

Introduction to Monte Carlo Event Generation

Lecture 2: Parton Showers

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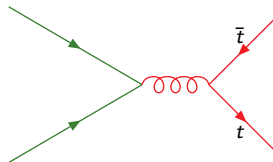
IPPP Durham

MCnet School: 5th August

Introduction

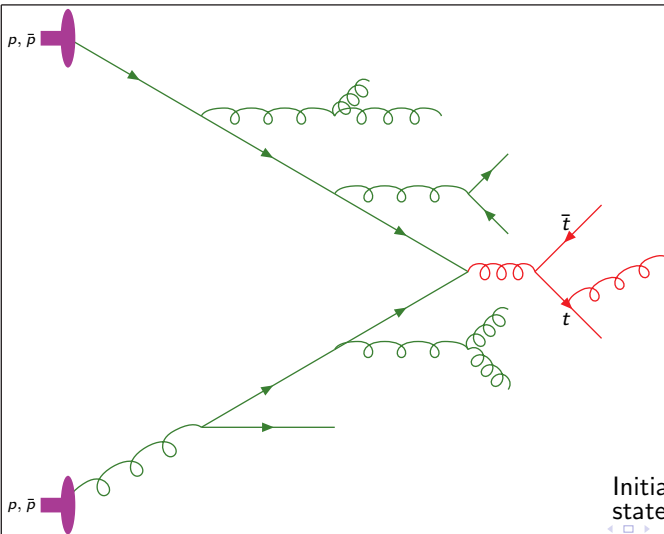
- In classical and quantum electrodynamics **accelerated charges radiate**.
- Similarly in QCD accelerated colour charges radiate.
- This gives a cascade of quarks and gluons, the **parton shower**

