Electron-Positron Pair Production from Multi-Photon Absorption

Christian Kohlfürst, Reinhard Alkofer, Holger Gies

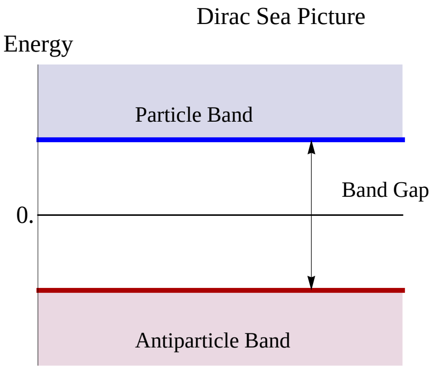
University of Graz
Institute of Physics

Physics in Intense Fields
Hamburg, July 10, 2013

Outline

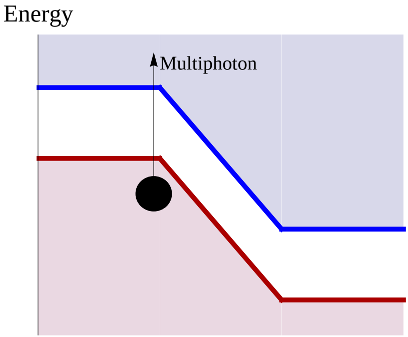
Outline

Dirac Sea Picture



- Blue: electron band, Red: positron band
- Measurement: Overcome band gap

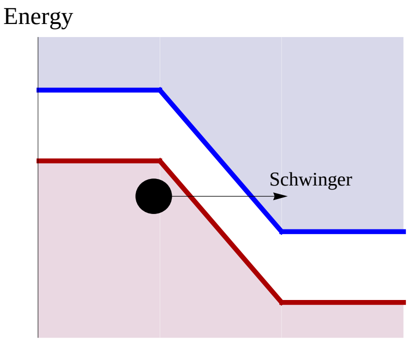
Multi-Photon Absorption



- Photon absorption $P \approx \left(\frac{eE\tau}{2m}\right)^{4m\tau}$
- Relies on photon energy

N. Narozhnyi: Sov. J. Nucl. Phys. 11(596), 1970

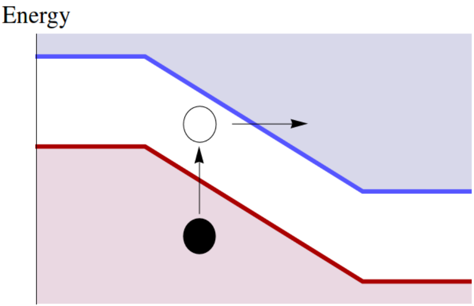
Schwinger Effect



- Electron tunneling $P \approx \exp(-\pi m^2/eE)$
- Relies on field strength $E_{cr} = 1.3 \cdot 10^{18}\, V/m$

F. Sauter: Z. Phys. 69(742), 1931
J. S. Schwinger: Phys. Rev. 82(664), 1951

Dynamical Pair Production



- Dynamically assisted pair production
- Combination of both effects

R. Schützhold et al.: Phys. Rev. Lett. 101:130404, 2008

Experiment

Objective

- Measure Schwinger effect

Problem

- No direct measurement in foreseeable future

Ideas

- Assistance by photon absorption
- Pulse shaping

Comparison to Atom Physics

Electric Field: $E(t) = \varepsilon \mathrm{Cos}(\omega t)$

	Atomic Physics	Pair Production
Keldysh-Parameter	$\gamma = \frac{\omega\sqrt{2E_{IP}}}{eE}$	$\gamma = \frac{\omega}{eE}$
Absorption($\gamma \gg 1$)	$P \sim \left(\frac{eE}{2\omega\sqrt{2E_{IP}}}\right)^{2(E_{IP}/\omega)}$	$P \sim (\frac{eE}{2\omega})^{4/\omega}$
Tunneling($\gamma \ll 1$)	$P \sim \exp(-\frac{2}{3}\frac{2(E_{IP})^{3/2}}{eE})$	$P \sim \exp(-\pi\frac{E_{cr}}{eE})$

