ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

The top quark and the SM stability

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Top 2013

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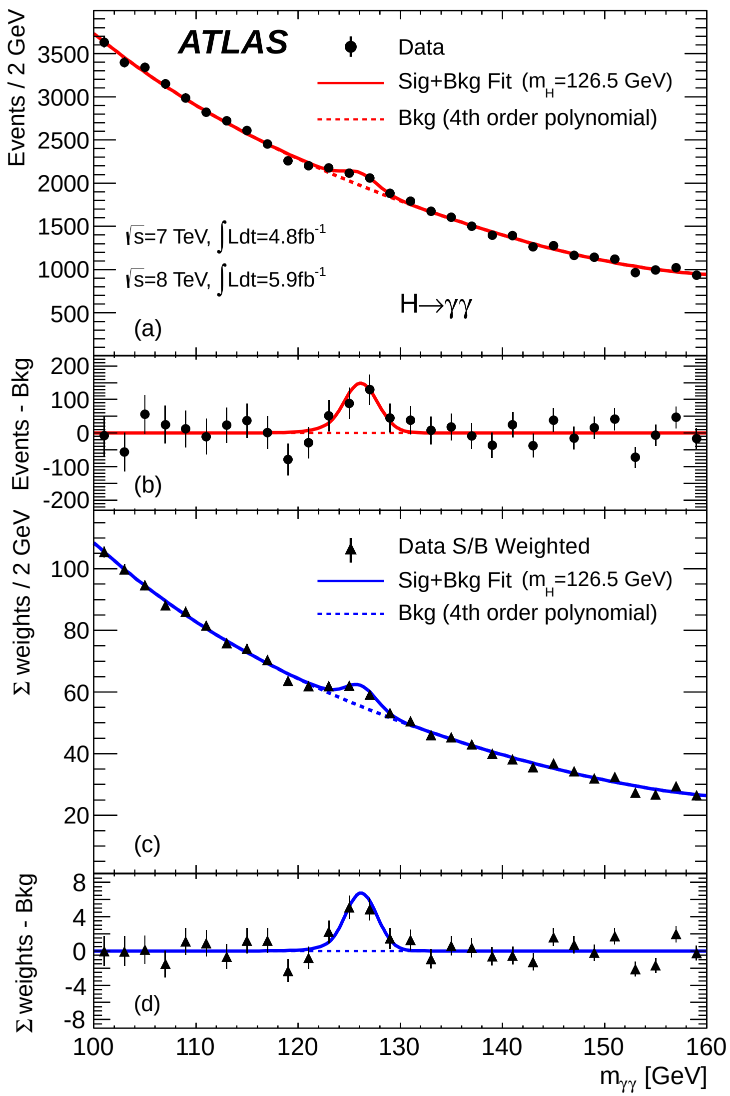
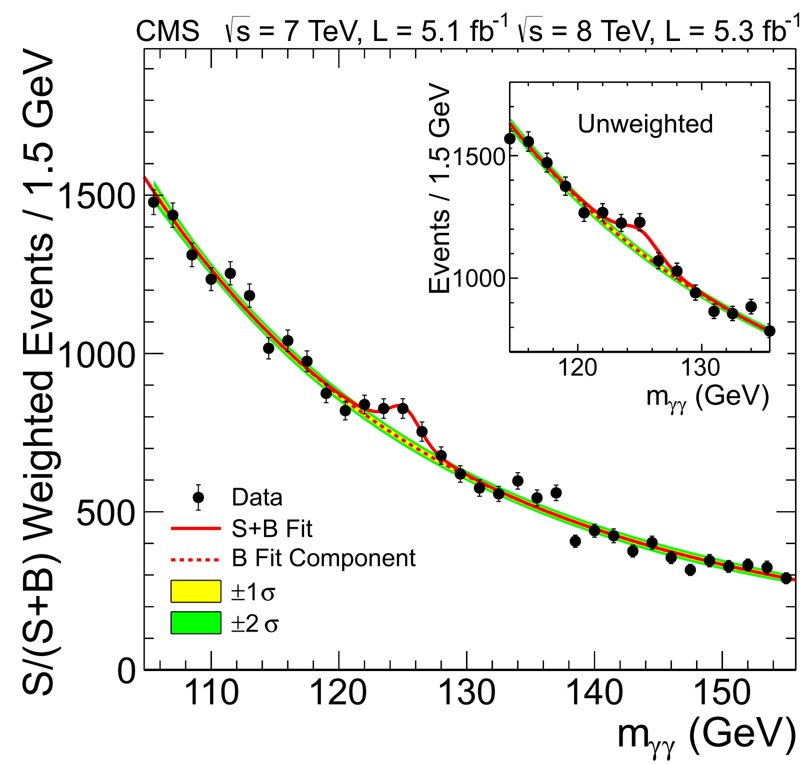
July 4, 2012, Higgs at ATLAS and CMS



CMS



Most recent data

According to CMS,

$$M_H = 125.7 \pm 0.4\,\mathrm{GeV},$$

According to ATLAS,

$$M_H = 125.5 \pm 0.6\,\mathrm{GeV}.$$

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The Standard Model is now a complete theory

The number of open questions remain. Perhaps, the most important one is:

Is there a new physics between electroweak and Planck scale ?

Yes

- proton decay yes (?)
- new physics at LHC yes (?)
- searches for DM WIMPS yes (?)
- searches for DM annihilation yes (?)
- searches for axions yes (?)

No

- proton decay no
- Higgs and nothing else at LHC
- searches for DM WIMPS no
- searches for DM annihilation no
- searches for axions no

The physics of top quark (and of the Higgs boson) plays an important role to answer this question

Outline

- Self-consistency of the Standard Model
- Top and Higgs: absolute stability bound
- Top and Higgs: asymptotically safe SM+gravity
- New physics between the Fermi and Planck scales?
- Top and Higgs: cosmological inflation
- Conclusions

Self-consistency of the SM

Within the SM the mass of the top quark and of the Higgs boson are the arbitrary parameters which can have any value.

