NLO and Parton Showers

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Plan of Talk

- Why NLO and Parton Showers?
- Review of Subtraction Formalism
- MC@NLO
- POWHEG
- An Example

LO

Simplest approximation for an observable - terms with lowest power of α_S that make a non-zero contribution to that observable.
In pp→X process, assuming only 1 set of partonic final-state flavours contributes, can write:

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 \, f_a(x_1) f_b(x_2) \int d\phi_n \, F(\phi_n) \frac{d\hat{\sigma}_{ab}}{d\phi_n}$$

Can evaluate the integrals by MC techniques $\rightarrow$

"Event Generator"-style program.

Fully automated e.g. HELAC-PHEGAS (A. Cafarella, C. Papadopoulos, M. Worek arXiv:0710.2427 [hep-ph]) MADGRAPH/MADEVENT (Alwall et al JHEP 0709:028,2007, arXiv:0706.2334 [hep-ph]).

Improvements

$\swarrow$ LO $\searrow$

NLO

- Include all terms at relative $O(\alpha_S)$.
- Diagramatically $\hat{\sigma}$ now contains tree·tree and tree·loop terms.
- Separately IRC divergent! Observable must be IRC safe.
- Can implement as MC code using e.g. subtraction formalism.
- Improves: description of "hard" processes (e.g. tail of p_t dist.), inclusive quantites (e.g. total σ).

PS

- Include part of $O(\alpha_S^n)$ for all n.
- Treat radiation in approximation holding in soft/collinear limit.
- Implemented as Markov process.
- By construction, doesn't affect values for inclusive observables.
- Improves: description of "soft" processes (e.g. peak of p_t dist.), exclusive quantites (eg. vetoing on radition).

$\searrow$ NLO + PS? $\swarrow$

NLO + PS

Goal: a “best of both worlds” program combining the advantages of NLO and PS Monte-Carlos. It should let us calculate (a class of) observables

- accurate to NLO in α_S (up to corrections due to shower cutoffs/hadronization) $\rightarrow$ better description of inclusive/hard quantities than PS.
- including the LL resummation of the PS $\rightarrow$ better description of soft/exclusive quantities than NLO.
- including hadronization effects by interfacing to a hadronization model to produce realistic final states (for input to detector simulation, etc.).

Double Counting

Structure of standard PS calculation:

$$\sigma_F = \text{LO} \to \text{PS} \to \text{Evaluate Observable}$$

Just replace LO$\to$NLO? Symbolically NLO$=$LO$+$REAL$+$VIRT. So

$$\text{NLO}\to\text{PS} = \text{LO}\to\text{PS} + \text{REAL}\to\text{PS} + \text{VIRT}\to\text{PS},$$

Two problems:

(1) REAL and VIRT are separately divergent - could fix using splitting/subtraction method, but still get left with large -ve weight events! Unpleasant.

(2) LO$\to$PS already contains a soft/collinear approximation to REAL$\to$PS and to VIRT$\to$black PS (in the Sudakov factor). We're including these bits twice: "Double Counting". Disastrous.

The challenge is to fix these problems.

The Problem of IRC Divergences

First, some preliminaries on NLO calculations. Consider $e^+e^- \rightarrow$hadrons. At LO (for an "n-jet-like observable") we have something like:

$$\sigma_F = \int d\phi_n \, F(\phi_n) \frac{d\sigma^B}{d\phi_n}.$$

At NLO this becomes

$$\sigma_F = \int d\phi_n \, F(\phi_n) \left(\frac{d\sigma^B}{d\phi_n} + \frac{d\sigma^V}{d\phi_n} \right) + \int d\phi_{n+1} \, F(\phi_{n+1}) \frac{d\sigma^R}{d\phi_{n+1}}.$$

But the two integrals are not separately finite! Indeed, working in $d = 4 - 2\epsilon$ dimensions we find:

$$\frac{d\sigma^V}{d\phi_n} \propto \frac{1}{\epsilon^2}; \qquad \int d\phi_{n+1}^{(4-2\epsilon)} \, F(\phi_{n+1}) \frac{d\sigma^R}{d\phi_{n+1}} \propto \frac{1}{\epsilon^2}.$$

This needs to be fixed if one wants to build an NLO MC. Two ways: splitting, subtraction.