A Monte Carlo Event

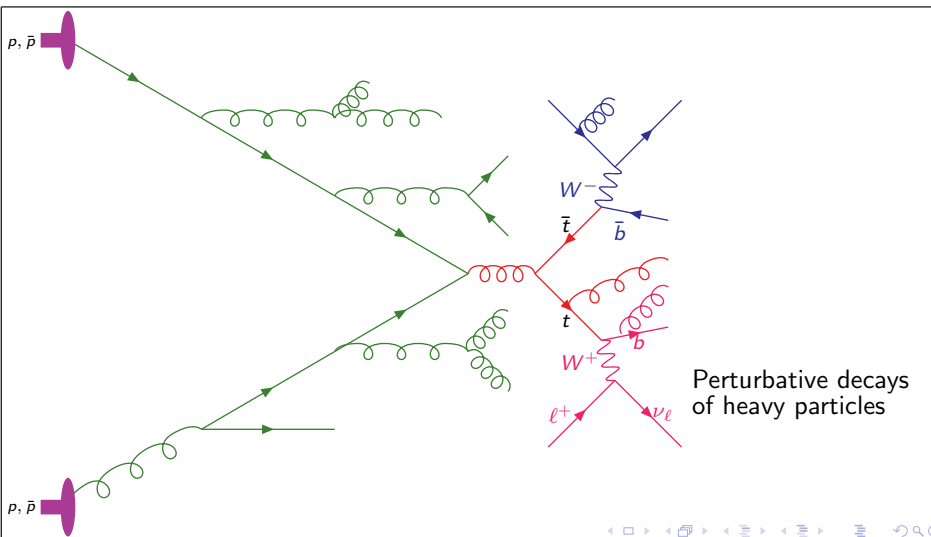


Hard Process, usually
calculated at leading order

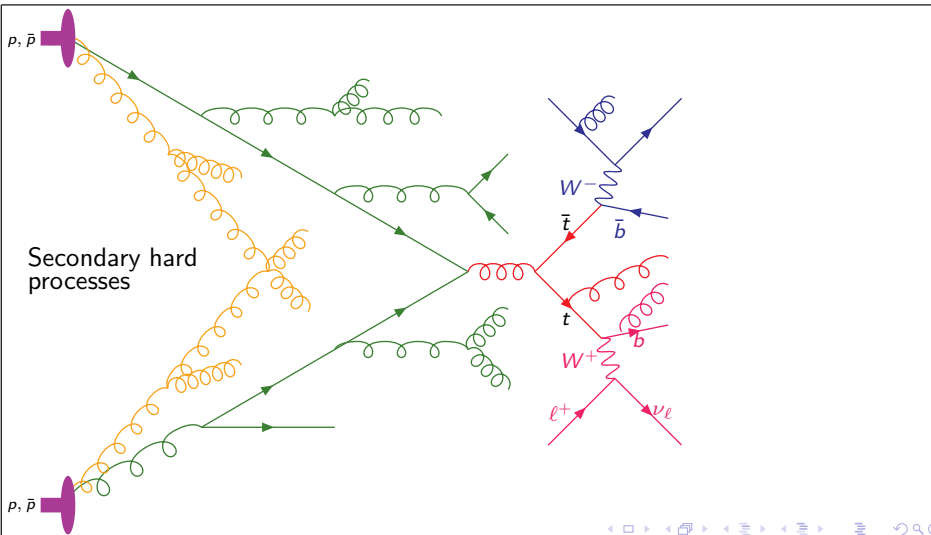
A Monte Carlo Event



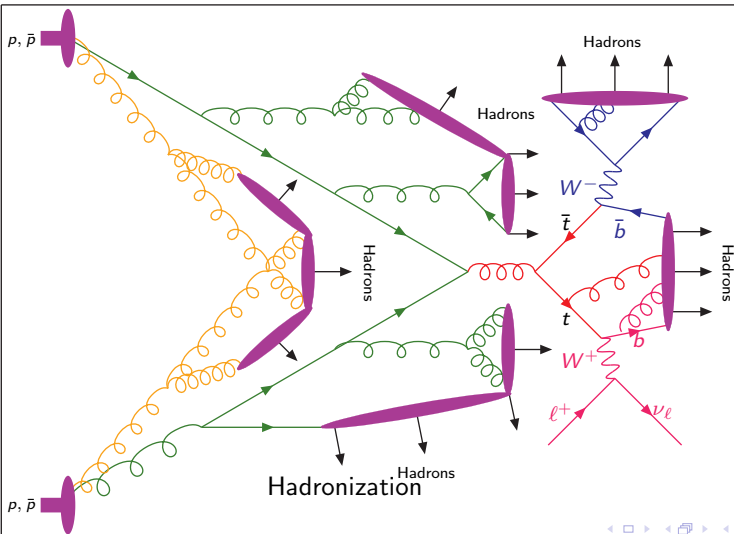
A Monte Carlo Event



A Monte Carlo Event



A Monte Carlo Event



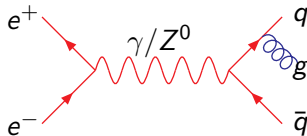
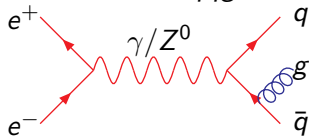


Plan

- Infrared divergences.
- Colinear Emission.
- Sudakov Form Factors.
- Soft emission and colour coherence.
- Initial-State radiation.
- Heavy quarks.
- Dipole cascades.
- Intrinsic p_{\perp}

Gluon Emission

- Let's start with the simplest possible gluon emission process, *i.e.* $e^+e^- \rightarrow q\bar{q}g$.



- The total cross section is

$$\sigma(e^+e^- \rightarrow q\bar{q}g) = \sigma_0 C_F \frac{\alpha_S}{2\pi} \int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)},$$

where $x_i \equiv 2p_i/\sqrt{s}$ and $\sigma_0 = \sigma(e^+e^- \rightarrow q\bar{q})$.

- Divergent at the edge of phase space as $x_{1,2} \rightarrow 1$ so that the total cross section is $\sigma = \infty$!

Gluon Emission

- Common feature of all perturbative QCD calculations.
- Configurations which are indistinguishable from the leading-order result are divergent.
- Physically there are two regions where this happens

1 Colinear limit: $x_1 \rightarrow 1$ at fixed x_2 or $x_2 \rightarrow 1$ at fixed x_1

$$2p_2 \cdot k = \frac{s x_2 x_3}{2} (1 - \cos \theta_{23}) = s(1 - x_1) \Rightarrow (1 - \cos \theta_{23}) = \frac{2(1 - x_1)}{x_2 x_3} \rightarrow 0.$$

2 Soft limit: $x_{1,2} \rightarrow 1$ at fixed $\frac{1-x_1}{1-x_2}$

$$E_g = \frac{\sqrt{s}}{2} x_3 = \frac{\sqrt{s}}{2} (1 - x_1 + 1 - x_2) \rightarrow 0.$$

- Both universal features of QCD matrix elements.

Collinear Limit

- If we take k parallel to p_2 ($\theta_{23} = 0$) we can define

$$p_2 = (1 - z)\bar{p}_2, \quad k = z\bar{p}_2, \quad \text{with } \bar{p}_2^2 = 0.$$

- In this limit the matrix element factorizes

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 \times \frac{g_s^2}{p_2 \cdot k} \times C_F \frac{1 + (1 - z)^2}{z}.$$

- As does the phase space

$$dx_1 dx_2 \longrightarrow \frac{1}{4} z(1 - z) dz d\theta_{23}^2.$$

Collinear Limit

- Putting this together

$$\sigma = \sigma_0 \int \frac{d\theta_{23}^2}{\theta_{23}^2} dz C_F \frac{\alpha_S}{2\pi} \frac{1 + (1-z)^2}{z}.$$

- The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting function is a universal probability distribution for the radiation of a collinear gluon in any process producing a quark.
- Exactly same form for anything proportional to θ^2 , e.g.
 - transverse momentum $k_{\perp}^2 = z^2(1-z)^2\theta^2$;
 - invariant mass $q^2 = z(1-z)\theta^2 E^2$.

such that

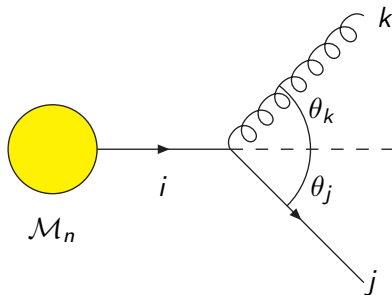
$$\frac{d\theta^2}{\theta^2} = \frac{dk_{\perp}^2}{k_{\perp}^2} = \frac{dq^2}{q^2}$$

Parton Shower

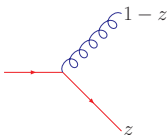
- The simulation of QCD radiation is based on this factorization, *i.e.*

$$d\sigma_{n+1} = d\sigma_n \frac{d\theta^2}{\theta^2} dz \frac{\alpha_S}{2\pi} P_{ji}(z)$$

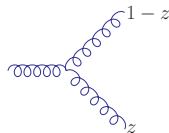
where the splitting function only depends on the spin and flavour of the partons.



