Introduction to Monte Carlo Event Generation
Lecture 5: Advanced Topics

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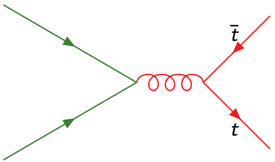
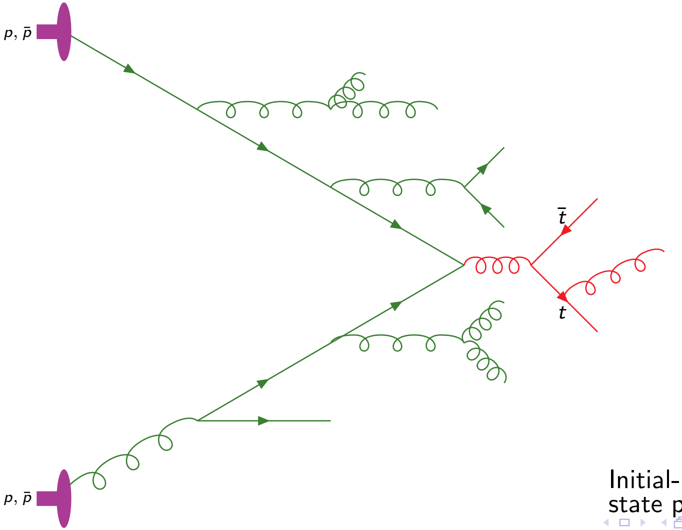
A Monte Carlo Event



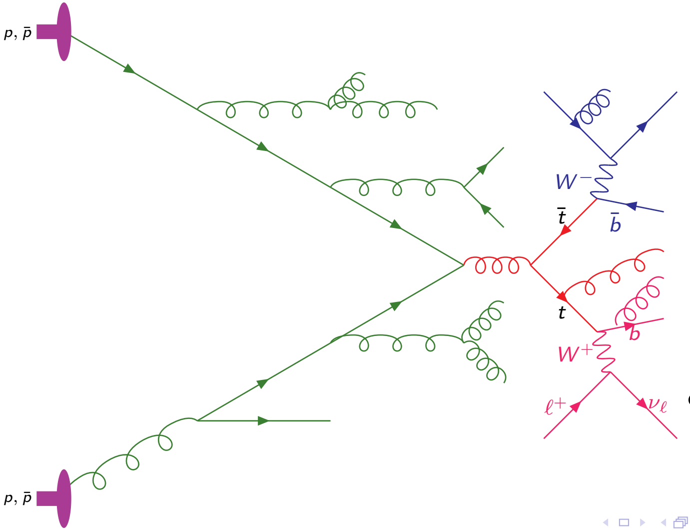
Hard Process, usually calculated at leading order

