# Introduction to Monte Carlo Event Generation
# Lecture 3: Hadronization

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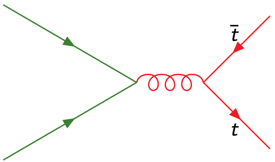
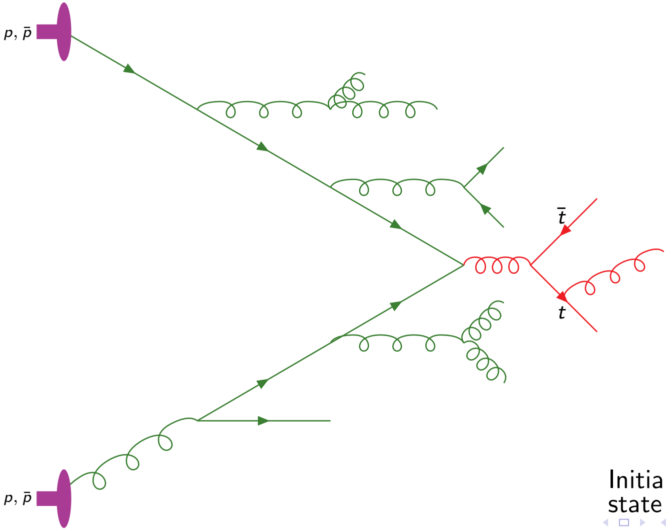
# A Monte Carlo Event



