Interpreting the top quark mass: theoretical and MC aspects

TOP 2013
Durbach, Sept 14-19 2013

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I will discuss two issues:

- theoretical ambiguities in interpreting the measured top mass as a pole mass
- dynamical issues relating the experimental observables used to extract mtop and the MC parameter mtop

Pole vs MSbar masses

$$m_{pole} = \overline{m} \times \left[1 + g_1 \frac{\bar{\alpha}}{\pi} + g_2 \left(\frac{\bar{\alpha}}{\pi}\right)^2 + g_3 \left(\frac{\bar{\alpha}}{\pi}\right)^3 \right]$$ where

$$\overline{m} = m_{MS}(m_{MS})$$

$$\bar{\alpha} = \alpha(\overline{m})$$

$$g_1 = \frac{4}{3}$$

$$g_2 = 13.4434 - 1.0414 \sum_k \left(1 - \frac{4}{3}\frac{\overline{m}_k}{\overline{m}}\right)$$

$$g_3 = 0.6527\, n_l^2 - 26.655\, n_l + 190.595$$

Melnikov, van Ritbergen, Phys.Lett. B482 (2000) 99

In the range m_{top} = 171 – 175 GeV, α_S is ~constant, and, using the 3-loop expression above,

$$m_{pole} = \overline{m} \times [1 + 0.047 + 0.010 + 0.003] = 1.060 \times \overline{m}$$

showing an excellent convergence. In comparison, the expansion for the bottom quark mass behaves very poorly:

$$m^b_{pole} = \overline{m}^b \times [1 + 0.09 + 0.05 + 0.04]$$

Assuming that after the 3rd order the perturbative expansion of m_{pole} vs m_{MS} start diverging, the smallest term of the series, which gives the size of the uncertainty in the resummation of the asymptotic series, is of O(0.003 * m), namely O(500 MeV), consistent with Λ_{QCD}

This same O(α_S^3) term gives also: $\overline{m}^{(3-loop)} - \overline{m}^{(2-loop)} = 0.49\,\mathrm{GeV}$

Meson vs hvy-Q masses

Heavy meson $\Longrightarrow$ (point-like color source) + (light antiquark cloud):
properties of "light-quark" cloud are independent of mQ for mQ$\rightarrow\infty$

$$m_M = m_Q + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_Q}$$

$$m_{M^*} = m_Q + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_Q}$$

$$\langle M|\,\bar{h}_Q\,(iD)^2 h_Q\,|M\rangle = -\lambda_1\,\mathrm{tr}\{\,\overline{\mathcal{M}}\,\mathcal{M}\,\} = 2M\,\lambda_1\,,$$
$$\langle M|\,\bar{h}_Q\,s_{\alpha\beta}G^{\alpha\beta}h_Q\,|M\rangle = -\lambda_2(\mu)\,\mathrm{tr}\{\,i\sigma_{\alpha\beta}\,\overline{\mathcal{M}}\,s^{\alpha\beta}\mathcal{M}\,\} = 2d_M M\,\lambda_2(\mu)\,,$$

$d_{M^*} = -1$, $d_M = 3$

See e.g. Falk and Neubert, arXiv:hep-ph/9209268v1

where $\bar{\Lambda},\ \lambda_1,\ \lambda_2$ are independent of m_Q

From the spectroscopy of the B-meson system:

$m(B^*) - m(B) = 2\,\lambda_2/m_b \Rightarrow \lambda_2 \sim 0.15$ GeV^2

QCD sum rules: $\lambda_1 \sim 1$ GeV^2

QCD sum rules: $\Lambda = 0.5 \pm 0.07$ GeV

thus corrections of $O(\lambda_{1,2}/m_{top})$ are of O(few MeV) and totally negligible

Separation between mQ and Λ is however ambiguous:
renormalon ambiguity on the pole mass:

$$\begin{aligned}\delta m_{pole} &= \frac{C_F}{2N_f|\beta_0|}\, e^{-C/2}\, m(\mu=m)\exp\left(\frac{1}{2N_f\beta_0\alpha(m)}\right)\\ &= \frac{C_F}{2N_f|\beta_0|}\, e^{-C/2}\, \Lambda_{QCD}\left(\ln\frac{m^2}{\Lambda_{QCD}^2}\right)^{\beta_1/(2\beta_0^2)},\end{aligned}$$

where $\beta_1 = -1/(4\pi N_f)^2 \times (102 - 38N_f/3)$ is the second coefficient of the β-function

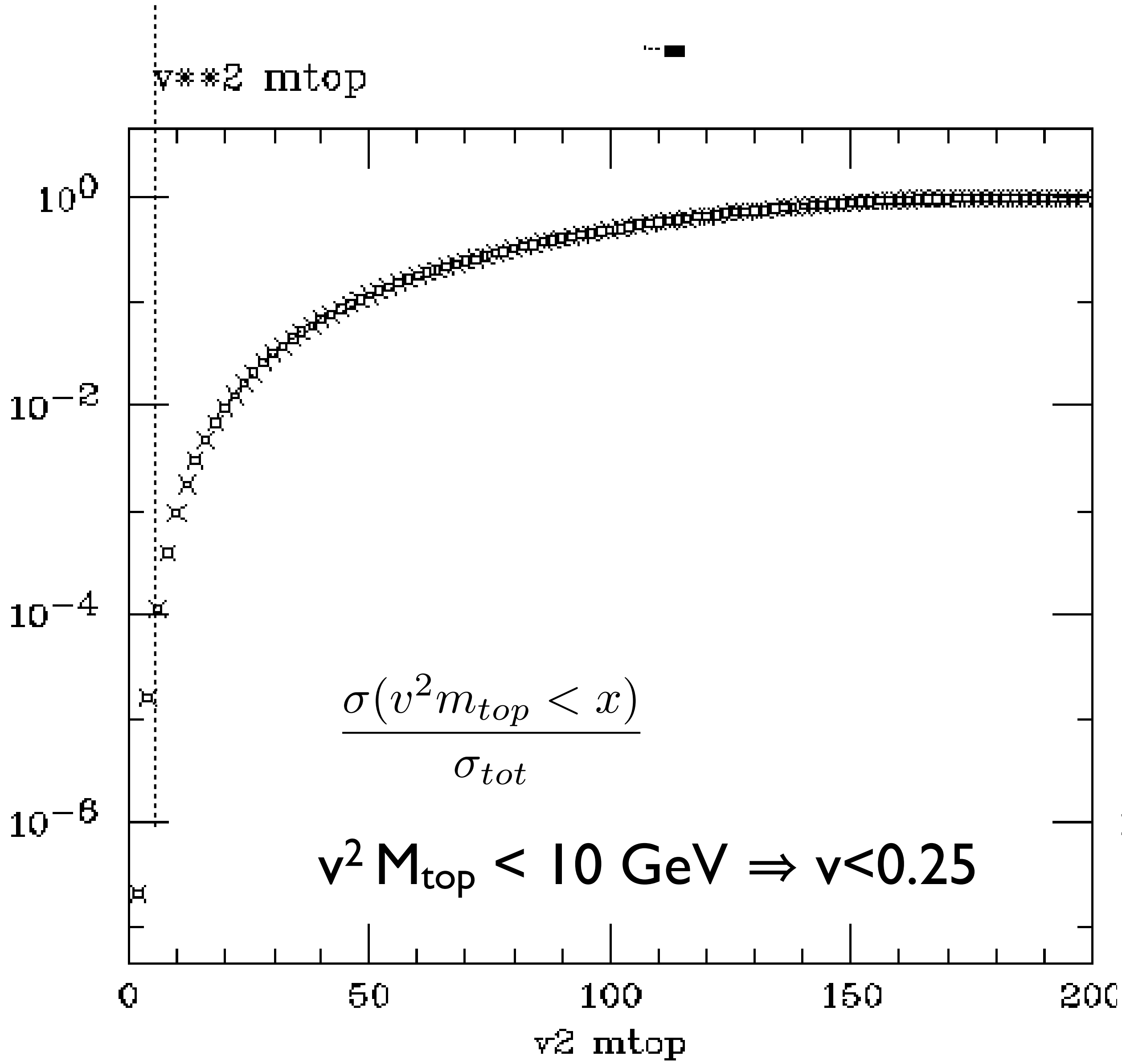
δm_{pole}=270 MeV for mtop.