A Monte Carlo Event



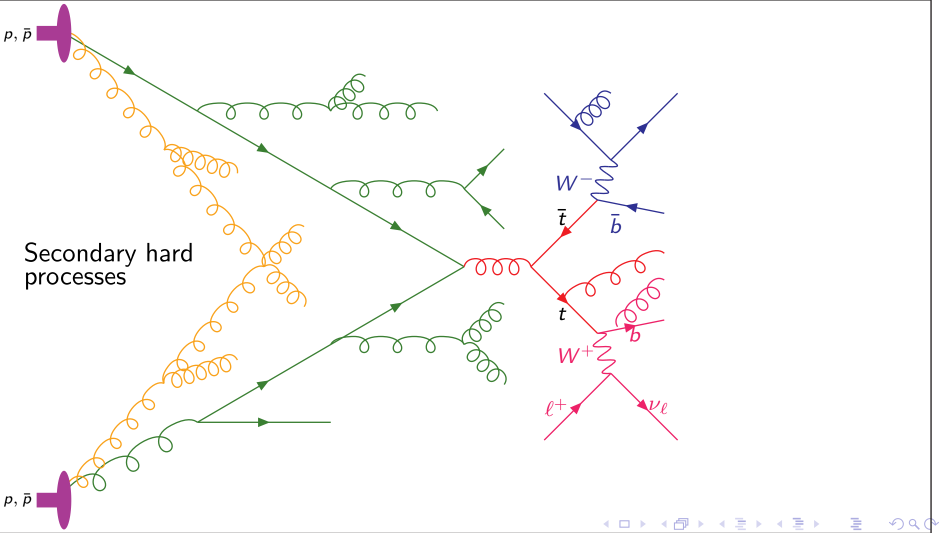
Initial- and final-state parton shower

A Monte Carlo Event

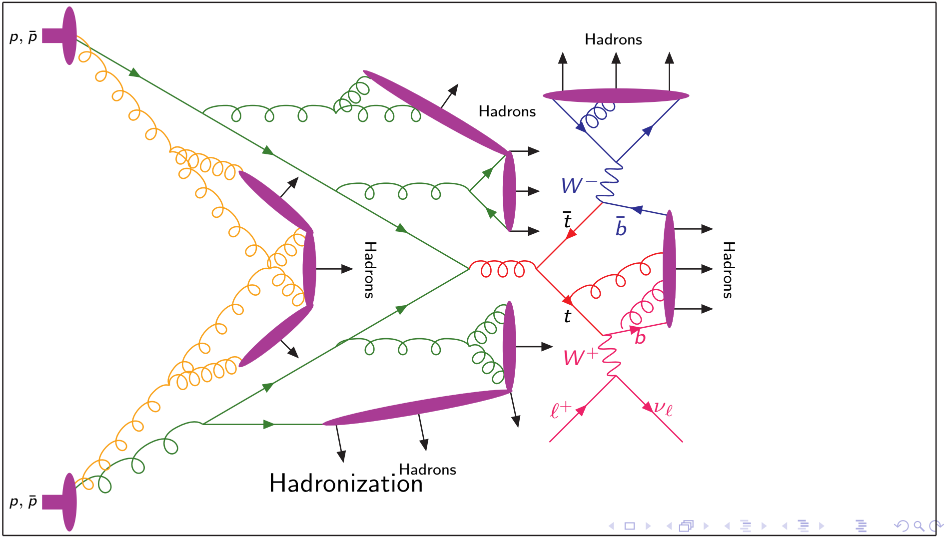


Perturbative decays of heavy particles

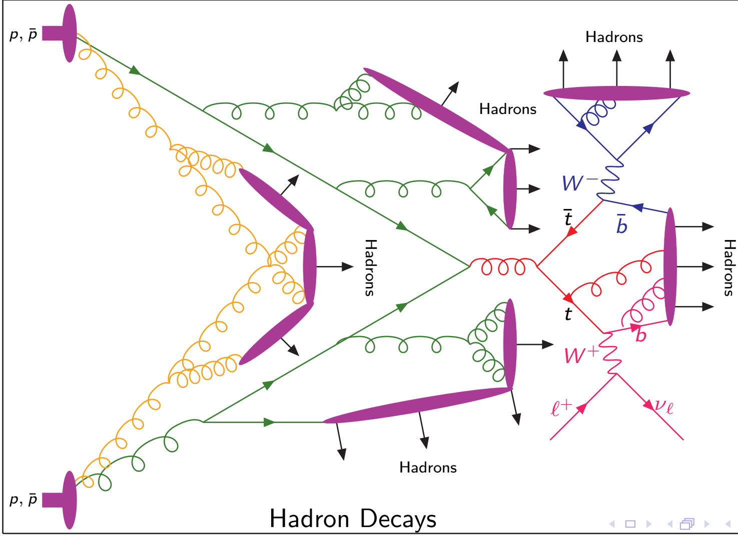
A Monte Carlo Event



A Monte Carlo Event



A Monte Carlo Event



Hadron Decays

- Analytic Jet Properties
- Monte Carlo Tuning
- BSM Simulation

Jet Properties

- Over the last four lectures we've looked at the theoretical basis of Monte Carlo event generators and the approximations and models involved.
- Usually we are interested in the numerical output of the simulation.
- However the sane ideas can be used to produce analytic results and approximations which are often useful.

Jet Properties

- Due to the steeply, $\sim \frac{1}{p_T^4}$, spectrum its vital we understand anything that can effect the transverse momentum of a jet.
- There are three main effects which we need to consider:
  1. the jet losing energy due to wide angle radiation;
  2. the jet losing energy due to producing hadrons at large angles which aren't included in the jet;
  3. the jet gaining energy by including particles from the underlying event.
- The first two are normally combined and called out-of-cone effects while the second is called underlying event.