L. V. Keldysh: Sov. Phys. JETP 20(1307), 1965

Outline

Quantum Vlasov Equation

$$\partial_t F(\mathbf{q},t) = W(\mathbf{q},t)\int_{t_{Vac}}^{t} dt' \, W(\mathbf{q},t')\,(1 - F(\mathbf{q},t'))\cos(2\theta(\mathbf{q},t,t')) \tag{1}$$

$$W(\mathbf{q},t) = \frac{eE(t)\,\varepsilon_\perp(\mathbf{q},t)}{\omega^2(\mathbf{q},t)}, \qquad \varepsilon_\perp^2(\mathbf{q},t) = m^2 + \mathbf{q}\perp^2, \qquad \mathbf{p} = \mathbf{q} - e\mathbf{A}$$

$$\theta(\mathbf{q},t,t') = \int_{t'}^{t} \omega(\mathbf{q},t'')\,dt'', \qquad \omega^2(\mathbf{q},t) = \varepsilon_\perp^2(\mathbf{q},t) + (q3 - eA(t))^2$$

- Non-Markovian integral equation, Rapidly oscillating term
- Fermions and Pauli statistics

S. A. Smolyansky et al. hep-ph/9712377 GSI-97-72, 1997

S. Schmidt et al.: Int.J.Mod.Phys. E7 709-722, 1998

Differential Equation

Auxiliary functions

$$G = \int_{t_{Vac}}^{t} dt' \, W(\mathbf{q}, t') \left(1 - F(\mathbf{q}, t')\right) \cos\left(2\theta(\mathbf{q}, t, t')\right) \tag{2a}$$

$$H = \int_{t_{Vac}}^{t} dt' \, W(\mathbf{q}, t') \left(1 - F(\mathbf{q}, t')\right) \sin\left(2\theta(\mathbf{q}, t, t')\right) \tag{2b}$$

Coupled differential equation

$$\begin{pmatrix} \dot{F} \\ \dot{G} \\ \dot{H} \end{pmatrix} = \begin{pmatrix} 0 & W & 0 \\ -W & 0 & -2\omega \\ 0 & 2\omega & 0 \end{pmatrix} \begin{pmatrix} F \\ G \\ H \end{pmatrix} + \begin{pmatrix} 0 \\ W \\ 0 \end{pmatrix} \tag{3}$$

J. C. R. Bloch et al.: Phys. Rev. D 60(116011), 1999

Pros and Cons of QKT

Positive Aspects

- Works for arbitrary time-dependent fields
- Insight into time evolution of system
- Gives momentum space distribution of particles
- Particle density easily calculable

Negative Aspects

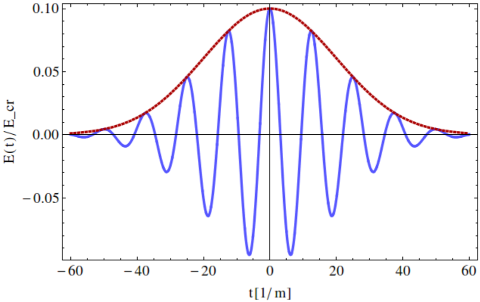
- Works only for spatial homogeneous fields
- No magnetic field present
- "Mean-Field" approximation

N.B.: Back-reaction and particle collisions can be included [1]

[1] J. C. R. Bloch et al.: Phys. Rev. D 60(116011), 1999

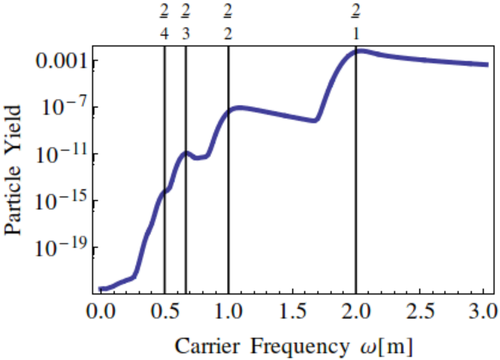
Outline

Electric Field



- Electric field: $E(t) = \varepsilon E_{cr} \mathrm{Exp}\left(-t^2/(2\tau^2)\right) \mathrm{Cos}(\omega t)$
- Photon energy: ω
- Field strength: ε