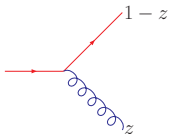
Splitting Functions



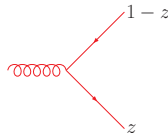
$$P_{q \rightarrow qg}(z) = C_F \frac{1+z^2}{1-z}$$



$$P_{g \rightarrow gg}(z) = C_A \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right]$$



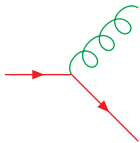
$$P_{q \rightarrow gq}(z) = C_F \frac{1+(1-z)^2}{z}$$



$$P_{g \rightarrow q\bar{q}}(z) = T_R [z^2 + (1-z)^2]$$

Parton Shower

- This expression is singular as $\theta \rightarrow 0$.
- What is a parton? (or what is the difference between a collinear pair and a parton).
- Introduce a resolution criterion, e.g. $k_{\perp} > Q_0$.
- Combine the virtual corrections and unresolvable emission



Resolvable Emission
Finite



Unresolvable Emission
Finite

- Unitarity: Unresolved + Resolved = 1

Sudakov Form Factor

- We can then exponentiate the real emission piece

$$\begin{aligned}
 \mathcal{P}(\text{unresolved}) &= 1 - \mathcal{P}(\text{resolved}), \\
 &= 1 - \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{\frac{Q_0^2}{q^2}}^{1 - \frac{Q_0^2}{q^2}} dz \frac{\alpha_S}{2\pi} P(z), \\
 &= \exp \left(- \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{\frac{Q_0^2}{q^2}}^{1 - \frac{Q_0^2}{q^2}} dz \frac{\alpha_S}{2\pi} P(z) \right).
 \end{aligned}$$

- The **Sudakov form factor** which is the probability of evolving between two scales and emitting no radiation.
- More strictly it is the probability of evolving from a high scale to the cut-off with no resolvable emission.

Sudakov Form Factor

- More formally, the probability of emission between dq^2 and $q^2 + dq^2$ is

$$d\mathcal{P} = \frac{dq^2}{q^2} \int_{\frac{Q_0^2}{q^2}}^{1 - \frac{Q_0^2}{q^2}} dz \frac{\alpha_S}{2\pi} P(z)$$

- We can then write a differential equation for the evolution of the probability of no-emission between Q^2 and q^2 , $\Delta(Q^2, q^2)$

$$d\Delta(Q^2, q^2) = \Delta(Q^2, q^2) d\mathcal{P} \Rightarrow \frac{d\Delta(Q^2, q^2)}{\Delta(Q^2, q^2)} = \frac{dq^2}{q^2} \int_{\frac{Q_0^2}{q^2}}^{1 - \frac{Q_0^2}{q^2}} dz \frac{\alpha_S}{2\pi} P(z)$$

giving

$$\Delta(Q^2, q^2) = \exp \left(- \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{\frac{Q_0^2}{k^2}}^{1 - \frac{Q_0^2}{k^2}} dz \frac{\alpha_S}{2\pi} P(z) \right)$$

Numerical Procedure

Radioactive Decay

- Start with an isotope.
- Work out when it decays by generating a random number $R \in [0, 1]$ and solving

$$R = \exp \left[-\frac{t}{\tau} \right],$$

where τ is its lifetime.

- Generate another random number and use the branching ratios to find the decay mode.
- Generate the decay using the masses of the decay products and phase space.
- Repeat the process for any unstable decay products.
- This algorithm is actually used in Monte Carlo event generators to simulate particle decays.

Parton Shower

- Start with a parton at a high virtuality, Q , typical of the hard collision.
- Work out the scale of the next branching by generating a random number $R \in [0, 1]$ and solving

$$R = \Delta(Q^2, q^2),$$

where q is the scale of the next branching.

- If there's no solution for $q > Q_0$ then stop.
- Otherwise work out the type of branching.
- Generate the momenta of the decay products using the splitting functions.
- Repeat the process for the partons produced in the branching.

Veto Algorithm

- Usually we cannot easily solve $\Delta(Q^2, q^2) = \mathcal{R}$.
- Instead we start by picking an overestimate $P_{\text{over}}(z) \geq P(z)$ which is easily invertible, *i.e.* we can calculate $H(z) = \int P_{\text{over}}(z) dz$ and $H^{-1}(z)$.
- Also overestimate of the integration region $z_{\text{min}}^{\text{over}} \leq z_{\text{min}}$ and $z_{\text{max}}^{\text{over}} \geq z_{\text{max}}$, and the maximum value of α_S , $\alpha_S^{\text{over}} \geq \alpha_S(p_{\perp}(q^2, z)) \forall z, q^2$.
- We now have an overestimate of the integrand of the Sudakov form factor, *i.e.*

$$F(k^2) = \frac{1}{k^2} \int_{z_{\text{min}}}^{z_{\text{max}}} dz \frac{\alpha_S}{2\pi} P(z) \rightarrow G(k^2) = \frac{1}{k^2} \int_{z_{\text{min}}^{\text{over}}}^{z_{\text{max}}^{\text{over}}} dz \frac{\alpha_S^{\text{over}}}{2\pi} P_{\text{over}}(z)$$

Veto Algorithm

- We can solve this to get a first trial value of q^2

$$\ln \mathcal{R} = - \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{z_{\min}^{\text{over}}}^{z_{\max}^{\text{over}}} dz \frac{\alpha_S^{\text{over}}}{2\pi} P_{\text{over}}(z) \Rightarrow q^2 = Q^2 \exp \left(\frac{\ln \mathcal{R}}{\int_{z_{\min}^{\text{over}}}^{z_{\max}^{\text{over}}} dz \frac{\alpha_S^{\text{over}}}{2\pi} P_{\text{over}}(z)} \right)$$

- However we cannot do simple accept/reject
- Instead generate a value of z using

$$z = H^{-1} [H(z_{\min}^{\text{over}}) + \mathcal{R}(H(z_{\max}^{\text{over}}) - H(z_{\min}^{\text{over}}))]$$

- We reject the emission if z is outside the true limits or with probability

$$\frac{F(q^2)}{G(q^2)} = \frac{\frac{\alpha_S}{2\pi} P(z)}{\frac{\alpha_S^{\text{over}}}{2\pi} P_{\text{over}}(z)} \geq \mathcal{R}$$

if z is inside the true limits but if the try is rejected start again with $Q^2 = q^2$ and generate another try.

