Merging Matrix Elements and Parton Showers

[for practitioners]

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Outline

improving QCD jet modelling through multi-leg tree-level calculations

- **The physics: multi-particle final states at the LHC**
- **The ingredients: parton shower & matrix element calculations**
- **The method: combining matrix elements and parton showers**
  - merging prescriptions: anatomy and systematics
  - sample applications: DY+jets, $t\bar{t}$+jets

Simulation of signal & background processes

main objectives for the LHC era

- reveal the mechanism of EWSB [discovery of the Higgs?, alternatives?]
- search for physics beyond the SM: weak scale SUSY, ED, W' & Z', ...

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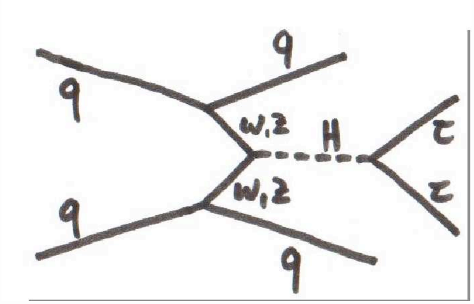
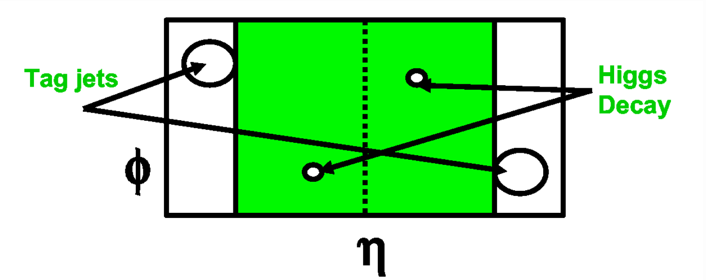
example: Higgs-boson production in weak boson fusion





➔ rapidity gap between two forward/backward tagged jets

➔ signal/background ratio depends on central jet veto

background challenge

- multi-jet final states with WBF like kinematics
- $V + n$-jets, $VV + n$-jets (QCD & EW), $t\bar{t} + n$-jets, $Wt + n$-jets

Simulation of signal & background processes

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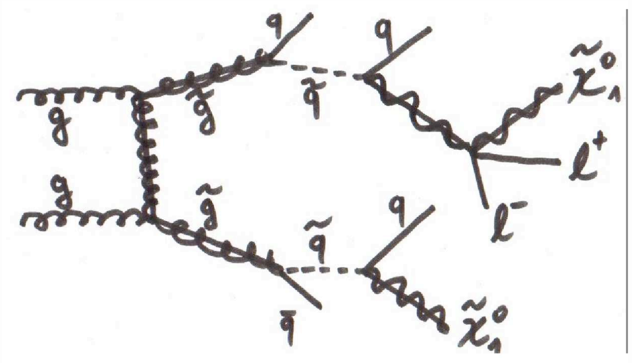
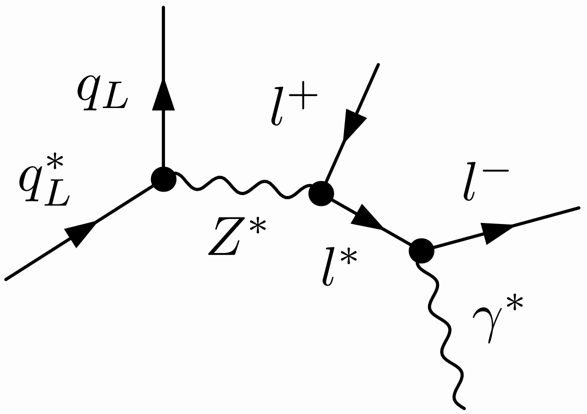
example: cascade decays of new heavy coloured states





➔ #-leptons + #-jets + $\not{E}_T$

➔ jet properties depend on nature of new physics [energies, flavours, edges]

background challenge

- SM(BSM) processes associated with (many) high-p_T jets
- QCD multi-jets, V+jets, VV+jets, VVV+jets, $t\bar{t}$+jets

Simulation of signal & background processes

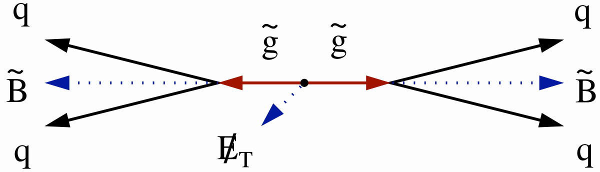
main objectives for the LHC era

- reveal the mechanism of EWSB [discovery of the Higgs?, alternatives?]
- search for physics beyond the SM: weak scale SUSY, ED, W' & Z', ...

example: gluino pair production for squeezed SUSY spectra

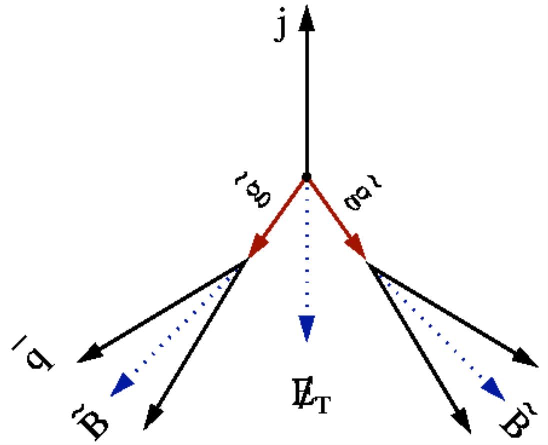
- mSugra searches: $m_{\tilde{g}} : m_{\tilde{B}} = 6 : 1 \Rightarrow$ 4 hard jets plus missing energy

➔ not looked for nearly degenerated scenarios $(m_{\tilde{g}} \sim m_{\tilde{B}})$



- softer decay jets, only modest $\not{E}_T$
- but, when accompanied by hard IS jet

  $\Rightarrow$ monojet signature with sizeable $\not{E}_T$

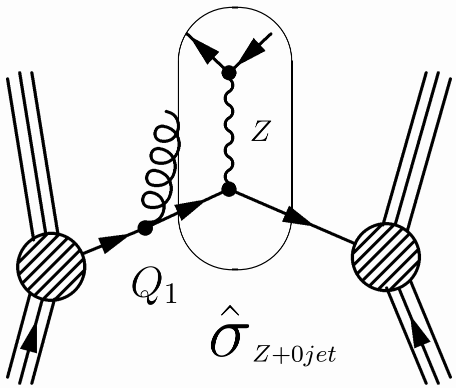


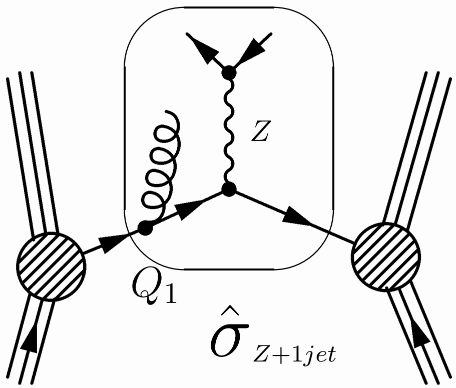
signal challenge

- rather light gluinos (process scale not too high)
- consistent modelling of additional hard jet(s)

objectives

- first few hardest emissions through tree-level ME [ME pros + PS pros]
- quantum interferences and correlations + universal hadronisation
- avoid double counting of phase-space configurations in PS & ME





Combining ME and PS

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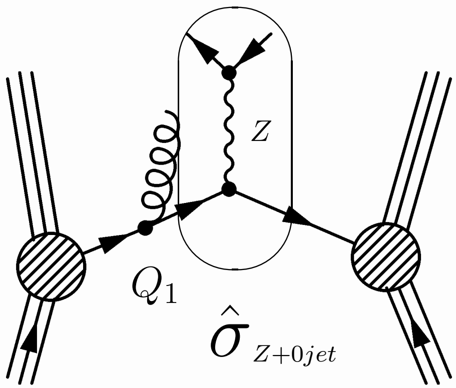
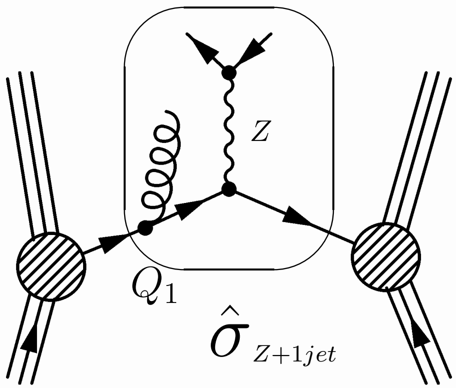




solution

- split multi-jet phase space in two regimes [k_T-measure $Q_{\rm cut}$ or $\Delta R_{\rm min} \& E_{T,{\rm min}}$]
  - jet seeds produced through tree-level MEs [hard partons]
  - jets evolved down to fragmentation scale by PS
- reweight MEs to get exclusive samples at resolution scale