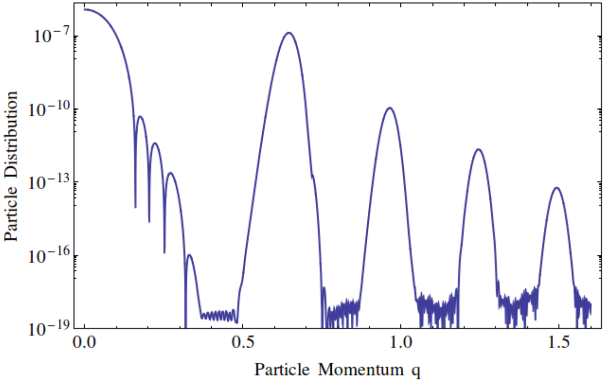
Carrier Frequency



Blue curve: $\varepsilon = 0.01, \tau = 12[1/m]$

- Resonant at n-photon frequencies
- The higher τ the more peaks at $2/n$ become visible

Particle Distribution



Blue curve: $\varepsilon = 0.1, \tau = 100[1/m], \omega = 0.4[m]$

- High peak around vanishing momentum
- Additional peaks at higher momenta q

Comparison to Atomic Physics

Above Threshold Ionization

- Multi-photon ionization above required ionization energy
- $E = (n+l)\hbar\omega - W$

P. Agostini et al.: Phys. Rev. Lett. 42, 1127-1130, 1979

Ponderomotive Energy

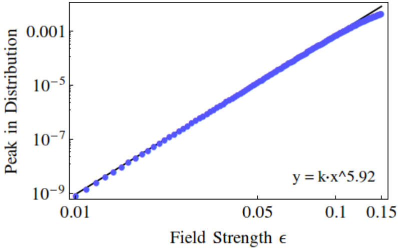
- Average oscillation energy of free electron in laser field
- $E = (n+l)\hbar\omega - E_{IP} - U_p$

Effective Mass

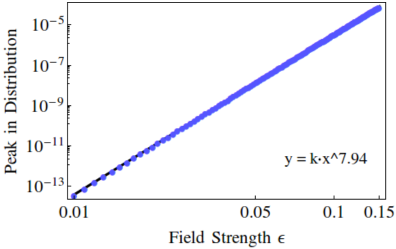
- Introduce $m^* = m\sqrt{1+2U_p}$
- $E = (n+l)\hbar\omega - 2m^*$

C. Harvey et al.: Phys. Rev. Lett. 109(100402), 2012

Multi-Photon Peaks



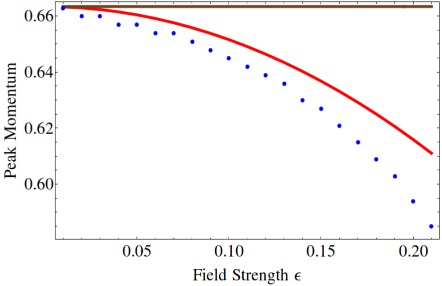
Blue: Peak height at $q = 0$

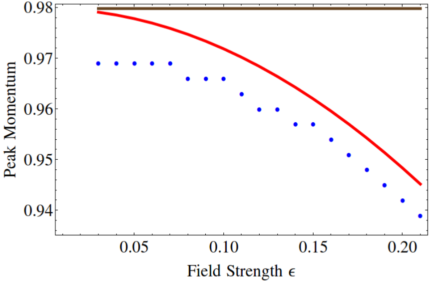


Blue: Peak height at first AT peak

- Power law behavior $\sim \varepsilon^{2l}$
- Configuration: $\omega = 2/3[m], \tau = 60[1/m]$

