PHYSICS AT THE TERASCALE

Monte Carlo School

DESY Hamburg, April 2008

Introduction to NLO calculations

Zakaria Merebashvili

II. Institut für Theoretische Physik
Universität Hamburg

Prerequisites

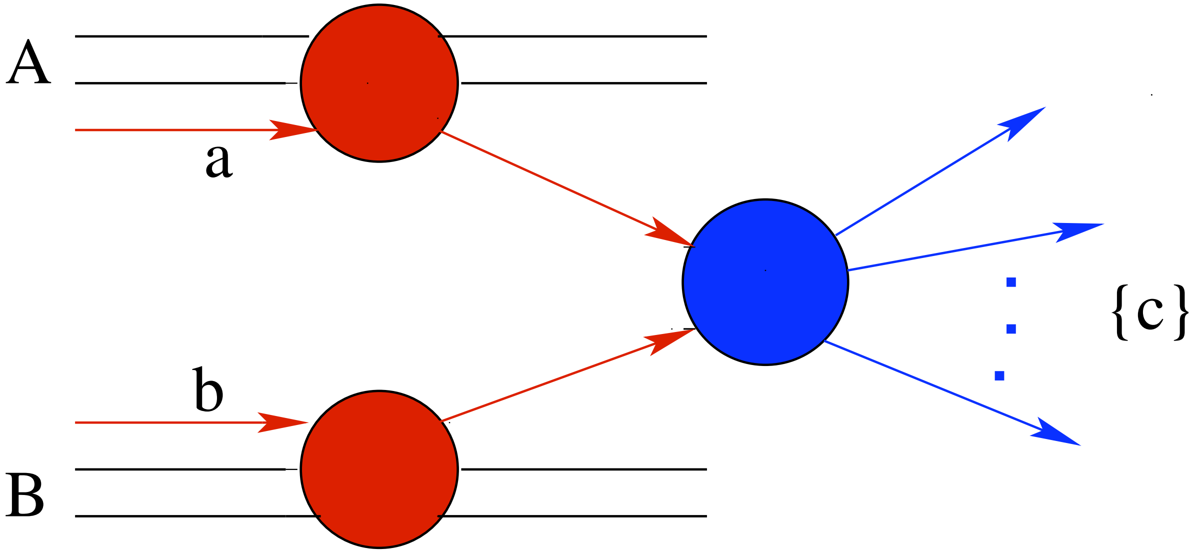
- You know what is the Lagrangian
- You have a notion of Feynman rules

Outline

- I will try to give a overview of NLO calculations in Standard Model
- I will talk about QCD corrections

Inclusive Cross Section in conventional/collinear theory

According to Feynman's Parton Model:



$$d\sigma_{AB\to\{c\}X}(S,\ldots) = \sum_{a,b}\int dx_a dx_b \; f_{a/A}(x_a) f_{b/B}(x_b) d\hat{\sigma}_{ab\to\{c\}}(g,s,\ldots)$$

Ingredients

▲ $f_{a/A}(x_a)$ – parton densities/distribution functions (PDF's)

- x_a is momentum fraction
- PDF's describe long distance effects, must be measured experimentally

▲ $d\hat{\sigma}_{ab\to c}(\alpha_s, s, \ldots)$ is partonic scattering cross section

- Calculate as *expansion* in $\alpha_s = g^2/(4\pi)$
- Partonic energy $s = x_a x_b S$

Factorization Theorem: factorized form of cross section is valid at each order of expansion in α_s

Remember: We discarded transverse degrees of freedom of partons.

What is Next-to-leading order

- If the first approximation/leading order (LO) is $\alpha_s^n \Longrightarrow$ NLO is α_s^{n+1}

- $\alpha_{em}(M_Z) = 1/137$ versus $\alpha_s(M_Z) \sim .12 \Longrightarrow$ QCD corrections are much larger $\Longrightarrow$ the most important ones

- Convergence of QCD perturbation series may not be good in the whole Phase Space

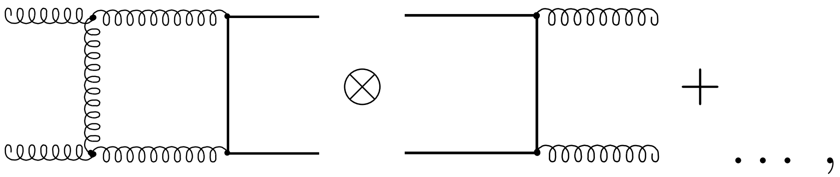
- Generally LO prediction is based on tree graphs