Subtraction

Recall that the divergence of the integral of $\frac{d\sigma^R}{d\phi_{n+1}}$ comes from soft and collinear singularities in the matrix element. Suppose we construct an "approximation" to $\frac{d\sigma^R}{d\phi_{n+1}}$ that shares these singularities, so that

$$\left(\frac{d\sigma^R}{d\phi_{n+1}} - \frac{d\sigma^A}{d\phi_{n+1}}\right)$$

has at worst integrable divergences (subtlety: away from divergences of Born ME!) We suppose $\frac{d\sigma^A}{d\phi_{n+1}}$ to be built out of pieces corresponding to various singular limits of the cross-section:

$$\frac{d\sigma^A}{d\phi_{n+1}} = \sum_\alpha \frac{d\sigma^{A(\alpha)}}{d\phi_{n+1}}$$

$$\frac{d\sigma^{A(\alpha)}}{d\phi_{n+1}} = \frac{d\sigma^B}{d\phi_n} \otimes_{c,s} \frac{dP}{d\phi_{rad}}$$

with $\phi_n = \phi_n^{(\alpha)}(\phi_{n+1})$ and likewise for ϕ_{rad}.

Subtraction II

Now we take the following term

$$\int d\phi_{n+1} \sum_\alpha F(\phi_n^{(\alpha)}(\phi_{n+1})) \frac{d\sigma^{A(\alpha)}}{d\phi_{n+1}} = \int d\phi_n F(\phi_n) \sum_\alpha \int d\phi_{rad} \frac{d\sigma^{A(\alpha)}}{d\phi_{n+1}}$$

and we **add and subtract** it to (from) our weighted cross-section like this:

$$\sigma_F = \int d\phi_n \, F(\phi_n) \left(\frac{d\sigma^B}{d\phi_n} + \frac{d\sigma^V}{d\phi_n} + \sum_\alpha \int d\phi_{rad} \frac{d\sigma^{A(\alpha)}}{d\phi_{n+1}} \right)$$

$$+ \int d\phi_{n+1} \left(F(\phi_{n+1}) \frac{d\sigma^R}{d\phi_{n+1}} - \sum_\alpha F(\phi_n^{(\alpha)}(\phi_{n+1})) \frac{d\sigma^{A(\alpha)}}{d\phi_{n+1}} \right).$$

It's possible to choose a subtraction term simple enough that

$$\frac{d\sigma^{Vf}}{d\phi_n} := \frac{d\sigma^V}{d\phi_n} + \sum_\alpha \int d\phi_{rad} \frac{d\sigma^{A(\alpha)}}{d\phi_{n+1}}$$

can be evaluated analytically and is **finite in** $d = 4$. For an IRC-safe F, the real integrand is also integrable throughout phase space $\rightarrow$ we can build a sensible NLO MC.

MC@NLO

Calculate the part of the NLO corrections already included by using a particular parton shower Monte-Carlo. Then change the starting conditions of the parton shower to include a set of events constructed to make up the missing part of the NLO corrections (some of these events might appear with negative weights). No change to the parton shower code itself is needed.

$$\text{LO}\rightarrow\text{PS} + (\text{REAL}-\text{REAL}_{PS})\rightarrow\text{PS} + (\text{VIRT}-\text{VIRT}_{PS})\rightarrow\text{PS}$$

Introduced: Frixione and Webber, JHEP 0206 (2002) 029 [hep-ph/0204244]
Latest Manual: Frixione and Webber, hep-ph/0612272

Rewriting NLO I

Start from the NLO cross-section written in the subtraction formalism with $f(x_1)f(x_2)\hat{\sigma}_{ab} \to \Sigma_{ab}$:

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 \left[\int d\phi_n \, F(\phi_n) \left(\frac{d\Sigma^B_{ab}}{d\phi_n} + \frac{d\Sigma^{V\,f}_{ab}}{d\phi_n} \right) \right.$$

$$+ \int d\phi_{n+1} \left(F(\phi_{n+1}) \frac{d\Sigma^R_{ab}}{d\phi_{n+1}} - \sum_\alpha F(\phi_n^{(\alpha)}) \frac{d\Sigma^{A(\alpha)}_{ab}}{d\phi_n} \right)$$

$$\left. + \int d\phi_n dz \left(F(\phi_n) \frac{d\Sigma^{C\,f1}_{ab}}{d\phi_n dz} + F(\phi_n) \frac{d\Sigma^{C\,f2}_{ab}}{d\phi_n dz} \right) \right]$$

Note we now have additional $\Sigma^{C\,f\,1,2}$ pieces, finite remnants of collinear subtraction terms that appeared when the initial state collinear singularities were absorbed into the pdfs.

Want to rewrite this so we only have one $n+1$-particle phase space integral: the ϕ_n are to be taken as functions (to be determined later!) of ϕ_{n+1}.

Rewriting NLO II

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 d\phi_{n+1} \left[F(\phi_{n+1}) \frac{d\Sigma^R_{ab}}{d\phi_{n+1}} + F(\phi_n) \frac{1}{\mathcal{I}_n} \left(\frac{d\Sigma^B_{ab}}{d\phi_n} + \frac{d\Sigma^{Vf}_{ab}}{d\phi_n} \right) \right.$$

$$\left. - \sum_{\alpha} F(\phi_n^{(\alpha)}) \frac{d\Sigma^{A(\alpha)}_{ab}}{d\phi_n} + \frac{1}{\mathcal{I}^C_n} \left(F(\phi^C_n) \frac{d\Sigma^{Cf1}_{ab}}{d\phi_n dz} + F(\phi^C_n) \frac{d\Sigma^{Cf2}_{ab}}{d\phi_n dz} \right) \right]$$

We have several different $n-$particle configurations here that have to be obtained by projecting from $\phi_{n+1} \to \phi_n$. By making a different, carefully chosen, change of variables in each term we can reduce this to just one projection:

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 d\phi_{n+1} \left\{ F(\phi_{n+1}) \frac{d\overline{\Sigma}^R_{ab}}{d\phi_{n+1}} + F(\phi_n) \left[\frac{1}{\mathcal{I}_n} \left(\frac{d\overline{\Sigma}^B_{ab}}{d\phi_n} + \frac{d\overline{\Sigma}^{Vf}_{ab}}{d\phi_n} \right) \right. \right.$$

$$\left. \left. - \sum_{\alpha} \frac{d\overline{\Sigma}^{A(\alpha)}_{ab}}{d\phi_n} + \frac{1}{\mathcal{I}^C_n} \left(\frac{d\overline{\Sigma}^{Cf1}_{ab}}{d\phi_n dz} + \frac{d\overline{\Sigma}^{Cf2}_{ab}}{d\phi_n dz} \right) \right] \right\}$$

Expanding the Shower

Expanding out general shower predictions to NLO gives us:

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 d\phi_{n+1} \left\{ F(\phi_{n+1}) \frac{d\overline{\Sigma}_{ab}^{R,PS}}{d\phi_{n+1}} + F(\phi_n) \left[\frac{1}{\mathcal{I}_n} \frac{d\overline{\Sigma}_{ab}^{B}}{d\phi_n} - \frac{d\overline{\Sigma}_{ab}^{R,PS}}{d\phi_{n+1}} \right] \right\}$$