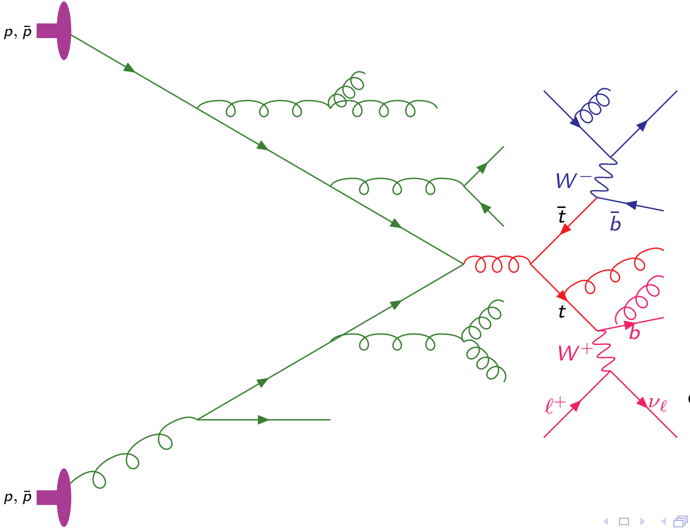

Hard Process, usually calculated at leading order

# A Monte Carlo Event



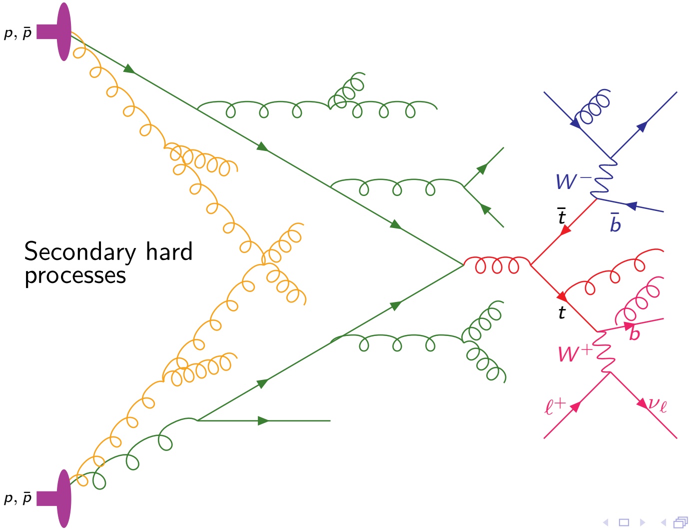

Initial- and final-state parton shower

# A Monte Carlo Event

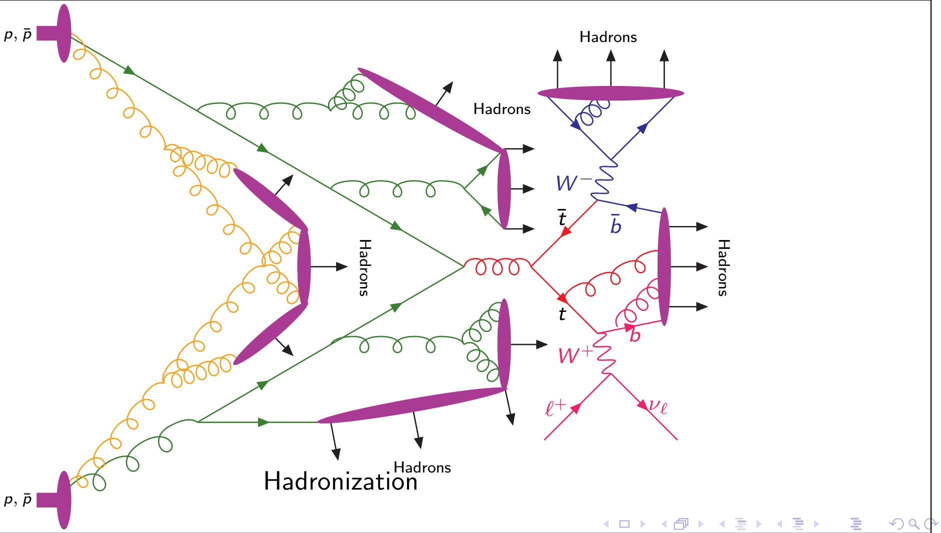



Perturbative decays of heavy particles

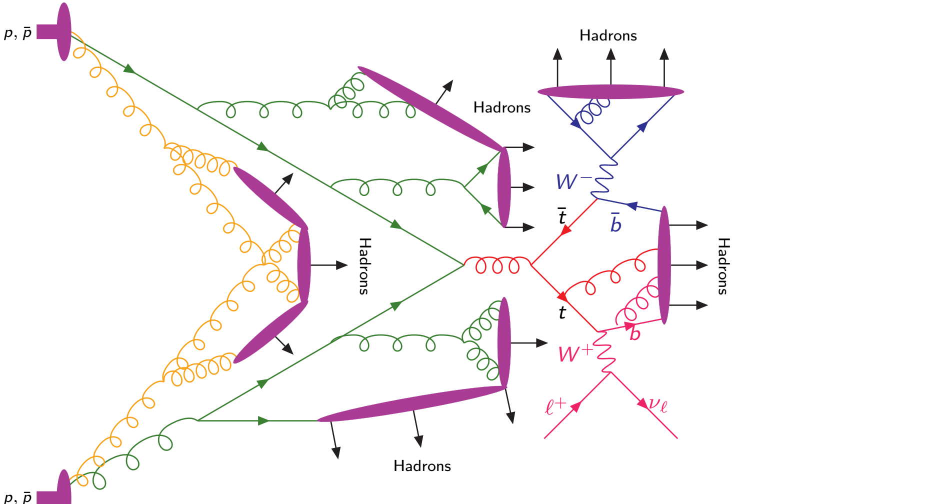
# A Monte Carlo Event



# A Monte Carlo Event



# A Monte Carlo Event



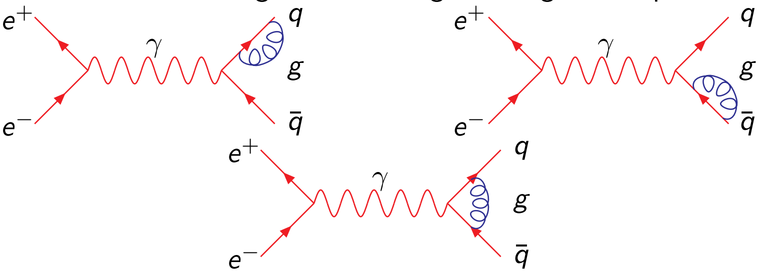

Hadron Decays

# Higher Order Corrections

- When we draw Feynman diagrams we are performing a perturbative expansion in the (hopefully) small coupling constant.
- Unfortunately the strong coupling often isn't very small, at the $Z^0$ mass, $\alpha_S(M_Z) = 0.118$.
- We therefore need to consider higher orders in the perturbative expansion.
- There are always two types of correction:
  - real gluon emission;
  - virtual gluon loops.

# Virtual Gluon Corrections

- There are three diagrams involving virtual gluon loops.




- This contribution is also divergent, but negative.
- This will cancel the real divergence to give a finite answer.