Jet Properties

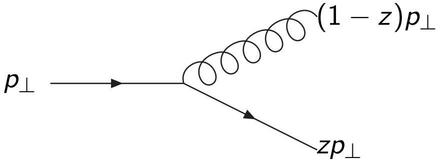
- Historically the data was corrected for these effects so it could be compared with fixed-order simulations.
- However Monte Carlo simulations include these effects so this data was useless for improving these simulations.
- Now correct the data for detector effects, but then apply the out-of-cone and underlying-event corrections to the theoretical prediction rather than the data.

Jet $p_\perp$

- In general the computation of jet properties in hadron–hadron collisions is extremely complicated, however for some quantities we can get estimates of various effects.
- The simplest of these is to estimate the change in the $p_\perp$ between a parton and the jet it forms.

Jet $p_\perp$: Perturbative correction

- Start by considering the change due to perturbative QCD radiation.
- A quark with transverse momentum $p_\perp$ radiates a gluon such that the quark carries a fraction z of its original momentum and the gluon a fraction $1 - z$



Jet $p_\perp$: Perturbative correction

- After the radiation the centre of the jet will be the parton with the highest transverse momentum after the branching, *i.e.*
  - the quark if $z > 1 - z$ or
  - the gluon if $z < 1 - z$.
- If the other parton is at an angular distance greater $\theta > R$ it's longer in the jet and the jet will have a smaller transverse momentum

$$\begin{aligned} \delta p_\perp &= (1-z)p_\perp - p_\perp &&= -zp_\perp && 1-z > z \\ \delta p_\perp &= zp_\perp - p_\perp &&= -(1-z)p_\perp && z > 1-z \end{aligned}$$

than the original parton.

Jet $p_\perp$: Perturbative correction

- We can use the splitting probabilities we derived earlier to compute the average transverse momentum loss

$$
\begin{aligned}
\langle p_\perp \rangle_q &= -\frac{C_F \alpha_S}{2\pi} p_\perp \int_{R^2}^{1} \frac{\mathrm{d}\theta^2}{\theta^2} \int_0^1 \mathrm{d}z \frac{1+z^2}{1-z} \min\{1-z, z\}, \\
&= -\frac{C_F \alpha_S}{2\pi} p_\perp \ln\left(\frac{1}{R^2}\right) \left[\int_0^{\frac{1}{2}} \frac{1+z^2}{1-z} z + \int_{\frac{1}{2}}^{1} \frac{1+z^2}{1-z} 1 - z\right], \\
&= -\frac{C_F \alpha_S}{\pi} p_\perp \ln\left(\frac{1}{R}\right) \left[2\ln 2 - \frac{3}{8}\right].
\end{aligned}
$$

- The loss of transverse momentum can be calculated for gluon jets in the same way using the gluon splitting functions giving

$$
\langle p_\perp \rangle_g = -\frac{\alpha_S}{\pi} p_\perp \ln\left(\frac{1}{R}\right) \left[C_A \left(2\ln 2 - \frac{43}{96}\right) + T_R n_f \frac{7}{48}\right].
$$

Jet $p_\perp$: Perturbative correction

- These calculations give

$$\frac{\langle p_\perp \rangle_q}{p_\perp} = -0.43\alpha_S \ln\frac{1}{R}, \qquad \frac{\langle p_\perp \rangle_g}{p_\perp} = -1.02\alpha_S \ln\frac{1}{R}.$$

- For a jet with $R = 0.4$ quark and gluon jets will have 5% and 11% less transverse momentum than the parent parton, respectively.
- Subject to significant finite R and higher order corrections.
- The result also depends on the precise details of the recombination scheme, for example SISCONE has a different recombination scheme where the centre of the cone is the direction of the sum of the partons and we require one parton to fall outside the cone.

Jet $p_\perp$: Hadronization correction

- The hadronization and underlying event corrections cannot be calculated from first principles.
- However we can use some simple models to gauge the size of the effects.
- One model for the effect of hadronization on event shapes in e^+e^- collisions, due to Dokshitzer and Webber, is to perform a perturbative calculation.
- Instead of stopping the calculation at some small energy scale μ_I because the strong coupling becomes non-perturbative continue the calculation into the infrared regime with a model of the strong coupling in this regime which does not diverge.