Self-consistency requirements for the Higgs mass (assuming that the mass of the top is fixed at $M_t = 173.2$ GeV, and all other parameters to their experimental values)

$$m_{\rm meta} \simeq 111 \text{ GeV (metastability bound)}$$

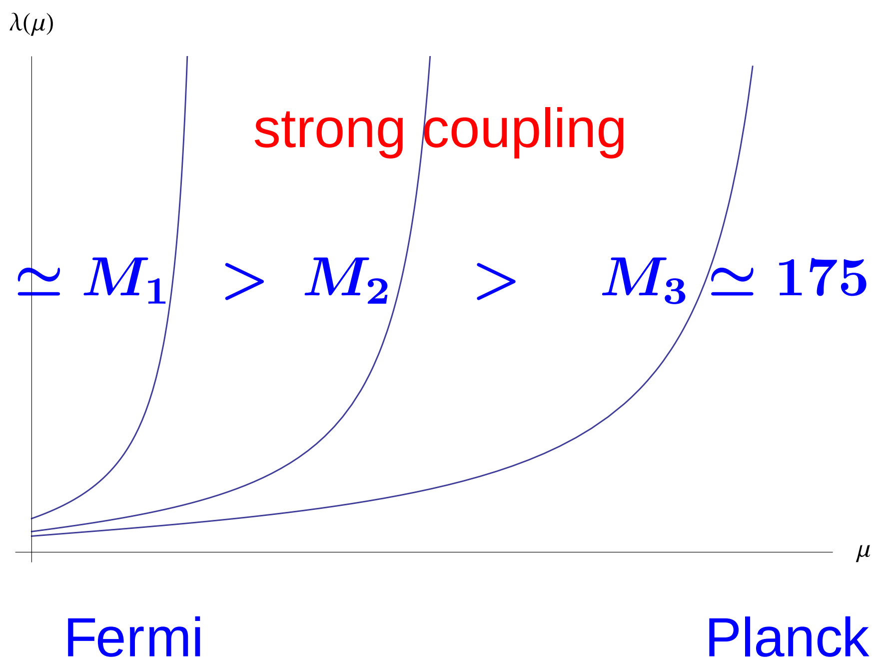
to

$$m_{\rm Landau} \simeq 1 \text{ TeV (triviality bound)}$$

Triviality bound

L. Maiani, G. Parisi and R. Petronzio ’77; Lindner ’85; T. Hambye and K. Riesselmann ’96;...

The Higgs boson self-coupling has a Landau pole at some energy determined by the Higgs mass. For $M_H \simeq m_{\mathbf{Landau}} \simeq 1$ TeV the position of this pole is close to the electroweak scale.



Higgs mass 1 TeV $\simeq M_1 \; > \; M_2 \; > \; M_3 \simeq 175$ GeV

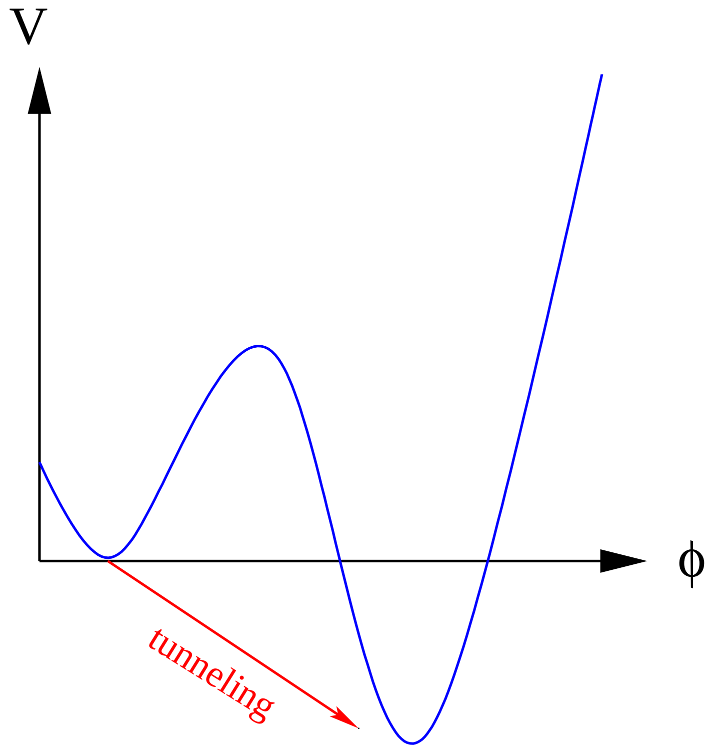
Triviality bound

If $m_H < m_{\mathrm{max}} \simeq 175$ GeV the Landau pole appears at energies higher than the Planck scale $E > M_P$.

LHC: The Standard Model is weakly coupled all the way up to the Planck scale

Metastability bound

Krasnikov '78, Hung '79; Politzer and Wolfram '79; Altarelli and Isidori '94; Casas, Espinosa and Quiros '94,'96;...; Ellis, Espinosa, Giudice, Hoecker, Riotto '09;...



The life-time of our vacuum is smaller than the age of the Universe if $m_H < m_{\mathbf{meta}}$, with $m_{\mathbf{meta}} \simeq 111$ GeV Espinosa, Giudice, Riotto '07

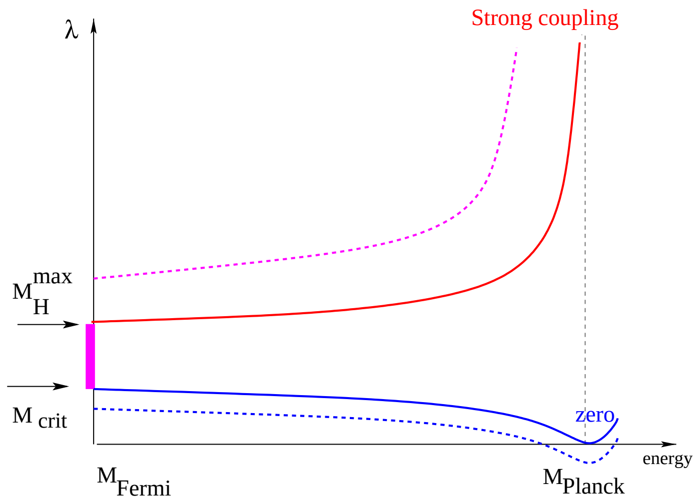
Metastability bound

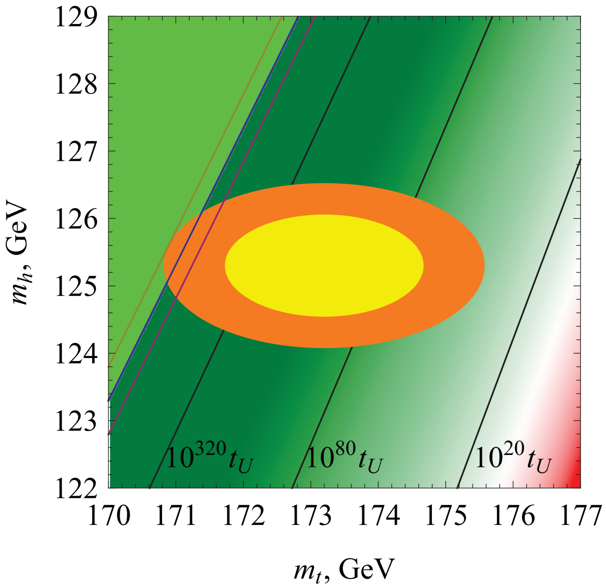
If the Higgs mass happened to be smaller than $m_{\mathbf{meta}} \simeq 111$ GeV, we would be forced to conclude that there must be some new physics beyond the SM, which stabilizes the SM vacuum.