# Total Cross Section

- To show this we need to regularize both the real and virtual cross sections and add them together. Should then be finite when we remove the regularization.
- The standard way of doing this is to work in $d = 4 - 2\epsilon$ dimensions where to regularize these infrared divergences $\epsilon < 0$.
- In this case

$$\begin{aligned}\sigma_{\mathrm{real}} &= \sigma_0 C_F \frac{\alpha_S}{2\pi} H(\epsilon)\left( \frac{4}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon)\right) \\ \sigma_{\mathrm{virtual}} &= \sigma_0 C_F \frac{\alpha_S}{2\pi} H(\epsilon)\left( -\frac{4}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right)\end{aligned}$$

- The sum is finite as $\epsilon \to 0$.

# Total Cross Section

- So putting it all together

$$R(e^+e^-) = R_0(e^+e^-)\left(1 + \frac{\alpha_s}{\pi}\right)$$

- One way of measuring the strong coupling

$$\alpha_S(m_Z) = 0.1226 \pm 0.0038.$$

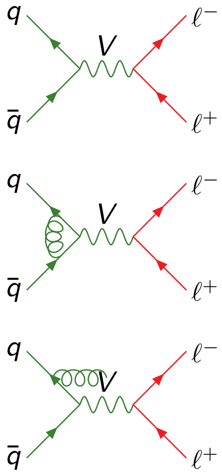
taken from the PDG.

- The second and third order corrections, and the results for the next-to-leading order corrections including quark masses are also known.
- So we can calculate the higher order corrections to the total cross section, what about more complicated observables?

# Higher Order Corrections

- As the two separate parts of the NLO cross section are infinite calculating the cross section numerical is a problem.
- However, we can use the universal properties to construct a subtraction counter-term which has the same singularities as the matrix element for real emission.
- Pick a counter-term which can be analytically integrated in $d = 4 - 2\epsilon$ dimensions and added to the virtual piece

$$
\begin{aligned}
\mathrm{d}\sigma &= B(v)\mathrm{d}\Phi_v + (V(v) + C(v,r))\mathrm{d}\Phi_r\mathrm{d}\Phi_v \\
&\quad + (R(v,r) - C(v,r))\mathrm{d}\Phi_v\mathrm{d}\Phi_r
\end{aligned}
$$



# Hadronization

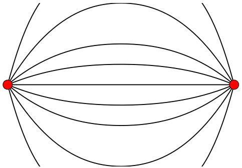
- Partons aren't physical particles: they can't propagate freely.
- We therefore need to describe the transition of the quarks and gluons in our perturbative calculations into the hadrons which can propagate freely.
- We need a phenomenological model of this process.
- There are three models which are commonly used:
  - Independent Fragmentation;
  - Lund String Model;
  - Cluster Model.

# Independent Fragmentation Model: Feynman-Field

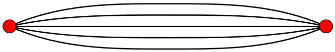
- The longitudinal momentum distribution is given by an arbitrary fragmentation function which is a parameterization of data.
- The transverse momentum distribution is Gaussian.
- The algorithm recursively splits $q \to q' + \mathrm{hadron}$.
- The remaining soft quark and antiquark are connected at the end.
- The model has a number of flaws:
    - strongly frame dependent;
    - no obvious relation with the perturbative physics;
    - not infrared safe;
    - not a confinement model;
    - wrong energy dependence.

# Confinement

- We know that at small distances we have asymptotic freedom and the force between a quark-antiquark pair is like that between an $e^+e^-$ pair.



- But at long distances the self interactions of the gluons make the field lines attract each other.



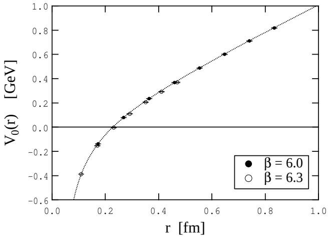
# Confinement

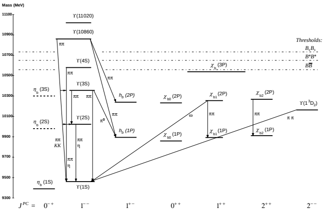
- Gives $1/r$ potential at short distances
- Linear potential at long distances and confinement.

$$V(r) \sim -\frac{4}{3}\frac{\alpha_S}{r} + \kappa r \sim -\frac{0.13}{r} + r$$

for $\alpha_S \sim 0.5$, $r$ in fm and $V$ in GeV.

- Either phenomenogically from quarkonium or lattice QCD.