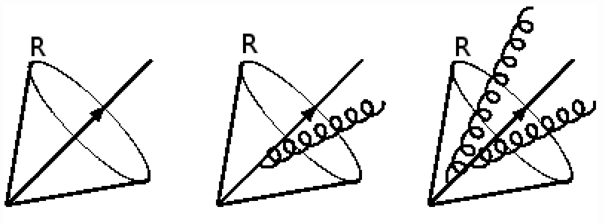
  $\Rightarrow$ allows to add samples of different ME jet multiplicities
- reject PS configurations taken into account by higher order ME

Combining ME and PS – the idea

Decomposition of the LO total inclusive cross section

$$\sigma_X|_{incl} = \sigma_{X+0}|_{excl}(Q_{cut}) + \sigma_{X+1}|_{excl}(Q_{cut}) + .. + \sigma_{X+n_{max}}|_{incl}(Q_{cut})$$

- $\sigma_{X+i}|_{excl}$ cross section for FS X plus exactly i jets at given resolution Q_{cut}
- $\sigma_{X+n_{max}}|_{incl}$ cross section for X plus at least n_{max} jets at resolution scale



- "hard" jets above Q_{cut} initiated by ME partons
- parton shower adds substructure to ME jets
- shower adds radiation below the smallest ME "emission scale"

merging commandments

- ensure full coverage of phase space without any double counting
- dependence on separation scale should be weak

Combining ME and PS – CKKW

The CKKW method [Catani,Krauss,Kuhn,Webber 2001; Krauss 2002]

- separate emission phase space through a k_T measure $Q_{\rm cut}$
- reweight matrix elements by (analytic) pseudo shower history
- veto shower emissions above the separation scale

CKKW pre-requisites

- k_t measure: $Q^2_{ij} = \min(p^2_{\perp i}, p^2_{\perp j}) \cdot R^2_{ij}/D^2 \quad \text{or} \quad Q^2_{iB} = p^2_{\perp i}$

$$R^2_{ij} = 2\left[\cosh(\eta_i - \eta_j) - \cos(\phi_i - \phi_j)\right]$$

- NLL Sudakov form factors

$$\Delta_q(Q^2, Q_0^2) = \exp\left\{-\int_{Q_0^2}^{Q^2} dq^2\, \Gamma_q(Q^2, q^2)\right\}, \quad \Delta_g(Q^2, Q_0^2) = \exp\left\{-\int_{Q_0^2}^{Q^2} dq^2\, \left[\Gamma_g(Q^2, q^2) + \Gamma_f(q^2)\right]\right\}$$

$$\Gamma_q = \frac{2C_F}{\pi}\frac{\alpha_S(q^2)}{q^2}\left(\ln\frac{Q^2}{q^2} - \frac{3}{4}\right), \quad \Gamma_g = \frac{2C_A}{\pi}\frac{\alpha_S(q^2)}{q^2}\left(\ln\frac{Q^2}{q^2} - \frac{11}{12}\right), \quad \Gamma_f = \frac{N_f}{3\pi}\frac{\alpha_S(q^2)}{q^2}$$

Combining ME and PS – CKKW

The CKKW algorithm

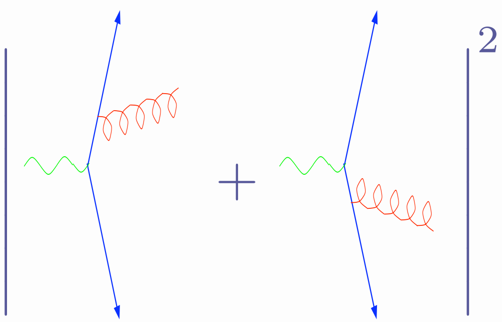
[as implemented in Sherpa, using built-in ME generator Amegic]

- evaluate MEs $X + 0, 1, .., n_{\max}-$jets at k_T-resolution Q_{cut} [regulator,μ_F,μ_R]
- select a jet multiplicity with probability

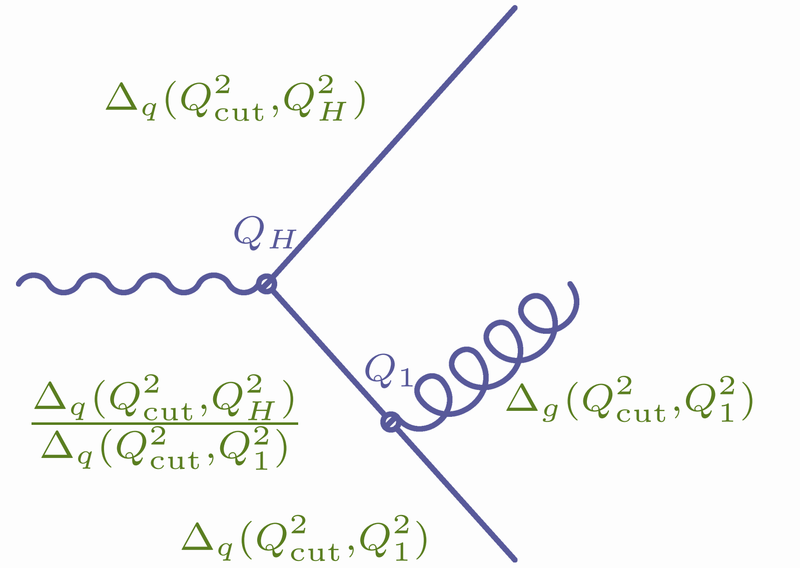
$$P_n = \frac{\sigma_n}{\sum_{i=0}^{n_{\max}} \sigma_i}$$

- generate final-state momenta p_i according to the ME
- reweight ME according to a reconstructed pseudo shower history
  - determine jet emission scales Q_{n},...,Q_1 with a k_T cluster algorithm
  - calculate corresponding (analytical) Sudakov weights
    - $\Delta_{q,g}(Q^2_{\mathrm{cut}}, Q^2_{\mathrm{prod}})$ for outgoing partons
    - $\Delta_{q,g}(Q^2_{\mathrm{cut}}, Q^2_{\mathrm{prod}})/\Delta_{q,g}(Q^2_{\mathrm{cut}}, Q^2_{\mathrm{dec}})$ for lines between $Q_{\mathrm{prod}} > Q_{\mathrm{dec}}$
  - recalculate α_S at each vertex in the tree at the corresponding k_T scale
- start initial- or final-state parton shower for all partons of the events
  - at scale where it was produced
  - veto on shower emissions above the scale Q_{cut}

Combining ME and PS – CKKW



shower history $\Longrightarrow$ exclusive event



$$W=\Delta_g(Q^2_{\rm cut},Q^2_1)[\Delta_q(Q^2_{\rm cut},Q^2_H)]^2\,\frac{\alpha_s(Q^2_1)}{\alpha_s(Q^2_{\rm cut})}$$

$\hookrightarrow$ inclusive events for $n_{\rm max}$

replace $Q_{\rm cut}$ by $Q_{n_{\rm max}}$

- general implementation for e^+e^- and hadron colliders in Sherpa
- $Q_{\rm cut}$ dependence cancels to (N)LL accuracy
- a variant, the Lönnblad scheme, implemented for Ariadne

Combining ME and PS – MLM

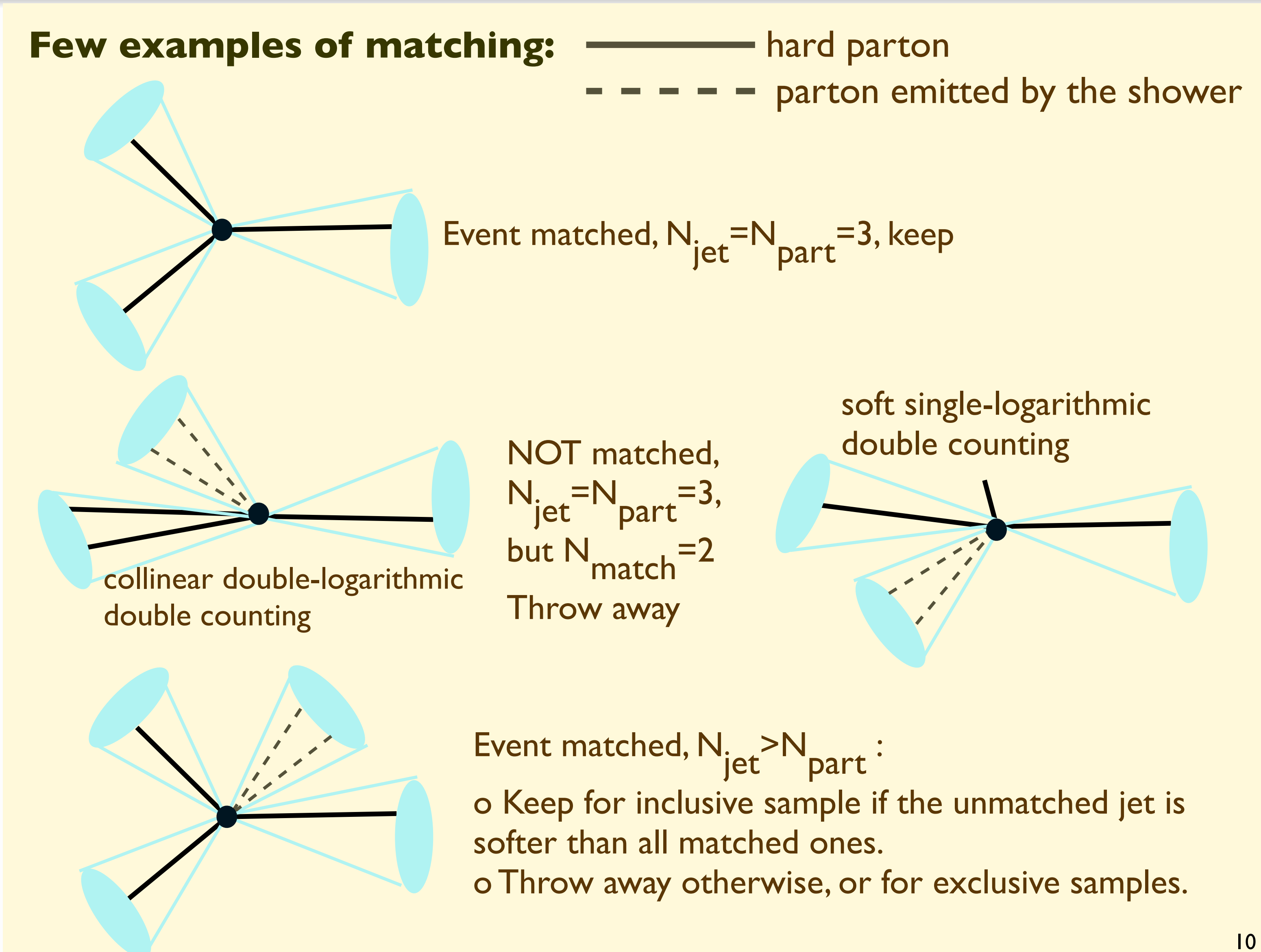
The MLM method [Mangano 2002]