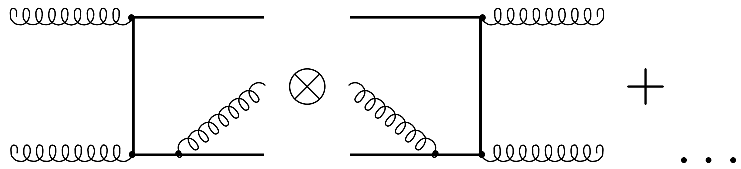
- Generally additional powers of coupling are obtained by inserting vertices due to gluon radiation and/or absorbtion

How to calculate NLO corrections

There are two categories of Feynman diagrams at NLO

Heavy flavor production: NLO QCD $\sim \alpha_s^3$



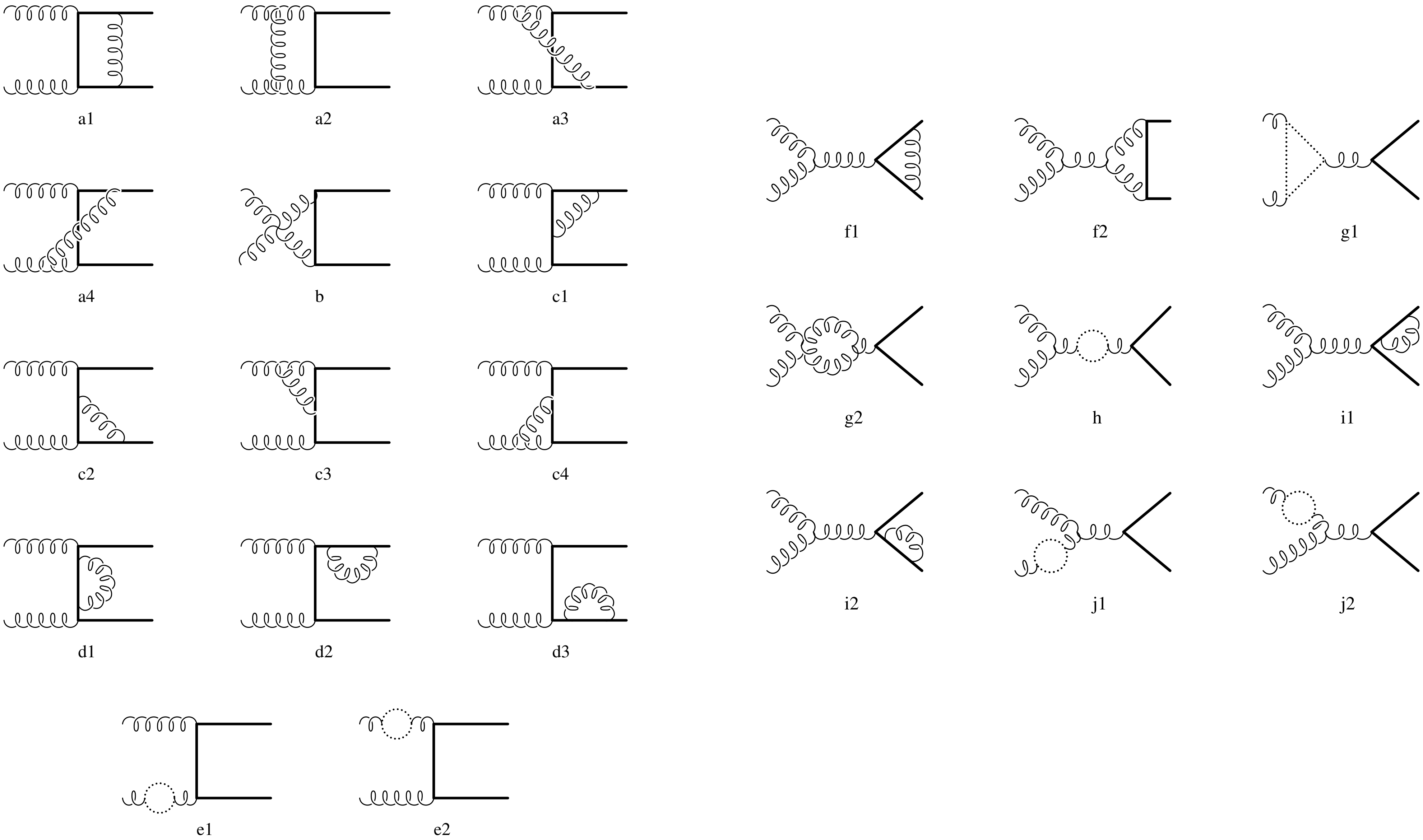


Virtual or One-loop diagrams:
gluon radiated and reabsorbed

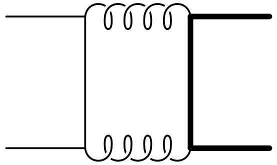
Real or Bremsstrahlung diagrams:
radiated and present in final state