Jet $p_\perp$: Hadronization correction

- They define

$$\mathcal{A}(\mu_I) = \frac{1}{\pi}\int_0^{\mu_I} \mathrm{d}k_\perp \alpha_S(k_\perp).$$

- This model can also be used to assess the size of the hadronization corrections for the jet transverse momentum.
- The hadronization is modelled by soft gluons with $k_\perp \sim \Lambda_{\mathrm{QCD}}$.
- In this case the transverse momentum loss is

$$\delta p_\perp = z p_\perp - p_\perp = -(1-z)p_\perp.$$

- As before the transverse momentum loss is

$$\langle p_\perp \rangle_q \quad = \quad -\frac{C_F}{2\pi} p_\perp \int \frac{\mathrm{d}\theta^2}{\theta^2} \int \mathrm{d}z \alpha_S \frac{1+z^2}{1-z}(1-z).$$

Jet $p_\perp$: Hadronization correction

- As we are dealing with soft gluons $z \sim 1$ so $1 + z^2 \simeq 2$.
- In this case we will not use a fixed value of α_S but need to evaluate it at the scale of the transverse momentum of the gluon with respect to the quark $k_\perp = p_\perp(1-z)\theta$. We also transform the integration variables to use $k_\perp$ and θ giving

$$\langle p_\perp \rangle_q = -\frac{2C_F}{\pi}\int_R^1 \frac{\mathrm{d}\theta}{\theta^2}\int_0^{\mu_I} \mathrm{d}k_\perp \alpha_S(k_\perp) = -\frac{2C_F\mathcal{A}}{R}.$$

- Using the coefficients from fits to the e^+e^- thrust distribution

$$\langle \delta p_\perp \rangle_q \sim -\frac{0.5\,\mathrm{GeV}}{R}, \qquad \langle \delta p_\perp \rangle_g \sim -\frac{1\,\mathrm{GeV}}{R}.$$

- The hadronization correction has a $\frac{1}{R}$ dependence on the size of the jet, unlike the $\ln\frac{1}{R}$ dependence of the perturbative radiation.

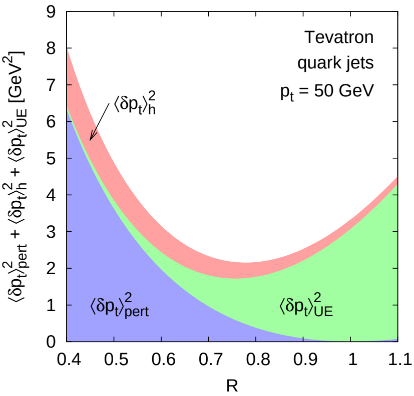
Jet $p_\perp$: Underlying event correction

- We can estimate the underlying event contribution by assuming there is $\Lambda_{\rm UE}$ energy per unit rapidity due to soft particles from the underlying event giving a correction to the transverse momentum of

$$\langle \delta p_\perp \rangle = \Lambda_{\rm UE} \int_{\eta^2+\phi^2<R^2} {\rm d}\eta \frac{{\rm d}\phi}{2\pi} = \Lambda_{\rm UE} \frac{R^2}{2}.$$

- This is a useful estimate although strictly the area of the jet is only πR^2 for the anti-k_T algorithm.

Jet $p_\perp$: Corrections



An example of the various contributions to the shift between the partonic and jet transverse taken from arXiv:0906.1833

Monte Carlo Tuning

- In principle we should be able to use the techniques from the statistics lectures to get the best fit MC parameters and their errors.
- However there are a myriad of problems:
  - impossible to run the event generators in a minimisation loop;
  - the best fit isn't statistically all the good;
  - theoretical biases and unphysical for many parameters;
  - should a parton shower approach be able to fit anything better than $\sim 5\%$?

Monte Carlo Tuning

- In principle we want to minimise the chi squared.
- Suppose we have a set of Monte Carlo parameters $\{\mathbf{p}\}$

$$\chi^2 = \sum_{\mathcal{O}} w_{\mathcal{O}} \sum_{b\in\mathcal{O}} \frac{\left(f^b(\mathbf{p}) - \mathcal{R}_b\right)^2}{\Delta_b^2},$$

  where $\mathcal{O}$ is the observable distribution, $f^b(\mathbf{p})$ the MC prediction for a given bin b, $\mathcal{R}_b$ the experimental measurement and Δ_b the error.
- A weight $w_{\mathcal{O}}$ is used to enhance the contribution of particular observables.