[as implemented for Alpgen, combined with Herwig or Pythia showers]

- evaluate the $n = 0, .., n_{\max}$-parton MEs with cone measure $R_{\min}$ & $E_{T,\min}$
- α_S at k_T scales
- generate unweighted events for each multiplicity
- perform showering by Herwig or Pythia [LHA interface]
- after showers (before hadronisation) run cone finder with R_{clus} and $E_{T,\text{clus}}$
- try to geometrically match ME partons with jets after showering
  - for each ME parton select the jet with minimal $\Delta R_{\text{j,parton}}$
  - if $\Delta R_{\text{j,parton}} < R_{\text{match}}$ the parton is “matched” [default R_{match} =1.5R_{clus}]
  - a jet can be matched to a single parton only
  - if all partons are matched, keep the event, else reject it

    $\Rightarrow$ this defines an inclusive sample, $n_{\text{jet}} \geq n_{\max}$

    $\Rightarrow$ for exclusive sample require in addition $n_{\text{jet}} = n$
- after matching combine exclusive and inclusive samples



- variants implemented for MadGraph & Helac/Phegas

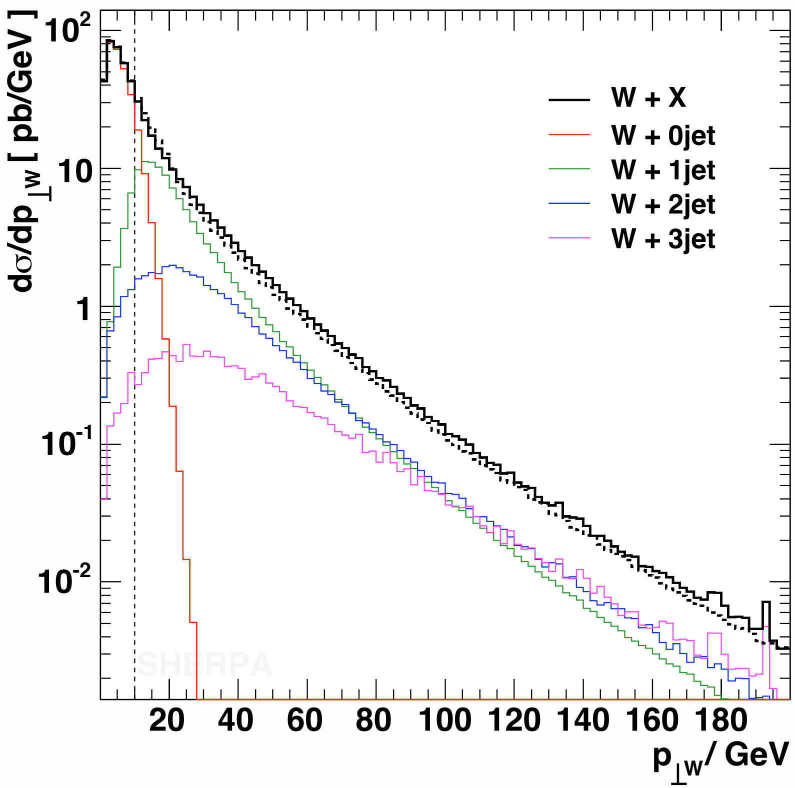
Common Systematics of Merging Approaches

- **all merging prescriptions have similar systematics**
  - residual dependence on separation cut(s) Q_{cut} or ΔR_{min}, $E_{T,min}$
  - variations with the number of ME legs taken into account
  - dependencies on the internal jet algorithm [cone vs. K_T, D parameter]
- **different choices for internal scales**
  - nodal scales for α_S-reweighting
  - choice of factorization scale [different defaults in the approaches]
  - parton shower starting conditions
- **choices for the merging parameter(s) triggered by the analysis**
  - to get jets from ME: $Q_{cut} \leq Q_{ana}$ ($\Delta R_{min} \leq \Delta R_{ana}$, $E_{T,min} \leq E_{T,ana}$)
  - n_{max} should preferably be larger/equal than the LO of your observable

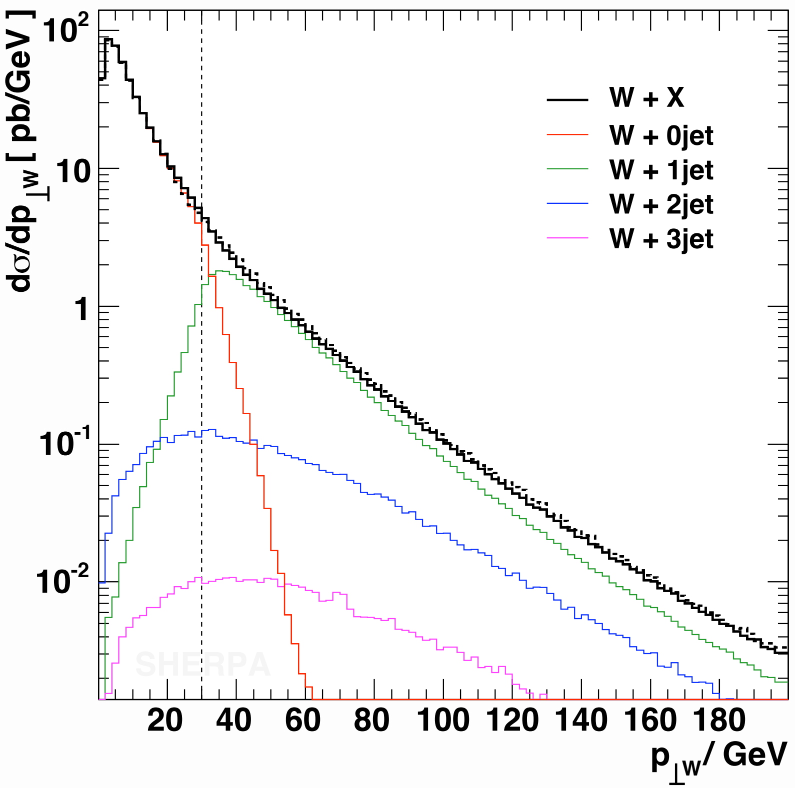
Sherpa consistency checks: variation of Q_{cut}

$p_\perp$ distribution of the W^- in $p\bar{p} \to e^- \, \bar{\nu}_e + X$ @ $\sqrt{s} = 1.96$ TeV

[Krauss, Schälicke, S., Soff, 2004]



Q_{cut}=10 GeV



Q_{cut}=30 GeV

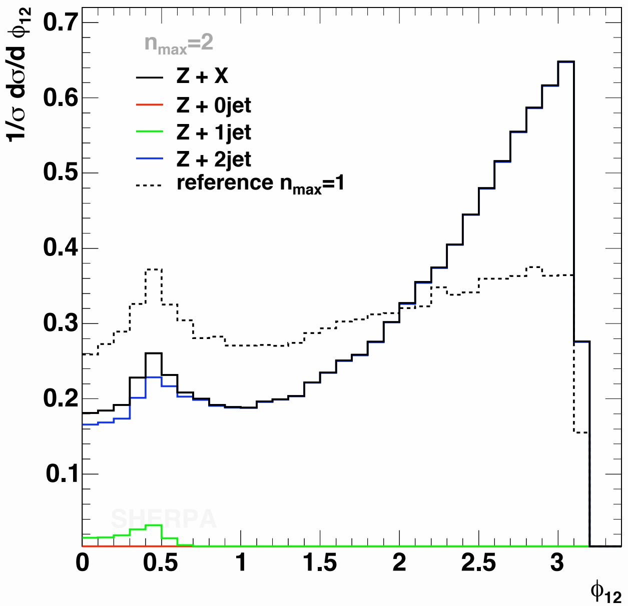
[dashed reference Q_{cut}=20 GeV]

