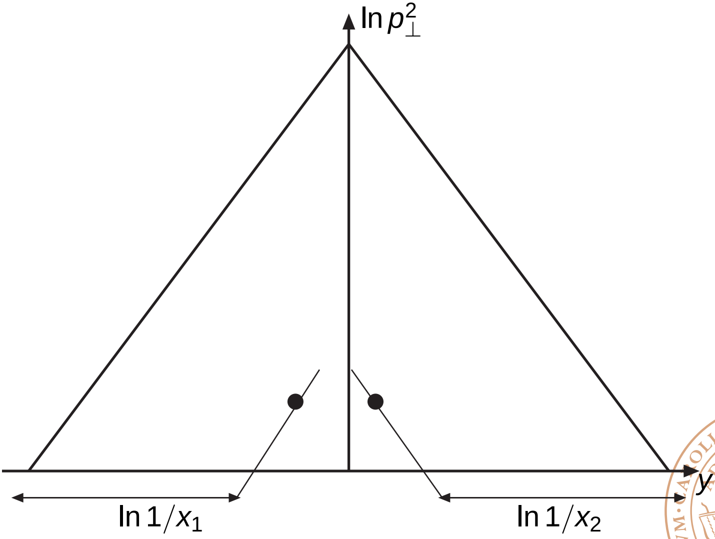
The $1/z$ pole in the gluon splitting function makes it possible to emit many initial-state gluons even for small Q^2.

We need to take into account unordered evolution.

Forward jets at HERA cannot be reproduced by DGLAP based initial-state parton showers.

ZEUS 1995


ZEUS data
ARIADNE 4.08
LEPTO 6.5
HERWIG 5.9
a)
b)
c)
d)
$d\sigma_{Det}/dE_{T,Jet}$ [nb/GeV]
$E_{T,Jet}$ [GeV]
$d\sigma_{Det}/dx_{Jet}$ [nb]
x_{Jet}
$d\sigma_{Det}/d\eta$ [nb]
η_{Jet}
$d\sigma_{Det}/d(E^2_{T,Jet}/Q^2)$ [nb]
$E^2_{T,Jet}/Q^2$


$\ln p_\perp^2$
y
$\ln 1/x_1$
$\ln 1/x_2$

Let's look at the unintegrated gluon density, which should be dominating. Starting from a (non-perturbative) gluon at some x_0 we get the contribution

$$\mathcal{G}(x,k_\perp^2) = \sum_n \prod_i^n \int \frac{dq_{\perp i}^2}{q_{\perp i}^2} dz_i \bar{\alpha} \tilde{P}(z_i, q_{\perp i}^2) \Theta(z_i, q_{\perp i}^2) \delta(x - x_0 \Pi z_i) \delta(k_\perp^2 - k_{\perp n}^2)$$

$\bar{\alpha}$ is a suitably scaled α_s
$\tilde{P}(z_i, q_{\perp i}^2)$ is the splitting function
$\Theta(z_i, q_{\perp i}^2)$ is some phase space limitation defining which emissions we want to include in the evolution.

For large $k_\perp$ and small x we can use the double leading logarithmic approximation with $\tilde{P}(z) \approx 1/z$ and $\Theta = \theta(q_{\perp i} - q_{\perp i-1})$

$$\mathcal{G}(x, k_\perp^2) = \sum_n \prod_i^n \int \frac{dq_{\perp i}^2}{q_{\perp i}^2} \frac{dx_i}{x_i} \theta(q_{\perp i} - q_{\perp i-1}) \theta(x_{i-1} - x_i) \delta(x - x_n) \delta(k_\perp^2 - k_{\perp n}^2)$$

which can be easily integrated to get the well known DLL result

$$\mathcal{G} \propto \exp(2\sqrt{\bar{\alpha} \ln k_\perp^2 \ln 1/x})$$

Using running coupling $\bar{\alpha} = \alpha_0 / \log(q_\perp^2/\Lambda^2)$ we would instead get

$$\mathcal{G} \propto \exp(2\sqrt{\alpha_0 \ln \ln k_\perp^2 \ln 1/x})$$

This corresponds to standard DGLAP evolution

In the limit of asymptotically small x and moderate $k_\perp$ we may use BFKL evolution. Here there is no upper limit on the $q_\perp$ of the emitted gluons and the splitting function

$$\tilde{P}(z, k_\perp^2) = \Delta_R(z, k_\perp^2)/z$$

corresponds to real gluon emissions from *Reggeized* gluons, where the Regge form factor corresponding to a sum over virtual diagrams:

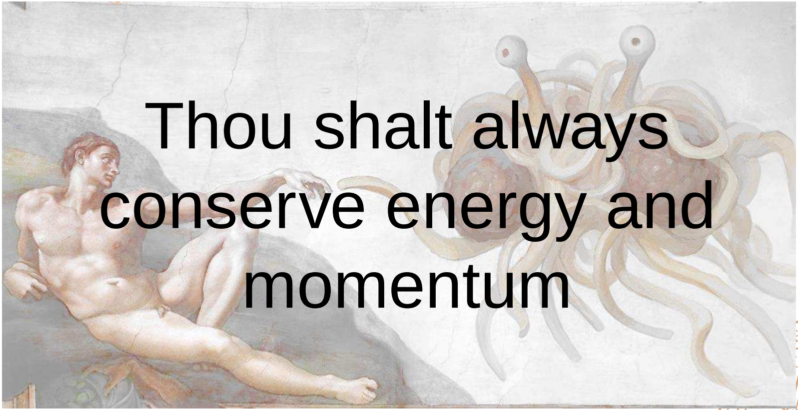
$$\Delta_R(z, k_\perp^2) = \exp\left(-\bar{\alpha}\int_{z_i}^{1}\frac{dz}{z}\int_{\mu^2}^{k_{\perp i}^2}\frac{dk_\perp^2}{k_\perp^2}\right)$$

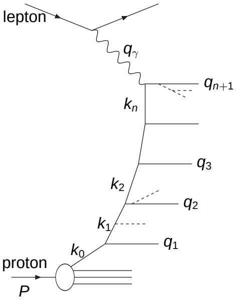
The integration is a bit more tricky, but is doable and the result is the well-known strong rise of the gluon

$$\mathcal{G} \propto x^{-\lambda} = x^{-4\ln 2\bar{\alpha}}$$

The next-to-leading logarithmic corrections to BFKL turns out to be massive. The main reason for this seems to be related to the lack of (transverse) momentum conservation when allowing for unlimited $q_\perp$ in the emissions.

The Seventh Commandment of Event Generation





In a parton shower scenario we typically want to separate between initial-state emissions which corresponds to the evolution of the parton densities, and final-state emissions which do not.

In CCFM evolution this done by defining all emissions not corresponding to a angular ordered final-state shower to be initial-state emissions.

CCFM

CCFM limits the initial-state emissions to have increasing opening angles (rapidity). In terms of the rescaled transverse momentum $\bar{q} = q_\perp/(1-z)$ we then get the phase space restriction

$$\Theta = \theta(\bar{q}_i - z_{i-1}\bar{q}_{i-1})$$

Starting from BFKL and resumming all emissions now treated as final-state will cancel parts of the Regge form factor giving

$$\frac{\Delta_R}{z_i} \longrightarrow \frac{\Delta_{ne}}{z_i} = \frac{1}{z_i} \exp\left(-\bar{\alpha}\int_{z_i}^{1}\frac{dz}{z}\int^{k_{\perp i}^2}\frac{d\bar{q}^2}{\bar{q}^2}\theta(\bar{q} - z\bar{q}_i)\right)$$

The angular ordering properly takes into account gluon coherence and also results in less infrared sensitivity.

Here we may also include the soft pole in the splitting function with a corresponding Sudakov form factor to conserve energy

$$\tilde{P} = \frac{\Delta_{ne}}{z} + \frac{\Delta_S}{1-z}$$

which means that for not so small x we recover the main features of DGLAP evolution.

