Radiation reaction from lightfront QED

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CHALMERS

Vetenskapsrådet

- Anton Ilderton
- Quantum and classical radiation reaction from lightfront QED, arXiv:1304.6842 [hep-th]

Introduction

1 Classical radiation reaction (RR)
- Problems with Abraham-Lorentz-Dirac equation
- Different classical equations

2 Derive RR from QED
- Use lightfront quantisation
- Use Hamiltonian formalism

3 Take the classical limit $\hbar \to 0$
- **Which equations are consistent with QED?**

Classical Equations

- Most classical equations have the form

$$\ddot{x}^\mu = f^{\mu\nu}\dot{x}_\nu + \frac{2}{3}\frac{e^2}{4\pi m}R_{\mu\nu}\dot{x}^\nu$$

- Lorentz force $f = eF_{\text{ext}}/m$

Radiation Reaction	$R_{\mu\nu}$
Abraham Lorentz Dirac (LAD)	$\dddot{x}\dot{x} - \dot{x}\dddot{x}$
Landau Lifshitz (LL)	$\dot{f} + (f^2\dot{x})\dot{x} - \dot{x}(f^2\dot{x})$
Eliezer Ford O'Connell (EFO)	$\frac{\mathrm{d}}{\mathrm{d}\tau}(f\dot{x})\dot{x} - \dot{x}\frac{\mathrm{d}}{\mathrm{d}\tau}(f\dot{x})$
Mo and Papas (MP)	$(f\ddot{x})\dot{x} - \dot{x}(f\ddot{x})$
Herrera (H)	$(f^2\dot{x})\dot{x} - \dot{x}(f^2\dot{x})$
Sokolov (S)	$q + m\dot{x}$

Classical Equations

$$\ddot{x}^\mu = f^{\mu\nu}\dot{x}_\nu + \frac{2}{3}\frac{e^2}{4\pi m}R_{\mu\nu}\dot{x}^\nu \qquad f = eF_{\text{ext}}/m$$

- We treat these equations by expansion in e^2
  - Coupling expansion in QED in Furry picture
  - Asymptotic series? Sen Zhang (2013)

- The Lorentz term is treated exactly
  - Makes it easy to distinguish RR from Lorentz
  - Detect RR with high intensity lasers?

    Di Piazza et al. Rev. Mod. Phys. 84 (2012), Harvey, Heinzl, Marklund PRD 84 (2011),

    Di Piazza, Hatsagortsyan, Keitel, PRL 102 (2009), Bulanov *et al.* PRE 84 (2011)

Classical Equations

- Equations to $\mathcal{O}(e^2)$:
  - $\ddot{x} = f\dot{x}$ in $R \implies$
  - LAD = LL = EFO and MP = H
- The difference is $\dot{f}\dot{x}$

	$R_\mu = R_{\mu\nu}x^\nu$
LAD	$\dddot{x} + \ddot{x}^2\dot{x}$
EFO	$\dot{f}\dot{x} + f\ddot{x} + \ddot{x}f\dot{x}\dot{x}$
LL	$\dot{f}\dot{x} + ff\dot{x} + (f\dot{x})^2\dot{x}$
MP	$f\ddot{x} + \ddot{x}f\dot{x}\dot{x}$
H	$ff\dot{x} + (f\dot{x})^2\dot{x}$

Plane Waves

- We use a plane wave background field
- Null wavevector $k^2 = 0$, $k_\mu = \omega n_\mu$
- Transverse polarisation vector $ka' = 0$
- Choose coordinates so $\phi := kx = \omega x^+$

$$eF^{\text{ext}}_{\mu\nu}(\phi) = k_\mu a'_\nu(\phi) - a'_\mu(\phi) k_\mu$$

- Depends only on ϕ
- $a'_\perp(\phi) = eE_\perp(\phi)$
  $a'(\pm\infty) = 0$

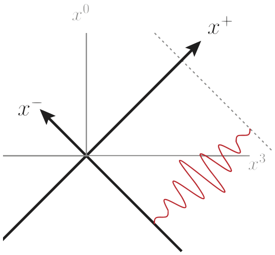


Figure : Plane wave

Classical Predictions

- Some equations have exact solutions in a plane wave

  Di Piazza Lett. Math. Phys. 83 (2008) 305, Sokolov JETP 109 (2009) 207
- Solution to zeroth order (Lorentz momentum)

$$\pi := p - a + \frac{2ap - a^2}{2kp} k$$

$$a_\perp(\phi) = \frac{e}{\omega} \int_{-\infty}^{\phi} \mathrm{d}\varphi \; E_\perp(\varphi)$$