➔ combines different multiplicity final states into fully inclusive sample

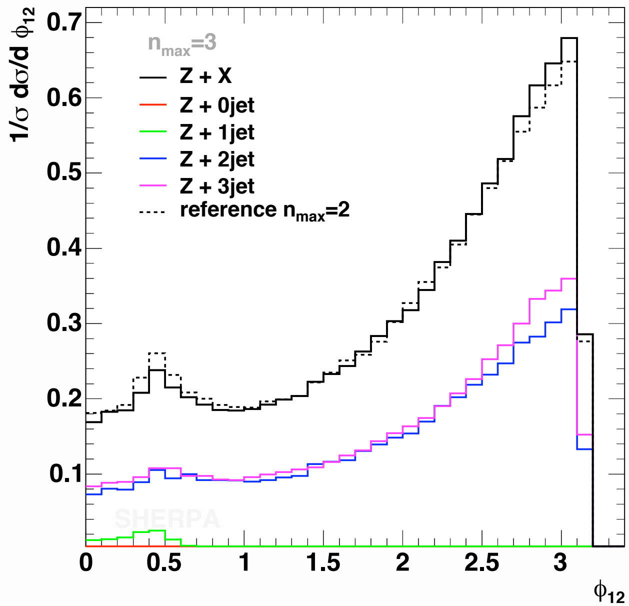
Sherpa consistency checks: variation of n_{max}

The $\Delta\phi$ separation of the two hardest k_T-jets in $pp \to e^+e^- + X$ @ LHC

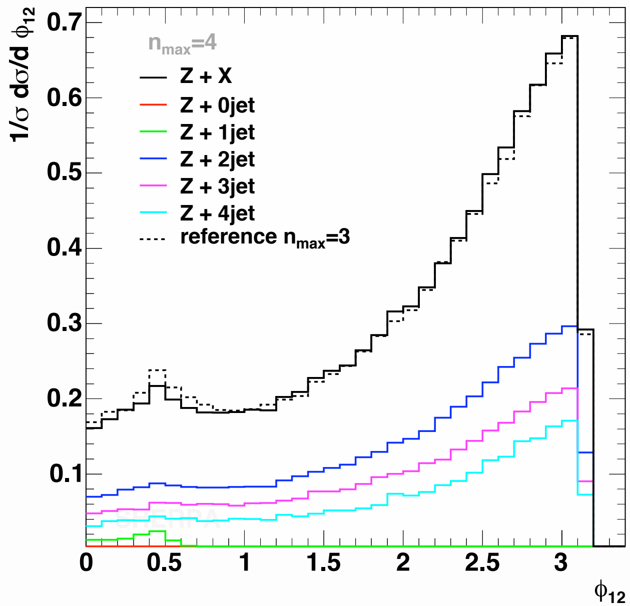
[Krauss, Schälicke, S., Soff, 2005]



$n_{\max}$=2



$n_{\max}$=3



$n_{\max}$=4

two-jet observable in DY

[dashed reference $\tilde{n}_{\max} = n_{\max} - 1$]

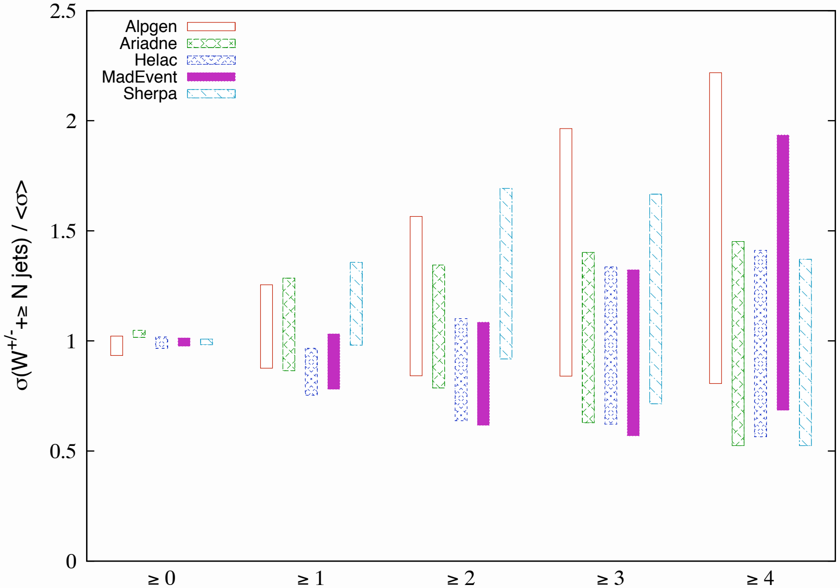
- standard shower accurracy $\sum_{n=2}^{\infty} \alpha_S^n \ln^{2n} Q^2/Q_0^2$, uncorrelated emissions
- merging approach yields $\alpha_S^2 \sum_{n=0}^{\infty} \alpha_S^n \ln^{2n} Q^2/Q_0^2$, full ME correlations

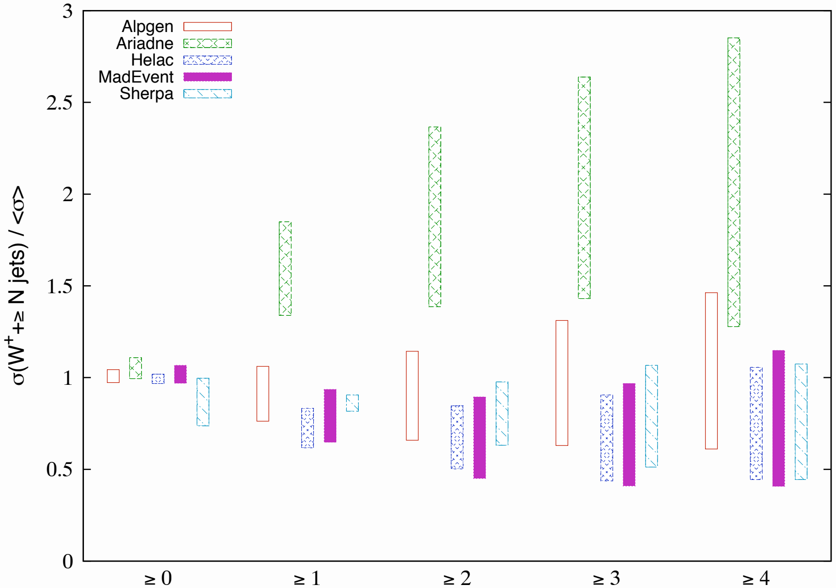
Comparative study of merging algorithms for W+jets

comparing implementations for ME+PS merging [Alwall et al. 2007]

- up-to 4 matrix element jets taken into account [hadron level, UE off]
- analysed for cone jets with $\Delta R = 0.7(0.4)$, $E_T = 10(20)$ GeV at TeV(LHC)
- attempt to estimate the systematics for each generator
  - residual dependence on merging params, $Q_{\rm cut}$, $E_{T,{\rm clus}}$, ($\Delta R_{\rm clus}$, $n_{\rm max}$)
  - scale dependencies: $\alpha_S(k_\perp$ vs. 0.5$k_\perp$ vs. 2$k_\perp$), Sherpa also μ_F