CASCADE implements CCFM in a backward evolution algorithm.

Linked Dipole Chains

The division between initial- and final-state emissions can be made in many ways. However it is reasonable to require that the final-state emissions do not change the basic propagators in the ladder too much.

In the Linked Dipole Chain (LDC) model the final-state emissions are coming from the dipoles between the gluons emitted in the initial-state. A suitable constraint on the initial state emissions turns out to be

$$\Theta = \theta(q_{\perp i} - \min(k_{\perp i-1}, k_{\perp i}))$$

This is a stronger restriction than in CCFM and summing up the contributions from final-state emissions will give us simply

$$\Delta_{ne}/z \longrightarrow 1/z$$

In this way, LDC becomes even less infrared sensitive, and the absence of a form factor makes it easy to include full DGLAP splitting functions (not only the singular parts) and even include the evolution of quarks.

Also LDC has been implemented in an event generator, LDCMC.

But we can also learn some qualitative lessons from the LDC formulation.

Looking at the limit of strongly ordered $k_\perp$, not only increasing but also decreasing, we find that the phase space restriction in LDC means that $q_{\perp i} \approx \max(k_{\perp i-1}, k_{\perp i})$. Also considering strongly ordered x we get for each emission

$$\bar{\alpha}\frac{dz_i}{z_i}\frac{dq^2_{\perp i}}{q^2_{\perp i}} \approx \bar{\alpha}\frac{dz_i}{z_i}\frac{dk^2_{\perp i}}{\max(k_{\perp i-1}, k_{\perp i})} = \bar{\alpha}\frac{dz_i}{z_i}\frac{dk^2_{\perp i}}{k^2_{\perp i}}\min\left(\frac{k_{\perp i}}{k_{\perp i-1}}, 1\right)$$

Comparing with the DLL approximation above which we can rewrite in terms of $\kappa = \log k_{\perp i}^2/\Lambda^2$ and $l_i = \log(1/x_i)$:

$$\mathcal{G}(l,\kappa) \propto \sum_n \prod_i^n \left\{ \bar{\alpha} \int^{\kappa} d\kappa_i \theta(\kappa_i - \kappa_{i-1}) \int^{l} dl_i \theta(l_i - l_{i-1}) \right\} = \sum_n \bar{\alpha}^n \frac{\kappa^n}{n!} \frac{l^n}{n!}$$

we now want to allow also for unordered κ, but we note that taking a step down in κ is punished exponentially by $k_{\perp i}^2/k_{\perp i-1}^2 = \exp(\kappa_{i-1} - \kappa_i)$.

Approximating the exponential suppression with a step function we get an approximate ordering in κ, $\theta(\kappa_i - \kappa_{i-1} + 1)$ and we have

$$\int^{\kappa} \prod_i^n d\kappa_i \theta(\kappa_i - \kappa_{i-1} + 1) \approx \frac{(\kappa + n)^n}{n!}$$

For large κ we recover the DLL result

On the other hand if κ is small we get from Sterlings formula

$$\frac{(\kappa + n)^n}{n!} \approx \frac{n^n}{n!} \approx e^n$$

and

$$\mathcal{G}(l, \kappa) \propto \sum_n \bar{\alpha}^n e^n \frac{l^n}{n!} \approx e^{\bar{\alpha} e l} = x^{-\lambda}$$

with $\lambda = e\bar{\alpha} \approx 2.72\bar{\alpha}$ which is remarkably close to the BFKL result $\lambda = 4\log 2\bar{\alpha} \approx 2.77\bar{\alpha}$.