Veto Algorithm

If we define $\mathcal{P}_n(q^2)$ to be probability we accept q^2 after rejecting n attempts then the probability of generating q^2 is $\sum_{n=0}^{\infty} \mathcal{P}_n(t)$, where

$$\mathcal{P}_0(q^2) = G(q^2) \Delta^{\text{over}}(Q^2, q^2) \frac{F(q^2)}{G(q^2)} = \Delta^{\text{over}}(Q^2, q^2) F(q^2)$$

$$\begin{aligned} \mathcal{P}_1(q^2) &= \int_{q^2}^{Q^2} dq'^2 G(q'^2) \Delta^{\text{over}}(Q^2, q'^2) \left[1 - \frac{F(q'^2)}{G(q'^2)} \right] G(q^2) \Delta^{\text{over}}(q'^2, q^2) \frac{F(q^2)}{G(q^2)} \\ &= F(q^2) \Delta^{\text{over}}(Q^2, q^2) \int_{q^2}^{Q^2} dq'^2 (G(q'^2) - F(q'^2)) \end{aligned}$$

...

$$\mathcal{P}_n(q^2) = \frac{1}{n!} F(q^2) \Delta^{\text{over}}(Q^2, q^2) \left[\int_{q^2}^{Q^2} dq'^2 (G(q'^2) - F(q'^2)) \right]^n$$

Veto Algorithm

■ Summing

$$\begin{aligned}
 \sum_{n=0}^{\infty} \mathcal{P}_n(t) &= F(q^2) \Delta^{\text{over}}(Q^2, q^2) \sum_{n=0}^{\infty} \left[\int_{q^2}^{Q^2} dq'^2 (G(q'^2) - F(q'^2)) \right]^n \\
 &= F(q^2) \Delta^{\text{over}}(Q^2, q^2) \exp \left[\int_{q^2}^{Q^2} dq'^2 (G(q'^2) - F(q'^2)) \right] \\
 &= F(q^2) \exp \left[- \int_{q^2}^{Q^2} dq'^2 G(q'^2) \right] \exp \left[\int_{q^2}^{Q^2} dq'^2 (G(q'^2) - F(q'^2)) \right] \\
 &= F(q^2) \Delta^{\text{over}}(Q^2, q^2)
 \end{aligned}$$

as required.

Monte Carlo Procedure

The key difference between the different Monte Carlo simulations is in the choice of the evolution variable.

- Evolution Scale

- Virtuality, q^2
- Transverse Momentum, k_\perp
- Angle, θ

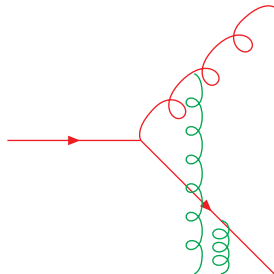
- Energy fraction, z

- Energy fraction
- Light-cone momentum fraction
- ...

Are all the same in the collinear limit.

Soft Emission

- We have only considered collinear emission. What about soft emission?
- Soft gluons come from all over the event.
- There is quantum interference
- Does this spoil the parton shower picture?



Soft Limit

- In the limit that $E_g \rightarrow 0$ the matrix element for the $e^+e^- \rightarrow q\bar{q}$ factorizes

$$\mathcal{M}_{q\bar{q}g} = \mathcal{M}_{q\bar{q}} g_s t_{ij}^a \left(\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right) \cdot \epsilon_A(k).$$

- Called the **Eikonal Current**.
- The matrix element squared therefore factorizes in this case

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 g_s^2 C_F \frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k}.$$

- In this case the phase space is

$$dx_1 dx_2 \longrightarrow \frac{2}{s} E_g dE_g d\cos\theta.$$

Soft Limit

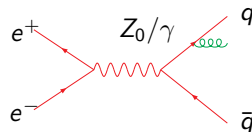
- So in the soft limit

$$\sigma = \sigma_0 \int C_F \frac{\alpha_S}{2\pi} \frac{dE_g}{E_g} d\cos\theta \frac{2(1 - \cos\theta_{qq})}{(1 - \cos\theta_{qg})(1 - \cos\theta_{gq})}.$$

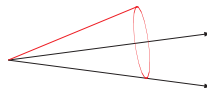
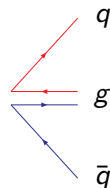
- Gives the **Dipole Radiation pattern**.
- Universal probability distribution to emit a soft gluon from any colour-connected pair of partons.
- **Only universal at the amplitude level**

Angular Ordering

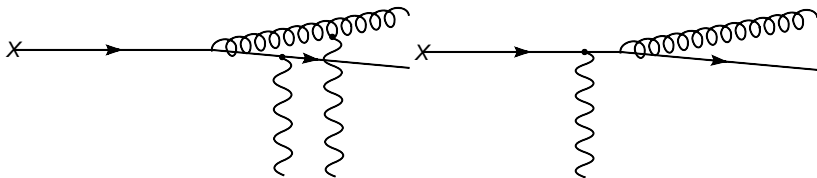
- In the soft limit the matrix element factorizes but at the amplitude level.
- The remarkable result is that if we take the large number of colours limit much of the **interference** is **destructive**.
- In particular if we consider the colour flow in an event.
- QCD radiation only occurs in a cone up to the direction of the colour partner.
- The best choice of evolution variable is therefore an angular one



Feynman Diagram



Colour Coherence



- Wide angle soft gluons cannot resolve the difference between a gluon and "colinear" quark and gluon with the same quantum numbers.
- Called **Colour Coherence**
- Angular ordering is one way of including this physics, but there are others.