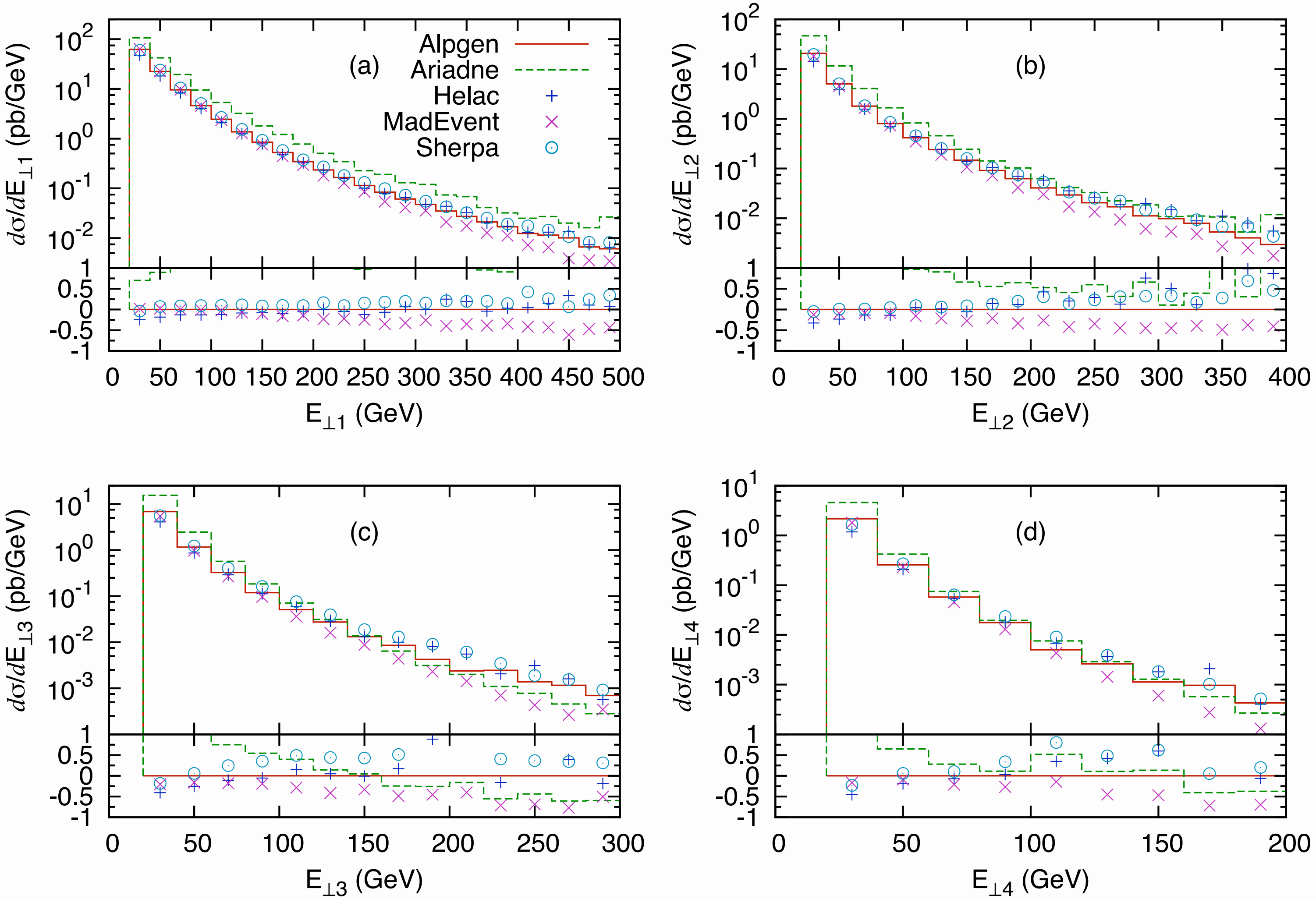
multi-jet rate variations @ Tevatron & LHC





Comparative study of merging algorithms for W+jets

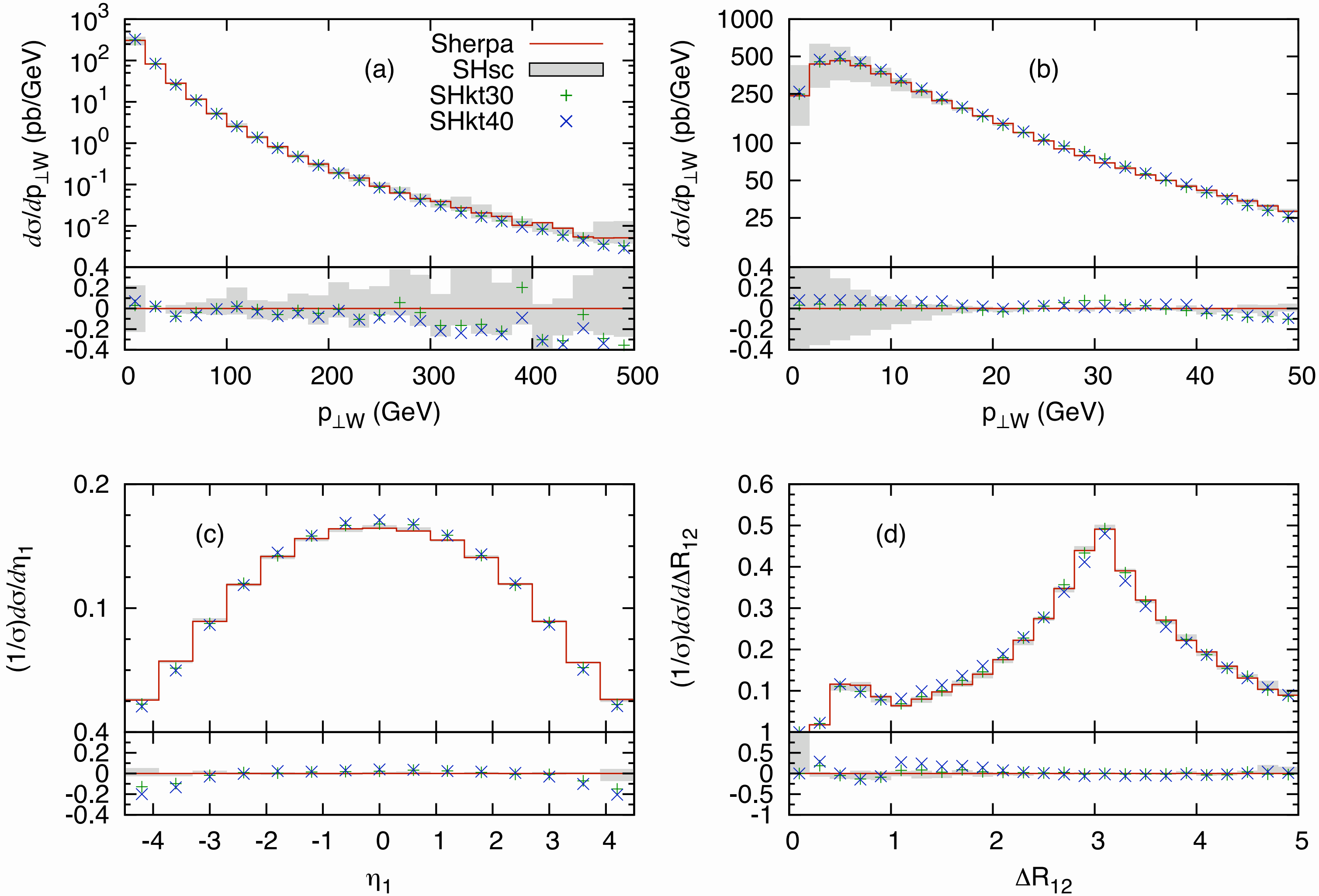
jet transverse energies @ LHC [Alpgen as reference in lower panels]