However, already since LEP we know that $m_H > m_{\mathrm{meta}}$ so that new physics is not needed from this point of view.

The main LHC result: SM is a consistent effective theory all the way up to the Planck scale

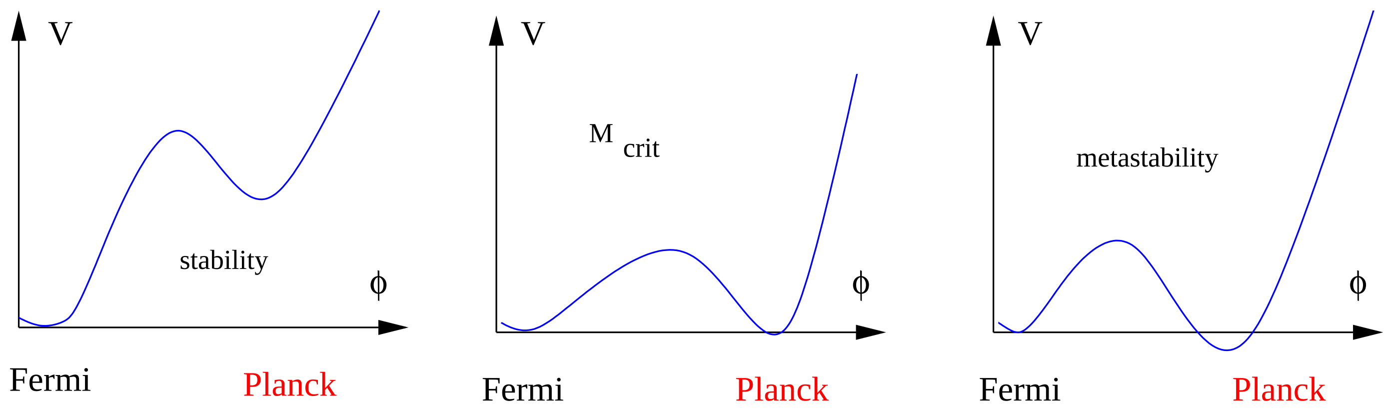
- No signs of new physics beyond the SM are seen
- $M_H < 175$ GeV : SM is a weakly coupled theory up to Planck energies
- $M_H > 111$ GeV: Our EW vacuum is stable or metastable with a lifetime greatly exceeding the Universe age.





Top and Higgs: absolute stability bound

At the same time, the combination of top-quark and Higgs boson masses is very close to the stability bound of the SM vacuum* (95'), to the Higgs inflation bound** (08'), and to asymptotic safety values for M_H and M_t *** (09'):



* Froggatt, Nielsen

** Bezrukov et al, De Simone et al

*** Wetterich, MS

Computation of absolute stability bound

- Choose the renormalisation scheme for the SM: $\overline{\mathrm{M}S}$
- Compute the effective potential $V(\phi)$ for the Standard Model in tree, one-loop, two-loop,... approximation. It will be a function of the scalar field and $\overline{\mathrm{M}S}$ parameters $\alpha_s(\mu),\ y_t(\mu),\ \lambda(\mu)$ etc.
- Find the relation between $\overline{\mathrm{M}S}$ parameters of the SM at low energy scale (e.g. $\mu = M_Z$ and experimentally measured quantities, such as masses of weak bosons, the Higgs and the pole top masses, etc in tree, one-loop, two-loop,... approximation.
- Make the renormalisation group improvement of the effective potential with the use of RG equations for the SM couplings in one-loop, two-loop, three-loop,... approximation.
- Find the parameters at which the effective potential has two degenerate minima:

$$V(\phi_{SM}) = V(\phi_1), \quad V'(\phi_{SM}) = V'(\phi_1) = 0,$$

Simplified procedure

Instead of computing effective potential, solve "criticality equations":

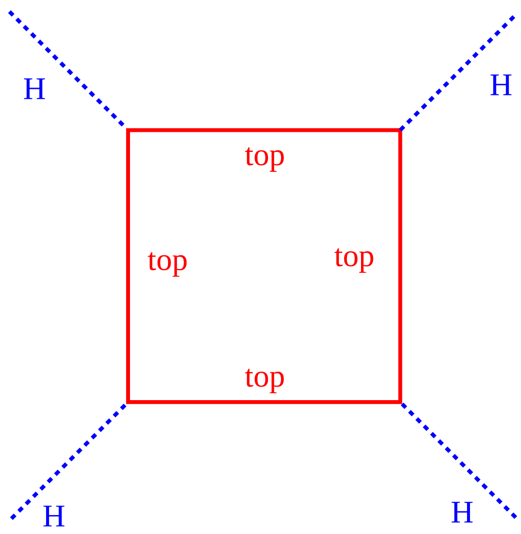
$$\lambda(\mu_0) = 0, \quad \beta_\lambda^{\mathrm{SM}}(\mu_0) = 0$$

The reason:

$$V(\phi) \propto \lambda(\phi)\phi^4 \left[1 + O\left(\frac{\alpha}{4\pi}\log(M_i/M_j)\right)\right],$$

where α is here the common name for the SM coupling constants, and M_i are the masses of different particles in the background of the Higgs field. If $O(\alpha)$ corrections are neglected two sets of equations are equivalent. Works with accuracy $\simeq 0.15\,\mathrm{GeV}$ for the masses of the Higgs and of the top.