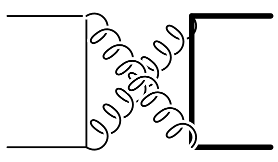
$$d\hat{\sigma}_{ab\to\{c\}}(g, s, \ldots) = \frac{d(\mathrm{PS})_{n_c}}{\mathrm{Flux}} \frac{1}{\mathrm{Spins\ Colors}} |M|^2_{ab\to\{c\}}$$

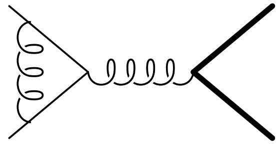
Gluon fusion subprocess: t– & s–channel graphs

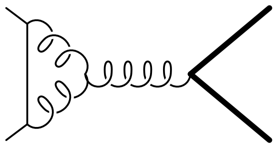


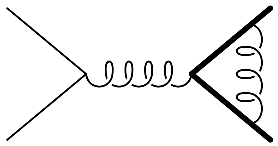
light quark-antiquark annihilation:

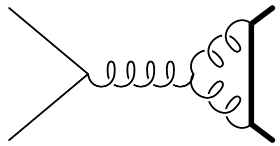

a

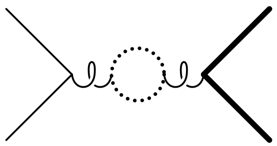

b

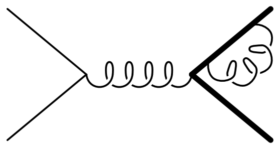

c

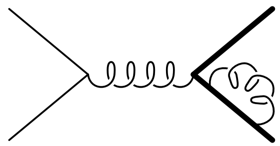

d

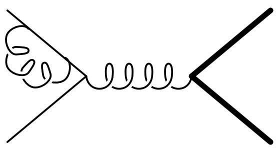

e

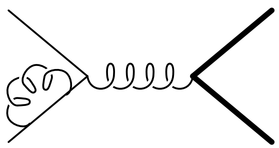

f


g


h


i


j


k

Partonic cross section: One-loop contributions

Feynman rules from the QCD Lagrangian :

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_I \\
\mathcal{L}_0 &= -\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)\left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}\right) - \frac{1}{2\alpha}\left(\partial^\mu A^a_\mu\right)^2 + \left(\partial^\mu \xi^{a*}\right)\left(\partial_\mu \xi^a\right) \\
&\quad + \overline{\Psi}_i (i\hat{\partial} - m)\Psi_i \\
\mathcal{L}_I &= -\frac{g}{2} f^{abc}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) A^{b\mu} A^{c\nu} - \frac{g^2}{4} f^{abe} f^{cde} A^a_\mu A^b_\nu A^{c\mu} A^{d\nu} \\
&\quad - g f^{abc}\left(\partial^\mu \xi^{a*}\right)\xi^b A^c_\mu + g\overline{\Psi}_i T^a_{ij}\gamma^\mu \Psi_j A^a_\mu
\end{aligned}
$$

Loop integrals diverge logarithmically (i.e. as $\frac{dx}{x}$) – seen by power counting.
The most useful tools for evaluating one-loop integrals:

Dimensional regularization & Feynman parameterization techniques etc…

Shifting space-time dimension to $n = 4 - 2\varepsilon \Rightarrow$ divergences appear as poles in

$$\frac{1}{\varepsilon} \quad \text{and} \quad \frac{1}{\varepsilon^2}$$

One-loop contributions: Ultraviolet Singularities

Some poles are due to one-loop integrals when loop momenta tend to infinity.
Renormalize the bare Lagrangian by rescaling fields and parameters:

fields	$A^a_\mu = Z_3^{1/2}\, A^a_{r\mu}$	$\xi^{a(*)} = \tilde{Z}_3^{1/2}\, \xi_r^{a(*)}$	$\Psi = Z_2^{1/2}\, \Psi_r$
parameters	$g = Z_g g_r$	$\alpha = Z_3 \alpha_r$	$m = Z_m m_r$

Inserting above into bare Lagrangian we obtain renormalized Lagrangian:

$$\mathcal{L}_r = \mathcal{L}_{r0} + \mathcal{L}_{rI} + \mathcal{L}_C$$

First two terms are obtained from the bare Lagrangian by adding subscript "r" to each field and parameter. In the $\mathcal{L}_C$ we collect all the renormalization constants Z_i:

One-loop contributions: Counterterms

$$\mathcal{L}_C = \frac{1}{2}(Z_3 - 1)\delta^{ab} A^a_{r\mu} \left(g^{\mu\nu}\Box - \partial^\mu \partial^\nu\right) A^b_{r\nu} - (\tilde{Z}_3 - 1)\delta^{ab}\xi^{a*}_r \Box \xi^b_r$$

$$+ (Z_2 - 1)\delta_{ij}\overline{\Psi}_{ri}(i\hat{\partial})\Psi_{rj} + (Z_2 Z_m - 1)\delta_{ij}\overline{\Psi}_{ri}(-m_r)\Psi_{rj}$$

$$-(Z_1 - 1)\frac{g_r}{2} f^{abc}\left(\partial_\mu A^a_{r\nu} - \partial_\nu A^a_{r\mu}\right) A^{b\mu}_r A^{c\nu}_r - (Z_4 - 1)\frac{g_r^2}{4} f^{abe} f^{cde} A^a_{r\mu} A^b_{r\nu} A^{c\mu}_r A^{d\nu}_r$$

$$- (\tilde{Z}_1 - 1) g_r f^{abc} \left(\partial^\mu \xi^{a*}_r\right) \xi^b_r A^c_{r\mu} + (Z_{1F} - 1) g_r \overline{\Psi}_{ri} T^a_{ij} \gamma^\mu \Psi_{rj} A^a_{r\mu}$$

and we have denoted

$$Z_1 \equiv Z_g Z_3^{3/2} \qquad \tilde{Z}_1 \equiv Z_g \tilde{Z}_3 Z_3^{1/2} \qquad Z_{1F} \equiv Z_g Z_2 Z_3^{1/2} \qquad Z_4 \equiv Z_g^2 Z_3^2$$

Basically, we got extra Feynman rules/terms (i.e. COUNTERTERMS) for every vertex and additionally for every propagator line.

One-loop contributions: Ultraviolet Singularities

We observe:

$$Z_g Z_3^{1/2} = \frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3} = \frac{Z_{1F}}{Z_2} = \frac{Z_4}{Z_1}$$

- This is Slavnov–Taylor identity $\Longrightarrow$ guarantees uniqueness of charge g_r and is a consequence of a local gauge symmetry.

- It can be proven that the renormalization performed is sufficient to remove the UV singularities to all orders. Such theories are known as Renormalizable Theories.

- The remaining singularities are of soft (Infrared, IR) and collinear (M) origin, or their combination.

Ultraviolet Singularities: Scheme choice

- In practice it is sufficient to derive the mass and propagator counterterms first (e.g. $Z_m, Z_2, Z_3, \tilde{Z}_3$) and just ONE vertex counterterm, say Z_{1F}. Then use ST identities to obtain the rest.

- Obviously, there is a freedom how we choose Z_i constants. The usual $\overline{\mathrm{MS}}$ scheme does the job.

- For massive quarks the On-Shell scheme is used. It is defined by conditions

$$\Sigma(\hat{p}, m)|_{\hat{p}=m} = 0 \text{ (fixes } Z_m) \qquad \frac{d\Sigma(\hat{p}, m)}{d\hat{p}}|_{\hat{p}=m} = 0 \text{ (fixes } Z_2)$$

The purpose: to be seen later ...

- Shortcut: 1. Perform mass renormalization. 2. Halve all the external self-energies. 3. Renormalize the coupling constant in the LO amplitude.

Ultraviolet Singularities: Renormalization scale

When we calculate loop integrals in DREG in $n = 4 - 2\varepsilon$ dimensions, originally dimensionless charge g aquires dimension of [g]=mass$^{(4-n)/2}$, e.g. formally

$$g \longrightarrow g(\mu_b) \qquad \text{and} \qquad g_r \longrightarrow g_r(\mu)$$

To keep charge dimensionless, we multiply it by arbitrary mass parameter:

$$g \longrightarrow g(\mu_b)\mu_b^{\varepsilon} \qquad \text{and} \qquad g_r \longrightarrow g_r(\mu)\mu^{\varepsilon}$$

Consequently

$$g = Z_g g_r \Longrightarrow g(\mu_b)\mu_b^{\varepsilon} = Z_g g_r(\mu)\mu^{\varepsilon}$$

Differentiating we get RGE for the running coupling constant at LO:

$$\frac{d\alpha_s(\mu^2)}{d\ln(\mu^2)} = -\frac{\beta_0}{4\pi}\alpha_s^2(\mu^2) + \mathcal{O}(\alpha_s^3) \qquad \text{with} \qquad \alpha_s = \frac{g_r^2(\mu)}{4\pi}$$

β_0 will include only light flavors because of decoupling of heavy quark !