➔ similar pattern for Tevatron & LHC energies: Tevatron studies desirable

Comparative study of merging algorithms for W+jets

Sherpa's uncertainty band @ LHC

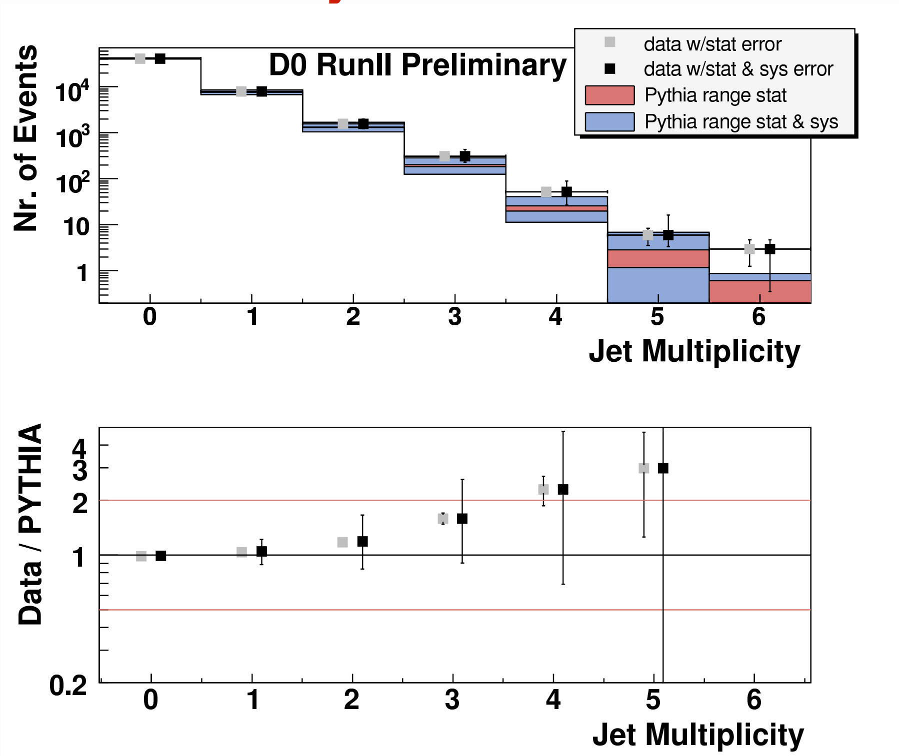


➔ again similar pattern for Tevatron

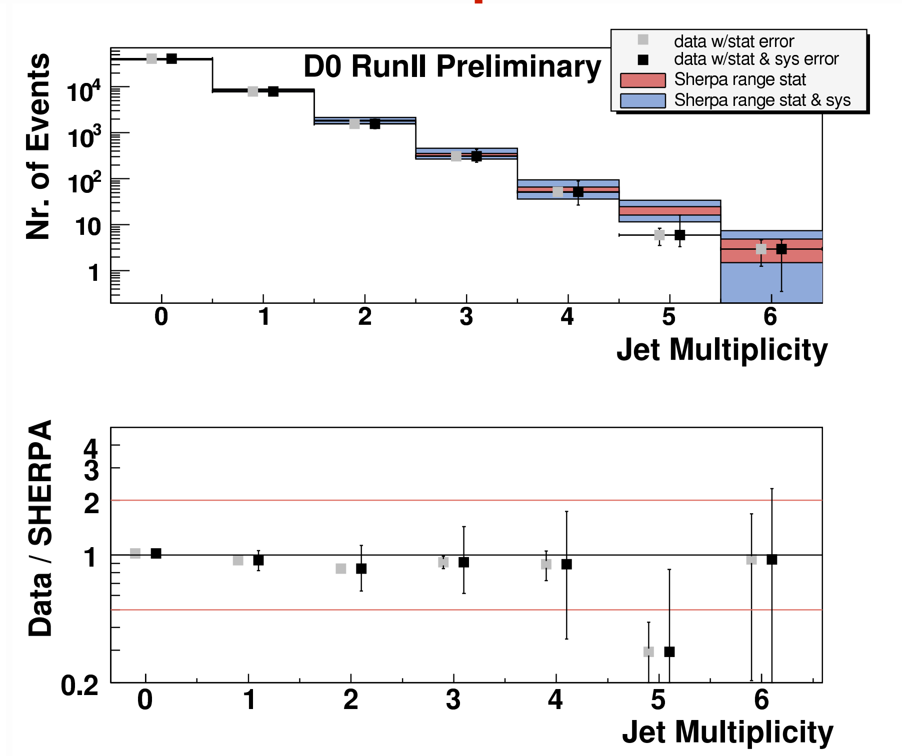
Comparison PS vs. ME+PS: DY at Tevatron

e^+e^-+jets @ Tevatron RunII: jet-multiplicities [DØ Note 5066]

Pythia-v6.2



Sherpa

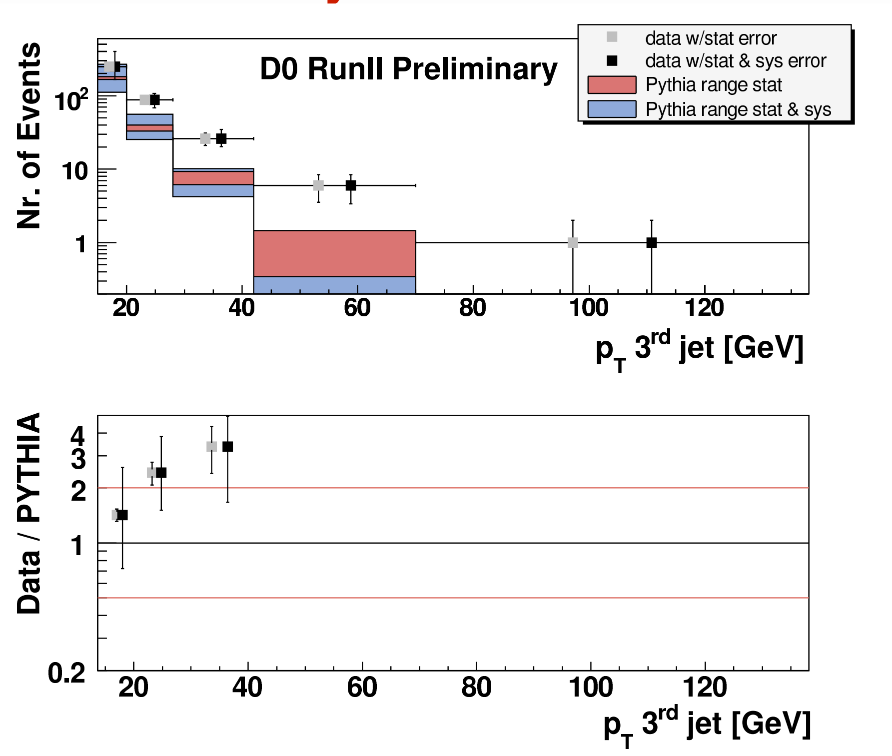


- inclusive samples normalised to total number of measured events
- pure shower approach seems to underestimate multi-jet contributions

Comparison PS vs. ME+PS: DY at Tevatron

e^+e^-+jets @ Tevatron RunII: $p_\perp$ of the third jet [DØ Note 5066]

Pythia-v6.2



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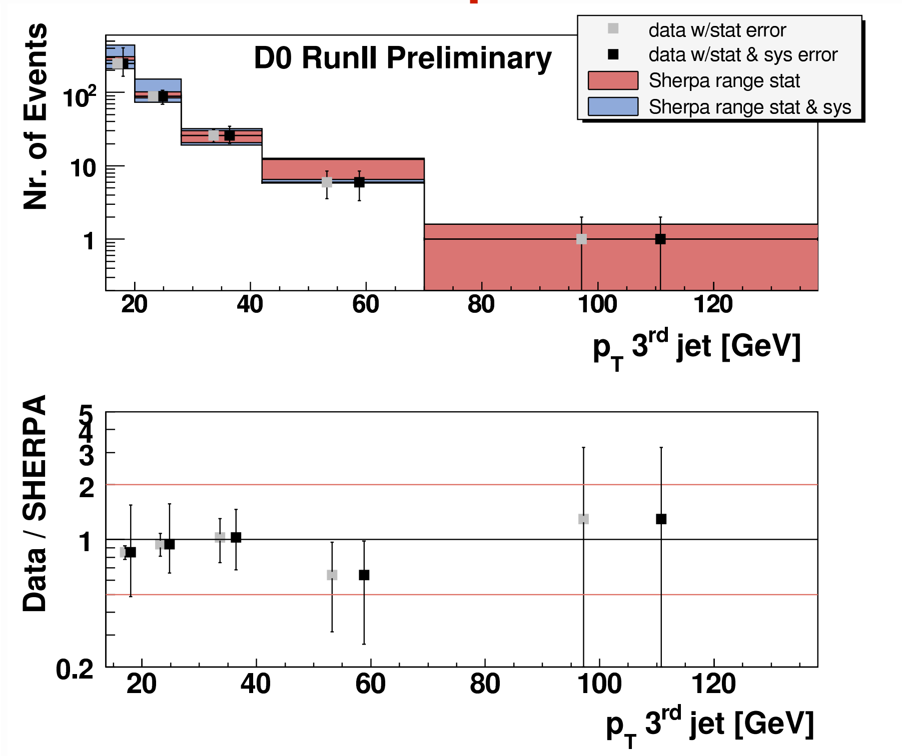


- the more hard jets you require the worse gets the leading-log approximation
- similar analyses done for Alpgen (MLM) within CDF

Alpgen + Herwig for $t\bar{t}$+X: consistency checks

inclusive $t\bar{t}$ production @ Tevatron & LHC [Mangano et al. 2006]

- ME generation Tevatron(LHC): $E_{T,min} = 20(30)$ GeV, $R_{\min} = 0.7(0.7)$
- matching parameters: $E_{T,\text{clus}} = 25(36)$ GeV, $R_{\text{match}} = 1.5 \times 0.7$
- inclusive xsecs: merged $t\bar{t} + 0, 1$ (S_1) vs. $t\bar{t} + 0, 1, 2, 3$ (S_3) hard partons