Accuracy of Parton Shower simulations

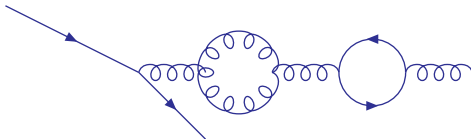
- Formally the parton shower is accurate to leading log.

$$\Delta(Q^2, Q_0^2) \approx \exp \left[-C_F \frac{\alpha_S}{2\pi} \ln^2 \left(\frac{Q^2}{Q_0^2} \right) \right]$$

- However Monte Carlo simulations include a number of subleading effects.
- The most important is the conservation of energy and momentum.
- Others include the choice of the scale for α_S .

Running Coupling

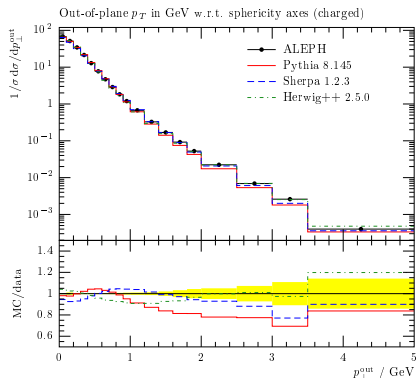
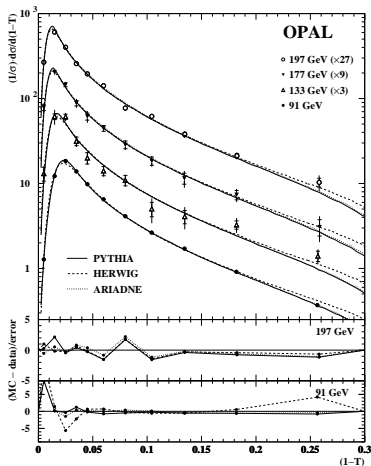
- Some of the higher order effects are included by



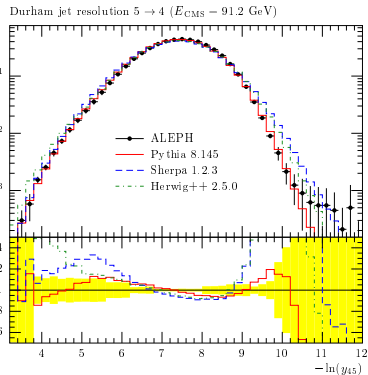
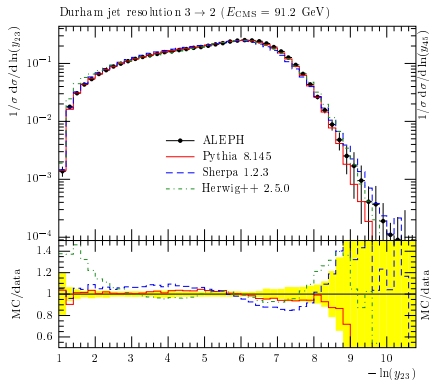
replacing $\alpha_S \rightarrow \alpha_S(k_\perp^2)$

- Gives more emission as $k^2 \rightarrow Q_0^2$. The phase space fills with soft gluons.
- Must avoid the Landau pole $K_\perp^2 \gg \Lambda^2$ so that Q_0^2 becomes a physical parameter.

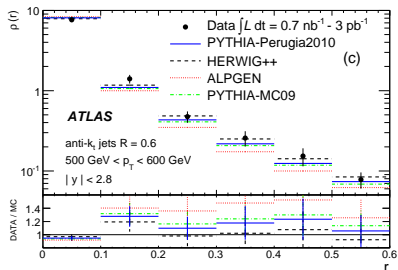
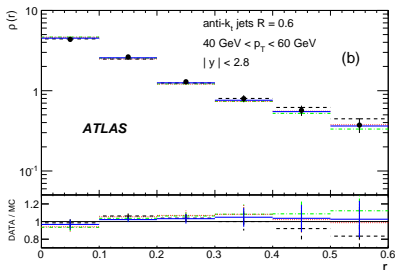
LEP Event Shapes



LEP Jet Resolution



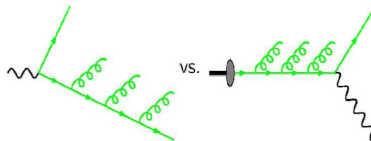
Jet Shapes



Phys. Rev. D83 052003 ATLAS (2011)

Hadron-Hadron Simulations

- In order to simulate hadron collisions we also need to simulate initial-state radiation.
- In principle this is similar to final-state radiation, but in practice there is a complication.
- For **final-state** radiation: One end of the evolution fixed, the scale of the hard collision.
- For **initial-state** radiation: Both ends of the evolution fixed, the hard collision and the incoming hadron.



- Use a different approach based on the evolution equations.