We can also get an estimate of where the transition between DGLAP and BFKL should occur, and obtain $\kappa \approx \lambda l$

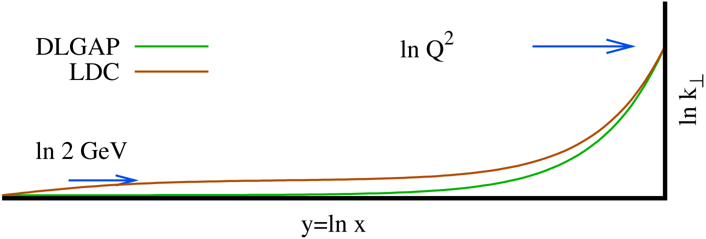
We can now also try to include a running coupling which means

$$\bar{\alpha} d\kappa \rightarrow \alpha_0 \frac{d\kappa}{\kappa} = \alpha_0 du$$

with $u = \log \kappa$.

Remembering the approximate phase space constraint $\theta(\kappa_i - \kappa_{i-1} + 1)$ we note that for large κ, one extra unit in κ is negligible in u and we recover the DLL situation, while for small κ the restriction basically vanishes and we get a random walk in κ.

We can therefore expect the typical evolution path, going backwards from the hard scale, to be DGLAP-like until the virtualities reach smaller values where it becomes BFKL-like.



This can be simulated with a DGLAP shower by adding an unnaturally large intrinsic $k_\perp$ (needed to describe $p_\perp$-distributions for prompt photons and W production).

The event generators CASCADE and LDCMC give consistent result.

However, the forward jet rates, a measurement designed to be impossible to reproduce without unordered evolution, is only reproduced by CASCADE and LDCMC if non-singular terms are omitted from the gluon splitting function. Using the full function

$$P_{gg}(z) = \frac{1}{z} + \frac{1}{1-z} + z(1-z) - 2$$

will underestimate forward jet rates by almost a factor 2.

Lately CASCADE does a bit better with specially tuned unintegrated PDFs.

The event generators CASCADE and LDCMC give consistent result.

However, the forward jet rates, a measurement designed to be impossible to reproduce without unordered evolution, is only reproduced by CASCADE and LDCMC if non-singular terms are omitted from the gluon splitting function. Using the full function

$$P_{gg}(z) = \frac{1}{z} + \frac{1}{1-z} + z(1-z) - 2$$

will underestimate forward jet rates by almost a factor 2.

Lately CASCADE does a bit better with specially tuned unintegrated PDFs.

The dipole shower in ARIADNE allows for un-ordered evolution (although not directly related to BFKL/CCFM/LDC) and reproduces forward jets quite nicely.

Tree-level matrix element generators are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Parton shower generators are not good for a handful hard, well separated partons, but good for many soft and collinear partons.

Why can't we simply combine the two?

Tree-level matrix element generators are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Parton shower generators are not good for a handful hard, well separated partons, but good for many soft and collinear partons.

Why can't we simply combine the two?

For one extra parton emission we can usually simply modify the splitting functions to reproduce the correct matrix element.

Tree-level matrix element generators are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Parton shower generators are not good for a handful hard, well separated partons, but good for many soft and collinear partons.

Why can't we simply combine the two?

For one extra parton emission we can usually simply modify the splitting functions to reproduce the correct matrix element.

For more partons we need CKKW.

Parton Shower

$$
\begin{aligned}
O_{+0} &= \sigma_0 \Delta_{S0} \\
O_{+1} &= \sigma_0 C_{11}^{PS} \alpha_s \Delta_{S1} \\
O_{+2} &= \sigma_0 C_{22}^{PS} \alpha_s^2 \Delta_{S2} \\
&\vdots
\end{aligned}
$$

Matrix Element

$$
\begin{aligned}
O_{+0} &= \sigma_0 \\
O_{+1} &= \sigma_0 C_{11}^{ME} \alpha_s \\
O_{+2} &= \sigma_0 C_{22}^{ME} \alpha_s^2 \\
&\vdots
\end{aligned}
$$

Comparing the α_s expansions the strategy should be obvious. Generate events with 1, 2, 3, ..., N extra hard jets according to tree-level matrix elements using some (large) cutoff. Then reweight with Sudakov form factors from the parton shower. Finally add parton shower to get events with more than N partons and with partons below the ME cutoff.