Classical Predictions

- Solutions to $\mathcal{O}(e^2)$ $\rightarrow$ effects due to radiation reaction

$$\Delta = \frac{2}{3}\frac{e^2}{4\pi m}\frac{kp}{m}$$

- Momentum for LAD

$$q(\phi)_{\mathsf{LAD}} = \pi(\phi) + \Delta\pi'(\phi) + \frac{\Delta}{m^2}\int^{\phi}\mathrm{d}\varphi\, a'^2(\varphi)\left(\pi(\varphi) - \frac{\pi(\varphi)\pi(\phi)}{kp}k\right)$$

- Position for LAD

$$kpx'_{\mathsf{LAD}} = q_{\mathsf{LAD}} - \pi\frac{\Delta}{m^2}\int a'^2$$

Classical Predictions

- Momentum

$$\left\{\begin{array}{c}\text{LAD}\\ \text{LL}\\ \text{EFO}\end{array}\right\} = \left\{\begin{array}{c}\text{MP}\\ \text{H}\\ \text{S}\end{array}\right\} + \Delta\pi'$$

- Lightfront time derivative of position

$$\left\{\begin{array}{c}\text{LAD}\\ \text{LL}\\ \text{EFO}\\ \text{S}\end{array}\right\} = \left\{\begin{array}{c}\text{MP}\\ \text{H}\end{array}\right\} + \frac{\Delta\pi'}{kp}$$

- $\Delta\pi'(\infty) = 0 \rightarrow$ **Need finite time results from QED**

Lightfront quantisation

- Plane wave → lightfront quantisation
- Interested in the classical limit → We can use sQED
- Lightfront coordinates

$$x^{\pm} = 2x_{\mp} = x^0 \pm x^3 \qquad x^{\perp} = \{x^1, x^2\}$$

- Lightfront Hamiltonian Neville and Rohrlich PRD 8 (1971)

$$H = \frac{1}{2}\int \mathrm{dx} - A_j \partial_{\perp}^2 A_j + |\mathcal{D}_{\perp}\Phi|^2 + m^2|\Phi|^2 + ejA - e^2A^2|\Phi|^2 + \frac{e^2}{2}\left(\frac{j_-}{\partial_-}\right)^2$$

$$\mathcal{D}_{\perp} = \partial_{\perp} + ia_{\perp}$$

- Treat background exactly → Furry picture
  - The background field is included in the "free" part
  - Fermions are dressed by the background field

Furry Picture

- Klein-Gordon with a background field

$$(\mathcal{D}^2 + m^2)\Phi = 0$$

- Solution

$$\Phi(x) = \int \mathrm{dp}\ b(\mathsf{p})\varphi_{\mathsf{p}}(x) + d^\dagger(\mathsf{p})\varphi_{-\mathsf{p}}(x)$$

- Volkov solutions describe Lorentz force

$$i\mathcal{D}_\mu \varphi_{\mathsf{p}}(x) = \pi_\mu(\phi)\varphi_{\mathsf{p}}(x)$$

- Commutation relations

$$\left[\Phi(x), \Phi^\dagger(y)\right]_{x^+=y^+} = -\frac{i}{4}\varepsilon(x^- - y^-)\delta^2(x^\perp - y^\perp)$$

$$\left[b(\mathsf{p}), b^\dagger(\mathsf{q})\right] = \left[d(\mathsf{p}), d^\dagger(\mathsf{q})\right] = 2p_-(2\pi)^3\delta^3(\mathsf{p} - \mathsf{q})$$

Expectation values

- Lightfront time evolution of a one electron state

$$|\,\psi; x^+\,\rangle = \mathcal{T}_+ e^{-i \int^{x^+} H_F} |\,\text{in}\,\rangle$$

- Obtain finite time RR from expectation values
  - e.g. momentum $\langle P^e_\mu\rangle(\phi) = \langle\,\psi; x^+\,|P^e_\mu|\,\psi; x^+\,\rangle$
  - IR finite

See also Krivitsky and Tsytovich, Sov.Phys.Usp. 34 (1991) 250, Johnson and Hu, PRD 65 (2002) 065015 and Higuchi and Martin, PRD 73 (2006) 025019.

