On the properties of the boost modes and existence of the Unruh effect

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Physics in Intense Fields (PIF2013)
DESY Hamburg 9-11 July 2013

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Number of citations of the paper W. G. Unruh, "Notes on black-hole evaporation", Phys. Rev. D 14, 870-892 (1976)

PHYSICAL REVIEW D VOLUME 14, NUMBER 4 15 AUGUST 1976

Notes on black-hole evaporation

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This paper examines various aspects of black-hole evaporation. A two-dimensional model is investigated where it is shown that using fermion-boson cancellation on the stress-energy tensor reduces the energy outflow to zero, while other noncovariant techniques give the Hawking result. A technique for replacing the collapse by boundary conditions on the past horizon is developed which retains the essential features of the collapse while eliminating some of the difficulties. This set of boundary conditions is also suggested as the most natural set for a preexistent black hole. The behavior of particle detectors under acceleration is investigated where it is shown that an accelerated detector even in flat spacetime will detect particles in the vacuum. The similarity of this case with the behavior of a detector near the black hole is brought out, and it is shown that a geodesic detector near the horizon will not see the Hawking flux of particles. Finally, the work of Berger, Chitre, Nutku, and Moncrief on scalar geons is corrected, and the spherically symmetric coupled scalar-gravitation Hamiltonian is presented in the hope that someone can apply it to the problem of black-hole evaporation.

INTRODUCTION

Hawking[1,2] has recently advanced arguments to suggest that black holes evaporate, due to quantum particle creation, and behave as though they had an effective temperature of $(8\pi M)^{-1}$, with M the mass of the black hole.

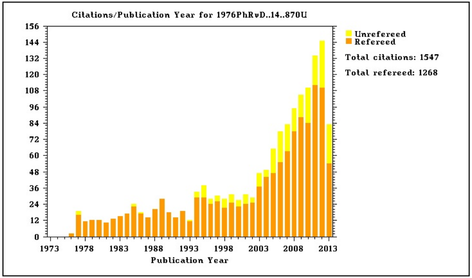
This paper is an attempt to investigate the assumptions which lead to this result, and so discover whether such an evaporation is physically reasonable or is the result of difficulty in defining physical particles on a non-flat-background spacetime.

Section I will examine the collapse of a spherical shell of matter to a black hole in a two-dimensional model. Here the field equations for massless fields are exactly solvable. Following the procedure used by Hawking gives a constant particle production rate long after collapse in this two-

the energy-momentum tensor would be canceled (i.e., the ω^4 divergences). The two-dimensional result suggests that black-hole evaporation may be linked to these divergent parts of the energy-momentum tensor.

Section II of the paper discusses an attempt to replace the difficult collapse problem with an equivalent formulation in full empty Schwarzschild spacetime. Here the collapsing body is replaced by boundary conditions on the past horizon of the black hole. In particular, the definition of positive frequency for normal modes which come out of the past horizon of the black hole is changed from the naive formulation. Instead of defining positive frequency for these modes via the timelike Killing vector, it is defined for the initial data on the past horizon via a null vector field which is of Killing type on the horizon (only).

The Feynman propagator is briefly investigated



The SAO/NASA Astrophysics Data System (ADS)

(http://www.adsabs.harvard.edu/), June 2013

Interest to Unruh effect had been growing essentially during 2000's

What the Unruh effect is? Analogy (paraphrasing)

QFT in EM background:

$$\left(i\gamma^i D_i - m\right)\Psi(x) = 0$$
$$D_i = \partial_i - ieA_i$$

Gauge transformations $U(1)$:

$A_i \to A_i + \frac{1}{e}\partial_i\Lambda(x)$,
$\Psi(x) \to e^{i\Lambda(x)}\Psi(x)$
$F_{ik} = \mathrm{inv}$

Total number of created pairs

e.g. $N_{e^-e^+} \propto \exp\left(-\frac{\pi E_S}{E}\right)$
is gauge-invariant.

QFT in gravitational background:

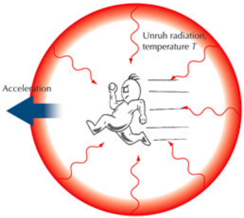
$$\left(\Box_g + m^2\right)\Phi(x) = 0$$
$$\Box_g = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}\left(g^{ik}\sqrt{-g}\frac{\partial}{\partial x^k}\right)$$

Analogue of gauge transformations in GR: general coordinate transformations $\mathrm{diff}(\mathcal{M})$: ${x'}^i = {x'}^i(x^1, \ldots x^D)$, $\Phi'(x') = \Phi(x)$

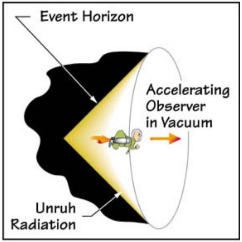
Particle content depends on the observer (its proper RF)!!! (even though $\bar{T}^{i'}_{k'} \propto \bar{T}^i_k = 0$)

Reveals (?!) even in Minkowski space (i.e., in the absence of "true" gravitational fields, that might cause pair creation).

What the Unruh effect is? - precise definition



picture from www.extreme-light-infrastructure.eu



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According to Unruh (1976), any (!!!) detector moving with a constant proper acceleration g in empty Minkowski spacetime (MS) responds as if it had been immersed in a thermal bath of Fulling particles at the Davies-Unruh temperature

$$T_{DU} = \frac{\hbar g}{2\pi c k}. \qquad (*)$$

More precisely, the Unruh effect means that from the point of view of a uniformly accelerated observer the usual QFT vacuum in MS occurs as a mixed state described by the thermal density matrix with the effective temperature (*).