Rivet

- Nowadays use Rivet for pretty much all comparisons of data and MC event generators.
- Allows us to calculate the Monte Carlo prediction to compare with a vast range of experimental analyses.
- However can take a long time to generate the answer for all the analyses we need with a given set of Monte Carlo parameters.
- Can't use it in a χ^2 minimisation

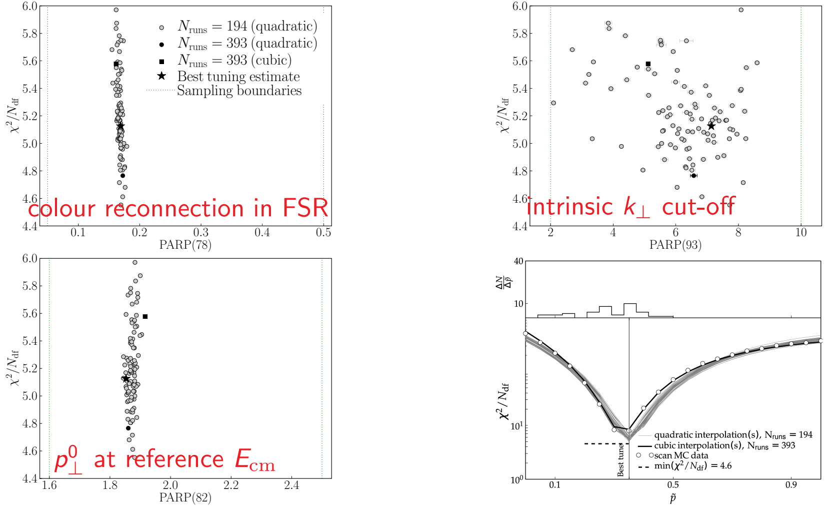
Professor

- Instead of running the event generator inside the χ^2 minimisation loop run a number of randomly selected points inside the parameter space.
- Professor can then interpolate between the anchor points to calculate the generator predictions for any parameter set inside the sampled parameter space.
- The use Minuit to minimise the interpolated generator response.

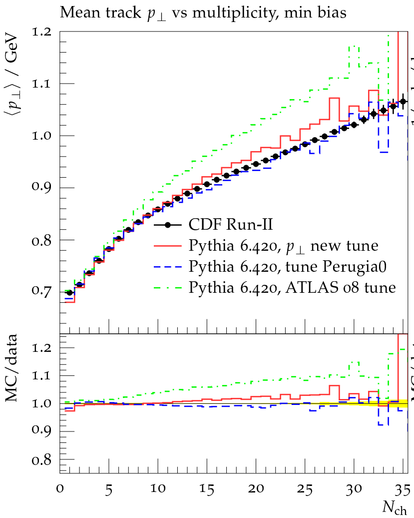
MC Tuning

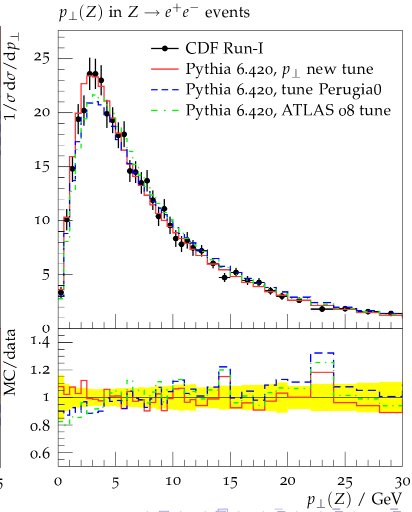
- Can be a bit of an art getting a reliable generator response.
- Often some parameters don't effect the result or want to be outside the physical region and have to be excluded.
- Given the χ^2 isn't really a χ^2 very hard to estimate uncertainties.
- Clear the standard $\Delta\chi^2 = 1$ is not a good measure, again a matter of opinion what is a sensible measure.