# Lund String Model

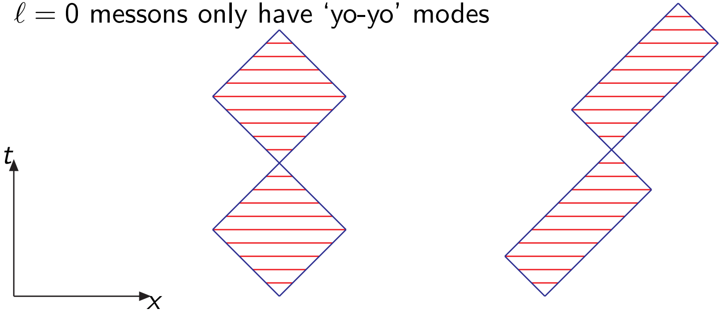
- Assume $\frac{1}{r}$ important for hadron structure but not production.
- In QCD the field lines seem to be compressed into a tube-like region, looks like a string.
- So we have linear confinement with a string tension,

$$F(r) \approx \mathrm{const} = \kappa \approx 1\,\mathrm{Gev/fm}.$$

- Separate the transverse and longitudinal degrees of freedom gives a simple description as a 1+1 dimensional object, the string, with a Lorentz invariant formalism.

# Mesons

- In the string model mesons are light $q\bar{q}$ pairs connected by a string.
- $\ell = 0$ messons only have 'yo-yo' modes




- Area law $m^2 = 2\kappa^2$.

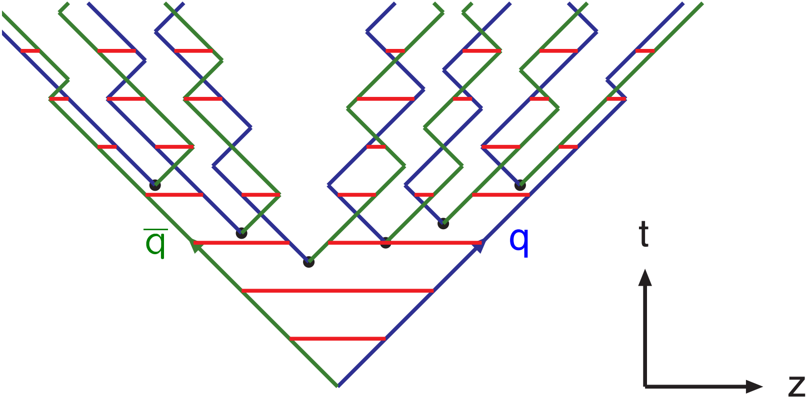
# Lund String Model

- Start by considering a $q\bar{q}$ pair produced in $e^+e^-$ annihilation.
- Ignore gluon radiation for the time being.
- $q$ and $\bar{q}$ joined by a string.
- $q\bar{q}$ pairs are created by tunnelling in the intense chromomagnetic field of the string.

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}x\mathrm{d}t} \propto \exp\left(-\pi\frac{m_q^2}{\kappa}\right)$$

- The string breaks into mesons long before the yo-yo point.
- Gives a simple but powerful picture of hadron production.

# Lund String Model



# Lund Fragmentation Function

- Fermi motion is a gaussian transverse momentum distribution
- The tunnelling probability becomes

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}x\mathrm{d}t} \propto \exp\left[-b\left(m_q^2 + p_\perp^2\right)\right]$$

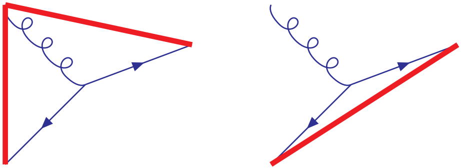
- The string picture constrains the fragmentation function
  - Lorentz invariance
  - Acausality
  - Left–right syymetry
- The function has the form

$$f(z) \propto z^{a_\alpha - a_\beta - 1}(1-z)^{a_\beta}$$

  where $a_{\alpha,\beta}$ are adjustable parameters for quarks $\alpha$ and $\beta$.
- $a$, $b$ and $m_q$ are the main tuneable parameters of the model.

# Three-jet Events

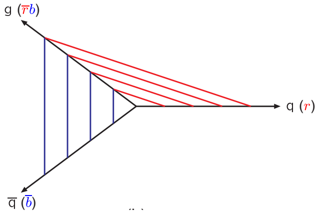
- So far we have only considered the hadronization of $q\bar{q}$ pairs, what about gluons?
- The gluon gives a kink on the string.