- An attempt to establish mapping between the perspectives of the inertial and uniformly accelerated observers in MS
- Most important feature: **universality** (independence on the structure and nature of a detector and the method applied for its acceleration)!

Two aspects of considerations of Unruh effect

Unruh problem (both original AND the successive considerations)

I. "QFT" aspect

Purely QFT (universal) level: abstract discussion of possible relations between the vacua and (multi-)particle states in the inertial and accelerated RFs, almost no reference to detectors is needed at all. **Sort of a new knowledge about the interplay between QFT and GR!?**

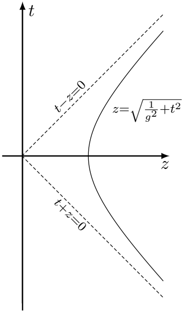
II. "Detector" aspect

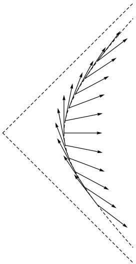
Consideration, listing and comparison of responses **of particular detectors** (Unruh-DeWitt, elementary particle, model atoms, etc.), sometimes constant proper acceleration is supported by a certain explicitly introduced external force. Calculations can be realized in inertial RF (!), **but universality can be never proved!**

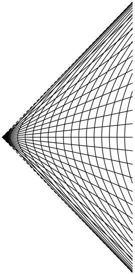
I. THE "QFT" ASPECT

"We emphasize that, although it is fine to interpret laboratory observables from the point of view of uniformly accelerated observers, **the Unruh effect itself does not need experimental confirmation any more than free quantum field theory does**." [L.C.B. Crispino, A. Higuchi, and G.E. Matsas, "The Unruh effect and its applications", RMP **80**, 787 (2008).]

What is an "accelerated observer" (uniformly accelerated rigid reference frame)?

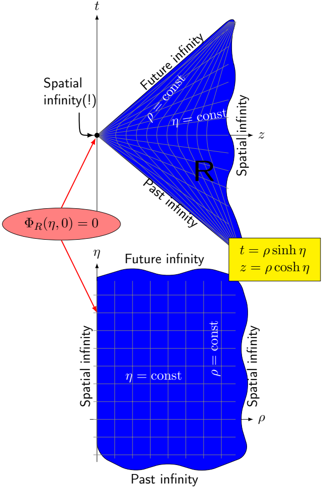






- Covers only a quarter (right Rindler wedge) of MS ☹.
- Explicitly, $t = \rho \sinh \eta$, $z = \rho \cosh \eta$ $(0 < \rho < \infty,\ -\infty < \eta < +\infty)$.
- Different world lines acquire different proper accelerations ☺: $g = 1/\rho$ (enforced by Born rigidity, M. Born, Ann. der Physik **335**, 1 (1909))
- The Jacobian $\frac{\partial(t,z)}{\partial(\eta,\rho)} = \rho = \sqrt{z^2 - t^2}$ - RF is singular at the event horizons $t \pm z = 0$ (!!).
- Metrics: $ds^2 = \rho^2 d\eta^2 - d\rho^2$, and $R_{ijkl} = 0$ except for the horizons (!), see A. Einstein and N. Rosen, Phys. Rev **48**, 73 (1935); A.I. Nikishov and V.I. Ritus, JETP **94**, 31 (1988).
- Further reading: C. Møller, "The Theory of Relativity", Oxford Univ. Press, London (1952); F. Rohrlich, Ann. Phys. **22**, 169 (1963); J.D. Hamilton, Am. J. Phys. **46**, 83 (1978).

Prehistory: quantization in R-wedge [S.A. Fulling (1973)]



- $ds^2 = \rho^2 d\eta^2 - d\rho^2$, $\hat{\Phi}_R(\eta, 0) = 0$
- $\left\{ \frac{1}{\rho^2} \frac{\partial^2}{\partial \eta^2} - \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) + m^2 \right\} \hat{\Phi}_R(\eta, \rho) = 0$
- $\Phi_\mu(\eta, \rho) = \frac{\sqrt{\sinh(\pi\mu)}}{\pi} K_{i\mu}(m\rho) e^{-i\mu\eta}, \quad \mu > 0$
- $\hat{\Phi}_R(\eta, \rho) = \int\limits_0^\infty d\mu \left\{ \Phi_\mu(\eta, \rho) \hat{c}_\mu + \Phi^*_\mu(\eta, \rho) \hat{c}^\dagger_\mu \right\}$
- $[\hat{c}_\mu, \hat{c}^\dagger_{\mu'}] = \delta(\mu - \mu'), \quad \hat{c}_\mu |0_R\rangle = 0$
- If $\hat{\Phi}_R(\eta, \rho) = \hat{\Phi}_M(t = \rho \sinh \eta, z = \rho \cosh \eta)$ in R, then

$$\hat{c}_\mu = \int\limits_{-\infty}^{+\infty} dp \, \frac{\left\{ e^{\pi\mu/2} \hat{a}_p + e^{-\pi\mu/2} \hat{a}^\dagger_p \right\}}{2\sqrt{\pi \varepsilon_p \sinh(\pi\mu)}} \left(\frac{\varepsilon_p + p}{m} \right)^{-i\mu},$$

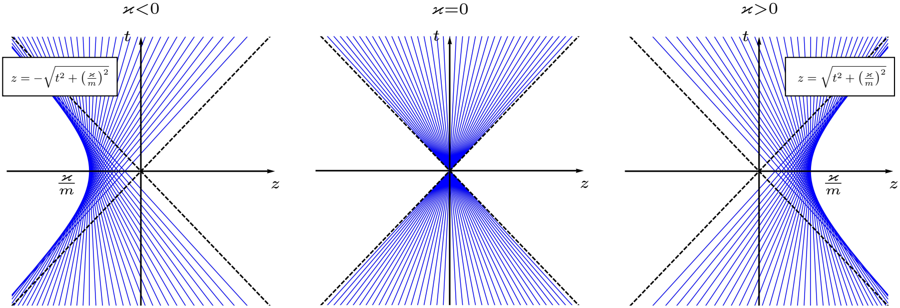
so that $|0_R\rangle \neq |0_M\rangle$. However, due to different boundary conditions the two problems are essentially different and hardly can be compared. Besides, only a half of degrees of freedom seems to be in use...