Monte Carlo Tuning

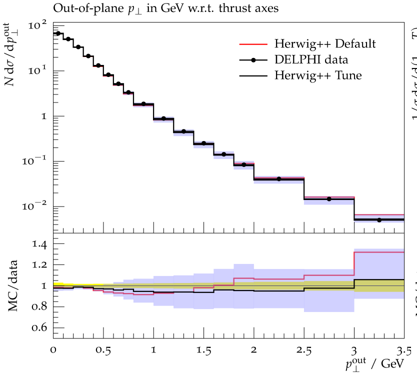


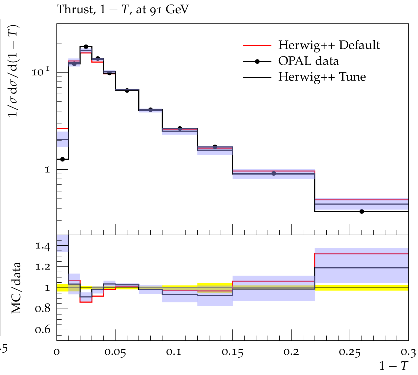
Monte Carlo Tuning



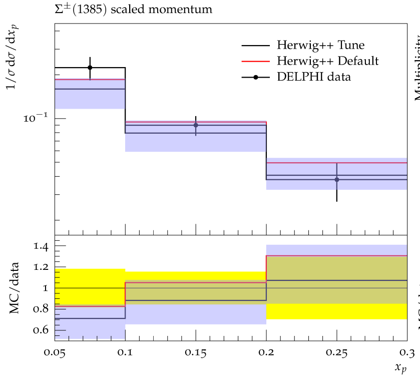


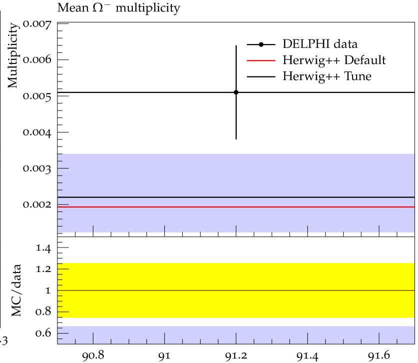
Uncertainties





Uncertainties





BSM Simulation

- As I emphasised in the first lecture the key thing for the simulation of BSM physics.
- However there are some interesting differences between simulating SM AND BSM models.
- Two general types of BSM model:
  1. Few, if any new particles, mainly changes SM predictions;
  2. Large numbers of new particles.
- Simulating the first is a lot like simulating the SM, *e.g.* $Z^0 \to Z'$
- Simulating the second can be more complicated.
- As can simulating models with new colour structures.

Simulating long decay chains

- In models such as SUSY the strongly interacting new particles would be produced followed by the decay into the lighter particles and eventually the LSP, *e.g.*

$$gg \to \tilde{q}_L \tilde{q}_L^* \quad \text{followed by} \quad \tilde{q}_L \to \mathrm{q}\tilde{\chi}_2^0 \to \mathrm{q}\tilde{\ell}_R^{\pm}\ell^{\mp} \to \mathrm{q}\ell^+\ell^-\tilde{\chi}_1^0$$

- The correlations due to the spins of the decaying particles can give important effects.

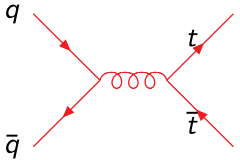
Spin Correlations

- Could calculate the production and decays as a $2 \to n$ process. This is difficult if:
  - We want to generate QCD radiation from the particles before they decay;
  - The particle has many decay modes;
  - There are many sequential decays;
  - There are many final state particles.
- For a Monte Carlo event generator we need an algorithm which allows
  - The production and decay to be generated separately,
  - Has a complexity which only grows linearly with the number of final-state particles.

Spin Correlations

- Rather than presenting the full algorithm in detail let's consider the example of $q\bar{q} \to t\bar{t}$ followed by semi-leptonic decay of the top quarks.
- First we generate the momenta of the top quarks in $q\bar{q} \to t\bar{t}$ using the polarized cross-section

$$\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda_1\ldots\lambda_n}$$



$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa'_1\kappa'_2;\lambda_t\lambda_{\bar{t}}}$$

Spin Correlations

- One of the outgoing quarks is selected at random and a spin density matrix calculated.

$$
\begin{aligned}
\rho_{\lambda_j\lambda'_j} &= \frac{1}{N_\rho}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\prod_{i\neq j} D^i_{\lambda_i\lambda'_i} \\
&\quad \mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_j\ldots\lambda'_n}
\end{aligned}
$$

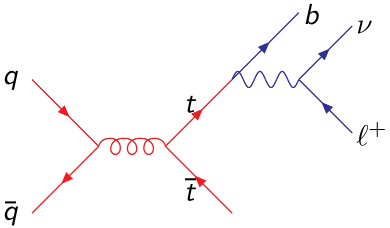
  In the initial step we take $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.
  The normalization is chosen so that $\mathrm{Tr}\rho = 1$.