Initial-State Radiation

There are two options for the initial-state shower:

■ Forward Evolution

- Start at the hadron with the distribution of partons given by the PDF.
- Use the parton shower to evolve to the hard collision.
- Reproduces the PDF by a Monte Carlo procedure.
- Unlikely to give an interesting event at the end, so highly inefficient.

■ Backward Evolution

- Start at the hard collision and evolve backwards to the proton guided by the PDF.
- Much more efficient in practice.

Initial-State Radiation

- The evolution equation for the PDF can be written as

$$t \frac{df_b(x, Q^2)}{dt} = \sum_a \int_x^1 \frac{dz}{z} f_a\left(\frac{x}{z}, Q^2\right) \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z) \quad \text{where} \quad t = \ln\left(\frac{Q^2}{\Lambda^2}\right)$$

or

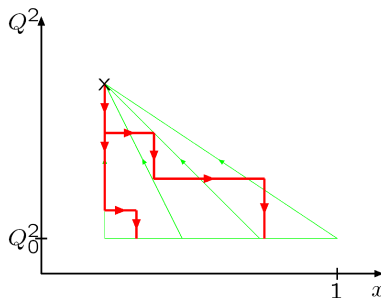
$$\frac{df_b(x, Q^2)}{f_b(x, Q^2)} = \frac{dt}{t} \sum_a \int_x^1 \frac{dz}{z} \frac{x' f_a(x', Q^2)}{x f_b(x, Q^2)} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z) \quad \text{where} \quad x' = \frac{x}{z}$$

- This can be written as a Sudakov form-factor for evolving backwards in time, i.e from the hard collision at high Q^2 to lower with

$$\Delta = \exp\left(-\frac{dt}{t} \sum_a \int_x^1 \frac{dz}{z} \frac{x' f_a(x', Q^2)}{x f_b(x, Q^2)} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z)\right).$$

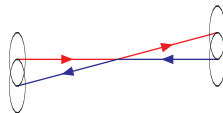
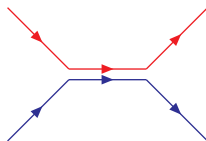
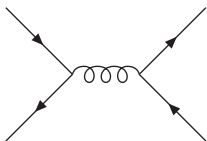
Backward Evolution

- The evolution equations give the PDFs at (x, Q^2) as a function of those at $(> x, < Q^2)$
- Backward evolution starts from the hard scattering at (x, Q^2) and work $\downarrow q^2$ and $\uparrow x$ towards the incoming hadron.



Hadron Collisions

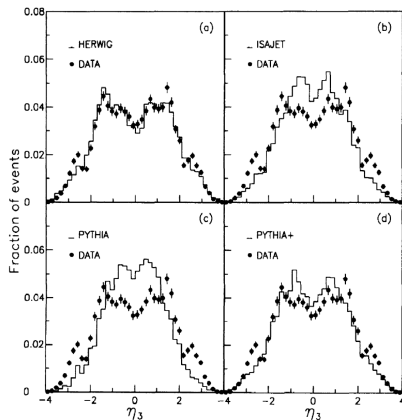
- The hard scattering sets up the initial conditions for the parton shower.
- Colour coherence is important here too.
- Each parton can only emit in a cone stretching to its colour partner.



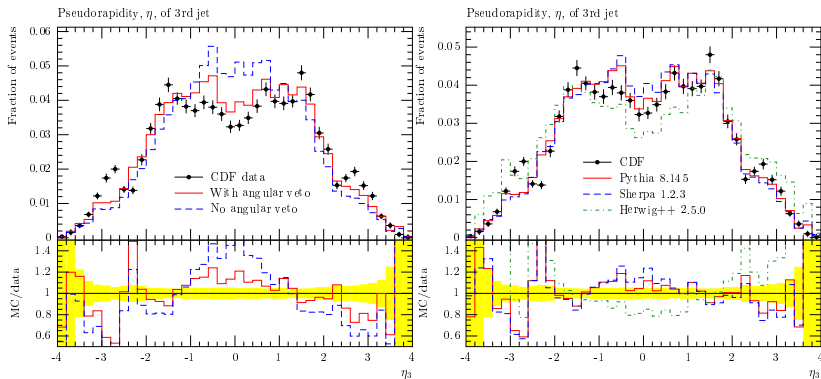
- Essential to fit the Tevatron data.

Colour Coherence

- Distributions of the pseudorapidity of the third jet.
- At the time only described by HERWIG which has complete treatment of colour coherence.
- PYTHIA+ had partial.
- Modern generators now all include coherence in some manner.

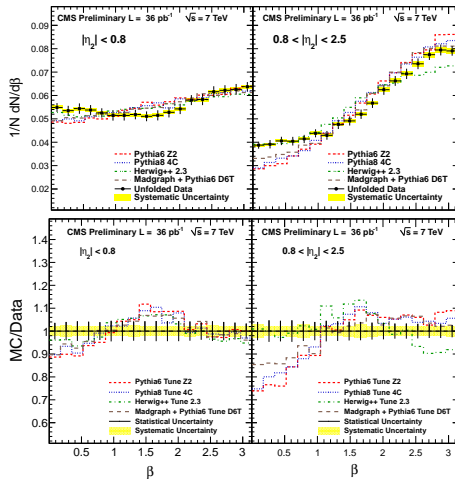


Colour Coherence



PRD50, 5562, CDF (1994)