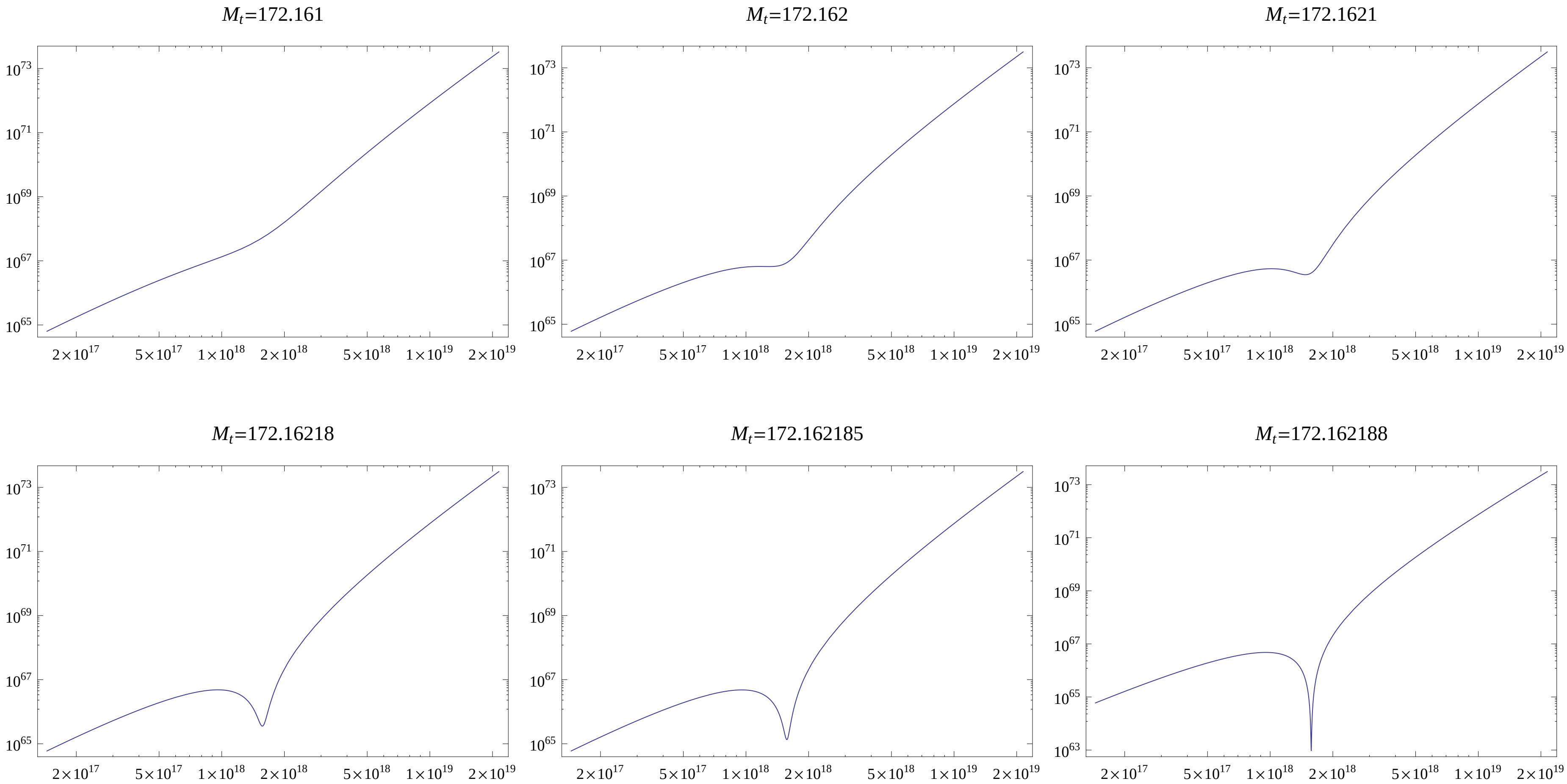
Top contribution



$$\delta V_t(\phi) \propto -y_t^4 \phi^4 \log(\phi^2/\mu^2)$$

Important - minus sign, t is a fermion!

Effective potential, $M_H = 125.7$ GeV



x axis: ϕ, GeV ; y axis : $V(\phi)$, GeV^4

Critical Higgs mass

Partial or complete two loop matching, three loop running:

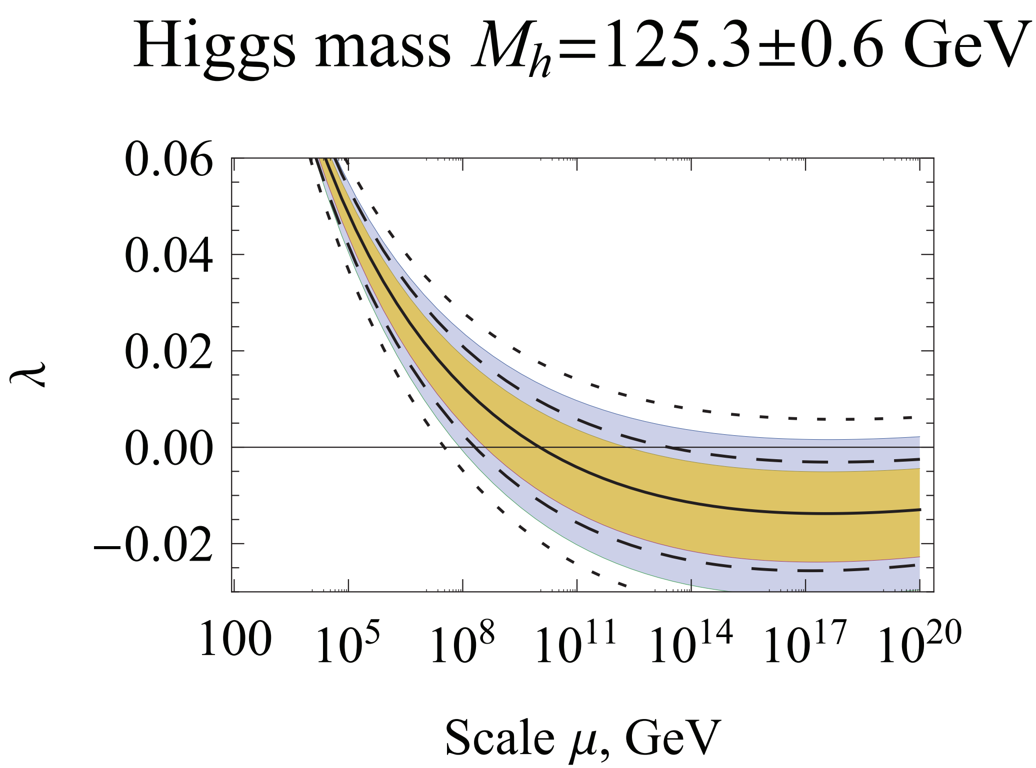
$$M_{crit} = [129.3 + \frac{y_t(M_t) - 0.9361}{0.0058} \times 2.0 - \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \times 0.5]\ \mathrm{GeV}$$