Renormalization scale: Asymptotic freedom

The analytic solution is

$$\alpha_s(\mu_r^2) \quad = \quad \frac{4\pi}{\beta_0 \ln \frac{\mu_r^2}{\Lambda_{\rm LO}^2}}, \qquad \Lambda_{\rm LO}^2 \equiv \mu_0^2 \exp\left[-\frac{4\pi}{\beta_0 \alpha_s(\mu_0^2)}\right]$$

Let us write

$$Z_g = 1 - \frac{g_s^2}{\varepsilon}\left\{\frac{\beta_0}{2} C(\mu_r^2) - \frac{1}{3} C(m^2)\right\} \qquad \text{where} \quad C(m^2) \equiv \frac{\Gamma(1+\varepsilon)}{(4\pi)^2}\left(\frac{4\pi\mu_r^2}{m^2}\right)^{\varepsilon}$$

▲ The two schemes are equal at $\mu_r = m$

▲ α_s must be the same in the two schemes at $\mu_r = m$, otherwise physical cross section would be different in the two schemes:

$$\alpha_s^{(n_{lf}+1)}(m) = \alpha_s^{(n_{lf})}(m)$$

This is a matching condition at flavour thresholds

Partonic cross section: Real contributions

- 3-particle PS:

$$(PS)_3 = \int \frac{d^n p_3}{(2\pi)^{n-1}} \frac{d^n p_4}{(2\pi)^{n-1}} \frac{d^n p_5}{(2\pi)^{n-1}} \delta_+(p_3^2 - m_3^2)\delta_+(p_4^2 - m_4^2)\delta_+(p_5^2 - m_5^2) \times (2\pi)^n \delta^n(p_1 + p_2 - p_3 - p_4 - p_5)$$

- Squared matrix element, as a function of scalar products of external momenta $(p_i p_j)$, $i, j = 1, 2, ..., 5$

- To calculate physical observable, we must integrate out the third parton, i.e. perform the phase space integrations

▲ We get IR singularities when momentum of the third parton tends to zero or collinear M singularities when two massless partons become collinear, e.g

$$\frac{1}{p_1 p_5} = \frac{1}{p_1^{(0)} p_5^{(0)} (1 - \cos\theta)}$$

Partonic cross section: Virtual + Real

▲ The IR (also combined with M) singularities cancel in the sum of Virtual and Real contributions for a inclusive observable.

Guaranteed by the Bloch-Nordsieck and Kinoshita-Lee-Nauenberg theorems

▲ There still remain collinear singularities due to initial state radiation.

These divergencies are universal, e.g. do not depend on the particular partonic subprocess.

▲ The remainining M singularities are reabsorbed into the original parton densities, $f_{a/A}(x_a)$ and $f_{b/B}(x_b)$.

Physical cross section

$$d\sigma_{AB\to\{c\}X}(S,\ldots) = \sum_{a,b}\int dx_a dx_b\, f_{a/A}(x_a, M_F^2) f_{b/B}(x_b, M_F^2) d\hat{\sigma}_{ab\to\{c\}}(g(\mu_r), s, \ldots)$$

where renormalized PDF's vs bare PDF's in the $\overline{\mathrm{MS}}$ scheme

$$f_i(x, M_F^2) = f_i(x) + \int_x^1 \frac{dy}{y} H_{ij}(\frac{x}{y}) f_j(y),$$

$$H_{ij}(x) = -\frac{1}{\varepsilon}\frac{\alpha_s}{2\pi} P_{ij}(x)\left(\frac{4\pi\mu_r^2}{M_F^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$P_{ij}(x)$ - Altarelli-Parisi splitting functions

M_F - factorization scale

Physical cross section at NLO: Properties

■ PDF's and parton cross-sections are scheme dependent
Physical observables are scheme independent

■ The only effect of a change in the renorm. and factoriz. scheme is to distribute radiative corrections differently between $d\hat{\sigma}$, PDF's and α_s.

■ Both scales are chosen based on a hard scale present in the process - e.g. m_Q, p_T. Any reasonable value is allowed

■ Variations in renormalization and factorization scales give a net change of a higher order in α_s.
One allows to estimate size of higher order corrections

■ For usual choices of scales, the central value of the NLO cross section generally increases compared to the LO one

Is NLO sufficient?