Colour Coherence

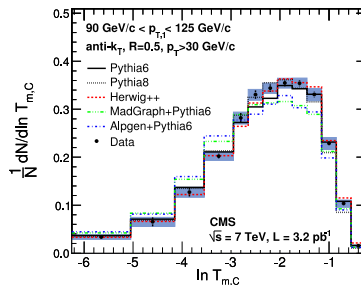
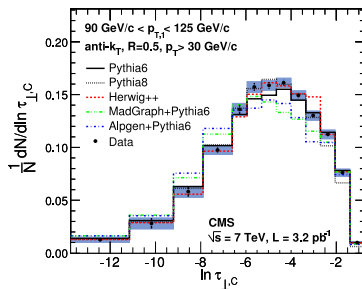


Hadronic Event Shapes

- In hadron collisions we can't use the same event shapes as e^+e^- collisions due to radiation along the beam direction.
- There are however a range of event shapes using transverse quantities, for example

$$\tau_{\perp,C} = 1 - \max_{\hat{n}_T} \frac{\sum_i |\vec{p}_{\perp,i} \cdot \hat{n}_T|}{\sum_i p_{\perp,i}}.$$

Hadronic Event Shapes



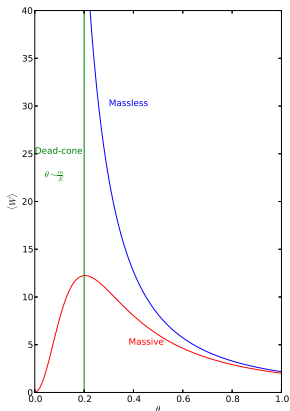
CMS Phys. Lett. B699 (2011) 48-67

Heavy Quarks

- The collinear singularity is regulated by the quark mass.

$$\begin{aligned}
 2p_2 \cdot k &= 2E_g(E_q - |p_q| \cos \theta_{23}) \\
 &= 2E_g |p_q| \left(\sqrt{1 + \frac{m^2}{|p_q|^2}} - \cos \theta_{23} \right)
 \end{aligned}$$

- Taking the azimuthal average of the soft radiation function gives a smooth suppression of radiation as $\theta \rightarrow 0$ starting from $\theta \sim \frac{m}{E}$.
- Historically implemented as a cut-off.



Heavy Quarks

- Better treatment involves the use of the quasi-collinear splitting functions [Catani et.al Phys.Lett. B500 149-160 \(2001\)](#)

$$d\mathcal{P} = \frac{\alpha_S}{2\pi} \frac{dq^2}{q^2 - m^2} P_{i\tilde{j} \rightarrow ij}(z, q^2)$$

where

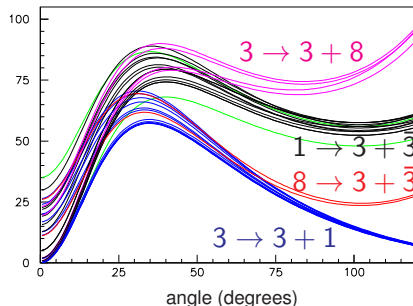
$$P_{q \rightarrow qg} = \frac{C_F}{1-z} \left[1 + z^2 - \frac{2(1-z)m_q^2}{q^2 - m_q^2} \right], \quad P_{g \rightarrow q\bar{q}} = T_R \left[1 - 2z(1-z) + \frac{2m_q^2}{q^2 - m_q^2} \right],$$

$$P_{\tilde{g} \rightarrow \tilde{g}g} = \frac{C_A}{1-z} \left[1 + z^2 - \frac{2(1-z)m_{\tilde{g}}^2}{q^2 - m_{\tilde{g}}^2} \right], \quad P_{\tilde{q} \rightarrow \tilde{q}g} = \frac{2C_F}{1-z} \left[z - \frac{(1-z)m_{\tilde{q}}^2}{q^2 - m_{\tilde{q}}^2} \right].$$

- Gives a smooth suppression as $\theta \rightarrow 0$.

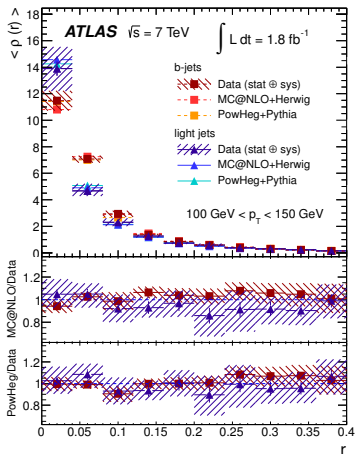
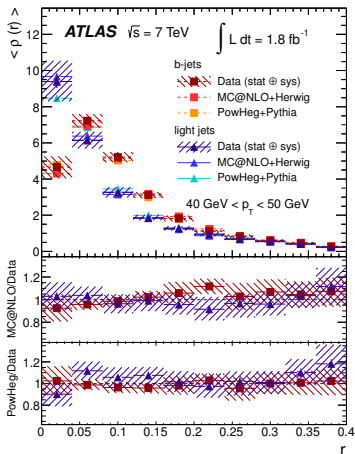
Heavy Quarks

- Only exact for either:
 - soft emission $E_{\text{gluon}} \rightarrow 0$;
 - radiation from scalars.
- In general the radiation depends on:
 - Gluon energy;
 - spins of radiating particles and colour partner;
 - colours of the particles;
 i.e. process-dependent mass corrections.