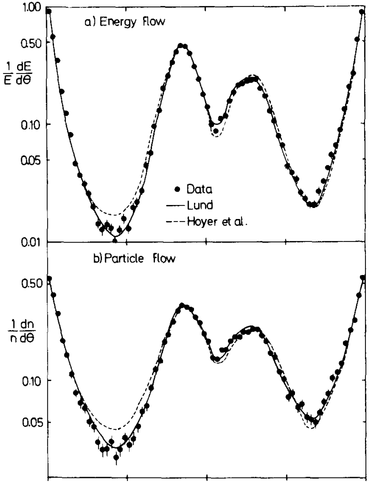


- the string effect
- The string model has an infrared safe matching with the parton shower.
- Gluons with $k_{\perp} < \frac{1}{\text{string width}}$ irrelevant.

# String Effect

- Less radiation between the quark and antiquark.
- Either non-perturbatively via the string model.
- Can get the same result perturbatively via colour coherence.



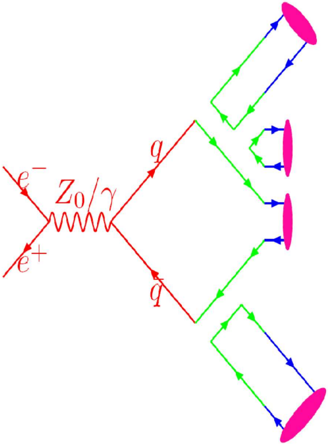



# Summary of the String Model

- String model strongly physically motivated.
- Very successful fit to data.
- Universal: fitted to $e^+e^-$ data little freedom elsewhere.
- How does motivation translate to prediction?
- $\sim$one free parameter per hadron/effect!
- Blankets too much perturbative information?
- Can we get by with a simpler model?

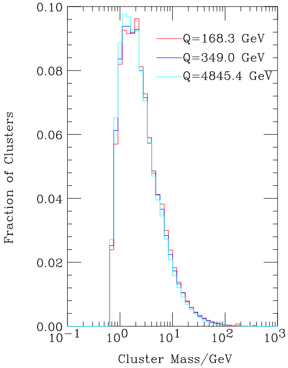
# Preconfinement

- In the planar approximation, large number of colours limit: Gluon = colour-anticolour pair
- We can follow the colour structure of the parton shower.
- At the end colour-singlet pairs end up close in phase space.
- Non-perturbatively split the gluons into quark-antiquark pairs.



# Preconfinement

- The mass spectrum of colour-singlet pairs is asymptotically independent of energy and the production mechanism.
- It peaks at low mass, of order the cut-off $Q_0$.
- Decreases rapidity for large cluster masses.



# Cluster Model

- Project the colour-singlet clusters onto the continuum of high-mass mesonic resonances (=clusters).
- Decay to lighter well-known resonances and stable hadrons using a pure 2-body phase-space decay and phase space weight.

$$W \propto (2s_1+1)(2s_2+1)\frac{2p^*}{m}$$

- The hadron-level properties are fully determined by the cluster mass spectrum, i.e. by the properties of the parton shower.
- Heavier hadrons, including baryons and strange hadrons supressed.
- The cut-off $Q_0$ is the crucial parameter of the model.

# Cluster Model: Problems

1. Tail of high-mass clusters for which cluster decay is not a good approximation.
   - Split heavy clustera into two lighter clusters along "string" direction.
   - $\sim 15\%$ of clusters in $e^+e^-$ collisions at $m_Z$ but gives$\sim \frac{1}{2}$ of the hadrons.
2. Sensitivity to particle content.
   - only include complete multiplets.
   - change model so adding new heavy particles doesn't effect decay of light clusters.
3. Leading hadrons are too soft
   - Perturbative quarks remember their direction

$$P(\theta^2) \sim \exp\left(-\frac{\theta^2}{2\theta_0^2}\right)$$

   - String like and extra parameter.
4. Problems with particle correlations.

# The "Beliefs"

- There are two main schools of thought in the event generator community.