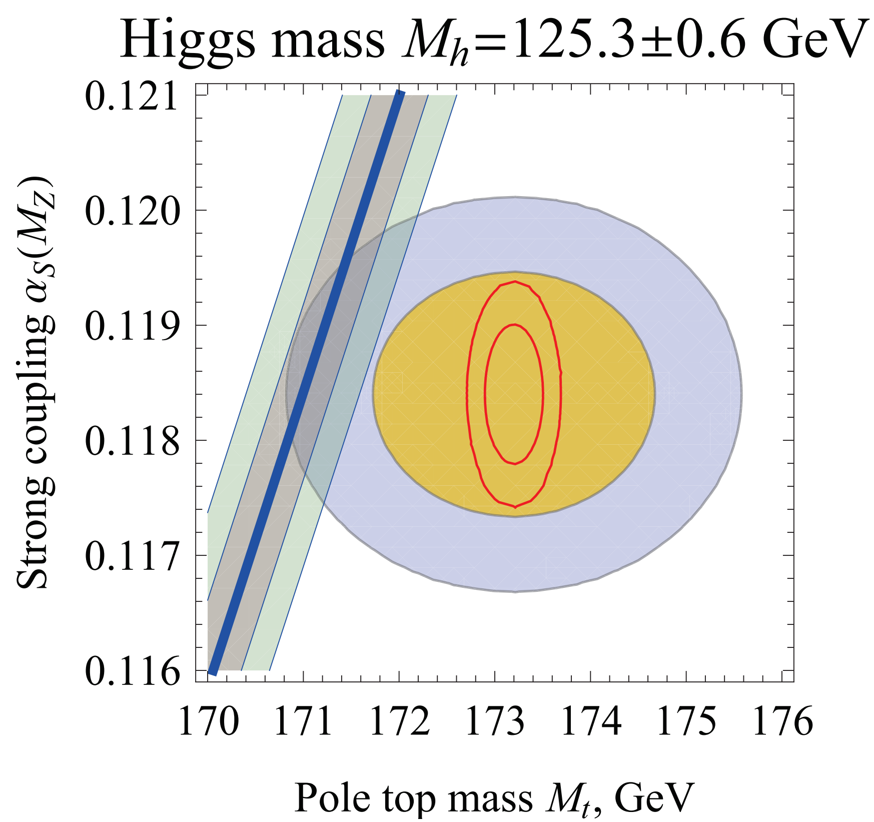
$y_t(M_t)$ - top Yukawa in $\overline{\mathbf{MS}}$ scheme

Matching at EW scale	Central value	theor. error
Bezrukov et al, $\mathcal{O}(\alpha\alpha_s)$	129.4 GeV	1.0 GeV
Degrassi et al, $\mathcal{O}(\alpha\alpha_s, y_t^2\alpha_s, \lambda^2, \lambda\alpha_s)$	129.6 GeV	0.7 GeV
Buttazzo et al, complete 2-loop	129.3 GeV	0.07 GeV

Chetyrkin et al, Mihaila et al, Bednyakov et al, 3 loop running to high energies

Comparison with experiment





errors in y_t: theory + experiment

Tevatron: $M_t = 173.2 \pm 0.51 \pm 0.71$ GeV

ATLAS and CMS: $M_t = 173.4 \pm 0.4 \pm 0.9$ GeV

$\alpha_s = 0.1184 \pm 0.0007$

We do not know whether our vacuum is stable or metastable!

Main uncertainty - top Yukawa coupling.

- Perturbation theory, $\mathcal{O}(\alpha_s^4)$. Estimate of Kataev and Kim: $\delta y_t/y_t \simeq -750(\alpha_s/\pi)^4 \simeq -0.0015$, $\delta M_{crit} \simeq -0.5$ GeV
- Non-perturbative QCD effects, $\delta M_t \simeq \pm\Lambda_{QCD} \simeq \pm 300$ MeV, $\delta M_{crit} \simeq \pm 0.6$ GeV
- 1 GeV experimental error in M_t leads to 2 GeV error in M_{crit}.
- Alekhin et al. Theoretically clean is the extraction of y_t from $t\bar{t}$ cross-section. However, the experimental errors in $p\bar{p} \to t\bar{t} + X$ are quite large, leading to $\delta M_t \simeq \pm 2.8$ GeV, $\delta M_{crit} \simeq \pm 5.6$ GeV.

Precision measurements of m_H, y_t and α_s are needed!

Is it a pure coincidence that the values of M_t and M_H are amazingly close to the critical values?

Or, this is a very important message about the structure of high energy theory?

Top and Higgs: asymptotically safe SM+gravity

Asymptotic safety = existence of non-Gaussian UV fixed point for gravity Weinberg '79. Though the theory is non-renormalizable, it is predictive and self-consistent.

To be true: all the couplings of the SM must be asymptotically safe or asymptotically free

Problem for:

- U(1) gauge coupling g_1, $\mu \frac{dg_1}{d\mu} = \beta_1^{\mathrm{SM}} = \frac{41}{96\pi^2} g_1^3$
- Scalar self-coupling λ, $\mu \frac{d\lambda}{d\mu} = \beta_\lambda^{\mathrm{SM}} =$

$$= \frac{1}{16\pi^2} \left[(24\lambda + 12h^2 - 9(g_2^2 + \frac{1}{3} g_1^2))\lambda - 6h^4 + \frac{9}{8} g_2^4 + \frac{3}{8} g_1^4 + \frac{3}{4} g_2^2 g_1^2 \right]$$

- Fermion Yukawa couplings, t-quark in particular h, $\mu \frac{dh}{d\mu} = \beta_h^{\mathrm{SM}} =$

$$= \frac{h}{16\pi^2} \left[\frac{9}{2} h^2 - 8g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \right]$$

Landau pole behaviour

Gravity contribution to RG running

Let x_j is a SM coupling. Gravity contribution to RG:

$$\mu \frac{dx_j}{d\mu} = \beta_j^{\mathrm{SM}} + \beta_j^{grav} \; .$$

On dimensional grounds

$$\beta_j^{grav} = \frac{a_j}{8\pi} \frac{\mu^2}{M_P^2(\mu)} x_j \; .$$

where

$$M_P^2(\mu) = M_P^2 + 2\xi_0 \mu^2 \; ,$$

with $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV, ξ_0 is some number

Asymptotic safety scenario for the SM model (MS, Wetterich) is realised when:

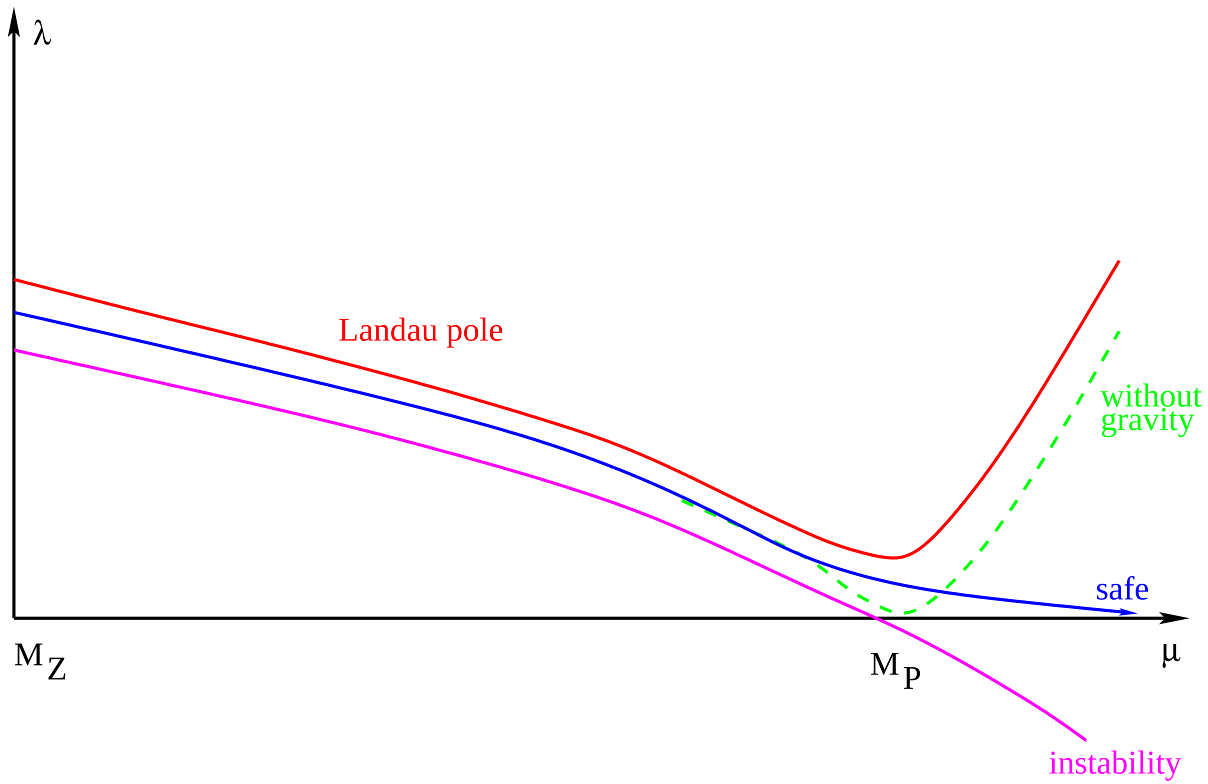
- Gravity contribution to gauge beta functions and that for the top Yukawa are negative, leading to asymptotic safety for U(1) and for y_t.
- If $a_\lambda > 0$ (according to Percacci and Narain '03) $\Longrightarrow$ Higgs mass prediction, $M_H = M_{crit}$
- If $a_\lambda < 0 \Longrightarrow M_{crit} < M_H < 175$ GeV

Computations of a_i: Robinson and Wilczek '05, Pietrykowski '06, Toms '07&'08, Ebert, Plefka and Rodigast '07, Narain and Percacci '09, Daum, Harst and Reuter '09, Zanusso et al '09, Folkerts, Litim and Pawlowski '11, Ellis, Mavromatos '12 ...

Suppose that indeed $a_1 < 0,\ a_h < 0,\ a_\lambda > 0$, what is found in a number of computations. Then the Higgs mass is predicted to be coming from solution of equation

$$\lambda(M_P) = 0$$

with uncertainty of few hundreds of MeV. Simultaneously, it is required that $\beta_\lambda(M_P) \ll 1$.

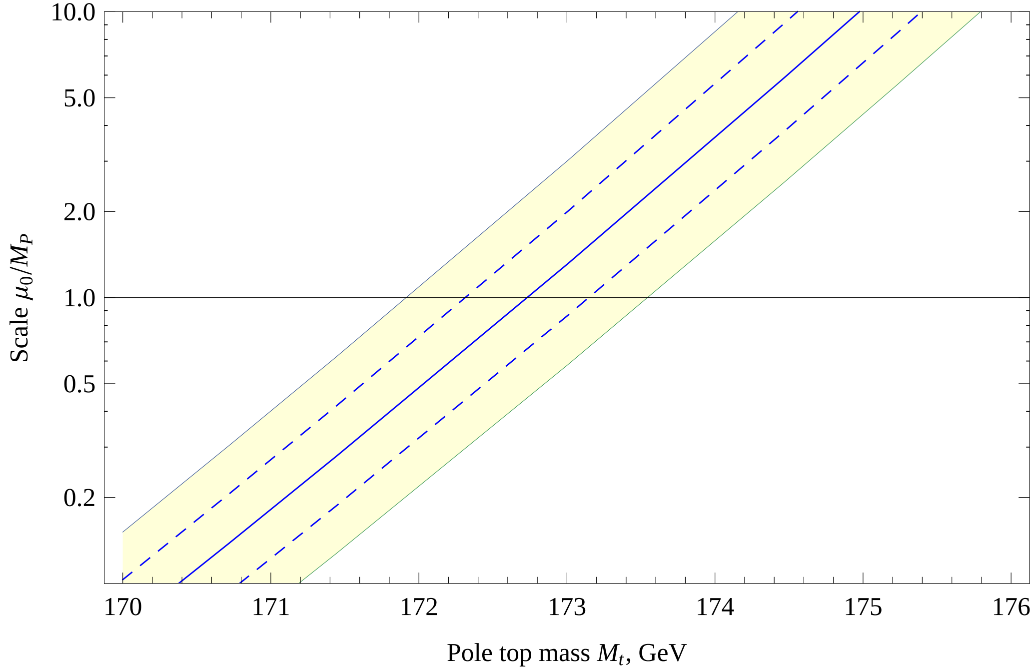


New Physics between the Fermi and Planck scales?