Boost states: classical considerations ($\varkappa \gg 1$)

- Free pointlike particle, classical case (motion along definite worldline $z(t)$)
- Energy in a uniformly accelerated (primed) RF:

$$\varepsilon' = mgz' = \frac{mg(z - \dot{z}t)}{\sqrt{1 - \dot{z}^2}} \eqqcolon \hbar\varkappa g = \text{const} \quad \text{(Clairaut's equation)}$$

- Boost number $\hbar\varkappa = z\varepsilon - tp = M^{zt}$ ($M^{ik} = x^i P^k - x^k P^i$ – 4D angular momentum) – similar to angular momentum (e.g. M^{xy})
- Regular and singular solutions:



Classically, particles with $\varkappa > 0$ never appear in the left wedge! (are confined between L-wedge and world line of accelerated observer, $1/g \leftrightarrows \varkappa/m$)

Boost modes: quantum considerations ("first quantization")

- For $\varkappa \lesssim 1$ the worldlines in the right wedge are confined to the region of size $\lesssim l_C = 1/m \quad \Longrightarrow \quad$ quantum considerations are required
- "First quatization": $p \to \hat{P}^t = -i\hbar\partial/\partial z$, $\varepsilon_p \to \hat{P}^z = i\hbar\partial/\partial t \quad \Longrightarrow \quad \varkappa \to \hat{\mathcal{B}} \equiv \hat{M}^{zt}/\hbar = i\left(z\partial/\partial t + t\partial/\partial z\right)$ – generator of Lorentz boosts in MS
- Poincaré algebra

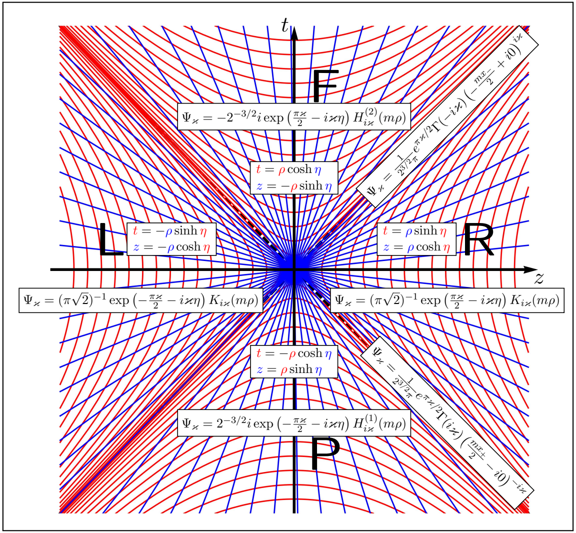
$$
\begin{aligned}
&[\hat{M}^{\mu\nu}, \hat{M}^{\rho\sigma}]_- = i\left(g^{\nu\rho}\hat{M}^{\mu\sigma} - g^{\mu\rho}\hat{M}^{\nu\sigma} - g^{\sigma\mu}\hat{M}^{\rho\nu} + g^{\sigma\nu}\hat{M}^{\rho\mu}\right), \\
&[\hat{M}^{\rho\sigma}, \hat{P}^{\mu}]_- = i\left(g^{\mu\sigma}\hat{P}^{\rho} - g^{\mu\rho}\hat{P}^{\sigma}\right), \quad [\hat{P}^{\mu}, \hat{P}^{\rho}]_- = 0.
\end{aligned}
$$

- $[\hat{\mathcal{B}}, \hat{P}^i\hat{P}_i - m^2]_- = 0$ – Lorentz invariance of free scalar field in MS. There exist solutions $\Psi_\varkappa(x)$ for KFG equation with $\hat{\mathcal{B}}\Psi_\varkappa = \kappa\Psi_\varkappa$ – the boost modes
- Take the plane wave *at rest* e^{-imt} and apply the projector $\hat{\Omega}_\varkappa = 2^{-1/2}\delta(\hat{\mathcal{B}} - \varkappa)$:

$$
\boxed{\Psi_\varkappa(x) = \hat{\Omega}_\varkappa e^{-imt} = \int_{-\infty}^{+\infty} \frac{dq}{2\pi\sqrt{2}} e^{-iq(\varkappa - \hat{\mathcal{B}})} e^{-imt} = \int_{-\infty}^{+\infty} \frac{dq}{2\pi\sqrt{2}} e^{im(z\sinh q - t\cosh q) - i\varkappa q}}
$$

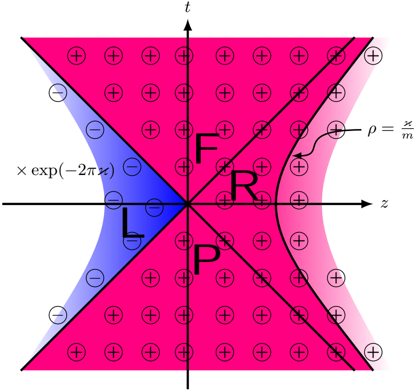
– orthonormal set of wave packets, each assembled from solely positive energy plane waves. Energy sign is preserved by boosts! q – "rapidity". Localized states, but the average energy $= \infty$ – infinite oscillations at horizons