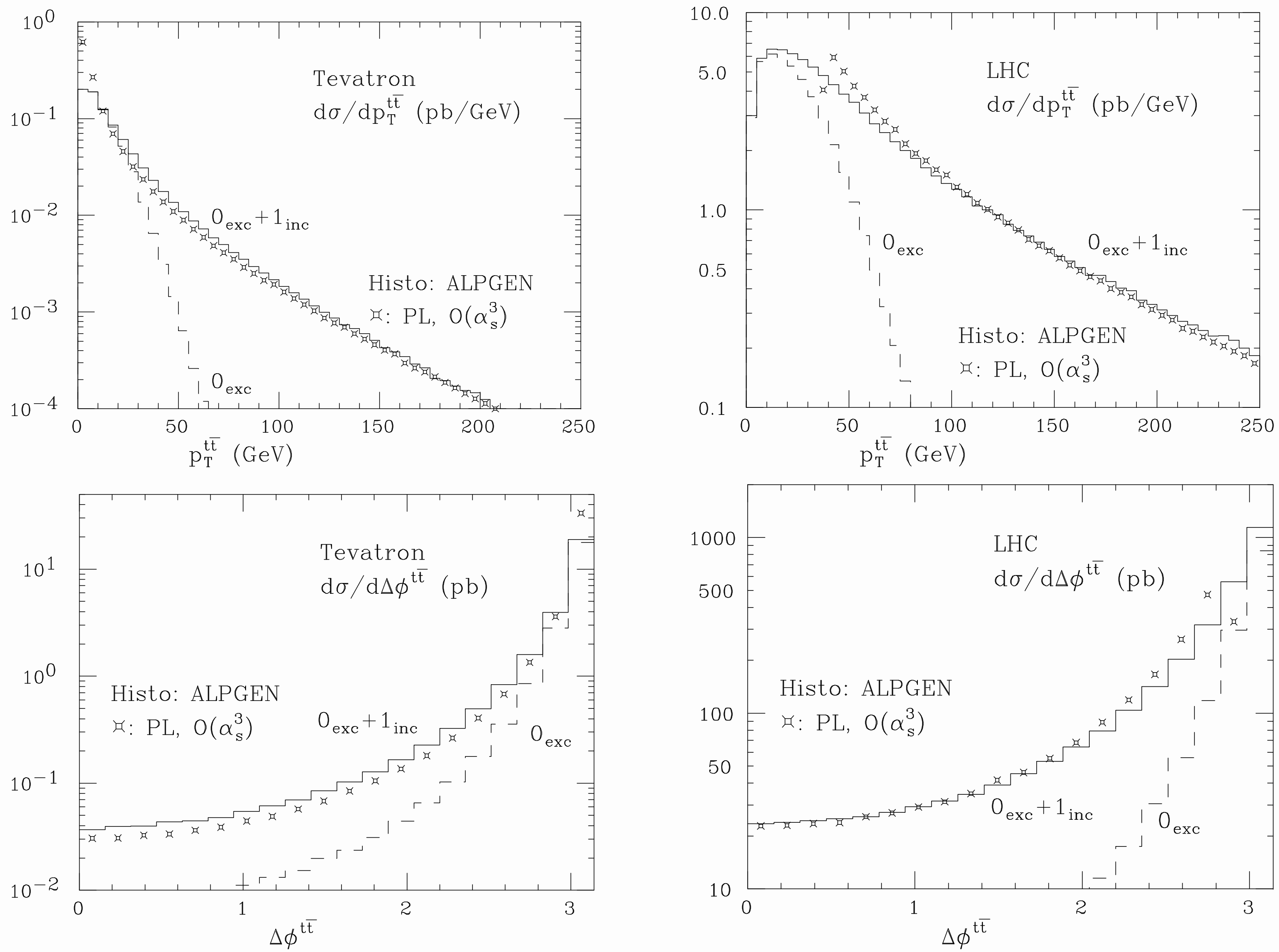
	Tevatron	LHC
0_{exc}	3.42	217
1_{inc}	0.78	252
total	**4.20**	**469**

	Tevatron	LHC
0_{exc}	3.42	216.6
1_{exc}	0.66	149.9
2_{exc}	0.09	65.8
3_{inc}	0.010	29.9
total	**4.18**	**462.2**

➔ naive leading order results for $t\bar{t}$+X Tevatron(LHC): **4.37(471) pb**

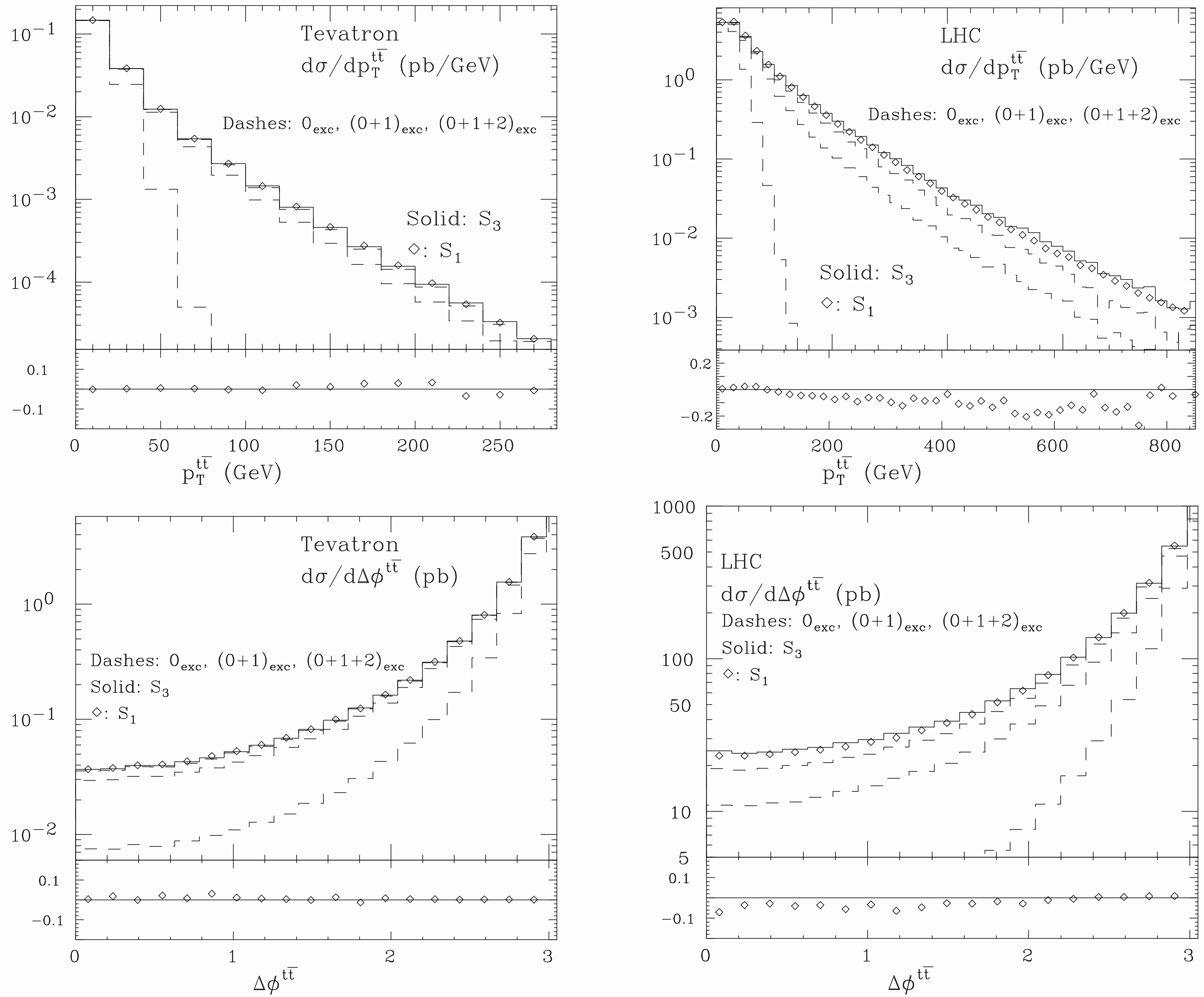
Alpgen + Herwig for $t\bar{t}$+X: consistency checks

$p_T^{t\bar{t}}$ and $\Delta\phi^{t\bar{t}}$ distributions @ Tevatron & LHC (S_1 vs. fixed order α_S^3)



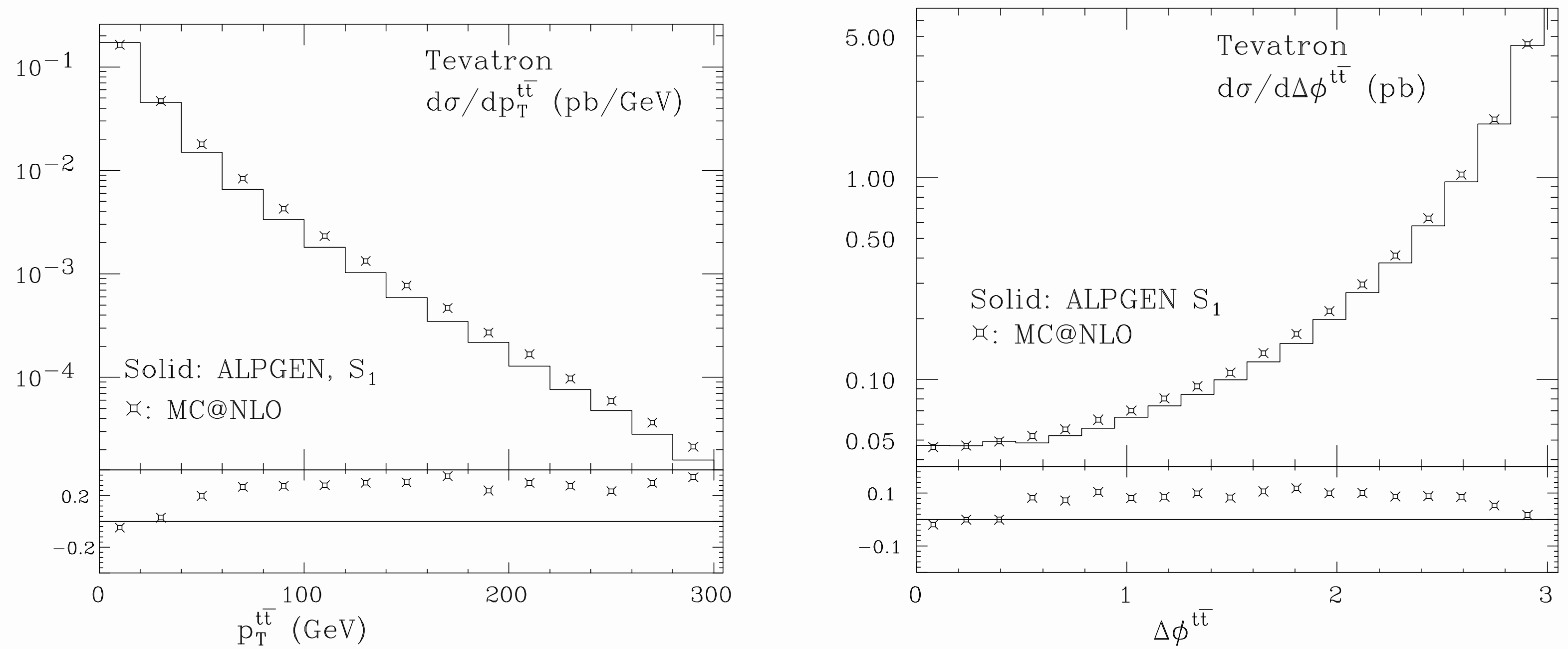
Alpgen + Herwig for $t\bar{t}$+X: consistency checks