The negative term is from the Sudakov factor - reduction in rate of $n-$parton events due to emission. Requires projection giving $\phi_n(\phi_{n+1})$ to act as inverse of MC splitting function! If this was true for our other projections, can easily read off that we get full NLO accuracy by adding to the shower:

$$F(\phi_{n+1}) \left(\frac{d\overline{\Sigma}_{ab}^{R}}{d\phi_{n+1}} - \frac{d\overline{\Sigma}_{ab}^{R,PS}}{d\phi_{n+1}} \right)$$

$$F(\phi_n) \left[\frac{1}{\mathcal{I}_n} \frac{d\overline{\Sigma}_{ab}^{Vf}}{d\phi_n} - \sum_{\alpha} \frac{d\overline{\Sigma}_{ab}^{A(\alpha)}}{d\phi_n} + \frac{1}{\mathcal{I}_n^C} \left(\frac{d\overline{\Sigma}_{ab}^{Cf1}}{d\phi_n dz} + \frac{d\overline{\Sigma}_{ab}^{Cf2}}{d\phi_n dz} \right) + \frac{d\overline{\Sigma}_{ab}^{R,PS}}{d\phi_{n+1}} \right]$$

Master Formula for MC@NLO

Can write general shower predictions as:

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 d\phi_n I_{PS}[F](\phi_n) \frac{d\Sigma_{ab}^B}{d\phi_n}$$

where I_{PS} represents the results of showering from the configuration ϕ_n and then evaluating the expectation of F. Now we add the corrections to this, bolting showers on to them as appropriate (they are already NLO, so this won't affect the NLO accuracy of the results because $I_{PS}[F](\phi) = F(\phi) + \mathrm{O}(\alpha_S)$). This gives:

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 d\phi_{n+1} \left\{ I_{PS}[F](\phi_{n+1}) \left(\frac{d\overline{\Sigma}_{ab}^R}{d\phi_{n+1}} - \frac{d\overline{\Sigma}_{ab}^{R,PS}}{d\phi_{n+1}} \right) + I_{PS}[F](\phi_n) \times \right.$$

$$\left. \left[\frac{1}{\mathcal{I}_n} \left(\frac{d\overline{\Sigma}_{ab}^B}{d\phi_n} + \frac{d\overline{\Sigma}_{ab}^{Vf}}{d\phi_n} \right) - \sum_{\alpha} \frac{d\overline{\Sigma}_{ab}^{A(\alpha)}}{d\phi_n} + \frac{1}{\mathcal{I}_n^C} \left(\frac{d\overline{\Sigma}_{ab}^{Cf1}}{d\phi_n dz} + \frac{d\overline{\Sigma}_{ab}^{Cf2}}{d\phi_n dz} \right) + \frac{d\overline{\Sigma}_{ab}^{R,PS}}{d\phi_{n+1}} \right] \right\}$$

If PS accurately reproduces singular behaviour of real-emission diagrams, both terms are separately finite.

Recipe

- Analyze PS to extract $\frac{d\overline{\Sigma}_{ab}^{R,PS}}{d\phi_{n+1}}$.
- This must reproduce the singular behaviour of the full $\frac{d\overline{\Sigma}_{ab}^{R}}{d\phi_{n+1}}$ so that

$$\sum_{ab} \int dx_1 dx_2 d\phi_{n+1} I_{PS}[F](\phi_{n+1}) \left(\frac{d\overline{\Sigma}_{ab}^{R}}{d\phi_{n+1}} - \frac{d\overline{\Sigma}_{ab}^{R,PS}}{d\phi_{n+1}} \right)$$

  is finite and can be evaluated by MC techniques, as can the other, $n-$particle term.
- To do this, produce samples of $n-/n+1-$particle events according to the weights appearing as coefficients of I_{PS} (a fraction of these weights are negative). One can unweight the events if desired, to give a collection of weight=1 events and weight=-1 “counter-events”.
- These event samples can then be fed into the standard PS code.
- The end results is a collection of fully showered, hadronized events (and counter-events).