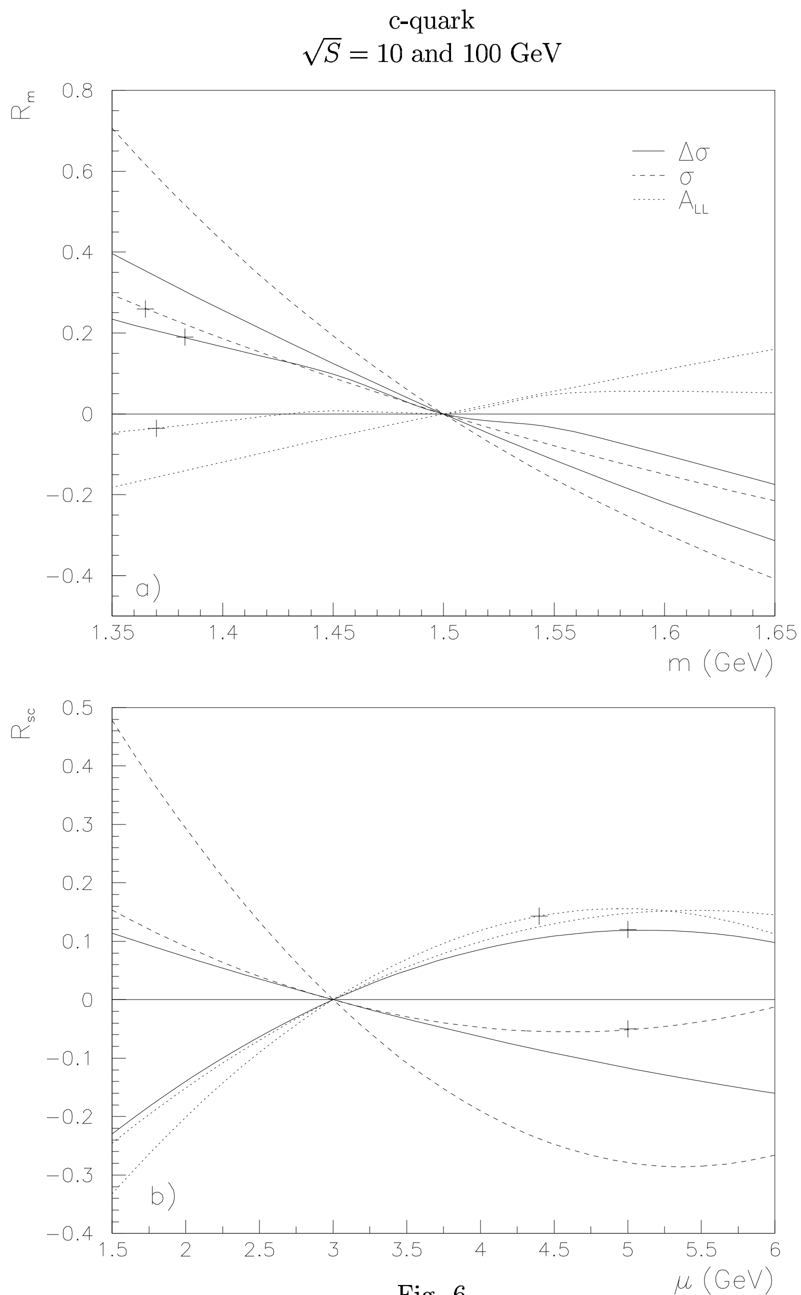
Consider heavy flavor photoproduction $\gamma + p \to Q(\overline{Q}) + X$ and define

$$R_m = \frac{\sigma(m_c) - \sigma(1.5\ \text{GeV})}{\sigma(1.5\ \text{GeV})}$$

$$R_{\text{scheme}} = \frac{\sigma(\mu) - \sigma(2m_c)}{\sigma(2m_c)}$$

σ - integrated cross section, $\mu = M_F$

from Z. Merebashvili et al, PRD 62, 114509 (2000)



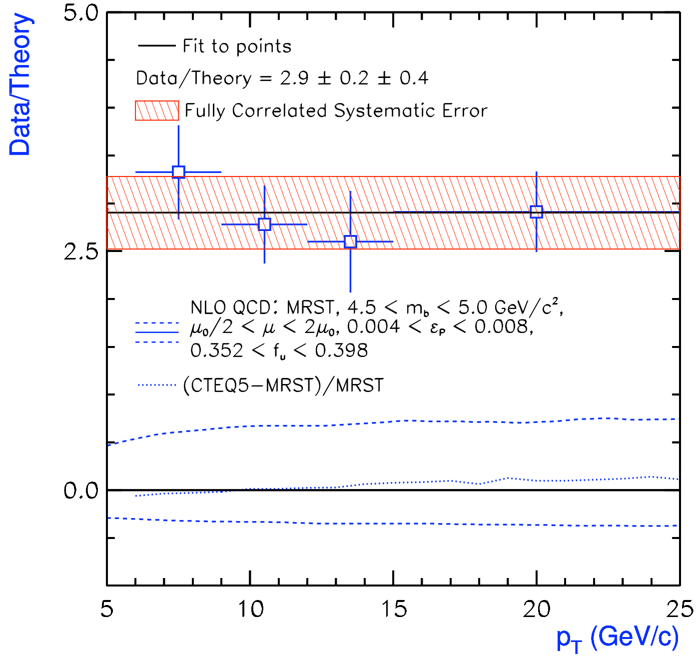
Is NLO sufficient?

- Theoretical errors due to uncertainty in heavy quark mass m:
  $\Delta m_c = \pm 0.05$ GeV $\Longrightarrow \Delta\sigma = \pm 10\% \div 20\%$ at c.m. $\sqrt{S} = 100 \div 10$ GeV

- Theoretical errors due to large uncertainty in choosing renormalization and factorization scales: $(\mu, M_F)/2, \times 2 \Longrightarrow \pm 15\% \div 50\%$

▲ Previous theoretical predictions consistently undershoot all the data by significant amounts. Experimental problem? Deficient QCD prediction? Appearance of new physics (indication for low-energy supersymmetry)?