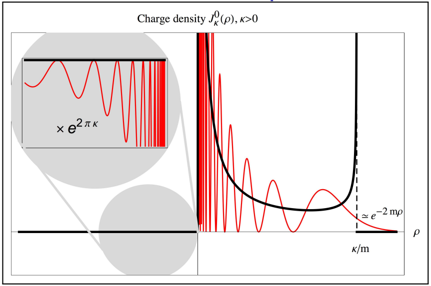
Boost modes: explicit expressions in the wedges of MS – analytical continuation of Fulling modes across the horizons



U. H. Gerlach, PRD **40**, 1037 (1989); N.B. Narozhny, A.M.F. et al, PRD **65**, 025004 (2002).

Boost modes ($\varkappa > 0$), visual image: charge distribution in locally uniformly accelerated RF in Minkowski Spacetime





- Black curve: "classical case"

$$n(z,t) = \int_{-\infty}^{+\infty} \delta(\varepsilon_p z - pt - \varkappa) \frac{dp}{2\pi\hbar}$$

- Current density for boost mode: $J^i_\varkappa = i\Psi^*_\varkappa(x) \frac{\overleftrightarrow{\partial}}{\partial x_i} \Psi_\varkappa(x)$, $J^{0'}_\varkappa = \frac{\partial x^{0'}}{\partial x^i} J^i_\varkappa$
- Charge is almost confined to the classically defined domain, except for the exponential [$\propto \exp(-2\pi\varkappa)$] tails - resulting later in "thermal" spectrum
- Charge density in the left wedge is negative!(vacuum polarization) "Thermal" factors?! V.I. Ritus, JETP **93**, 211 (2001).

Boost modes: important properties

$$\boxed{\Psi_\varkappa(x) = \int\limits_{-\infty}^{+\infty} \frac{dq}{2\pi\sqrt{2}} e^{im[z\sinh q-(t-i0)\cosh q]-i\varkappa q}}$$

- Conjugate solutions: $e^{-imt} \xrightarrow{\hat{\Omega}_\varkappa} \Psi_\varkappa$, $e^{+imt} \xrightarrow{\hat{\Omega}_\varkappa} \Psi^*_\varkappa$. Apart from normalization, Ψ_μ and $\Psi^*_{-\mu}$ $(\mu > 0)$ are different branches of analytical continuation of Fulling mode Φ_μ across the horizons.
- $\boxed{\Psi^*_\varkappa(-x) = \Psi_{-\varkappa}(x)}$ (PT-transformation).
- $e^{-i\hat{\mathcal{B}}q}\Psi_\varkappa(x) = \Psi_\varkappa(x(q)) = \Psi_\kappa(t\cosh q + z\sinh q, t\sinh q + z\cosh q)$. Let $q = q' + iq''$.
  Then $\mathrm{Im}\, t(q'+iq'') = \mathrm{Im}\,[t(q')\cos q'' + iz(q')\sin q''] = z(q')\sin q'' < 0$ for $-\pi < q'' < 0$ in R $(z(q') > 0)$ or for $0 < q'' < \pi$ in L $(z(q') < 0)$. On the other hand, $x(q = \pm i\pi) = -x$. Hence,

$$\boxed{\Psi_\varkappa(-x) = e^{\mp\pi\varkappa}\Psi_\varkappa(x), \quad x \in R(L)}$$

- By turn, $Q_L[\Psi_\varkappa] = -e^{-2\pi\varkappa}Q_R[\Psi_\varkappa]$, or (assuming $Q_\Sigma = Q_L + Q_R = 1$):
  $Q_L[\Psi_\varkappa] = -\dfrac{1}{e^{2\pi\varkappa}-1}$, $Q_R[\Psi_\varkappa] = \dfrac{e^{2\pi\varkappa}}{e^{2\pi\varkappa}-1}$ – "thermal" factors?!

Boost modes: "second quantization"

- Field operator: $\hat{\Phi}_M(x) = \int\limits_{-\infty}^{+\infty} d\varkappa \left\{ \Psi_\varkappa(x)\hat{b}_\varkappa + \Psi^*_\varkappa(x)\hat{b}^\dagger_\varkappa \right\}$, where $[\hat{b}_\varkappa, \hat{b}^\dagger_{\varkappa'}]_- = \delta(\varkappa - \varkappa')$ – boost annihilation/creation operators.
- $\hat{M}^{zt} = \int\limits_{-\infty}^{+\infty} \varkappa \hat{b}^\dagger_\varkappa \hat{b}_\varkappa \, d\varkappa$ ("secondly quantized" boost gets diagonalized)
- Relation with the commonly used "plane wave" operators:

$$\hat{b}_\varkappa = \int\limits_{-\infty}^{+\infty} \frac{dp}{\sqrt{2\pi\varepsilon_p}} \left(\frac{\varepsilon_p + p}{m}\right)^{i\varkappa} \hat{a}_p, \quad \hat{a}_p = \frac{1}{\sqrt{2\pi\varepsilon_p}} \int\limits_{-\infty}^{+\infty} d\varkappa \left(\frac{\varepsilon_p + p}{m}\right)^{-i\varkappa} \hat{b}_\varkappa$$

  – no admixture of creation operators appears!
- $\hat{b}_\kappa|0_M\rangle = 0$ – Minkowski vacuum remains unchanged (...no creation of real particles is present...) in boost representation!