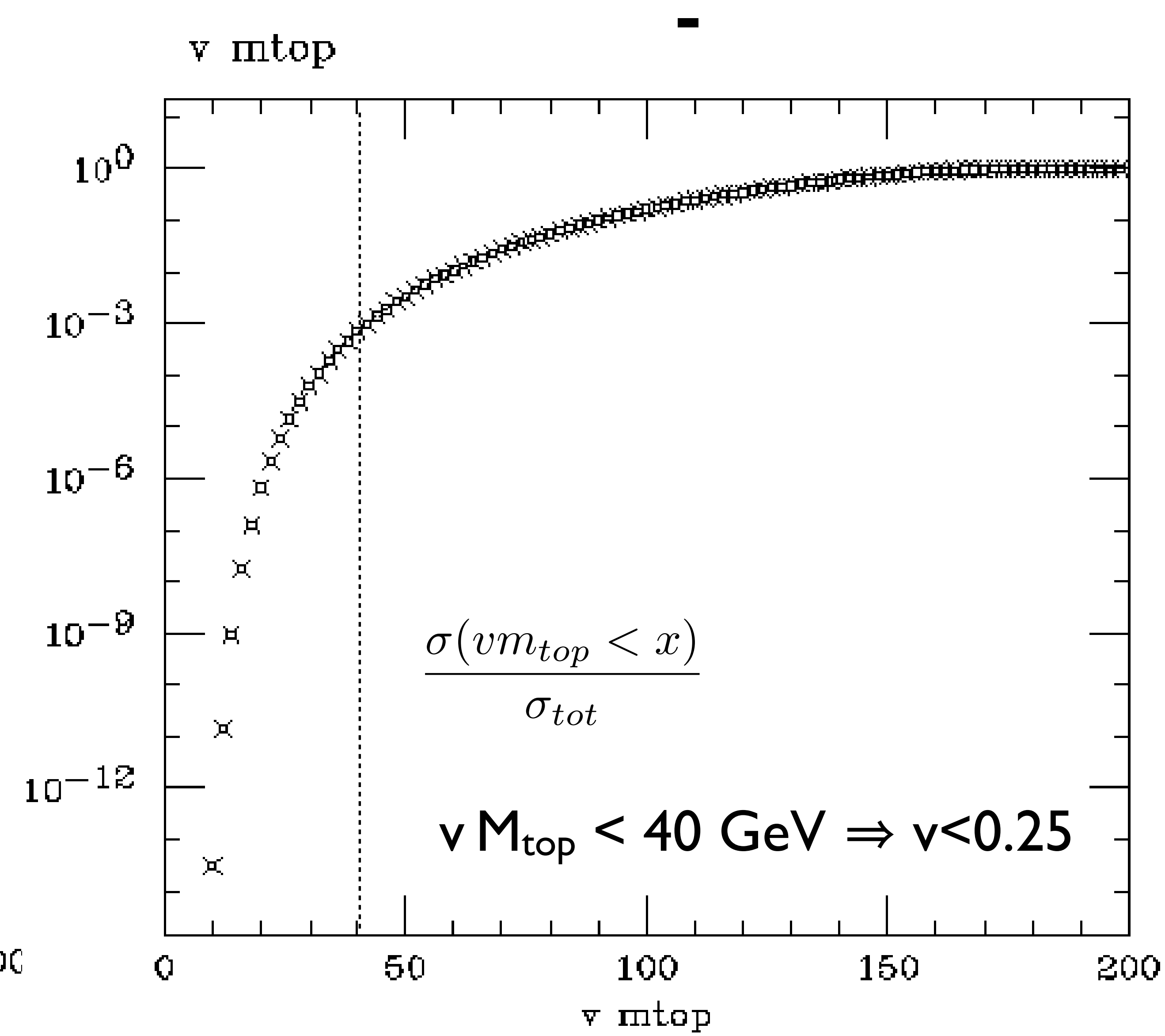
This is smaller than the difference between MSbar masses obtained using the 3-loop or 2-loop MSbar vs pole mass conversion.

It would be very interesting to have a 4-loop calculation of MSbar vs m_{pole}, to check the rate of convergence of the series, and improve the estimate of the m_{pole} ambiguity for the top

Beneke and Braun, Nucl. Phys. B426, 301 (1994)
Bigi et al, 1994

Impact of IR sensitive phase-space regions on σ(tt)

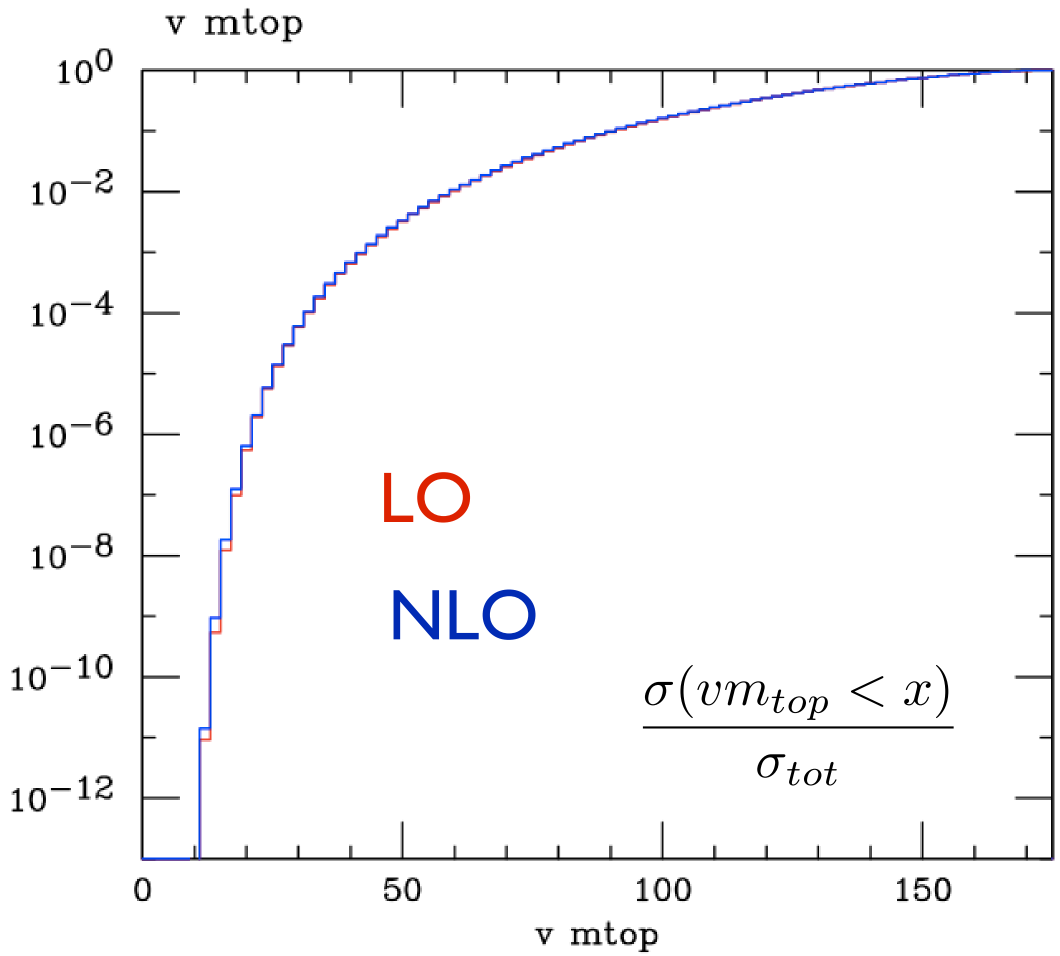




$$\frac{1}{\sigma}\,\frac{\Delta\sigma}{\Delta m} \sim 0.03\,\mathrm{GeV}^{-1}$$

$v^2 M_{top} < 10$ GeV

The region possibly sensitive to IR effects, v^2M_{top}<10 GeV, or v<0.25, contributes only 10^{-3} of the total rate.
Uncertainties of the order of 100% in the description of this region only change the extraction of M_{top} from the total rate at the level of 30 MeV



The impact of Coulomb corrections (which first appear at NLO) is confined to values of v that contribute very little to the total cross section

⇒ no evidence that the relation between m_{pole}(top) and total tt cross section in pp(bar) collisions is subject to the same IR problems that enter as main systematics in the extraction of m_{top} from the threshold scan in e^+e^-

All in all I believe that it is justified to assume that MC mass parameter is interpreted as m_{pole}, within the ambiguity intrinsic in the definition of m_{pole}, thus at the level of ~250-500 MeV