Multi-Photon Peaks and Effective Mass





Blue: First AT peak, QKT

Red: Effective Mass Estimate

Brown: Constant Mass Estimate

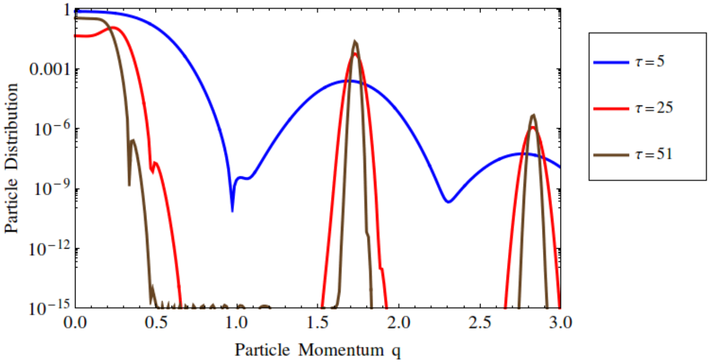
Blue: Second AT peak, QKT

Red: Effective Mass Estimate

Brown: Constant Mass Estimate

- Concept of effective mass meaningful
- Same qualitative behavior
- Configuration: $\omega = 0.4[m], \tau = 100[1/m]$

Pulse Length



- Similar to double slit in time
- Configuration: $\omega = 2[m], \varepsilon = 0.21$

E. Akkermans et al.: Phys. Rev. Lett. 108, 030401, 2012

Outline

Summary

- Quantum Kinetic Theory is capable of describing multi-photon processes
- Direct analogies between atomic physics and pair production
- Effective mass effects are observable

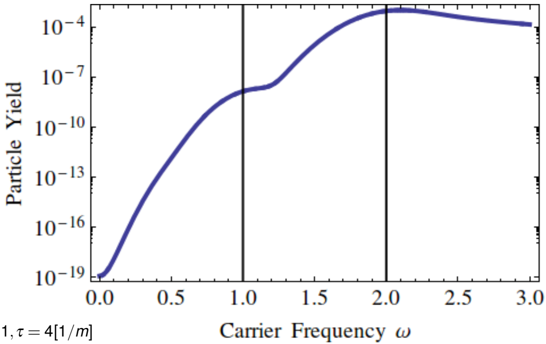
Outlook

- Understand full particle spectrum
- More analogies between atomic physics and pair production
- Include polarization degree of freedom
- Include back-reaction(effect of internal electric field)

Thank you!

supported by FWF Doctoral Program on **Hadrons in Vacuum, Nuclei and Stars** (FWF DK W1203-N16)

Carrier Frequency

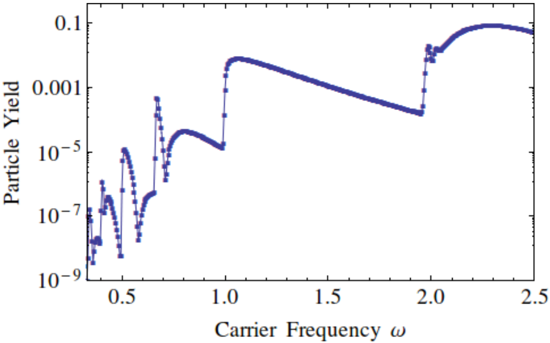


Blue curve: $\varepsilon = 0.01, \tau = 4[1/m]$

Black lines: $\omega = 2[m],\ \omega = 1[m]$

- Resonant at n-photon frequencies

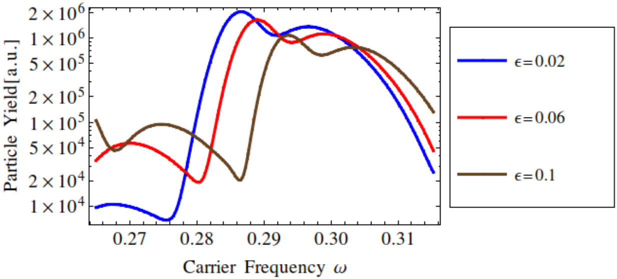
Carrier Frequency



Blue curve: $\varepsilon = 0.01, \tau = 100[1/m]$

- Resonant at n-photon frequencies

Carrier Frequency



General: Yield normalized via power law behaviour ε^{2l}

- Shift due to higher field strength
- Configuration: $\tau = 100[1/m], \omega_0 \sim 2/7 = 0.285714$