$p_T^{t\bar{t}}$ and $\Delta\phi^{t\bar{t}}$ distributions @ Tevatron & LHC (S_1 vs. S_3)



Alpgen + Herwig for $t\bar{t}$+X: comparison with MC@NLO

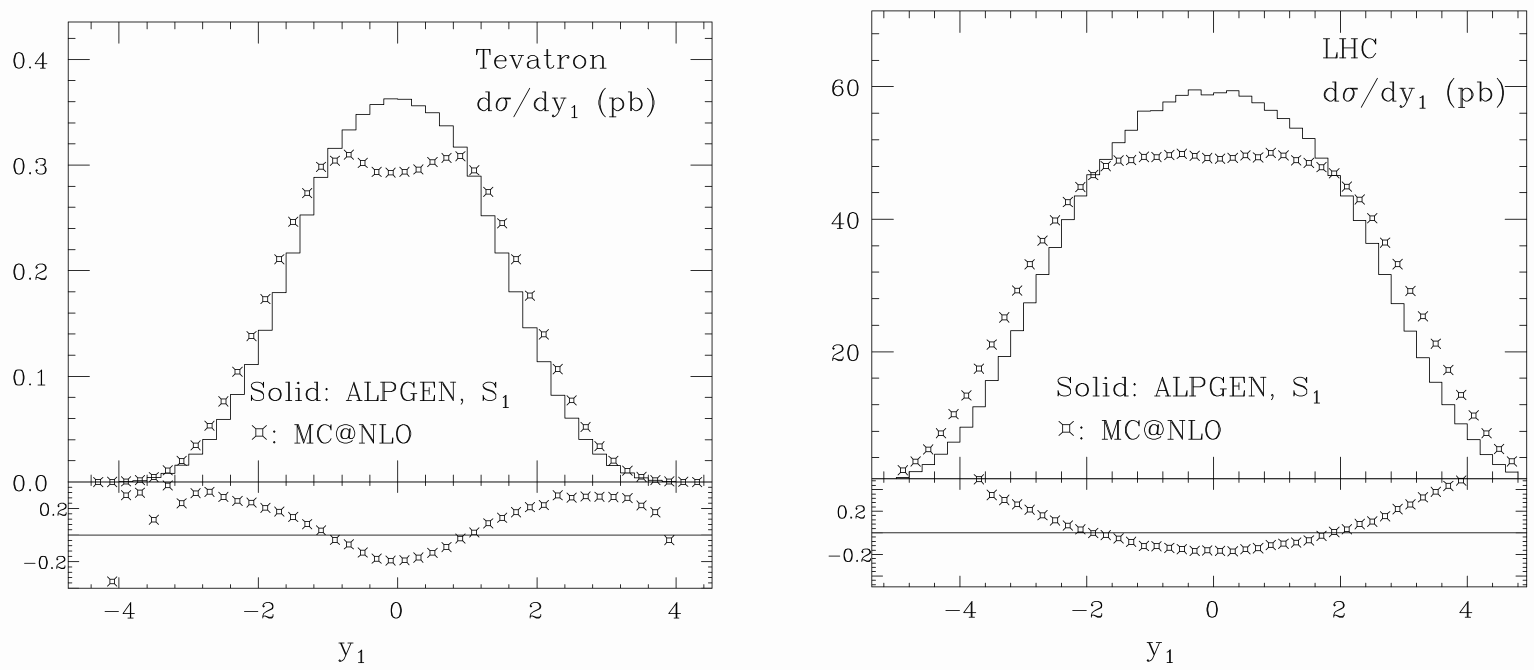
$p_T^{t\bar{t}}$ and $\Delta\phi^{t\bar{t}}$ distributions @ Tevatron (S_1)



- K-factor of 1.36 applied for Alpgen
- good agreement for inclusive quantities (both at Tevatron & LHC)
- but, ...

Alpgen + Herwig for $t\bar{t}$+X: comparison with MC@NLO

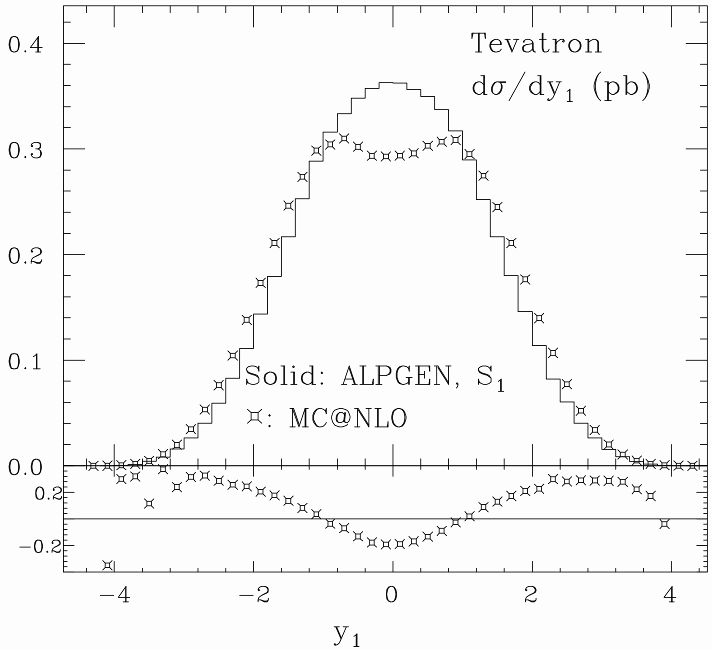
rapidity distribution of hardest jet @ Tevatron & LHC

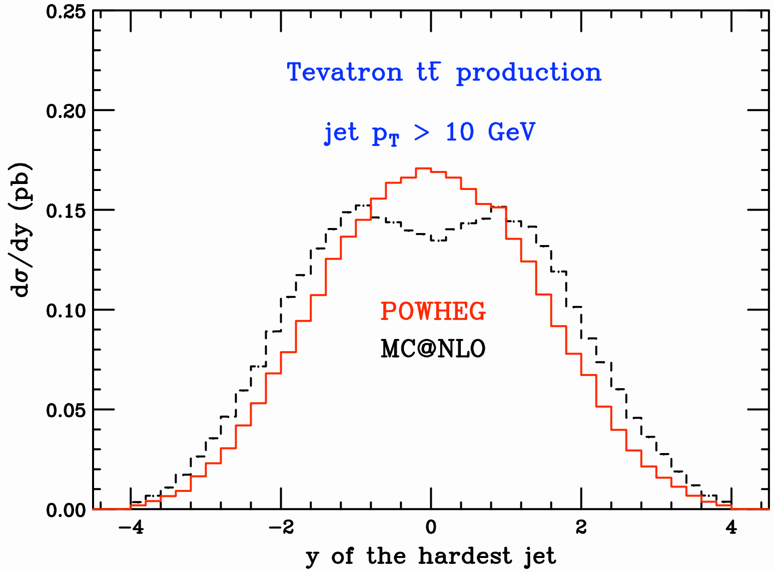


- significant difference in shape around $y_1 \approx 0$

Alpgen + Herwig for $t\bar{t}$+X: comparison with MC@NLO

rapidity distribution of hardest jet @ Tevatron & LHC





- significant difference in shape around $y_1 \approx 0$
- fixed order result looks like Alpgen [confirmed by POWHEG, $t\bar{t}$+jet @ NLO]
- effect of shower dead-cone in MC@NLO [new mass treatment in Herwig++]

Summary/Outlook

The LHC physics programme requires a detailed understanding of QCD

improved theoretical modelling of QCD jets in Monte Carlos

- merging higher-order matrix elements and parton showers
  - matrix elements for final states with many hard partons
  - inclusive samples independent on parton level generation
  - hadronisation, UE for realistic events
- improved theo. predictions for multi-jet events [SM backgrounds, Higgs signals]
- merging techniques can improve BSM analyses [Alwall, Wacker]

other directions

- MC at NLO: MC@NLO, POWHEG [Frixione, Webber 2002; Nason, Ridolfi 2006]
- new parton shower formalism(s) [Dinsdale et al.; Krauss, S.; Krauss, Winter 2007]

future steps

- towards automatisation of NLO + shower
- MC at NLO + tree-level merging