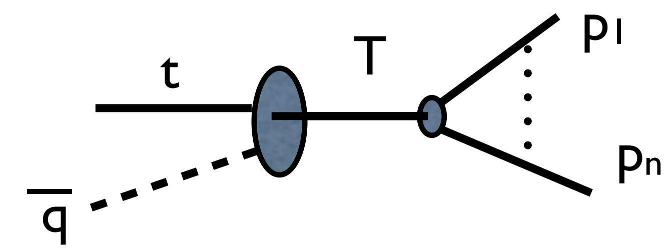
Definition of m_{top} from top decays

If Γ_{top} were < 1 GeV, top would hadronize before decaying. Same as b-quark

But Γ_{top} is > 1 GeV, top decays before hadronizing. Extra antiquarks must be added to the top-quark decay final state in order to produce the physical state whose mass will be measured

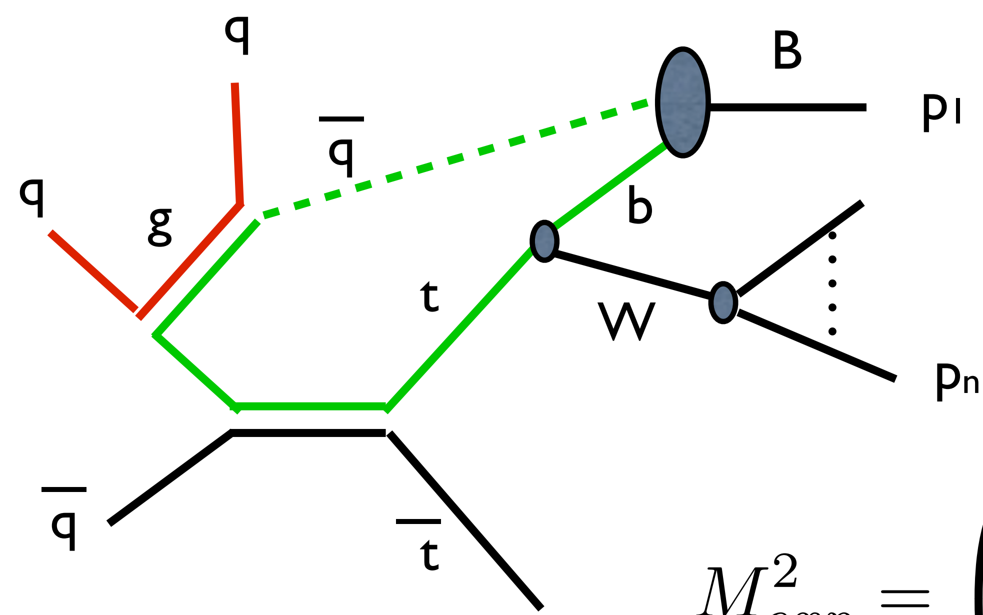
As a result, M_{exp} is not equal to m^{pole}_{top}, and will vary in each event, depending on the way the event has evolved.

The top mass extracted in hadron collisions is not well defined below a precision of $O(\Gamma_{top})$~ 1 GeV



$$m_T^2 = \left(\sum_{i=1,\dots,n} p_i \right)^2$$

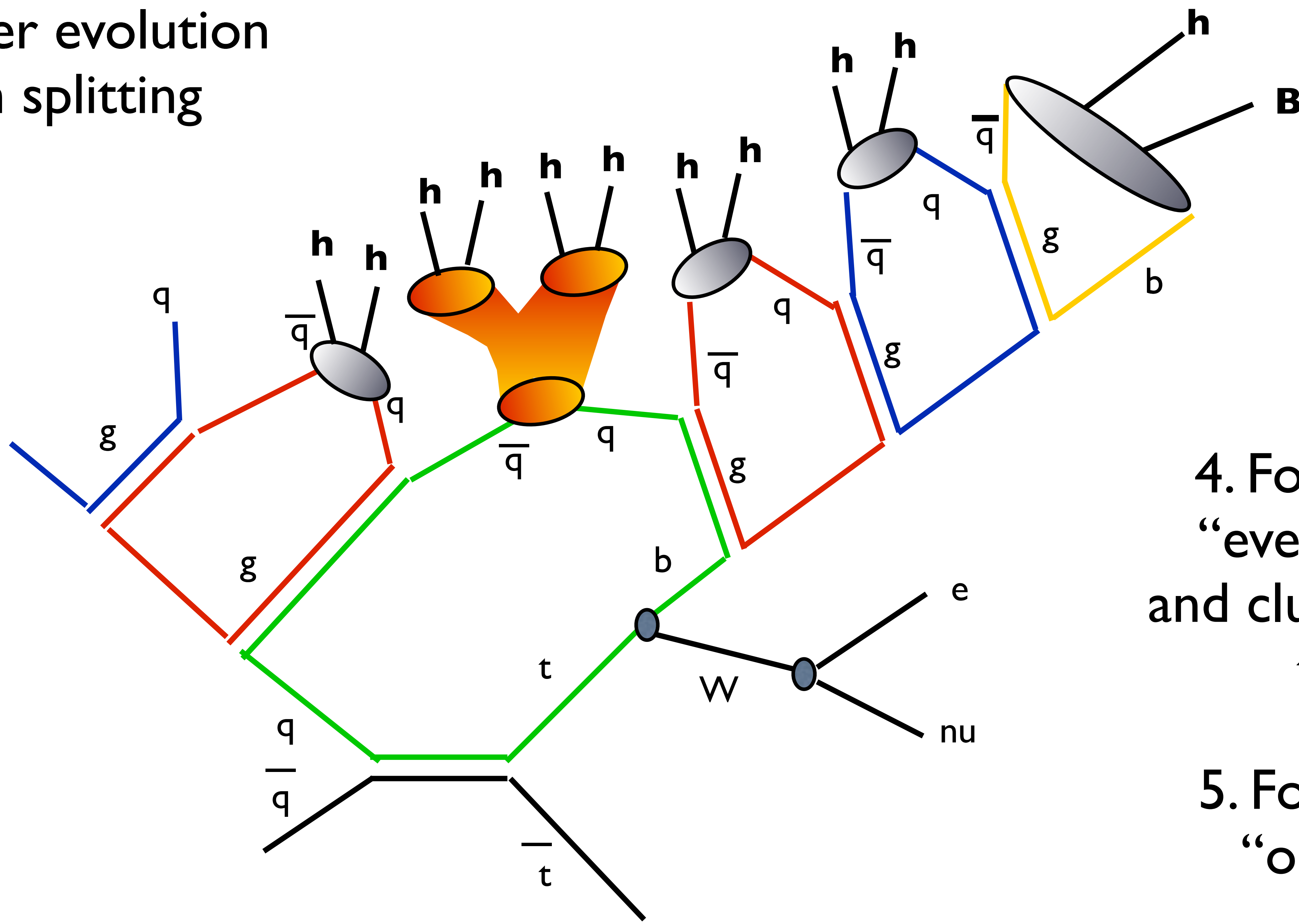
$m_t = F_{lattice/potential\ models}\ (m_T, \alpha_{QCD})$



$$M_{exp}^2 = \left(\sum_{i=1,\dots,n} p_i \right)^2$$

Goal:
- correctly quantify the systematic uncertainty
- identify observables that allow to validate the theoretical modeling of hadronization in top decays
- identify observables less sensitive to these effects