From two equations

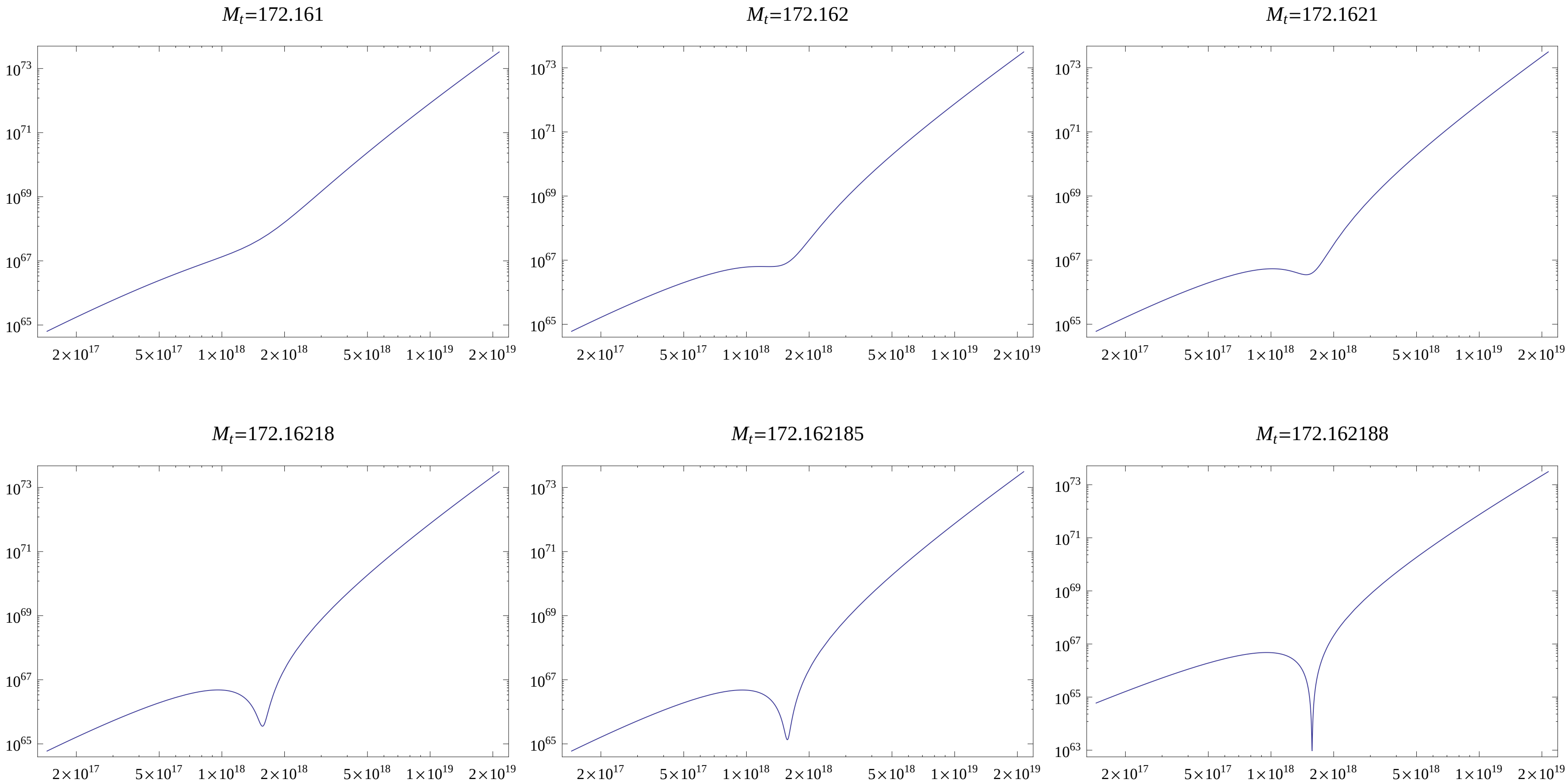
$$\lambda(\mu_0) = 0, \quad \beta_\lambda^{\mathrm{SM}}(\mu_0) = 0$$

one can determine not only the Higgs mass, but also the scale μ_0.



μ_0 determined by the EW physics gives the Planck scale, $\mu_0 \simeq M_P$!

Effective potential, $M_H = 125.7$ GeV



x axis: ϕ, GeV ; y axis : $V(\phi)$, GeV^4

Fermi scale is determined by the Planck scale (or vice versa)?

This relation is generically spoiled if new physics exists between the Fermi and Planck scales.

$$\Downarrow$$

Argument in favour of absence of new physics scales between Fermi and Planck.

Top and Higgs: cosmological inflation

Inflation: solution to a number of problems:

- Horizon: Why the universe is so uniform and isotropic?
- Structure formation : What is the origin of cosmological perturbations and why their spectrum is almost scale-invariant?
- Flatness : Why $\boldsymbol{\Omega_M + \Omega_\Lambda + \Omega_{\mathrm{rad}}}$ is so close to 1 now and was immensely close to 1 in the past?

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Yes: Higgs inflation

Bezrukov, MS

Higgs inflation

non-minimal coupling of Higgs field to gravity

$$S_G = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R - \frac{\xi h^2}{2} R \right\}$$

Jordan, Feynman, Brans, Dicke,...

Consider large Higgs fields h.

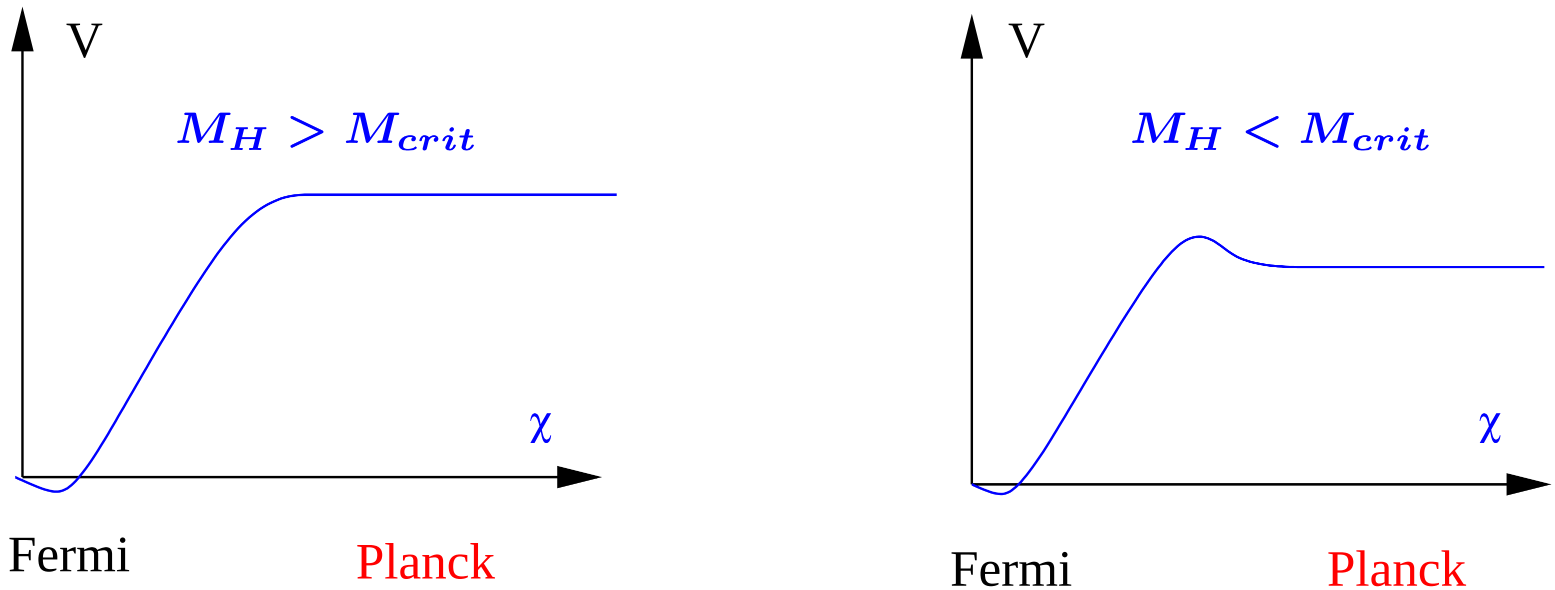
- Gravity strength: $M_P^{\text{eff}} = \sqrt{M_P^2 + \xi h^2} \propto h$
- All particle masses are $\propto h$