Boost modes: completeness, singularity at horizons and the "zero" mode

- Covariant completeness condition: decomposition of the Wightman function $\Delta^{(+)}(x,x') \doteqdot i\langle 0_M|\hat{\Phi}_M(x)\hat{\Phi}_M(x')|0_M\rangle$. Due to translational invariance of free field in MS, $\Delta^{(+)}(x,x') \equiv \Delta^{(+)}(x-x')$.
- We have

$$\Delta^{(+)}(x-x') = i\int_{-\infty}^{+\infty} d\varkappa\, \Psi_\varkappa(x)\Psi^*_\varkappa(x') = i\int_{-\infty}^{+\infty} d\varkappa\, \Psi_\varkappa(x-x')\Psi^*_\varkappa(0) \qquad (*)$$

Since $\Delta^{(+)}(\cdot)$ is Lorentz invariant, and (*) gives it's decomposition into irreducible representations, we must have $\Delta^{(+)} \propto \Psi_0$ and $\Psi_\varkappa(0) \propto \delta(\varkappa)$. With our normalization,

$$\boxed{\Delta^{(+)}(x) = \frac{i}{\sqrt{2}}\Psi_0(x), \quad \Psi_\varkappa(0) = \frac{1}{\sqrt{2}}\delta(\varkappa)}$$

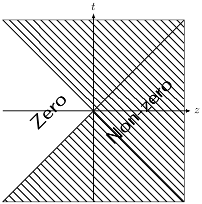
- At horizons $x_\pm \to 0$,

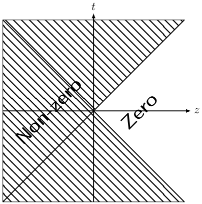
$$\Psi_\varkappa(x) = \frac{1}{\sqrt{2}}\delta(\varkappa) \underbrace{- \frac{1}{2\pi\sqrt{2}} e^{\pi\varkappa/2}\Gamma(\mp i\varkappa)\left(\mp\frac{mx_\pm}{2} \pm i0\right)^{\pm i\varkappa} + \dots}_{\text{more regular terms}}$$

- The zero mode coincides with Wightman function and provides the essential [the only] contribution to the field operator at the horizons [origin]!($\neq 0$ spectral measure)

Unruh modes and Unruh quantization: $\Psi_\varkappa \mapsto R_\mu, L_\mu \ (\mu > 0)$

$$R_\mu(x) = \frac{\Psi_\mu(x) - e^{-\pi\mu}\Psi^*_{-\mu}(x)}{\sqrt{1-e^{-2\pi\mu}}} \qquad L_\mu(x) = \frac{\Psi^*_{-\mu}(x) - e^{-\pi\mu}\Psi_\mu(x)}{\sqrt{1-e^{-2\pi\mu}}}$$



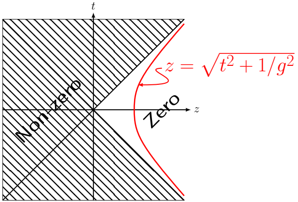


$$\hat{\Phi}_M = \int\limits_{-\infty}^{+\infty} d\varkappa \left\{\hat{b}_\varkappa \Psi_\varkappa + \hat{b}^\dagger_\varkappa \Psi^*_\varkappa\right\} \overset{?!}{=} \int\limits_{|\varkappa|>0} d\varkappa \left\{\hat{b}_\varkappa \Psi_\varkappa + \hat{b}^\dagger_\varkappa \Psi^*_\varkappa\right\} =$$

$$= \int\limits_0^\infty d\mu \left\{ \hat{b}_\mu \Psi_\mu + \hat{b}^\dagger_\mu \Psi^*_\mu \right\} + \int\limits_0^\infty d\mu \left\{ \hat{b}_{-\mu} \Psi_{-\mu} + \hat{b}^\dagger_{-\mu} \Psi^*_{-\mu} \right\} =$$

$$= \int\limits_{-\infty}^{+\infty} d\mu \left\{\hat{r}_\mu R_\mu + \hat{r}^\dagger_\mu R^*_\mu + \hat{l}_\mu L^*_\mu + \hat{l}_\mu L^*_\mu\right\}$$

[QFT aspect of] Unruh effect: sketch of derivation



Uniformly accelerated observer is confined to the right wedge and is incapable of detection of L–particles.
Elimination of [summing over] the (non-observable) degrees of freedom of the L–particles [information loss] is equivalent to introduction of a thermal density matrix with Davies-Unruh temperature $T_{DW} = g/(2\pi)$ [g – proper acceleration]?!

- $$\hat{r}_\mu = \frac{\hat{b}_\mu - e^{-\pi\mu}\hat{b}^\dagger_{-\mu}}{\sqrt{1 - e^{-2\pi\mu}}}, \quad \hat{l}_\mu = \frac{\hat{b}_{-\mu} - e^{-\pi\mu}\hat{b}^\dagger_{\mu}}{\sqrt{1 - e^{-2\pi\mu}}},$$
$$[\hat{r}_\mu, \hat{r}_{\mu'}]_- = [\hat{l}_\mu, \hat{l}_{\mu'}]_- = \delta(\mu - \mu')$$
- Average number of R–particles in Minkowski vacuum:
$$\bar{N}_R = \langle 0_M | \hat{r}^\dagger_\mu \hat{r}_\mu | 0_M \rangle = \frac{\delta(\mu' - \mu)|_{\mu'=\mu}}{e^{2\pi\mu} - 1}$$
- $$\delta(\mu' - \mu)|_{\mu'=\mu} = \int_{-\infty}^{+\infty} \frac{d\eta}{2\pi} e^{i(\mu-\mu)\eta} = \frac{g\Delta\tau}{2\pi}$$
$\tau = \eta/g$ – proper time; $\omega = g\mu$ – (dimensional) Fulling energy of R– particle in right wedge
- Thermal bath of **real** Fulling-Rindler particles:
$$\boxed{\frac{\Delta \bar{N}_R}{\Delta\tau} = \int_0^\infty \frac{d\omega}{2\pi} \frac{1}{e^{2\pi\omega/g} - 1}}$$