- This normalization is therefore the same as the matrix element used in the previous step.
- The decay mode of the particle is selected using the branching ratios.

Spin Correlations

- The momenta of the particles produced in the n-body decay are generated using

$$\rho_{\lambda_0\lambda'_0}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda_1\ldots\lambda_n}$$



$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*q\bar{q}\to t\bar{t}}_{\kappa'_1\kappa'_2;\lambda'_t\lambda_{\bar{t}}} \mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda'_t}$$

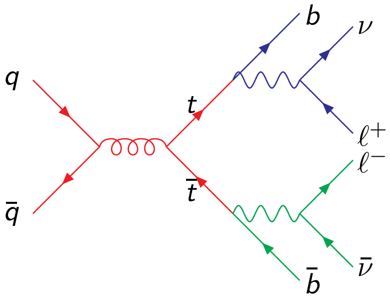
Spin Correlations

- In principle we could now continue down the decay chain, but here all the particles produced are stable.
- When all the particles in a decay have been developed a decay matrix is calculated

$$D_{\lambda_0\lambda'_0} = \frac{1}{N_D}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda'_1\ldots\lambda'_n}\prod_{i=1,n} D^i_{\lambda_i\lambda'_i}$$

  The normalization is chosen so that $\mathrm{Tr}D = 1$.
- Select another particle from the hard process. The whole procedure is repeated for this particle using the calculated decay matrices for those particles which have been decayed rather than the identity.
- In this case this means we decay the second quark.
- As before the normalization of the spin density matrix cancels the matrix element used for the previous step.

Spin Correlations



$$
\begin{aligned}
|\mathcal{M}|^2 = &\ \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*q\bar{q}\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t'\lambda_{\bar{t}}'} \\
&\ \mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda_t'}\mathcal{M}^{\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}}\mathcal{M}^{*\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}'}
\end{aligned}
$$

- This is the narrow width matrix element for the process.

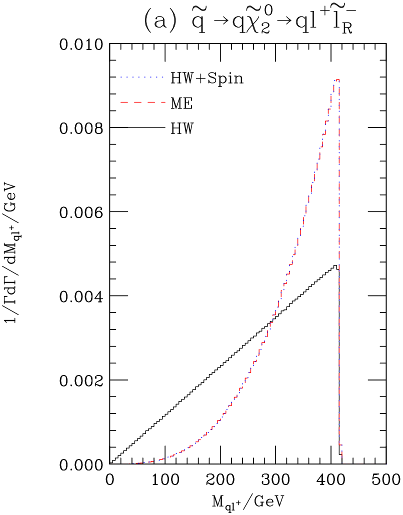
Squark Decay

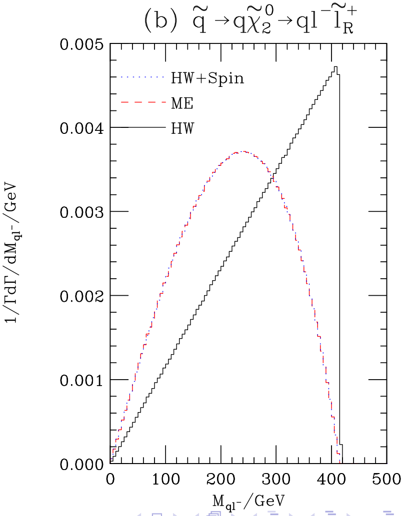
- In many models the decay chain

$$\tilde{q}_L \to q\tilde{\chi}^0_2 \to q\tilde{\ell}^{\pm}_R\ell^{\mp} \to q\ell^+\ell^-\tilde{\chi}^0_1$$

  is important as by measuring edges in the $\ell^+\ell^-$, $q\ell$ and $q\ell^+\ell^-$ distributions the masses of $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$, $\tilde{\ell}_R$ and $\tilde{q}$ can be measured.
- Here the spin correlation algorithm must calculate the spin density matrix for $\tilde{q}_L \to q\tilde{\chi}^0_2$ and then use this to perform the decay $\tilde{\chi}^0_2 \to \tilde{\ell}^{\pm}_R\ell^{\mp}$.
- The effects are most visible in the $q\ell$ mass distribution.