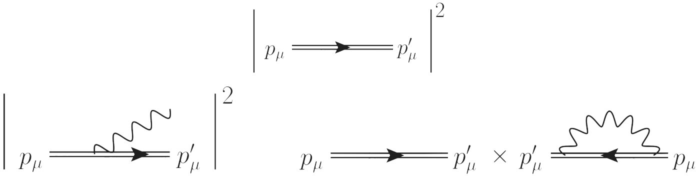


Figure : Diagrams illustrating $\mathcal{O}(e^2)$ contributions to expectation values. Lorentz force, nonlinear Compton scattering and loop

Momentum

- Momentum tensor → Electron + photon momentum

$$T_{\mu\nu} \to T^{e}_{\mu\nu} + T^{\gamma}_{\mu\nu}$$

$$T^{e}_{\mu\nu} = (D_\mu \Phi)^\dagger D_\nu \Phi + (D_\nu \Phi)^\dagger D_\mu \Phi - g_{\mu\nu}\left(|D\Phi|^2 - m^2|\Phi|^2\right)$$

$$D_\mu = \partial_\mu + ieA_\mu + ia_\mu$$

- Momentum operator

$$P^{e}_{\mu} = \int \mathrm{dx}\ T^{e}_{-\mu} = \int \mathrm{dp}\ \pi_\mu(\phi) b^\dagger(\mathsf{p}) b(\mathsf{p}) + e \ldots$$

Momentum

- Expectation value of momentum operator

$$\langle P^e_\mu \rangle = \pi_\mu - \frac{\alpha}{2\pi\epsilon}\pi_\mu(\phi) + \frac{\alpha}{2\pi\epsilon}\frac{m^2}{kp}k_\mu + e^2 \cdot \text{finite}$$

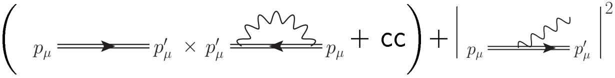
  - Transverse dim reg — Casher PRD 14 (1976) 452.
- Operator renormalisation
  - $a = 0$: $\langle P^e \rangle_\mu = p_\mu - \frac{\alpha}{2\pi\epsilon}p_\mu + \frac{\alpha}{2\pi\epsilon}\frac{m^2}{kp}k_\mu \xrightarrow{\text{renormalise}} p_\mu$
- Multiplicative and mass renormalisation

Mustaki et al PRD 43 (1991) 3411.

$$P^e_\mu \to \left(1 + \frac{\alpha}{2\pi\epsilon}\right)P^e_\mu \qquad m \to m - \frac{\alpha}{2\pi\epsilon}$$

Classical limit

- Classical limit $\hbar \to 0$
- Terms proportional to $1/\hbar$ cancel



See also Holstein & Donoghue, PRL 93 (2004), Higuchi & Martin, PRD 70 (2004)

$$\lim_{\hbar \to 0} \langle P^e_\mu \rangle = q_\mu(\phi) \quad \text{as in LAD, LL, EFO}$$

Ilderton and Torgrimsson. Quantum and classical radiation reaction from lightfront QED, arXiv:1304.6842 [hep-th]

Position

- Use position or current operator
- Position Operator

$$\mathsf{X}(\phi) = \int \mathrm{dx}\, \mathsf{x}\, j_{-}(\phi)$$

- Corresponds to the Newton-Wigner position operator

  See also Higuchi and Martin PRD 73 (2006) 025019

- Classical limit

$$\lim_{\hbar \to 0} \langle X^{\mu} \rangle = x^{\mu}(\phi) \qquad \text{as in LAD, LL, EFO, S}$$

Conclusions

- We have derived RR directly from QED
- Need finite time $\rightarrow$ Hamiltonian formalism
- Plane waves $\rightarrow$ lightfront quantisation
- **LAD, LL and EFO are consistent with QED to $\mathcal{O}(e^2)$**
- Distinguish between them by extending our method to $\mathcal{O}(e^4)$
  - Many diagrams and many terms to calculate