Unruh effect: objections

- Interpretation [particle interpretation of R – and L – modes] in Unruh effect appeals to the Fulling quantization in R-wedge. However, (double) Fulling quantization is based on boundary conditions which differ from those in Minkowski spacetime [P.K. Silaev and O.A. Khrustalev, Teor. Mat. Fiz. **91**, 217 (1992); V. A. Belinskii et al, JETP Lett. **65** 902 (1997); A.M.F. et al, PLA **254**, 126 (1999)].
- In the chain of equalities

$$\hat{\Phi}_M = \int_{-\infty}^{+\infty} d\varkappa \left\{ \hat{b}_\varkappa \Psi_\varkappa + \hat{b}_\varkappa^\dagger \Psi_\varkappa^* \right\} \overset{?!}{=} \int_{|\varkappa|>0} d\varkappa \left\{ \hat{b}_\varkappa \Psi_\varkappa + \hat{b}_\varkappa^\dagger \Psi_\varkappa^* \right\} = \ldots$$

$$\ldots = \int_{-\infty}^{+\infty} d\mu \left\{ \hat{r}_\mu R_\mu + \hat{r}_\mu^\dagger R_\mu^* + \hat{l}_\mu L_\mu^* + \hat{l}_\mu L_\mu^* \right\}$$

establishing transition to Unruh quantization, on second step one has to substitute

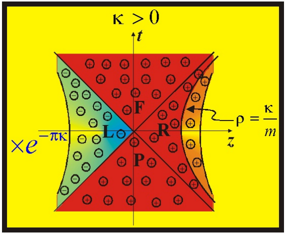
$$\int_{-\infty}^{+\infty} d\varkappa \ldots \longrightarrow \text{P.v.} \int_{-\infty}^{+\infty} d\varkappa \ldots$$

Thus the zero mode is being leaved omitted!However, at the origin and horizons

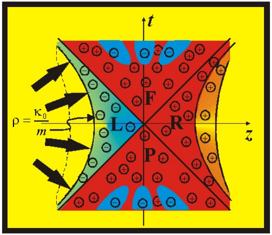
$$\Psi_\varkappa|_{\text{horizons}} = \frac{1}{\sqrt{2}} \delta(\varkappa) \text{ [+ possible regular terms].}$$

So that, such an operation may be correct only if the light cone is being cut out from MS. However, the resulting spacetime would have nothing to do with MS [N.B. Narozhny, A.M.F. et al, PRD **65**, 025004 (2002); *ibid* **70**, 048702 (2004)]!

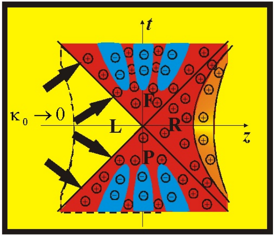
Possible pictorial explanation of production of **real R– particles** in Unruh effect



Only virtual pairs



Virtual+real pairs



Real pairs

By attempting to press the negative-charge tail of virtual boost particles out of the left wedge (e.g., using a mirror constraining the field there), real particles are created. The magnitude of the effect is $\sim e^{-2\pi\mu}$ $(\mu = \kappa \gg 1)$

Mathematical subtleties: comment by Fulling and Unruh

S.A. Fulling, W.G. Unruh, PRD **70**, 048701 (2004):

In response to the authors' Reply [16]: We acknowledge that the statement on p. 879 of [2] is technically incorrect when applied to distributions. However, the conclusions of [2] and the arguments of this Comment do not depend on that claim. The Cauchy problem for the wave equation is well posed within certain function spaces, such as smooth functions of compact support (in the roles of initial and final data). Such a function can be expanded in the boost eigenfunctions; the expansion is an integral, not a sum, so it is unaffected by the omission of one point of the spectrum. In this sense, the paragraph of [2] in question is correct in concluding "The expansion of Eq. (2.12) is then valid in the full Minkowski space-time." In principle one need never use (2.12) in Minkowski space except with smooth test functions, and hence the omission of the zero boost mode will not affect the calculations.

- The quantum field $\hat{\Phi}(x)$, from the point of view of mathematics, is an "operator-valued *distribution* (generalized function)". In particular, the operator $\hat{\Phi}^2(x)$ is poorly defined on any dense subspace of the Fock space of quantum states $|\cdot\rangle$.
- Any QFT must in fact be formulated in terms of merely "smeared fields"

$$\hat{\Phi}_M[f] \doteq \int_{\text{MS}} f(x)\hat{\Phi}_M(x)\, d^2x, \quad f \in \mathcal{T},$$

and their polynomial algebra, where $\mathcal{T}$ is a suitable functional space of "test functions".

- Seemingly, the origin and the horizons (where there indeed are some technical difficulties), being of "zero measure", do not contribute to the smeared fields (are "smeared out"), and thus can be safely ignored?!