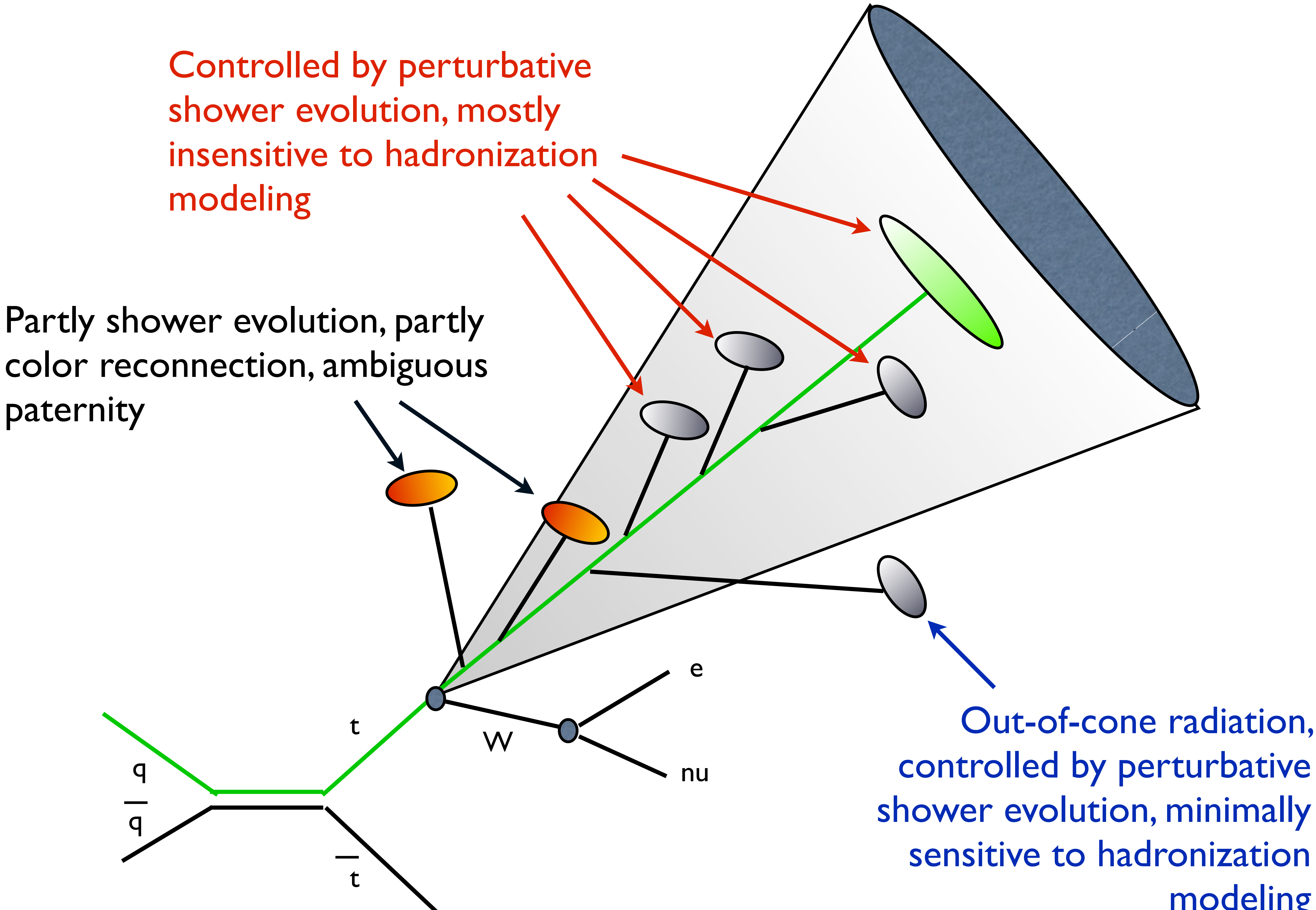
1. Hard Process
2. Shower evolution
3. Gluon splitting



4. Formation of "even" clusters and cluster decay to hadrons

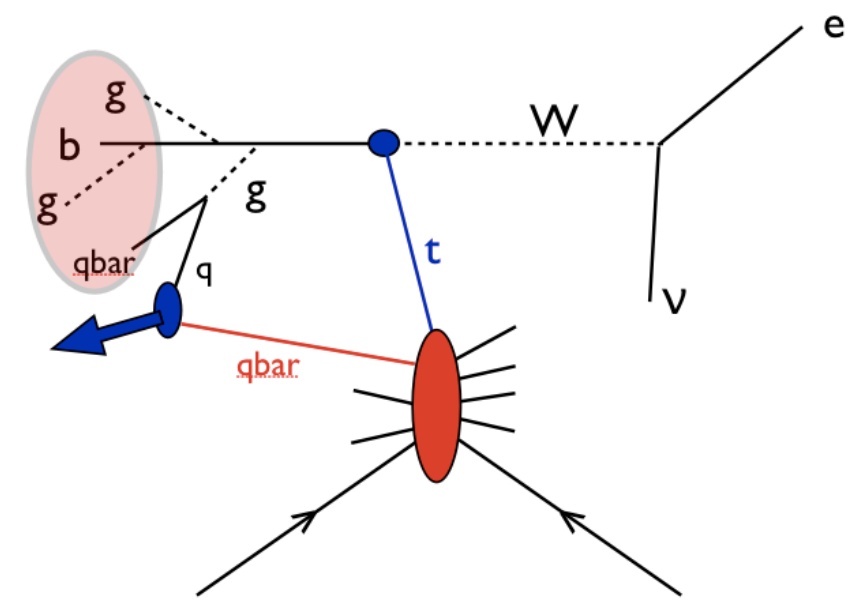
5. Formation of "odd" cluster

6. Decay of "odd" clusters, if large cluster mass, and decays to hadrons



Controlled by perturbative shower evolution, mostly insensitive to hadronization modeling
Partly shower evolution, partly color reconnection, ambiguous paternity
Out-of-cone radiation, controlled by perturbative shower evolution, minimally sensitive to hadronization modeling
e
t
W
nu
q
$\bar{q}$
$\bar{t}$

Relevant dynamical effects that influence the kinematics and mass reconstruction



$$e' = \gamma_W \, (e + \beta_W \, p \cos\theta)$$

e' = electron energy in the top rest frame (TRF)
e,p = electron energy/momentum in W rest frame (WRF)
θ = electron decay angle in the WRF
$\gamma_W = E_W(TRF)/m_W$ $\beta_W = p_W(TRF)/E_W(TRF)$

$$E_W = \frac{m_t^2 + m_W^2 - m(b - \mathrm{recoil})^2}{2m_t} \qquad p_W = \sqrt{E_W^2 - m_W^2} = p_{b-recoil}$$

⇒ the electron kinematics depends on the structure of the recoil b-jet

The structure of the recoil b-jet is determined by:
- its perturbative evolution (which gives the jet a mass)
- the non-perturbative hadronization (which requires combination with a source of colour, coming from the rest of the event)

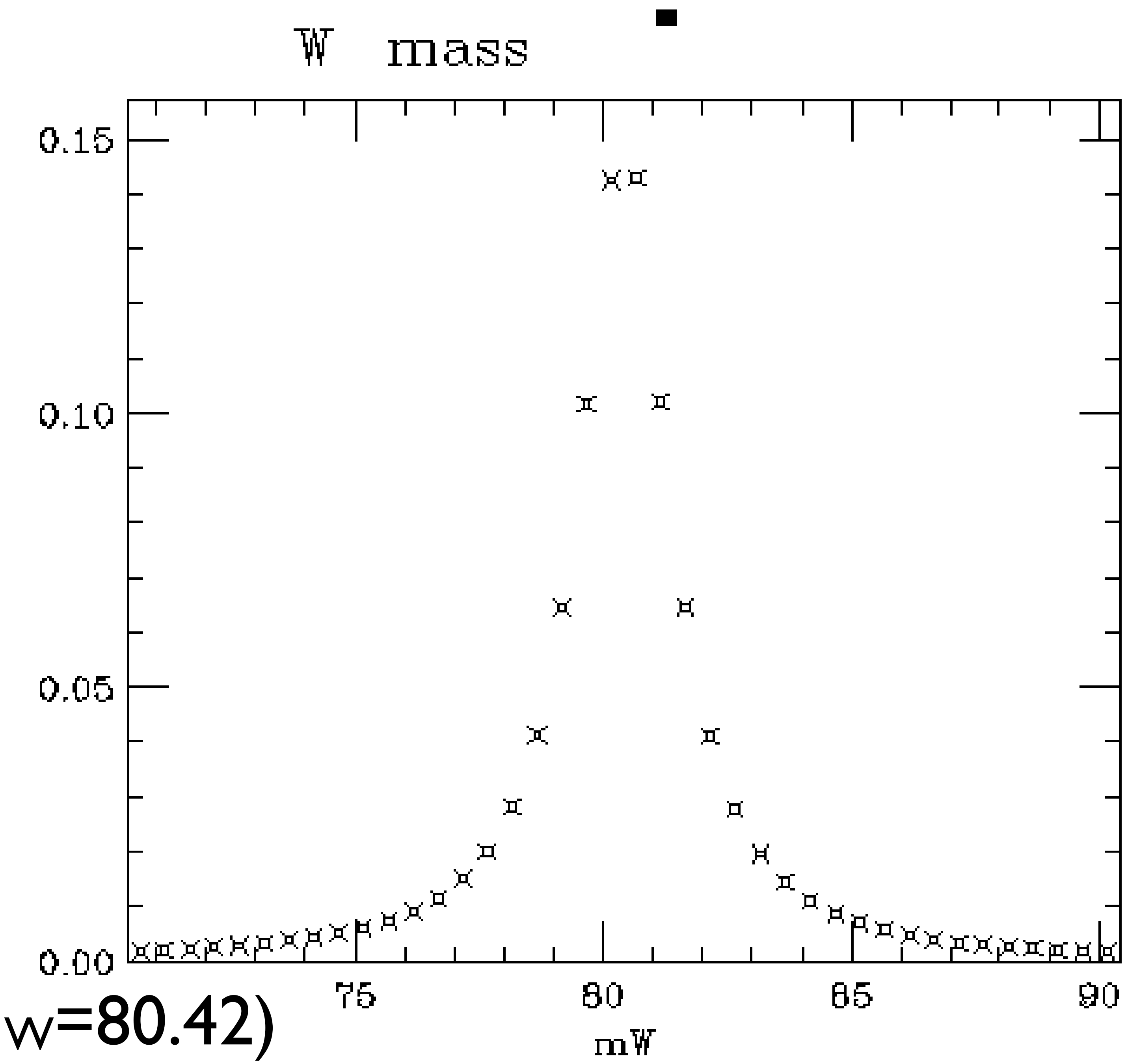
However, since Γ_W=2.5 GeV, the W decays *before* the b-jet enters the hadronization phase, so the dynamics of the W should not be sensitive to hadronization issues related to the b-jet