For $h > \frac{M_P}{\sqrt{\xi}}$ (classical) physics is the same (M_W / M_P^{eff} does not depend on h)!

Existence of effective flat direction, necessary for successful inflation.

Inflation and the Higgs mass

Radiative corrections to inflationary potential: Higgs inflation works only for $\lambda(M_P/\sqrt{\xi}) > 0$. Numerically, $M_H > M_{crit}$ with extra theoretical uncertainty of $\delta M_H \sim 1$ GeV.



χ - canonically normalized Higgs field in Einstein frame.

Analysis of higher dimensional operators and radiative corrections: Higgs inflation occurs in the weak coupling regime and is self-consistent. Bezrukov et al. The smaller the ξ the better $\Longrightarrow$ argument for $M_H \simeq M_{crit}$ rather than $M_H > M_{crit}$.

Inflaton potential and observations

If inflaton potential is known one can make predictions and compare them with observations.

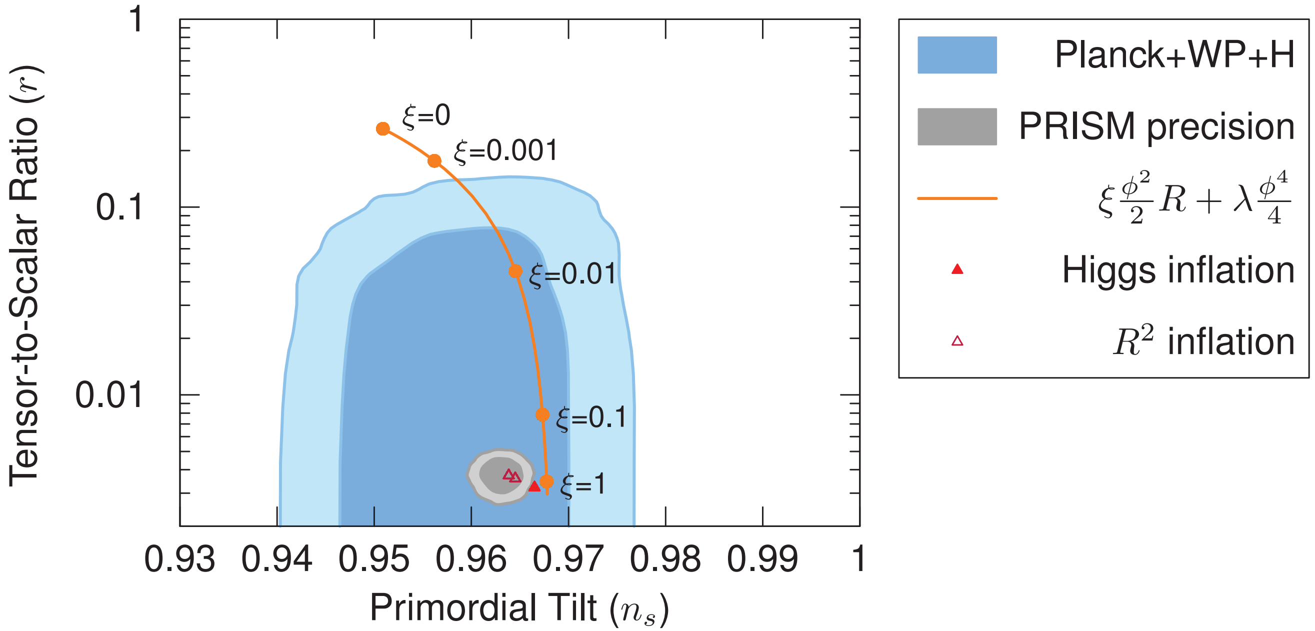
- $\delta T/T$ at the WMAP normalization scale ~ 500 Mpc.
- The value of spectral index n_s of scalar density perturbations

$$\left\langle \frac{\delta T(x)}{T}\frac{\delta T(y)}{T}\right\rangle \propto \int \frac{d^3k}{k^3} e^{ik(x-y)} k^{n_s-1}$$

- The amplitude of tensor perturbations $r = \frac{\delta\rho_s}{\delta\rho_t}$

These numbers can be extracted from WMAP observations of cosmic microwave background. Higgs inflation: one new parameter, $\xi \Longrightarrow$ two predictions. From WMAP normalization $\xi \sim 700$.

CMB parameters—spectrum and tensor modes



- Higgs inflation: $T_{reh} \sim 10^{13-14}$ GeV, $N \simeq 58$

Perturbations are Gaussian, in accordance with Planck.

Conclusions

- LHC experiments provide a strong evidence that the SM is a self-consistent effective theory all the way up to the Planck scale.
- The case of $M_H = M_{crit}$ is very peculiar: if this is indeed the case, this is an indication for the absence of new energy scales between the Fermi and Planck scales
- The relation $M_H = M_{crit}$ may come from asymptotic safety scenario for the SM and from the Higgs inflation
- To have a decisive statement, we should know:
  - Higgs mass with highest possible precision (LHC, $\mathbf{200}$ MeV?)
  - Top Yukawa coupling with accuracy $\mathbf{5 \times 10^{-4}}$ ($\delta M_t \simeq \mathbf{100}$ MeV) (future e^+e^- collider? LHC with new theory input?)
  - α_s with uncertainty $\delta\alpha_s \simeq \mathbf{2 \times 10^{-4}}$