Mathematical subtleties: smeared fields [reply]

- In spite of many attempts, as by now, it has been not yet demonstrated that the rigorous axiomatic (or algebraic) approach [based on the polynomial algebra of smeared fields] is exactly equivalent (i.e. can be used instead of) the usual QFT. In particular, it is not enough clear how to treat such important quantities, as Lagrangian, Stress-Energy Tensor (energy, momentum,...), etc.
- Singularities at the light cone are the essential ingredients of a usual QFT. Exclusion of zero mode is equivalent to exclusion of a Wightman function [$\Delta^{(+)}(x) \propto \Psi_0(x)$].
- In usual QFT [LSZ-formalizm], matrix elements between the physically realizable [wave packet] Fock states are already good functions, and in principle need not be necessarily smeared further over space and time. Smearing over $\varkappa$ vs smearing over spacetime?
- The discussion is about whether or not

$$\lim_{\varepsilon\to 0} \int\limits_{|\varkappa|<\varepsilon} d\varkappa \left\{ \hat{b}_\varkappa \Psi_\varkappa[f] + \text{h.c.} \right\} \to 0. \quad (*)$$

However, an order of limits (which depends on physics) is crucial here, because for each ε there always exist $f_\varepsilon \in \mathcal{T}$, such that the matrix elements of (*) remain arbitrary large.

- [In reply to L.C.B. Crispino, A. Higuchi, G.E.A. Matsas, RMP **80** 787 (2008)] The completeness relation is of the form:

$$\Delta^{(+)}(x-x') = i\int\limits_{-\infty}^{+\infty} d\varkappa\, \Psi_\varkappa(x)\Psi^*_\varkappa(x') = i\int\limits_{-\infty}^{+\infty} d\varkappa\, \Psi_\varkappa(x-x')\Psi^*_\varkappa(0) \qquad (**)$$

(the last step is due to translation invariance, i.e. is always possible in MS). From here it is evident that smearing over x and x' does not effect the contribution of the zero mode:

$$\Delta^{(+)}[f,f'] \risingdotseq \int\limits_{\text{MS}\times\text{MS}} d^2x\, d^2x'\, \Delta^{(+)}(x-x')f(x)f'(x') =$$

$$= i\int\limits_{\text{MS}\times\text{MS}} d^2x\, d^2x'\, f(x)f'(x') \int\limits_{-\infty}^{+\infty} d\varkappa\, \Psi_\varkappa(x-x')\Psi^*_\varkappa(0) =$$

$$= i\int\limits_{-\infty}^{+\infty} d\varkappa\, \Psi_\varkappa[f*f']\Psi^*_\varkappa(0)$$

($f*f'$ – some convolution). Hence, either zero mode all the same can not be ignored even under smearing, or translation invariance gets lost.

Mathematical subtleties: few words on algebraic approach

- In fact, there is also another subtlety: strictly speaking, Unruh density matrix does not exist at all (relevant partition function diverges). This is just an indication that Unruh quantization (if was correct) would be unitary inequivalent to the usual one ["plane waves", "boost modes", etc.] in Minkowski space.
- Such situations often arise in quantum statistical mechanics if the system is not immersed into a box but is rather considered in thermodynamic limit $V \to \infty$, $\bar{N} \to \infty$, $n = \bar{N}/V =$ const and $T =$ const.
- The problem can be regularized by introducing a box, which isolates the horizons (this at the same time allows to ignore the zero mode as well) [e.g., U. Gerlach, PRD **40**, 1037 (1989)]... but the problem obviously becomes about a "double Rindler wedge", i.e. unrelated to Minkowski spacetime.
- In principle, there is no need for such a regularization, since the situation can be well handled by different alternative formalisms (thermofield dynamics, etc.). One of them is algebraic approach.

- In algebraic approach, thermal states are treated as Kubo-Martin-Schwinger [KMS] states, this is some sort of "screw periodicity" of general non-equilibrium Green functions in imaginary time with period $2\pi\beta$ [this can be reformulated on a language of "algebraic states" as well]. For usual cases [e.g., infinitely extending non-relativistic Bose-gas], these KMS states bear all the pleasant and well known features of usual thermal states.
- The Bisogniano-Wichmann (BW) theorem states that the [algebraic] vacuum state of a field [satisfying some reasonable set of axioms, e.g. Wightman axioms] in Minkowski spacetime, being restricted to the "algebra of observables localized entirely in the right wedge", satisfies the KMS condition with Davies-Unruh temperature with respect to local Rindler time. The proof follows line of arguments very similar to those for a famous CPT theorem.
- It is a common belief [after G. Sewell: Ann. Phys. (NY) **141** 201 (1982)] that the BW theorem provides "a mathematically rigorous, universal and general proof" for existence of the Unruh effect, thus avoiding all the potential technical problems.

- However, a technically mandatory restriction to the "algebra of observables localized entirely in the right wedge" means restriction to only those test functions, which rapidly enough decay when approaching the horizons. This is obviously the same problem as formulated above. In the whole algebra of observables of the field in MS there exist the observables, that "catch on" the horizons or the origin, and for them the KMS property does not reveal [A.M.F. et al, PLA **254**, 126 (1999); N.B. Narozhny, A.M.F. et al, PRD **65**, 025004 (2002)]. Related issue: thermal properties take origin in non-simply connected topology of Wick rotated Rindler wedge [S.M. Christensen and M.J. Duff, Nucl. Phys. B **146**, 11 (1978); W. Troost and H. Van Dam, Nucl. Phys. B **152**, 442 (1979)].
- In addition, unlike the Unruh quantization, the algebraic approach proves only a formal KMS property, but tells nothing about the "particle content of the thermal bath" [N.B. Narozhny, A.M.F. et al, PRD **65**, 025004 (2002), A. Arageorgis et al, Phil. of Science **70**, 164 (2003)]. In particular, this derivation leaves totally unclear whether the "thermal bath" is indeed populated by "real" thermal Fulling-Rindler quanta or is just simulated by vacuum noise.