W-width effects

$$|M|^2 \, d(PS) \; \propto \; \beta_W \; m_{e\nu} \; (t \cdot \nu)(b \cdot e) \frac{dm^2_{e\nu}}{(m^2_{e\nu} - M^2_W)^2 + M^2_W \Gamma^2_W}$$

The decay dynamics (matrix element, shower evolution of the b quark, phase space) do not distort significantly the BW spectrum.

In particular, the average value of the invariant mass of the W decay products is consistent with m_W



$\langle m_W \rangle$ = 80.39 GeV (input m_W=80.42)

$\sqrt{\langle (m_W - \langle m_W \rangle)^2 \rangle}$ = 2.47 GeV

Remark

The leptonic endpoint appears as extremely robust against perturbative and non-perturbative effects, and in particular it is decoupled from the b-jet hadronization uncertainties, due to the short W lifetime.

Consider e.g. the end-point of the electron spectrum, in the top rest frame. Simple algebra gives:

$$e'_{max} = \frac{m_t}{2} \left(1 - \frac{m_b^2}{m_t^2 - m_W^2} + O(m_b^4/m_t^4)\right)$$

NB: $\mathbf{m_b}$ here refers to the mass of the full b jet recoiling against the W

Therefore:

using $m_b \sim 15$ GeV, i.e. the average mass of a b jet in top decays

$$\Delta e'_{max} = 0.5 \, \Delta m_t$$

$$\Delta e'_{max} = -\frac{1}{1-(m_W/m_t)^2} \left(\frac{m_b}{m_t}\right)^2 \Delta m_b \sim -10\,\mathrm{MeV} \, \frac{\Delta m_b}{\mathrm{GeV}}$$

Thus a mismodeling of the b-recoil mass of **1 GeV** leads to an error on m_{top} from the lepton endpoint of **20 MeV:**

Δm_{top} (e'_{max}) = – 20 MeV (Δm_b/GeV)

A similar calculation for the average value of e', ⟨e'⟩ = $E_W/2$, gives a larger, but still moderate, sensitivity:

Δm_{top} (⟨e'⟩) = – 200 MeV (Δm_b/GeV)

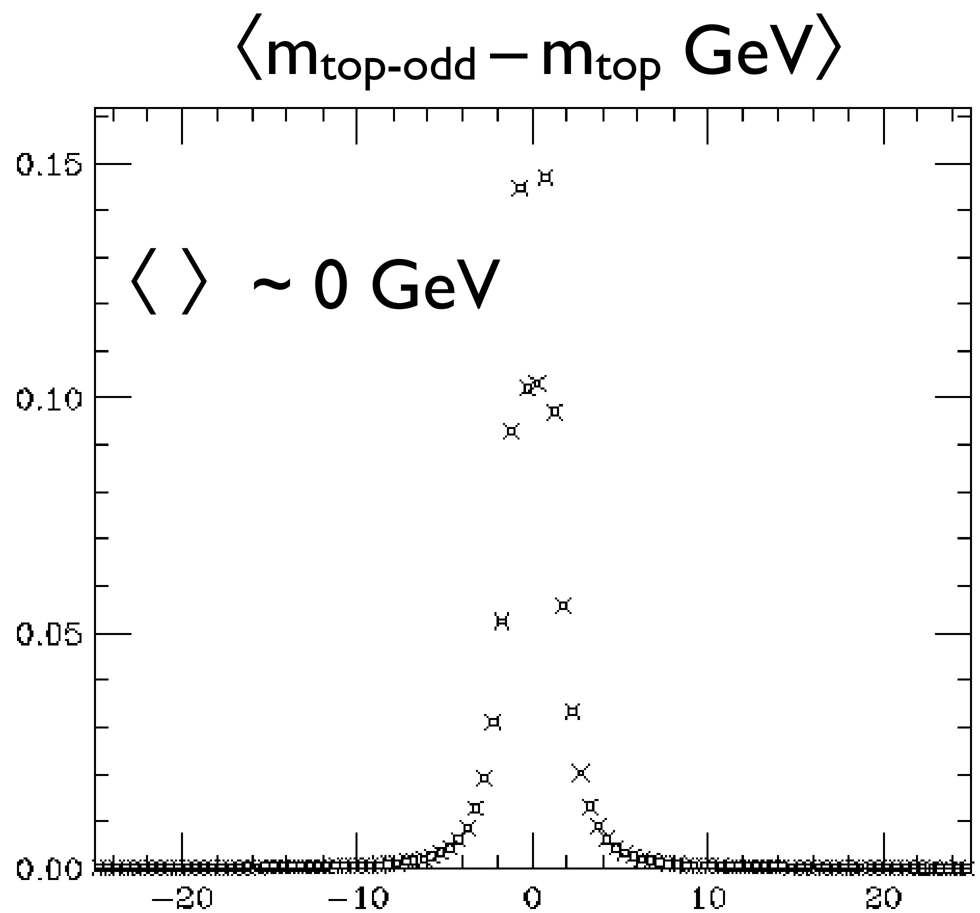
Remark

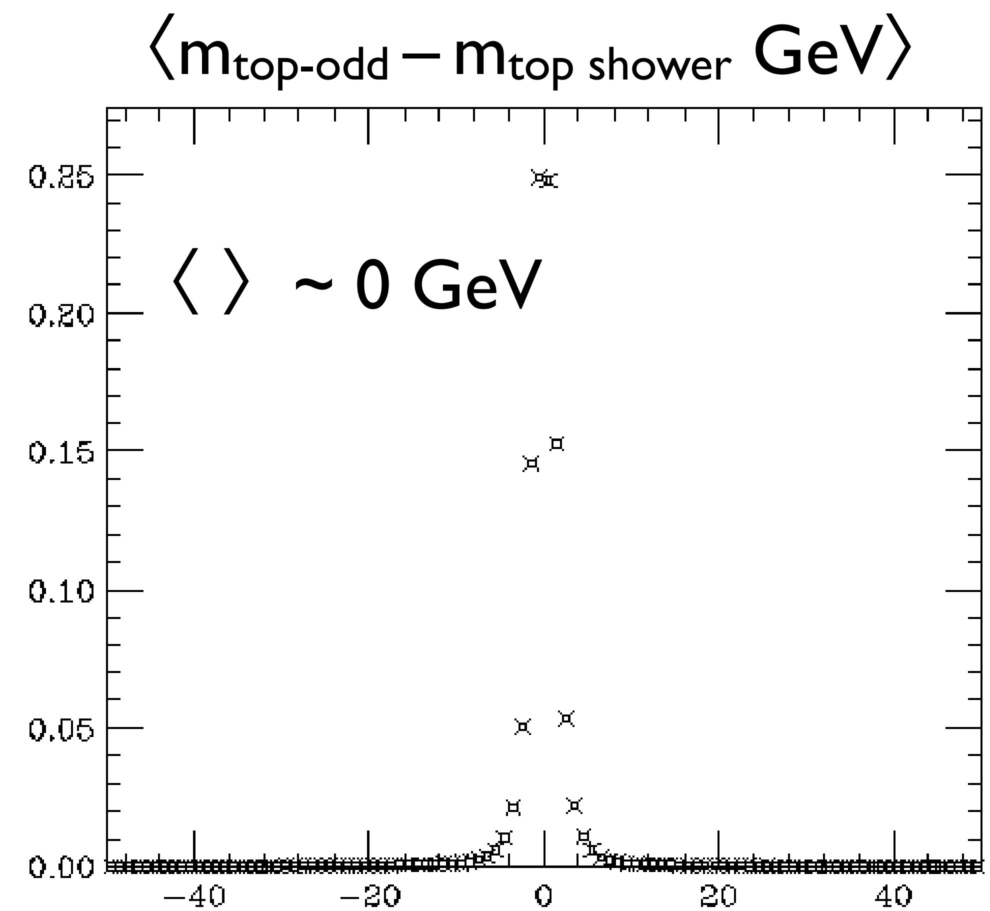
Unfortunately pt(lep) is not Lorentz invariant in haronic collisions, and any other LI quantity (e.g. $p_{lep} \cdot p_{bot}$) looses part of this robustness because fo the dependence on the evolution of the rest of the event