POWHEG

"POsitive Weight Hardest Emission Generator"

Treat the hardest emission first in a shower-like approximation designed to reproduce the NLO results along with a Sudakov factor. Then allow any remaining emissions to be handled by a PS code (which may need a veto to prevent harder events being generated).

LO→POWHEG→PS

Introduced: Nason, JHEP 0411 (2004) 040 [hep-ph/0409146]
Detailed description: Frixione, Nason, Oleari, arxiv:0709.2092 [hep-ph]

Dividing up the Real Emission

In POWHEG one divides up the real emission in a way which parallels the division of the subtraction term

$$\frac{d\Sigma^{R}_{ab}}{d\phi_{n+1}} = \sum_{\alpha} \frac{d\Sigma^{R(\alpha)}_{ab}}{d\phi_{n+1}}$$

so that

$$\frac{d\Sigma^{R(\alpha)}_{ab}}{d\phi_{n+1}} \sim \frac{d\Sigma^{A(\alpha)}_{ab}}{d\phi_{n+1}}.$$

For example in the case of Catani-Seymour subtraction one can write

$$\frac{d\Sigma^{R(\alpha)}_{ab}}{d\phi_{n+1}} = \frac{\mathcal{D}_{\alpha}}{\sum_{\beta} \mathcal{D}_{\beta}} \frac{d\Sigma^{R}_{ab}}{d\phi_{n+1}}.$$

The $\overline{B}$ Function

Start again from the NLO cross-section

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 \left[\int d\phi_n \, F(\phi_n) \left(\frac{d\Sigma_{ab}^{B}}{d\phi_n} + \frac{d\Sigma_{ab}^{V\,f}}{d\phi_n} \right) + \int d\phi_{n+1} \left(F(\phi_{n+1}) \frac{d\Sigma_{ab}^{R}}{d\phi_{n+1}} \right.\right.$$

$$\left.\left. - \sum_{\alpha} F(\phi_n^{(\alpha)}) \frac{d\Sigma_{ab}^{A(\alpha)}}{d\phi_n} \right) + \int d\phi_n dz \left(F(\phi_n) \frac{d\Sigma_{ab}^{C\,f1}}{d\phi_n dz} + F(\phi_n) \frac{d\Sigma_{ab}^{C\,f2}}{d\phi_n dz} \right) \right]$$

Split up the subtracted real-emission term as follows:

$$F(\phi_{n+1}) \frac{d\Sigma_{ab}^{R(\alpha)}}{d\phi_{n+1}} - F(\phi_n^{(\alpha)}) \frac{d\Sigma_{ab}^{A(\alpha)}}{d\phi_n} = F(\phi_n^{(\alpha)}) \left(\frac{d\Sigma_{ab}^{R(\alpha)}}{d\phi_{n+1}} - \frac{d\Sigma_{ab}^{A(\alpha)}}{d\phi_{n+1}} \right)$$

$$+ \frac{d\Sigma_{ab}^{R(\alpha)}}{d\phi_{n+1}} \left(F(\phi_{n+1}) - F(\phi_n^{(\alpha)}) \right)$$

The $\overline{B}$ Function II

Lump everything bar the last term together into a function $\overline{B}$ so that

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 d\phi_n F(\phi_n) \overline{B}_{ab}(\phi_n) +$$

$$\sum_{ab} \sum_{\alpha} \int dx_1 dx_2 d\phi_n d\phi_{rad} \frac{d\Sigma_{ab}^{R(\alpha)}}{d\phi_{n+1}} \left(F(\phi_{n+1}) - F(\phi_n) \right)$$

where $d\phi_{n+1}$ has been factorized as $d\phi_n d\phi_{rad}$.