To obtain Sudakov form factors we need to have an ordered set of emission scales. This can be done by applying a jet clustering algorithm to the parton state generated with the Matrix Element.

Alternatively we can make a shower reconstruction (answering the question how would my parton shower have generated this partonic state?)

The Sudakovs can then be calculated analytically or by making trial parton shower emissions from intermediate states in the shower reconstruction, remembering that the Sudakov is a no-emission probability

When adding the parton shower we must make sure we do not double-count and add shower emissions which could also have been generated by the matrix element. (Fourth commandment)

Also we must not under-count and miss phase space regions not covered by the matrix element. (Second commandment)

The solution is to do a full parton shower, starting from the highest possible scale, but to veto emissions which are above the matrix element cutoff.

Special care must be taken for the highest parton multiplicity state generated by the matrix element. There we must only veto emissions which are above the lowest reconstructed scale.

Parton Shower | **NLO Matrix Element**

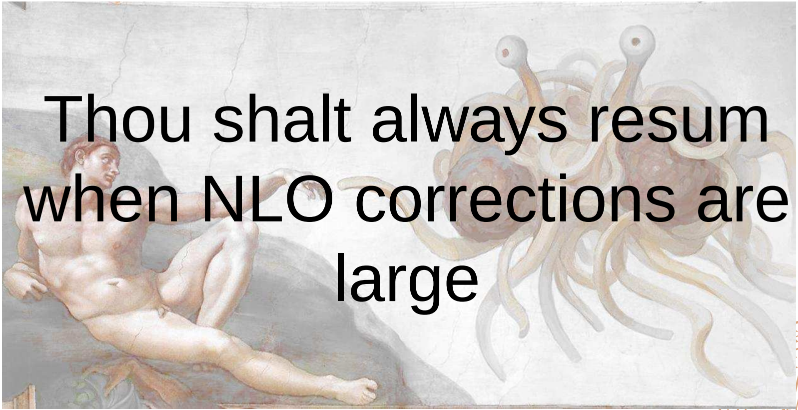
$$
\begin{aligned}
O_{+0} &= \sigma_0 \Delta_{S0} \\
&= \sigma_0 (1 + C_{10}^{PS} \alpha_s + \ldots) \\
O_{+1} &= \sigma_0 C_{11}^{PS} \alpha_s \Delta_{S1}
\end{aligned}
\qquad
\begin{aligned}
O_{+0} &= \sigma_0 (1 + C_{10}^{ME} \alpha_s) \\
O_{+1} &= \sigma_0 C_{11}^{ME} \alpha_s
\end{aligned}
$$

Two main strategies

- MC@NLO: Subtract the approximate PS term from the full ME, simply add PS
- POWHEG: Exponentiate $C_{10}^{ME} \alpha_s$ and add PS below first emission.

(more on matching and merging on Wednesday)

The Eighth Commandment of Event Generation



Parton Shower Generators

- PYTHIA: DGLAP-based $k_\perp$ and Q^2 ordering
  `http://home.thep.lu.se/~torbjorn/Pythia.html`
- HERWIG: DGLAP-based angle ordering
  `http://projects.hepforge.org/herwig`
- SHERPA: DGLAP-based Q^2 ordering, CKKW
  soon kT-ordered dipoles
  `http://projects.hepforge.org/sherpa`
- CASCADE: Initial-state CCFM shower
  `http://projects.hepforge.org/cascade`
- ARIADNE/LDC: Dipole shower
  (not quite suitable for LHC yet)
  `http://home.thep.lu.se/~leif/ariadne`
- ...

Outline of Lectures

- Lecture I: Basics of Monte Carlo, the event generator strategy, matrix elements, LO/NLO, . . .
- Lecture II: Parton showers, Sudakov formfactors, initial/final state, angular ordering, $k_\perp$-factorization, . . .
- Lecture III: Underlying events, multiple interactions, minimum bias, pile-up, hadronization, decays, . . .