Use of pt(lep) can become feasible if we can absolute confidence in the description of the top momentum spectrum

To simplify to the bare bones the problem, the analysis is best done considering $e^+e^- \rightarrow$ttbar, and t$\rightarrow$b ℓ ν

$\sqrt{S_{e+e-}}$=350 GeV

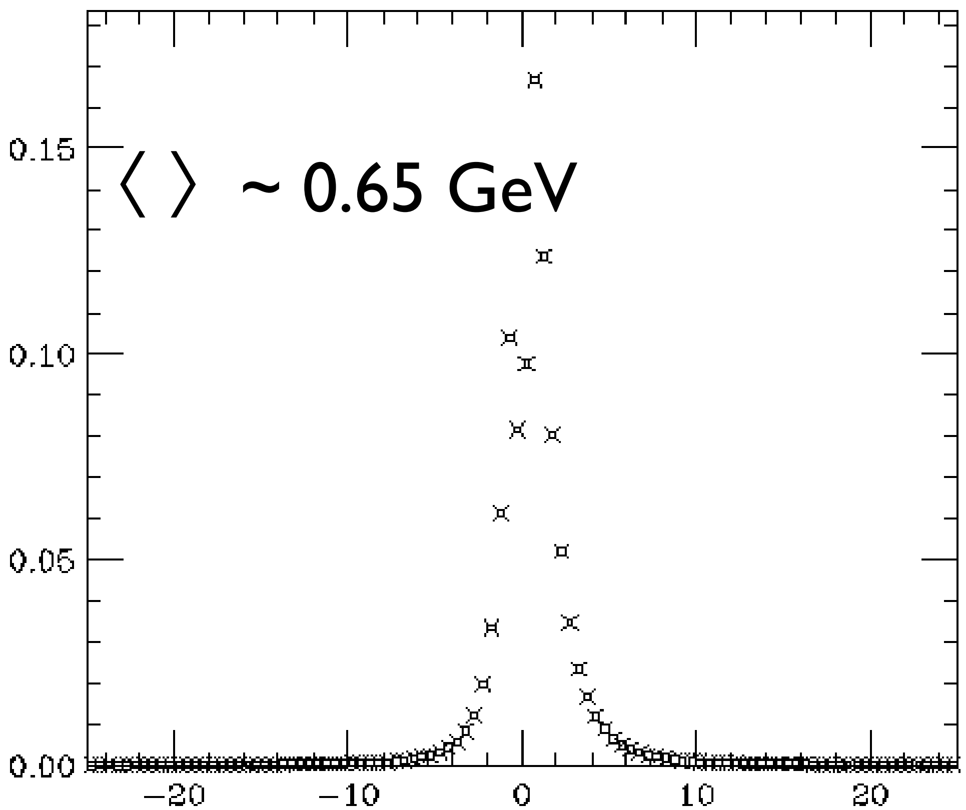




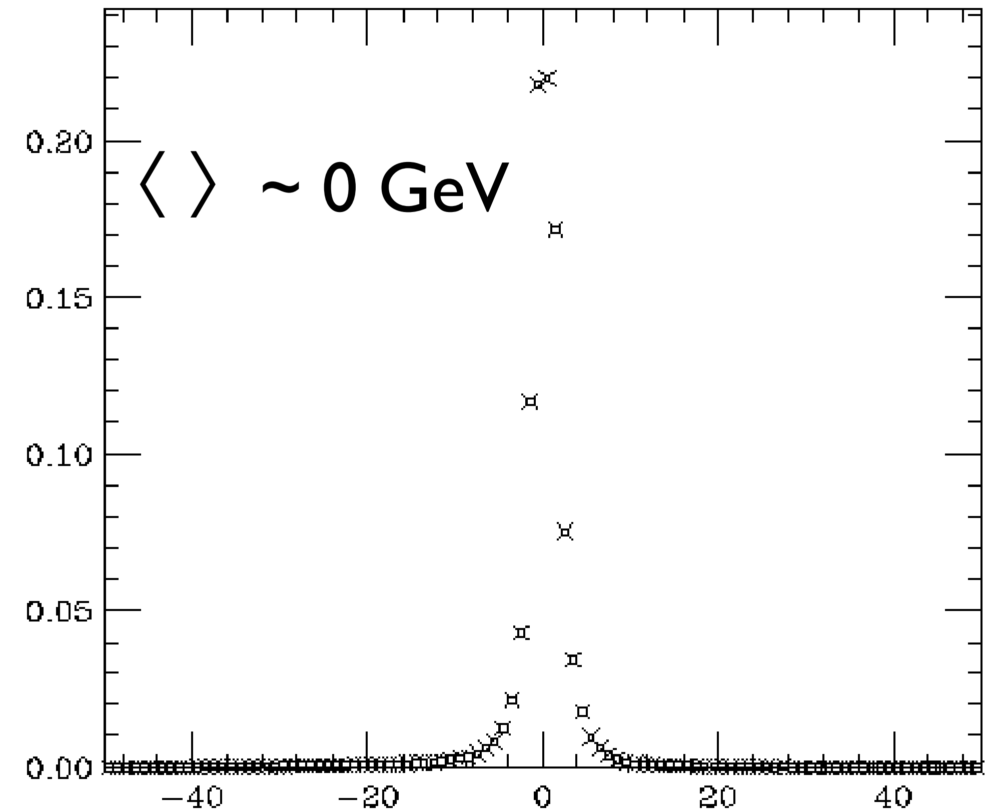
m(top-odd): top recostructed summing all even+odd clusters with at least one component from the b-jet evolution
m(top shower): top reconstructed using the partons form the b shower evolution (i.e. no hadronization of b jet)