PYTHIA

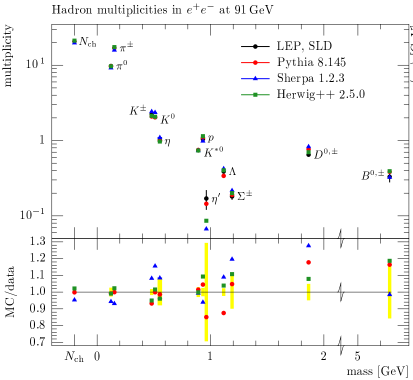
- Hadrons are produced by hadronization. You must get the nonperturbative dynamics right.
- Better data has required improvements to the perturbative simulation.

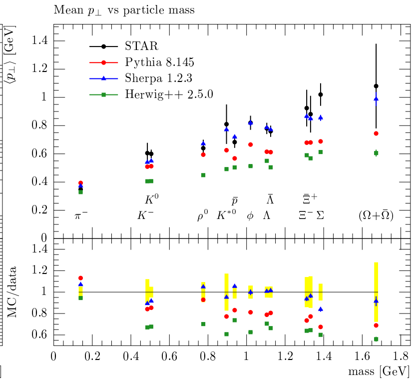
HERWIG

- Get the perturbative physics right and any hadronization model will be good enough
- Better data has required changes to the cluster model to make it more string-like.

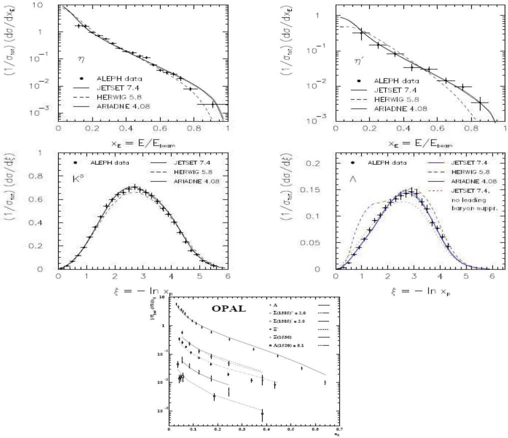
- There ain't no such thing as a good parameter-free description.

# Hadrochemistry



# Identified Particle Spectra

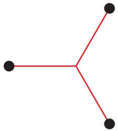


# The facts?

- Independent fragmentation doesn't describe the data, in particular the energy dependence.
- All the generators give good agreement for event shapes.
- HERWIG has less parameters to tune the flavour composition and tends to be worse for identified particle spectra.
- Baryon production is often a problem.

# Baryon Production

- All the models have some problems with baryon production.
- In the Lund model baryons are picture as quark quarks attached to a common centre, a colour source/sink



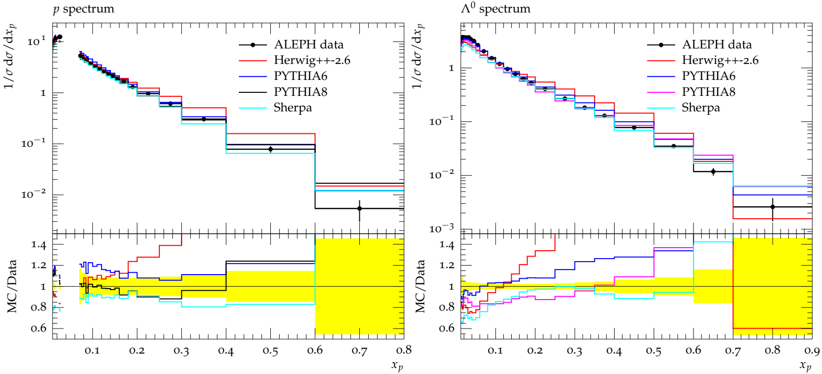
- At large separation two of the quarks are tightly bound, a diquark.

- The diquark is treated as a colour antitriplet $(3 \otimes 3 = \bar{3} \oplus 6)$

# Baryon Production

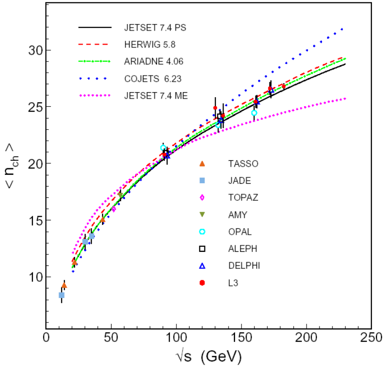
- Two quarks can tunnel nearby in phase space: baryon–antibaryon pair
- In the string model either use diquarks, with an extra parameter for each diquark.
- or the pop-corn model.
- In the cluster model allow diquarks to be produced in cluster decay (always) or non-pertrubative gluon splitting (allowed in some variants).