Problem Statement

Quantum electrodynamic

Only QED-Lagrangian and Dirac equation is known

$$\mathscr{L} = \bar{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi - \frac{1}{4}\mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu} \tag{4}$$

$$i\gamma^{\mu}\partial_{\mu}\psi - m\psi = e\gamma_{\mu}A^{\mu}\psi \tag{5}$$

Covariant derivative $D_{\mu} = \partial_{\mu} + ieA_{\mu}$
Field strength tensor $\mathscr{F}_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$

Open question

How to obtain statistical quantity $F(\mathbf{q}, t)$ from particle description?

Quantisation

Vector Potential

Spatial-independent vector potential in one direction

$$A_\mu = (0, A(t)\mathbf{e_3}) \tag{6}$$

- Very strong fields $\rightarrow$ regarded as classical
- Mean field approximation(background field)

S. Schmidt et al.: Int. J. Mod. Phys. E 7(709), 1998

F. Hebenstreit: Dissertation, 2011

Quantisation

Dirac Field

Fully quantized

$$\Psi \sim \chi^{+} a(q) + \chi^{-} b^{\dagger}(q) \tag{7}$$

Equation of motion

$$\left(\partial_t^2 + m^2 + (q_3 - eA(t))^2 + ieE(t)\right) \chi^{\pm} = 0 \tag{8}$$

Creation/Annihilation Operators

- Operators $a(q)$, $b^{\dagger}(q)$ hold information about particle statistics
- Fermions $\rightarrow$ anti-commutation relations

Operator Transformation

Hamiltonian

- Non-vanishing off-diagonal elements
- Diagonalization by Bogoliubov transformation
- Switching to quasi-particle picture

$$A(\mathbf{q},t) = \alpha(\mathbf{q},t)\, a(\mathbf{q}) - \beta^*(\mathbf{q},t)\, b^\dagger(-\mathbf{q}) \tag{9a}$$

$$B^\dagger(-\mathbf{q},t) = \beta(\mathbf{q},t)\, a(\mathbf{q}) + \alpha^*(\mathbf{q},t)\, b^\dagger(-\mathbf{q}) \tag{9b}$$

Bogoliubov coefficients have to fulfill $|\alpha(\mathbf{q},t)|^2 + |\beta(\mathbf{q},t)|^2 = 1$

Particle Distribution

One-particle distribution function

$$F(\mathbf{q},t) = \langle A^{\dagger}(\mathbf{q},t)\, A(\mathbf{q},t)\rangle \tag{10}$$

Fulfills equation of motion

$$\partial_t F(\mathbf{q},t) = S(\mathbf{q},t) \tag{11}$$

- Gives distribution in momentum space
- Time-dependent quantity
- Interpretation as electron/positron distribution for $t \to \pm\infty$ only

Magnetic Fields

Pure Magnetic Fields

- Particle creation not possible

Collinear EM-Fields

- Space-dependent vector potential $A_\mu(x,t) = (0,0,xB,A(t))$
- Additional constraints compared to pure electric case
- Quantised particle distribution
  $\varepsilon_\perp^2 \to m^2 + eB(2n+1+(-1)^s)$

N. Tanji: Ann. Phys. 8(324), 2009

Internal Electric Field

- Internal electric field $E_{\text{int}}(t)$ due to electron-positron pairs:

$$\mathbf{E}(t) = \mathbf{E}_{\text{ext}}(t) + \mathbf{E}_{\text{int}}(t) \quad \rightarrow \quad \dot{\mathbf{E}}(t) = -\mathbf{j}_{\text{ext}}(t) - \mathbf{j}_{\text{int}}(t)$$

Internal Electric Field

- Internal electric field $E_{\text{int}}(t)$ due to electron-positron pairs:

$$\mathbf{E}(t) = \mathbf{E}_{\text{ext}}(t) + \mathbf{E}_{\text{int}}(t) \quad \rightarrow \quad \dot{\mathbf{E}}(t) = -\mathbf{j}_{\text{ext}}(t) - \mathbf{j}_{\text{int}}(t)$$