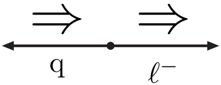
Squark Decay



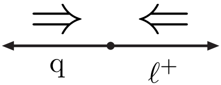


Squark Decay

- The quark will always be left-handed and the lepton right-handed



spin 1



spin 0

- As we can't experimentally distinguish quarks and antiquarks the average is observed.
- However some hopes of measurement as more squarks than antiquarks produced at the LHC.

Weird QCD

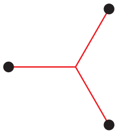
- In the SM:
  - baryon number is conserved;
  - only have particles in the 3, $\bar{3}$ and 8 representations of $SU(3)$.
- In BSM models
  - baryon number need not be conserved;
  - there can be particles in the 6, $\bar{6}$, 10, ... representations of $SU(3)$.

Hadronization

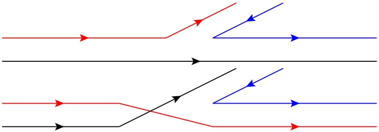
- In some models there are coloured particles whose lifetime exceeds the hadronization timescale$\sim \frac{1}{\Lambda_{\rm QCD}}$.
- Not a problem if the particles are in the fundamental representation of $SU(3)$.
  - Hadrons and hadronization should be the same as for the Standard Model.
  - Use a constituent quark model to estimate the masses of the "mesons" ($\tilde{t}\bar{q}$).
- If in the adjoint representation more problems
  - "Glueballs" ($\tilde{g}g$) can also be formed, no understanding of the production mechanism or relative rate of "glueballs" and "mesons" ($q\tilde{g}\bar{q}$)
  - Models for the masses of the hadrons.
- In all cases if they decay use a spectator model partonic decay.
- If they are long lived the interactions in the detector must also be modelled.

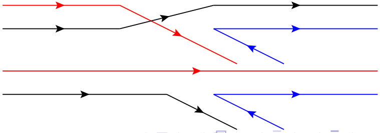
Weird Colours

- If baryon number is violated three colours connected to a colour source or anticolours to a sink.



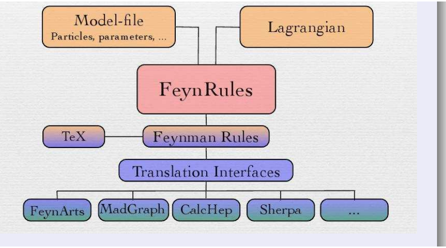
- Have to randomly select the colour baryon for perturbative radiation
- Either collapse to quarks to a diquark or have a junction in the hadronization.
- High representations just need to treat them as a number of fundamental lines





FeynRules

- Portable, transparent & reproducible implementation of (almost arbitrary) new physics models.
- In most programs new models are defined by the new particles, their properties and interactions.



- Output to the standard ME generators (MADGRAPH, SHERPA, WHIZZARD)
- Various models already implemented & validated for a list: http://feynrules.phys.ucl.ac.be
- More recently intermediate UFO stage to allow general communication between Feynman rule and ME calculation

Summary

- The hard process is well understood and calculable in fixed-order perturbative QCD.
- We have fairly good understanding of the parton shower, based on perturbative QCD with approximations
- Hadronization is less understood. It is modelled but the data constrains the models and the extrapolation to the LHC is reliable.
- We understand the underlying event the least. Modelled but many features pf the models are only weakly constrained by data and the extrapolation is often ad-hoc.
- Always ask "What physics is dominating my effect?"

Torbjorn Sjostrand's Final Words of Warning

Final Words of Warning

[. . .] The Monte Carlo simulation has become the major means of visualization of not only detector performance but also of physics phenomena. So far so good. But it often happens that the physics simulations provided by the Monte Carlo generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data.
[. . .] I am prepared to believe that the computer-literate generation (of which I am a little too old to be a member) is in principle no less competent and in fact benefits relative to us in the older generation by having these marvelous tools. They do allow one to look at, indeed visualize, the problems in new ways. But I also fear a kind of "terminal illness", perhaps traceable to the influence of television at an early age. There the way one learns is simply to passively stare into a screen and wait for the truth to be delivered. A number of physicists nowadays seem to do just this.

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