# Baryon Production



# Universality

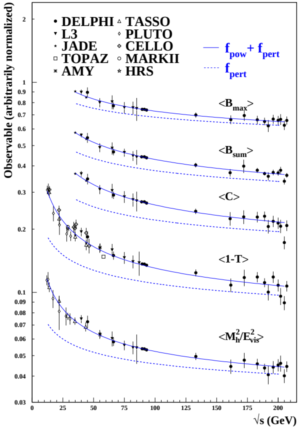
- Evolution to a universal, low hadronization scale ensures the hadronization parameters are universal.
- Don't need to retune at each energy.
- Only have to tune the new hadron specific parameters in hadronic collisions.



# Local Hadron-Parton Duality

- Analytic approach, continue shower evolution to $Q \sim \Lambda_{\rm QCD}$, or lower with a non-perturbative model for $\alpha_S$.
- Describes the momentum spectra $\frac{{\rm d}n}{{\rm d}x_p}$ but not identified particles.
- Gives the correction power corrections (renormalons), *e.g*

$$\langle 1 - T \rangle = a\alpha_S(E) + b\alpha_S^2(E) + \frac{c}{E}$$



# Hadron Properties

- Hadronization produces hadrons so we need both the hadron properties: quark content; spin; mass; width; etc..
- and to decide which hadrons to produce.
- Many of the hadrons produced during hadronization (primary hadrons) are unstable so we also need to know how they decay to secondary hadrons.
- Not just a matrer of typing in the PDG:
  - not all resonances in a given multiplet have been measured;
  - measured branching fractions rarely add up to exactly 100%;
  - measured branching fractions rarely exactly respect isospin;
- Also need to make a lot of choices for the matrix elements to describe the various decay modes.

# Hadron Properties

- Often not even numerical values for partial widths.
- Particles decaying into final-states which are'nt allowed for on-shell masses, *e.g.* $h'_1 \to K\bar{K}^*$.
- In some cases the choice of decay modelling effects the decay tables, *e.g.* $a_1 \to \rho\pi$ vs. $a_1 \to \pi\pi\pi$.

**$h_1(1170)$** $I^G(J^{PC}) = 0^-(1^{+-})$

**$h_1$(1170) DECAY MODES**

| | Mode | Fraction ($\Gamma_i/\Gamma$) |
|---|---|---|
| $\Gamma_1$ | $\rho\pi$ | seen |

**$h_1(1380)$** $I^G(J^{PC}) = ?^-(1^{+-})$

OMITTED FROM SUMMARY TABLE
Seen in partial-wave analysis of the $K\bar{K}\pi$ system. Needs confirmation.

**$h_1$(1380) DECAY MODES**

| | Mode |
|---|---|
| $\Gamma_1$ | $K\bar{K}^*(892)+$ c.c. |

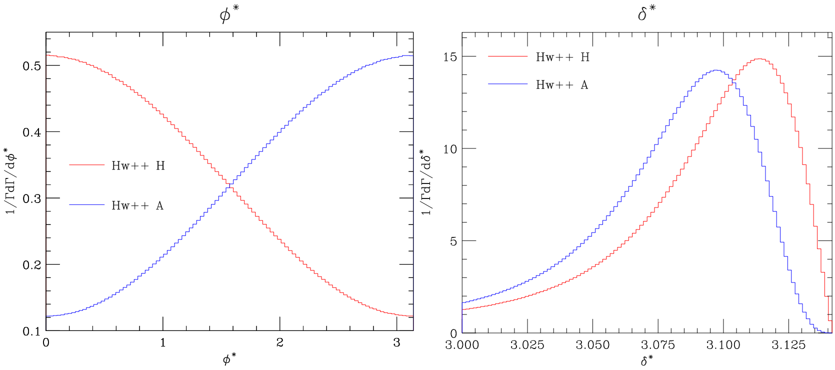
**$a_1$(1260)** [k] $I^G(J^{PC}) = 1^-(1^{++})$

| $a_1$(1260) DECAY MODES | Fraction ($\Gamma_i/\Gamma$) | $p$ (MeV/$c$) |
|---|---|---|
| $(\rho\pi)_{S-\text{wave}}$ | seen | 353 |
| $(\rho\pi)_{D-\text{wave}}$ | seen | 353 |
| $(\rho(1450)\pi)_{S-\text{wave}}$ | seen | † |
| $(\rho(1450)\pi)_{D-\text{wave}}$ | seen | † |
| $\sigma\pi$ | seen | – |
| $f_0(980)\pi$ | not seen | 179 |
| $f_0(1370)\pi$ | seen | † |
| $f_2(1270)\pi$ | seen | † |
| $K\bar{K}^*(892)+$ c.c. | seen | † |
| $\pi\gamma$ | seen | 608 |