Next we need the POWHEG Sudakov factor

$$\Delta_{ab}(\phi_n, p_T) = \exp \left\{ -\sum_{\alpha} \frac{\int d\phi_{rad}\, \theta(k_T^{(\alpha)}(\phi_{n+1}) > p_T) \frac{d\Sigma_{ab}^{R(\alpha)}}{d\phi_{n+1}}}{\frac{d\Sigma_{ab}^{B}}{d\phi_n}} \right\}$$

where ϕ_{n+1} is determined from ϕ_n and ϕ_{rad} and satisfies $\phi_n^{(\alpha)}(\phi_{n+1}) = \phi_n$.

POWHEG Cross Section

In POWHEG one starts by generating events (with n or $n+1$ particles) according to cross-sections following from the Sudakov factor:

$$\frac{d\Sigma^{POWHEG}}{d\phi_n} = \sum_{ab} \overline{B}_{ab}(\phi_n)\Delta_{ab}(\phi_n, p_T^0)$$

$$\frac{d\Sigma^{POWHEG}}{d\phi_{n+1}} = \sum_{ab} \overline{B}_{ab}(\phi_n) \sum_{\alpha} \Delta_{ab}(\phi_n^{(\alpha)}, k_T^{(\alpha)}(\phi_{n+1})) \frac{\theta(k_T^{(\alpha)}(\phi_{n+1}) > p_T^0)\, \frac{d\Sigma_{ab}^{R(\alpha)}}{d\phi_{n+1}}}{\left.\frac{d\Sigma_{ab}^{B}}{d\phi_n}\right|_{\phi_n = \phi_n^{(\alpha)}}}.$$

The key point is that calculating an observable with these events will give NLO accuracy:

$$\sigma_F = \int dx_1 dx_2 \left(d\phi_n\, F(\phi_n) \frac{d\Sigma^{POWHEG}}{d\phi_n} + d\phi_{n+1} F(\phi_{n+1}) \frac{d\Sigma^{POWHEG}}{d\phi_{n+1}} \right)$$

can be written to NLO as...

POWHEG Cross Section II

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 d\phi_n \, \overline{B}_{ab}(\phi_n) \left(F(\phi_n) \left[1 - \frac{\sum_\alpha \int d\phi_{rad} \frac{d\Sigma_{ab}^{R(\alpha)}}{d\phi_{n+1}} \theta}{\frac{d\Sigma_{ab}^{B}}{d\phi_n}} \right] + \right.$$

$$\left. F(\phi_{n+1}) \frac{\sum_\alpha \int d\phi_{rad} \frac{d\Sigma_{ab}^{R(\alpha)}}{d\phi_{n+1}} \theta}{\frac{d\Sigma_{ab}^{B}}{d\phi_n}} \right)$$

But this can be rearranged into the familiar form:

$$\sigma_F = \sum_{ab} \int dx_1 dx_2 d\phi_n F(\phi_n) \overline{B}_{ab}(\phi_n) +$$