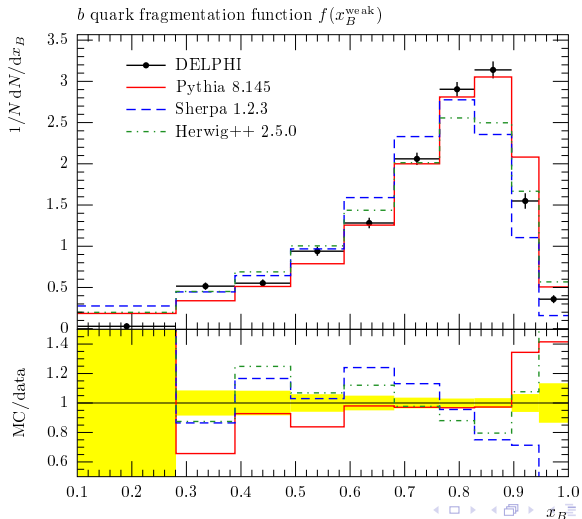


Nucl.Phys. B603 297-342 Norrbin & Sjostrand (2001)

Jet Shapes for bottom quark jets



b-fragmentation function



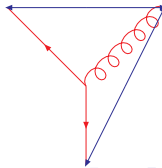
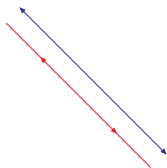
The Colour Dipole Model

- Conventional parton showers: start from collinear limit, modify to incorporate soft gluon coherence
- Colour Dipole Model: start from soft limit Emission of soft gluons from colour-anticolour dipole universal (and classical):

$$d\sigma \sim \sigma_0 \frac{1}{2} C_A \frac{\alpha_S(k_\perp)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} dy,$$

where $y =$ is the rapidity of the emitted particle.

- After emitting a gluon, colour dipole is split:



The Colour Dipole Model

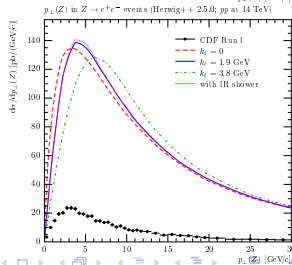
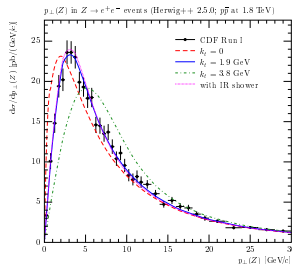
- Subsequent dipoles continue to split.
- Conventional parton-showers $1 \rightarrow 2$ parton splittings.
- CDM one dipole to two dipoles, $2 \rightarrow 3$ partons.
- Problems with the treatment of initial=state radiation.
- The hadronic remnant forms a dipole with scattered quark.
- But as the remnant is an extended object there is a suppression.

Dipole Cascades

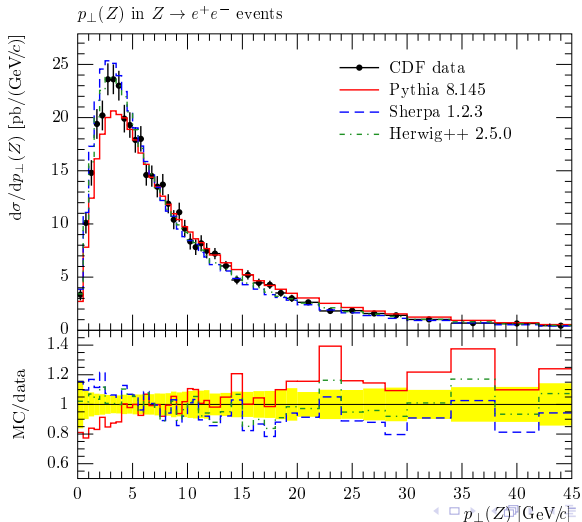
- Most new shower algorithms are based on a dipole picture.
 - However most split the dipole into two pieces, one for radiation from each of the partons forming it, as in Catani-Seymour subtraction.
 - The partner is used to absorb recoil and define the radiation pattern.
- Dipole Vinica, ARIADNE
Split PYTHIA6.3, PYTHIA8, Sherpa and Herwig++ dipole showers

Primordial p_{\perp}

- The partons inside the proton have some motion with $p_{\perp} \sim \frac{1}{1 \text{ fm}}$.
- Intrinsic p_{\perp} is essential to describe the low p_{\perp} behaviour of Drell-Yan.
- Particularly important at the Tevatron as no perturbative radiation in some events.
- Less important at the LHC, pp and higher energy.



Primordial p_{\perp}



Older Programs

- **PYTHIA 6**: two showers

- q^2 ordering with veto of non-ordered final state emission and partial implementation of angular ordering in initial state;
- p_\perp -ordered parton showers, interleaved with multi-parton interactions and dipole-style recoil.

Matrix element for first emission in many processes and a large range of hard processes.

- **HERWIG6**: complete implementation of colour coherence; NLO evolution for large x ; smaller range of hard processes.
- **ARIADNE**: complete implementation of colour dipole model; best fit to HERA data; interfaced to PYTHIA for hard processes.

Modern Programs

- **PYTHIA8**: new program with many of the same features as **PYTHIA6**, many obsolete features removed.
- **SHERPA**: new program built from scratch; either older q^2 or newer p_\perp -ordered dipole showers; multi-jet matching scheme (CKKW) and NLO built in.
- **Herwig++**: new program with similar parton shower to **HERWIG** (angular ordered) plus quasi-collinear limit and recoil strategy based on colour flow; spin correlations.

Summary

- Accelerated colour charges radiate gluons.
- As the gluon is also coloured this leads to a cascade of gluons.
- Modern parton shower algorithms are sophisticated implementations of perturbative QCD.
- Allows us to evolve from the scale of the hard collision to the hadronization scale.
- However we then need non-perturbative hadronization models.