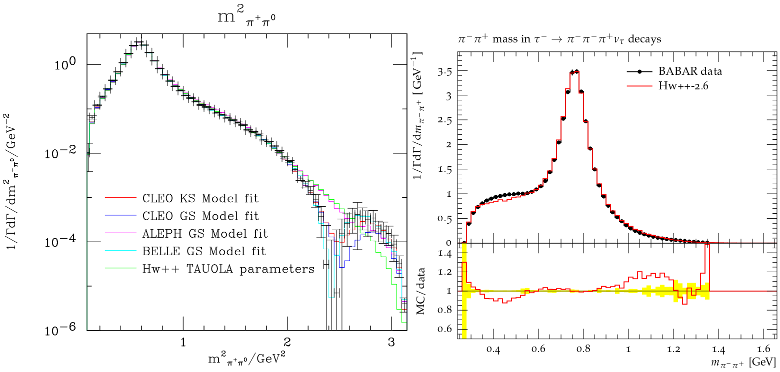
# Hadron Decays

- FORTRAN event generators typically used external packages:
  - TAUOLA $\tau$ lepton decays;
  - PHOTOS QED radiation in decays;
  - EVTGEN hadron, especially $B$ meson decays.
- Originally expected more of this in the new generation of programs.
- But better modelling requires passing more information between the different stages of event generation.
- Also many problems with interfaces.
- Net result: better simulation of hadron and $\tau$ lepton decays in all the new event generators and less use of external packages.

# $H^0, A^0 \to \tau^+\tau^- \to \pi^+\pi^-\nu_\tau\bar{\nu}_\tau$

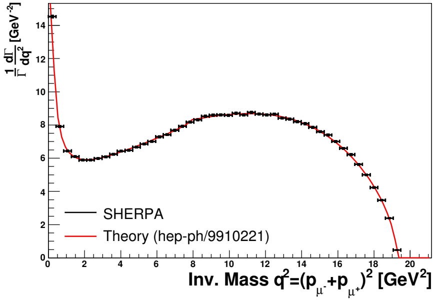


# $\tau \to \rho(a_1)\nu_\tau \to \pi\pi(\pi)\nu_\tau$

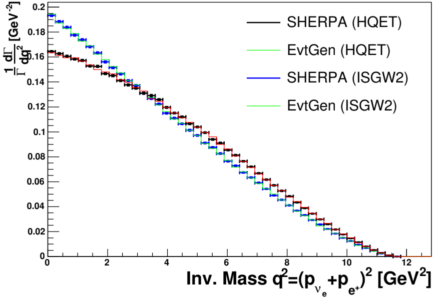


# $B$ decays

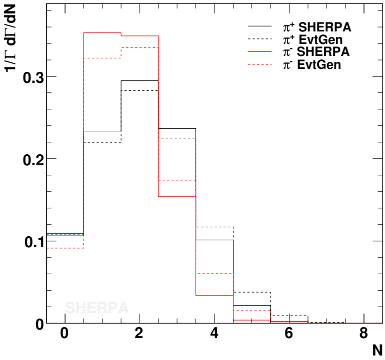
$B^+ \to K^{*+}\mu^+\mu^-$

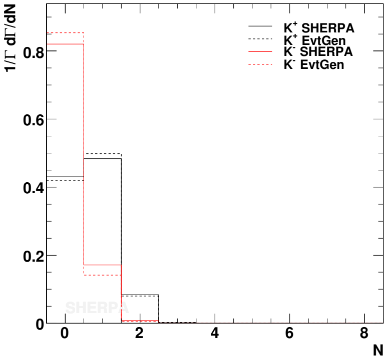



$B \to \bar{D}\ell\nu_\ell$



# Inclusive Observable for $B^+$ decay



# Summary

- Hadronization is described by non-perturbative models.
- Modern hadronization models give a good description of a wide range of processes.
- The parameters are universial allowing predictions once they are tuned to data.
- Don't forget about the hadron properties and decays.