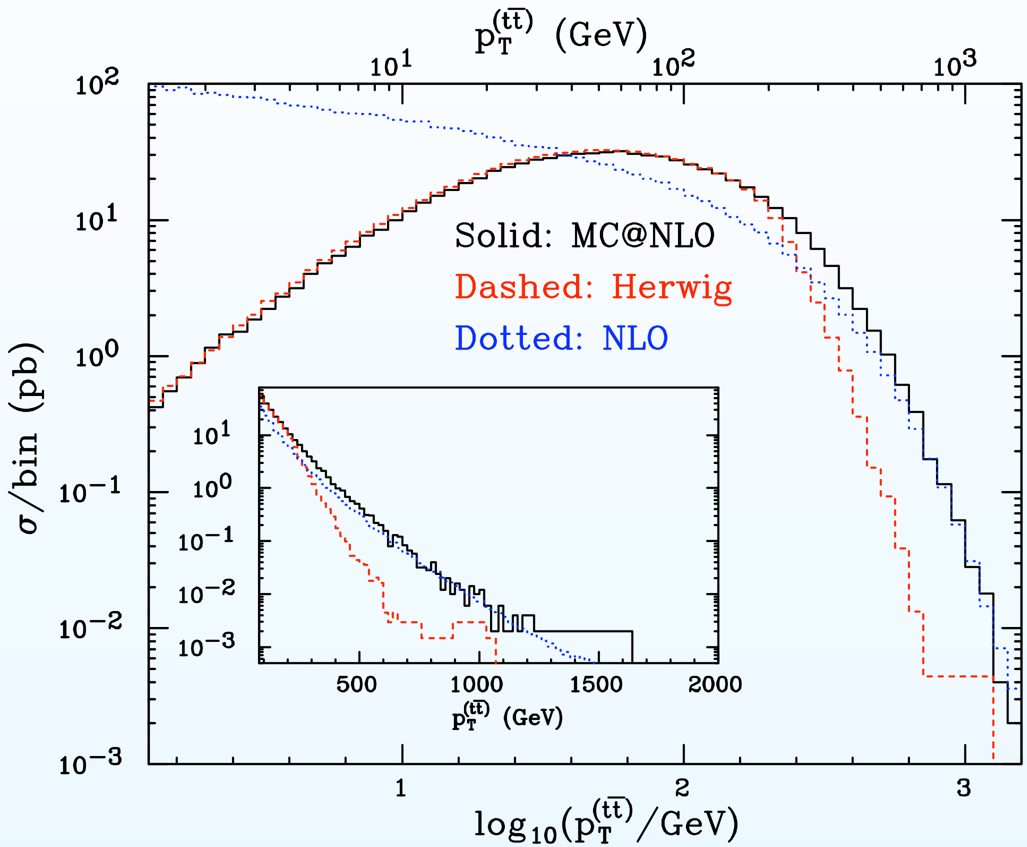
$$\sum_{ab} \sum_{\alpha} \int dx_1 dx_2 d\phi_n d\phi_{rad} \frac{d\Sigma_{ab}^{R(\alpha)}}{d\phi_{n+1}} \left(F(\phi_{n+1}) - F(\phi_n) \right) \theta(k_T > p_T^0).$$

For IR safe F, the theta function can be neglected provided p_T^0 is small, and then we simply obtain the NLO result for σ_F!

Recipe

- Generate a Born configuration according to the weight $\overline{B}$.
- Select a hard emission according to the POWHEG Sudakov factor, using standard parton shower techniques.
- Pass the event on to the PS code, requiring that there be no more emissions with k_T greater than that produced by POWHEG. If the PS is k_T-ordered this is easy - the starting scale just needs to be set appropriately. If not, a veto may need to be applied. Problems can also occur in angular ordered showers because the POWHEG emission will prevent some later emissions that should be allowed to occur. In principle this could be fixed by adding an extra "truncated shower" to replace the missing radiation.

A Quick Example



Looking at $pp \to t\bar{t} + X$.

Define p_T of $t\bar{t}$ pair as:
$\vec{p}_T^{(t\bar{t})} = \vec{p}_T^{(t)} + \vec{p}_T^{(\bar{t})}$

$\sqrt{s} = 14$TeV.

From S. Frixione, P. Nason, B.R. Webber JHEP 0308 (2003) 007 [hep-ph/0305252].

Contrasts

- Both MC@NLO and POWHEG provide examples of programs that consistently combine NLO calculations with parton showers, providing fixed-order NLO accuracy with a resummation of logarithms that allows exclusive quantities to be sensibly predicted.
- MC@NLO produces some ($\sim$10%) negatively-weighted events - this means more events are needed to attain the same statistical accuracy as would be in a MC with only positive weights, but the small number of events means it's not a serious problem. For POWHEG this issue doesn't arise.
- MC@NLO has been implemented for a larger number of processes than POWHEG.
- In its current implementations, MC@NLO is tied to using HERWIG as its parton shower, because as we saw it needs to be provided with the NLO expansion of the shower, which is not trivial to obtain.
- Note that MC@NLO and POWHEG aren't just different approaches to calculating the same thing: they differ in the approximations they make beyond the specified NLO/LL accuracy.