II. THE “DETECTOR” ASPECT

It is absolutely of no surprise that accelerating detector in vacuum is generally getting excited and radiates, because clearly an external source is applied to the system [e.g. Padmanabhan (1985); Padmanabhan &Singh (1987)]. **For discussions of Unruh effect, in question is only universality of its response: independence of the structure and realization of a detector, method applied for its acceleration, etc.** The existing literature is rather controversial about these issues.

Detector type	*Unruh behavior*		*Comments*
	Yes	*No*	
Model: dragged two-level (Unruh-DeWitt) detector in vacuum of scalar field (with linear or non-linear response)	Unruh (1976);. . . tons of papers. . . ; DeBièvre &Merkli (2006) [non-perturbative analysis]	–	No evidence for existence of realization: relativistic theory of compound systems is yet poorly developed; pointlike object moving along a prescribed trajectory (compatibility with uncertainty principle?) No evidence for taking into account inertia forces properly (stability and rigidity are enforced by model). In addition, detector is of non-Glauber type – there is no confidence of detection particles rather than vacuum noise [Svaiter&Svaiter (1992)]. See also Rarbrecht&Prokopec CQG **23** 3917 (2006); Costa&Piazza, NJP **11**, 113006 (2009)

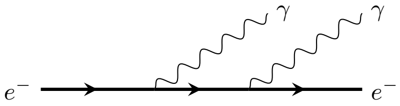
Detector type	*Unruh behavior*		*Comments*
	Yes	*No*	
Unruh-DeWitt detector supported in a stationary (e.g. uniform) gravitational field	–	Louko&Satz (2008)	Violation of equivalence principle or supports our idea that something goes wrong at horizons? See also Grishchuk, Zel'dovich&Rozhanskii (1987); Rosu (1999); Ginzburg&Frolov (1987); Sonego&Westman (2004); Kleinert (2009); Singleton&Wilborn (2011).
Dragged harmonic oscillator (exactly solvable model)	Grove (1986); Raine et al (1991); Massar et al (1992,1996); Ford&O'Connel (2005)	Shih-Yuin Lin & B.L. Hu (2007) [non-perturbative solution]	Negative result of Lin & Hu is due to back-reaction. Harmonic potential is an oversimplification (no destruction is possible); is any radiation observed by inertial observer?
Charged elementary particles accelerated by electric field	Parentani& Massar (1997); Gabriel, et al (1998); Gabriel & Spindel (2000)	Nikishov & Ritus (1988)	Model and interpretation are misleading: electric field creates pairs – no stable vacuum is present at all

Detector type	*Unruh (universal) behavior*		*Comments*
	Yes	*No*	
Fermion-boson system uniformly accelerated by specially configured stationary scalar field	–	A.M.F. et al (2002)	Unlike the previous set up, in the proposed configuration vacuum remains stable with respect to scalar background
Heavy ions accelerated by electric field	–	V.D. Mur et al (1998)	Ionization probability in non-relativistic regime differs from "thermal response". In relativistic case, for any realistic parameters detector gets destructed long before possible equilibration
Dragged atomic detectors dipole-coupled to EM vacuum	Marzlin &Audretsch (1997); Zhi-Ying& Hong-Wei (2008)	Passante (1998); Zhu et al (2007)	Model is similar to Unruh-DeWitt (challenging the same veriety of questions), numerical estimates for realistic setup have been not discussed. States "insensitive to Unruh effect" in Λ-configuration?

Many important contributions and several also controversial research directions represented in the literature are left out of the scope [e.g., "circular Unruh effect" – spin flips of electrons in storage rings and, more generally, interplay between "QFT" and "detector" aspects for uniform rotation].

Attempts to "experimental verification" of Unruh effect

- No matter whether the Unruh effect exists or not, it must be clear that it is all the same solely about a perspective of a non-inertial (uniformly accelerated) observer in a sense that any physical phenomenon can be considered and explained in inertial reference frame without any references to Unruh effect. The only way to prove it experimentally is to accelerate the lab up to $\sim 10^{22}$cm/sec^2.
- For example, this is absolutely incorrect (for many reasons) to claim the existence of some novel "Unruh radiation" $I_U \propto T_{DU}^4 \sim g^4$ for an electron in the laser field and the more so to contrast it to the Larmor radiation $I_L \propto g^2$!!!
- However, recently a lot of papers started to appear with the proposals to "observe the Unruh effect experimentally", in particular by observing radiation from electrons driven by high-power laser fields.
- An example of such proposals is an offer to explore two-photon emission



- If performed and succeeded, such sort of experiments can be used for verification of 2-order IFQED calculation [e.g., Lötstedt& Jentschura, PRL **103**, 110404 (2009)], rather than support speculations on Unruh effect.

Conclusion

- Fulling-Unruh quantization scheme assumes a boundary condition at the origin and horizons, which is evidently absent in Minkowski spacetime.
- Equivalently, the set of right and left Unruh modes is incomplete (does not contain a singular zero mode), and hence can not be used for quantization in MS.
- Need for operation in terms of smeared fields is not evident, however neither this nor application of BW theorem are capable for rehabilitation of the Unruh approach.
- As such, no convincing field-theoretical grounds for existence of the Unruh effect have in fact been adduced.
- Considerations of response of uniformly accelerated detectors are by now too controversial. One of the problems is lack of consistent quantum relativistic theory of compound systems.
- A popular idea of "experimental verification" of Unruh effect seems to be senseless.