- Internal electromagnetic current $j_{\text{int}}(t)$ in the **e3** direction:

$$j_{\text{int}}(t) = 4e \int \frac{d^3k}{(2\pi)^3} \frac{p3(t)}{\omega(\mathbf{q},t)} F(\mathbf{q},t) + \frac{4}{E(t)} \int \frac{d^3k}{(2\pi)^3} \omega(\mathbf{q},t) \dot{F}(\mathbf{q},t)$$

Internal Electric Field

- Internal electric field $E_{\text{int}}(t)$ due to electron-positron pairs:

$$\mathbf{E}(t) = \mathbf{E}_{\text{ext}}(t) + \mathbf{E}_{\text{int}}(t) \quad \rightarrow \quad \dot{\mathbf{E}}(t) = -\mathbf{j}_{\text{ext}}(t) - \mathbf{j}_{\text{int}}(t)$$

- Internal electromagnetic current $j_{\text{int}}(t)$ in the **e3** direction:

$$j_{\text{int}}(t) = 4e \int \frac{d^3k}{(2\pi)^3} \frac{p3(t)}{\omega(\mathbf{q},t)} F(\mathbf{q},t) + \frac{4}{E(t)} \int \frac{d^3k}{(2\pi)^3} \omega(\mathbf{q},t) \dot{F}(\mathbf{q},t)$$

- Conduction current stays regular for all momenta!
- Polarization current exhibits a logarithmic UV-divergence
- Charge renormalization
- Properly renormalized internal electromagnetic current:

Internal Electric Field

- Internal electric field $E_{\text{int}}(t)$ due to electron-positron pairs:

$$\mathbf{E}(t) = \mathbf{E}_{\text{ext}}(t) + \mathbf{E}_{\text{int}}(t) \quad \rightarrow \quad \dot{\mathbf{E}}(t) = -\mathbf{j}_{\text{ext}}(t) - \mathbf{j}_{\text{int}}(t)$$

- Internal electromagnetic current $j_{\text{int}}(t)$ in the **e3** direction:

$$j_{\text{int}}(t) = 4e \int \frac{d^3k}{(2\pi)^3} \frac{p3(t)}{\omega(\mathbf{q},t)} F(\mathbf{q},t) + \frac{4}{E(t)} \int \frac{d^3k}{(2\pi)^3} \omega(\mathbf{q},t) \dot{F}(\mathbf{q},t)$$

- Conduction current stays regular for all momenta!
- Polarization current exhibits a logarithmic UV-divergence
- Charge renormalization
- Properly renormalized internal electromagnetic current:

$$4e \int \frac{d^3k}{(2\pi)^3} \frac{p3(t)}{\omega(\mathbf{q},t)} \left[F(\mathbf{q},t) + \frac{\omega^2(\mathbf{q},t)}{eE(t)\,p3(t)} \dot{F}(\mathbf{q},t) - \frac{e\dot{E}(t)\,\varepsilon_\perp^2}{8\omega^4(\mathbf{q},t)\,p3(t)} \right]$$

Scattering picture

- $[\partial_t^2 + \omega^2(\mathbf{q},t)]g(\mathbf{q},t) = 0$
- 1-dimensional scattering problem

$$\mathscr{H}\psi(x) = \left[-\frac{\hbar^2}{2m}\partial_x^2 + V(x)\right]\psi(x) = E\psi(x)$$

Scattering picture

- $[\partial_t^2 + \omega^2(\mathbf{q}, t)] g(\mathbf{q}, t) = 0$
- 1-dimensional scattering problem

$$\mathscr{H}\psi(x) = \left[-\frac{\hbar^2}{2m}\partial_x^2 + V(x) \right] \psi(x) = E\psi(x)$$

- Formal similarity $\rightarrow$ Scattering potential: $V(t) \sim -\omega^2(\mathbf{q}, t)$
- Reflection coefficient $\leftrightarrow$ Produced pairs