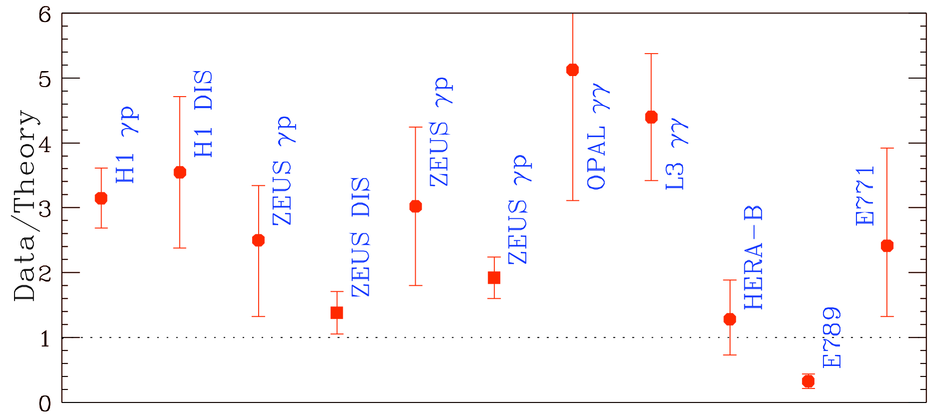
- CDF's comparison of B^+ experimental data with the NLO QCD (CDF collaboration, hep-ph/0111359)



Average ratio for differential cross section is

$2.9 \pm 0.2(\text{stat} \oplus \text{syst}_{\text{pt}}) \pm 0.4(\text{syst}_{\text{fc}})$

- Comparison of b rates from fixed-target [HERA-B], HERA [H1, ZEUS] and LEP [L3] experiments with the NLO QCD (from S. Frixione hep-ph/0211434)



Ratios data/theory for b rates

Charm and Bottom Production

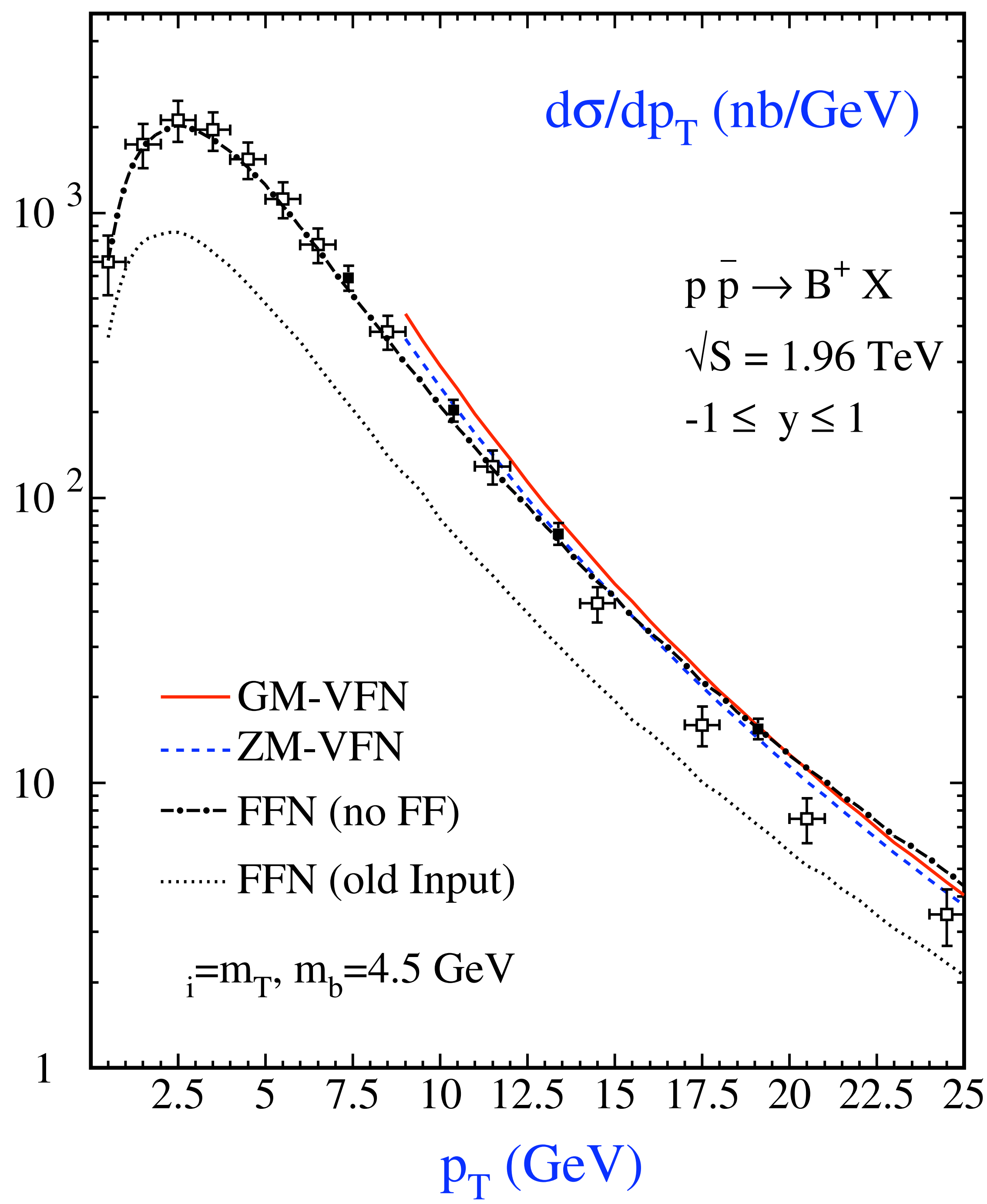
▲ Recent work – good description at NLO level:

- M. Cacciari et al., (2004)
- B. A. Kniehl et al., (2008)

Different treatment of *large mass logarithms*

$$m_T = \sqrt{p_T^2 + m^2}$$

Recent CDF data: D. Acosta et al., (2005); A. Abulencia et al., (2007)



Necessity of NNLO calculation for hadronic heavy flavor production

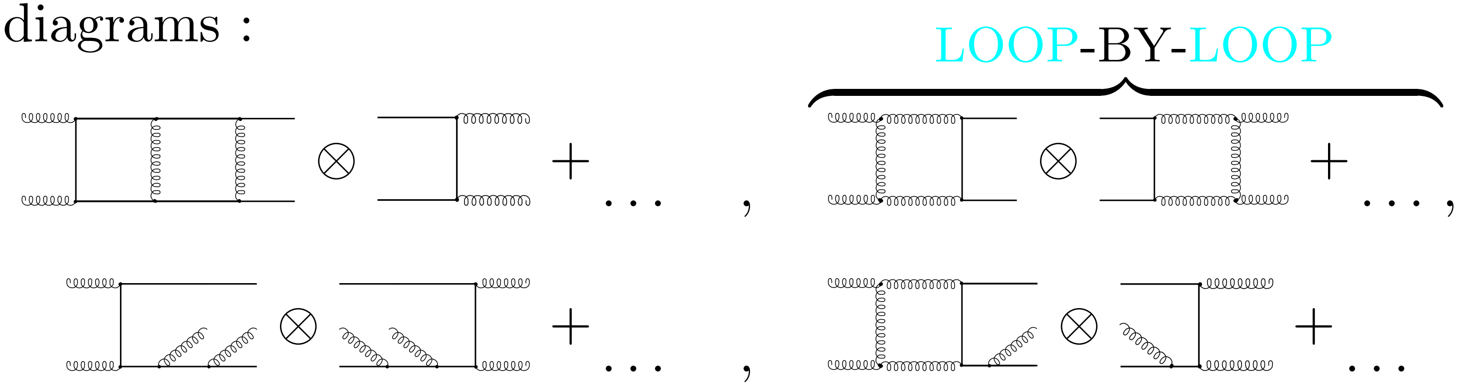
- Theoretical errors due to possible choices of renormalization and factorization scales are expected to be greatly reduced at NNLO QCD

- The increasing precision of present and forthcoming experiments requires a corresponding precision of theoretical predictions

- Are various resummations in different corners of phase space reliable ?

NNLO QCD $\sim {\alpha_s}^4$

Generic diagrams :



- Massless QCD subprocesses done mainly by N. Glover and collaborators
- Massive case is much more involved, new class of functions enter
- Expansion up to $\mathcal{O}(\varepsilon^2)$ in Dimensional Regularization is required. Imaginary contributions are also needed

The term LOOP-BY-LOOP

$$\Big(\underbrace{\frac{a_{-2}}{\varepsilon^2}+\frac{a_{-1}}{\varepsilon}+a_0}_{known}+\underbrace{a_1\varepsilon+a_2\varepsilon^2}_{newer}+\dots\Big)\Big(\frac{a_{-2}}{\varepsilon^2}+\frac{a_{-1}}{\varepsilon}+a_0+a_1\varepsilon+a_2\varepsilon^2+\dots\Big)^*$$

Bits and pieces of the NNLO calculation are now being assembled

- W. Bernreuther et al., Nucl. Phys. **B706**, 245 (2005);
  Two–loop heavy quark vertex form factor

- M. Czakon, arXiv:0803.1400 [hep-ph];
  Two-loop virtual amplitudes numerically

- J.G. Körner, Z. Merebashvili and M. Rogal, arXiv:0802.0106 [hep-ph];
  Loop-by-loop amplitudes analytically