$\sqrt{s_{e+e-}}$=400 GeV

$\langle m_{top\text{-}odd} - m_{top}\ \text{GeV} \rangle$

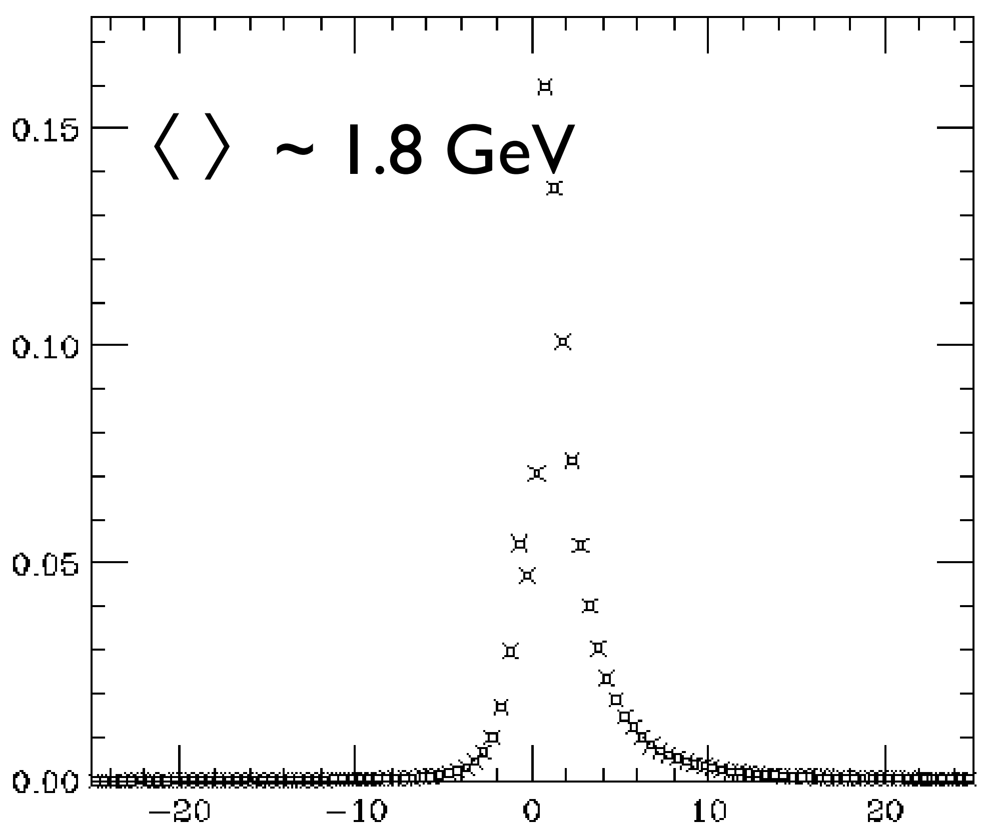


$\langle m_{top\text{-}odd} - m_{top\ shower}\ \text{GeV} \rangle$

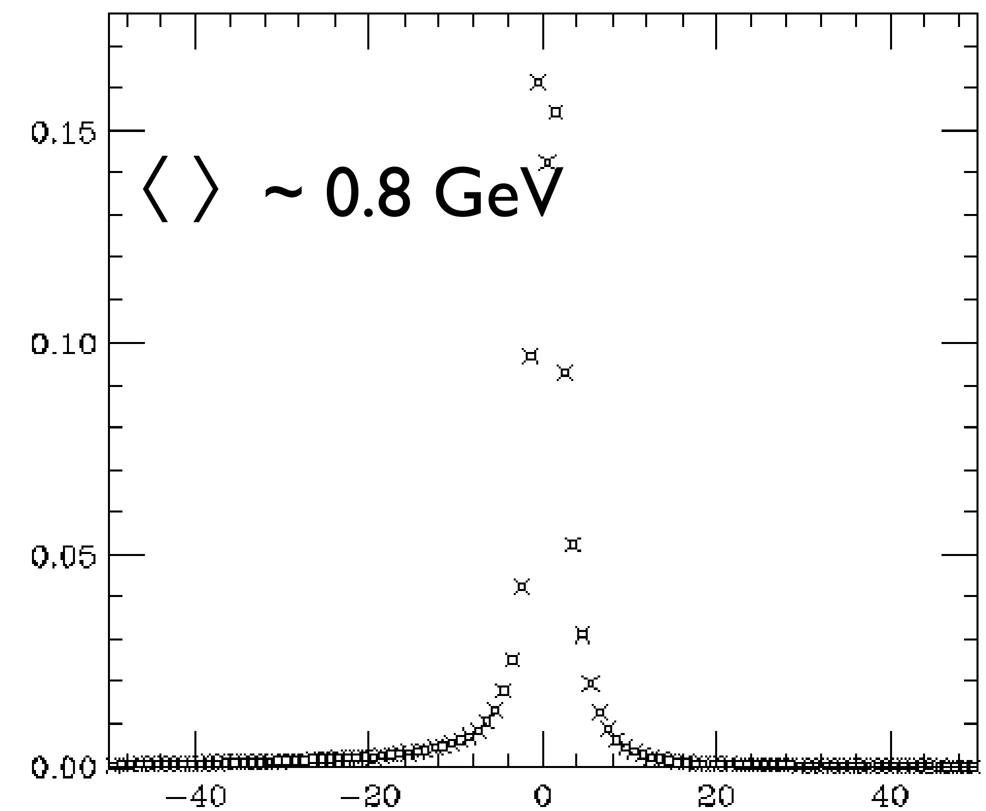


$\sqrt{S_{e+e-}}$=500 GeV

$\langle m_{top\text{-}odd} - m_{top}$ GeV$\rangle$

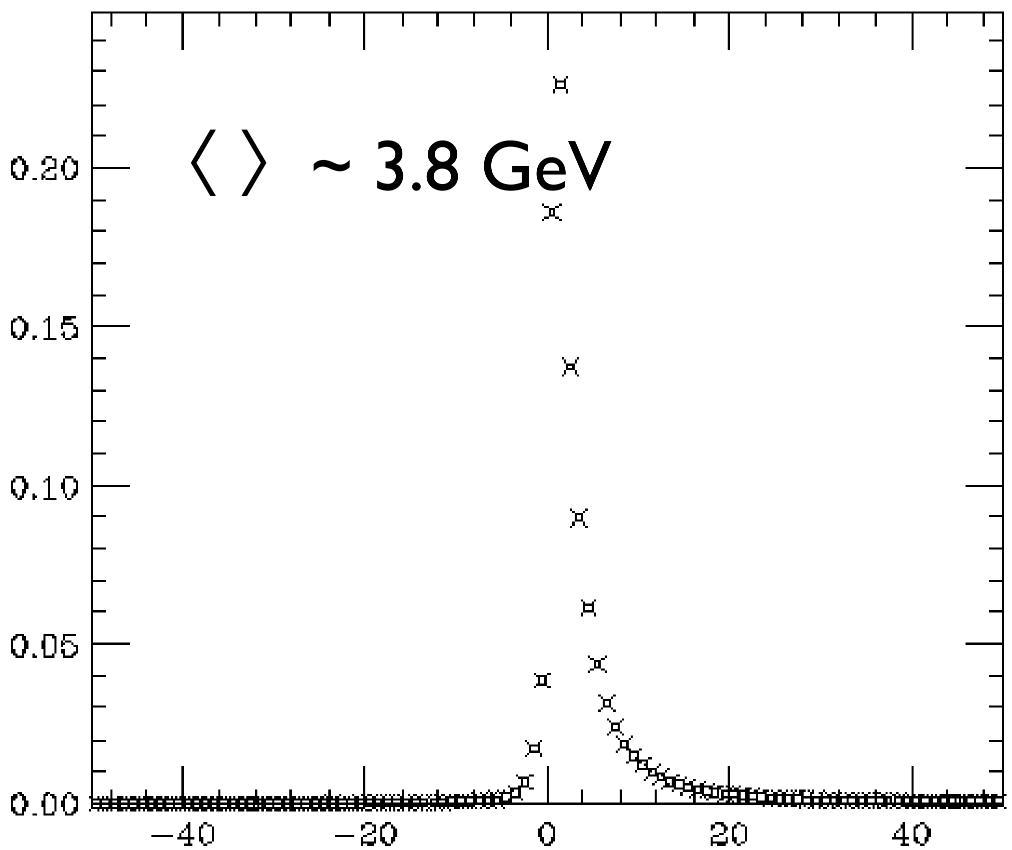


$\langle m_{top\text{-}odd} - m_{top\ shower}$ GeV$\rangle$

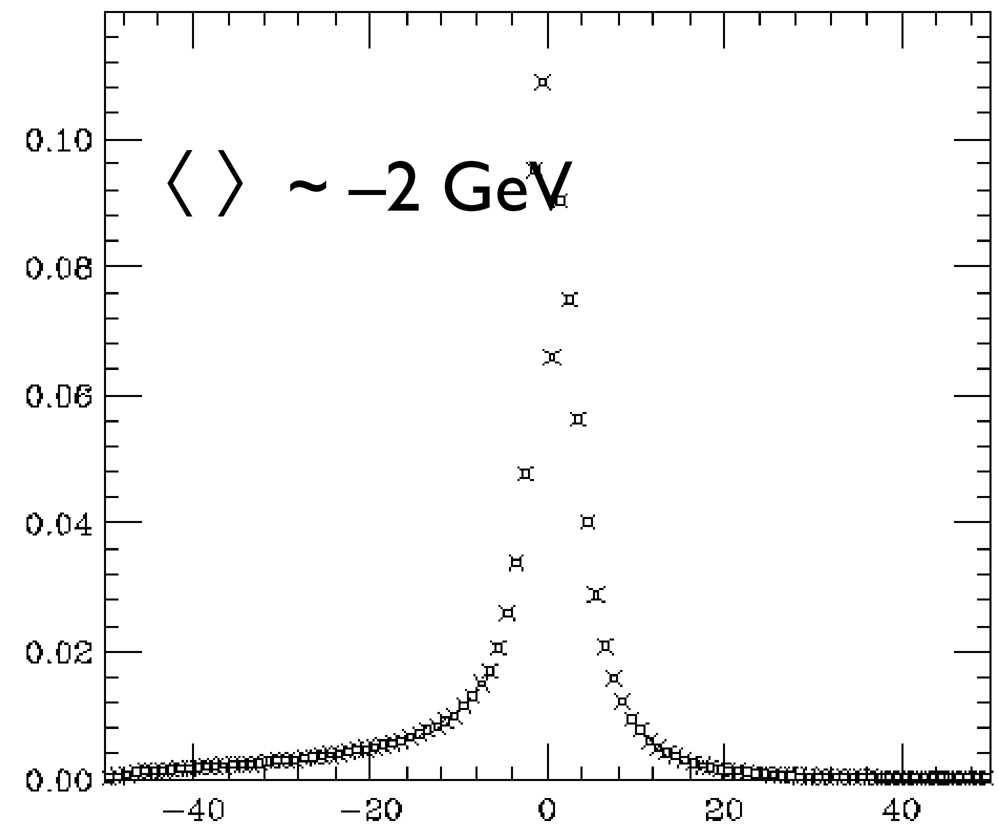


$\sqrt{S_{e+e-}}$=700 GeV

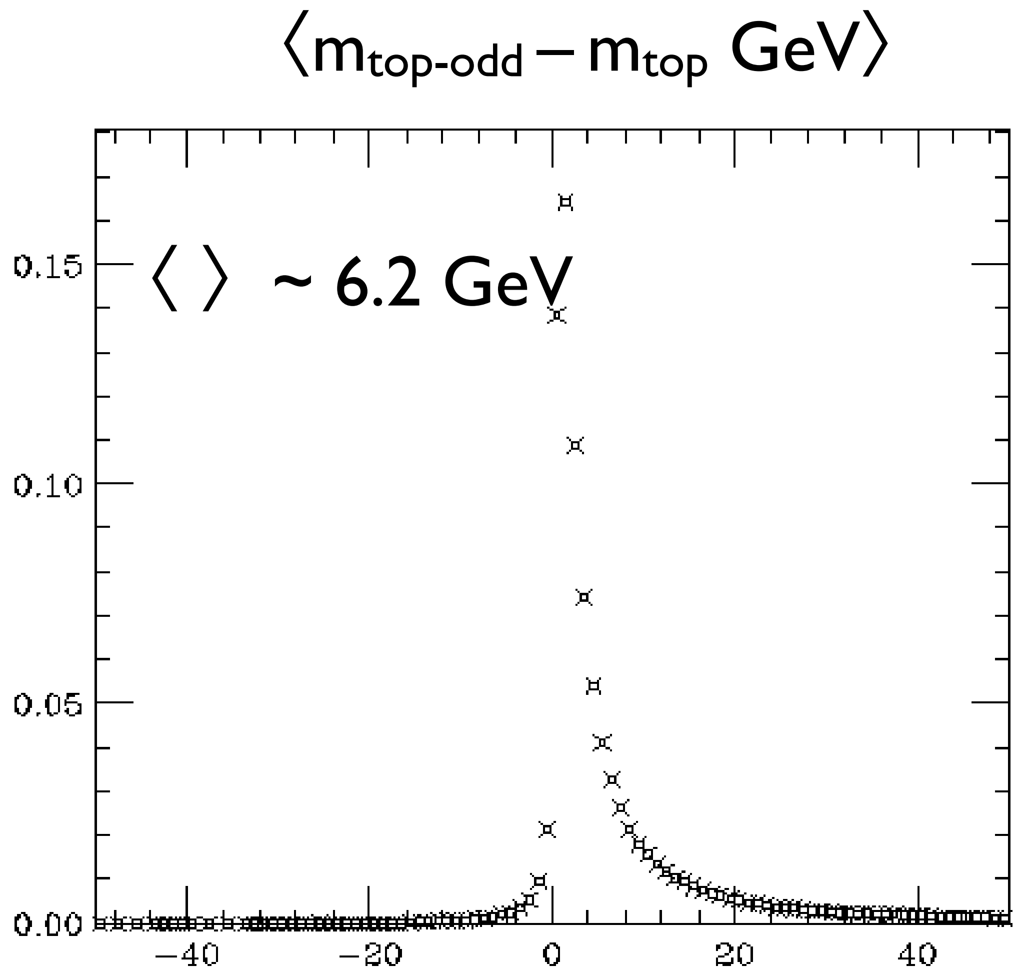
$\langle m_{top\text{-}odd} - m_{top}$ GeV$\rangle$

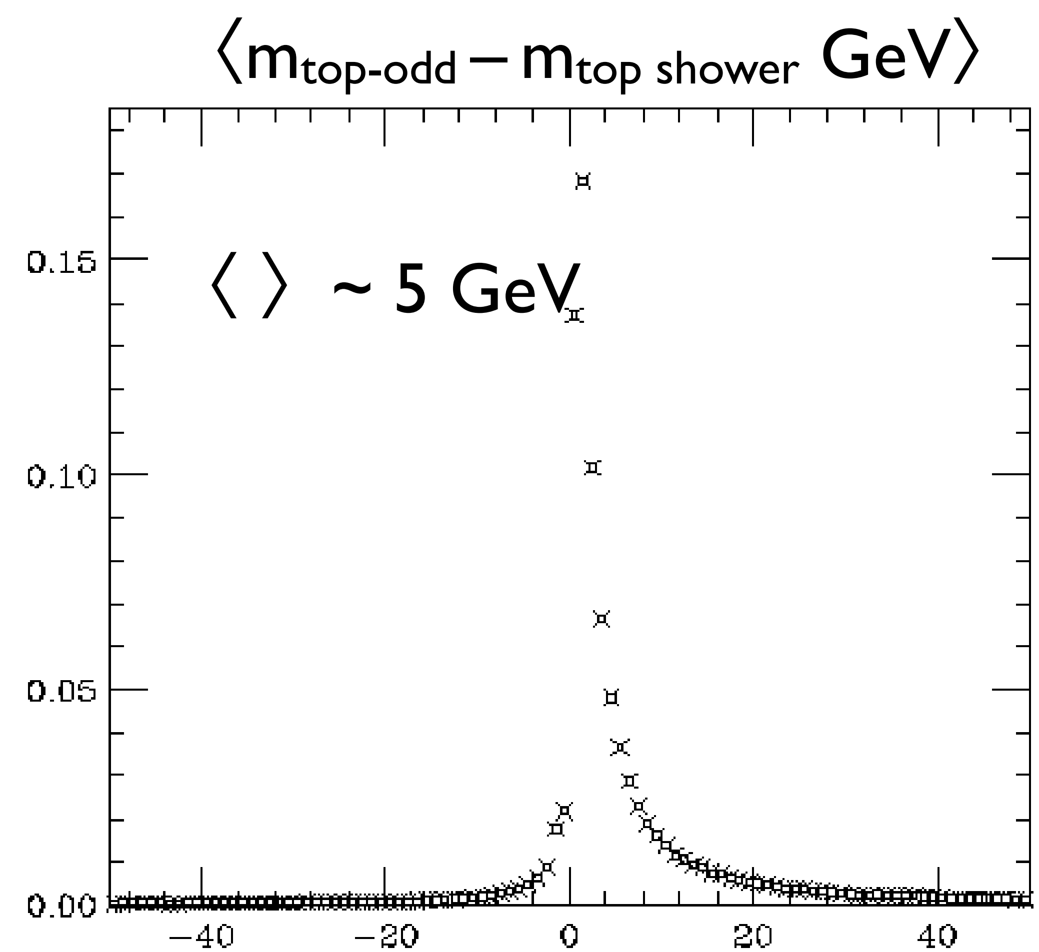


$\langle m_{top\text{-}odd} - m_{top\ shower}$ GeV$\rangle$



pp at 8 TeV


⟨mtop-odd – mtop GeV⟩
⟨ ⟩ ~ 6.2 GeV
0.15
0.10
0.05
0.00
−40
−20
0
20
40


⟨mtop-odd – mtop shower GeV⟩
⟨ ⟩ ~ 5 GeV
0.15
0.10
0.05
0.00
−40
−20
0
20
40

Conclusions

- To the level of 250-500 MeV, it is justified to consider m_{MC}=m_{pole}
- Dynamics "on the W side" extremely stable against all that happens on the b-side: try to exploit lepton endopoints, or other related observables
- Absolute effects of b-jet recombination in the few-GeV range, most of it controlled by perturbative effects, thus unaffected by NP uncertainties