QCD with external magnetic fields

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Magnetic fields and Quantum Chromodynamics

- early universe $\sqrt{eB} \simeq 2$ GeV

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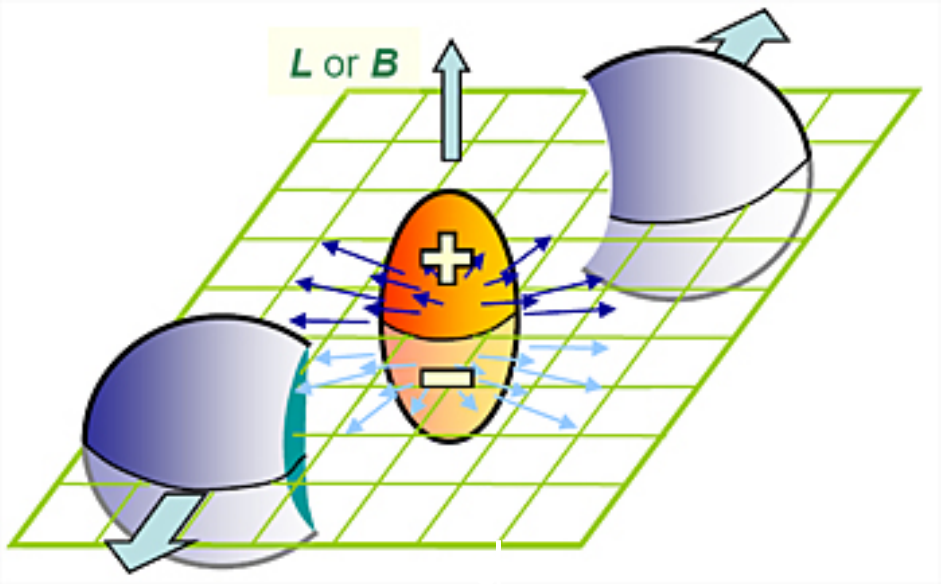
  non-central collisions
  charged spectators
  B perp. to reaction plane



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magn. fields as probes for our understanding of nonperturbative QCD

Setting

- quarks couple to electromagnetism: $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$
  neutral gluons do not $\leftarrow$ indirect effects via strong coupling
- strong electric field $\Rightarrow$ pair creation
- magnetic field $\Rightarrow$ Landau orbits

idealized situation:

- constant external magnetic field in Euclidean space (finite T)
  gluons, but no photons (no QED)
- accessible to numerical simulations on a lattice ▸ example
  magnetic fields quantized and bounded (like momenta) 't Hooft '79
  state-of-the-art: $\sqrt{eB} = 0.1 \ldots 1$ GeV

What to expect?

- free particles in magnetic fields

Dirac equation with magnetic field via $A_y = Bx$:

$$-\not{D}^2 = -\partial_t^2 - \partial_z^2 - \partial_x^2 - (\partial_y + qBx)^2 + qB\,\sigma_{12} \qquad \sigma_{12} \propto [\gamma_1, \gamma_2]$$

free waves harm. oscillator spin

$$\lambda^2 = p_t^2 + p_z^2 + |qB|(2n+1) + qB(2s)$$ Landau '30

$$p_t, p_z \in \mathbb{R} \quad n = 0, 1, \ldots \qquad s = \pm 1/2$$ Euler, Heisenberg '35

lowest Landau level: $\lambda = 0$ (massless case)

strong fields: dimensional reduction to 1+1 dimensions?

degeneracy of all Landau levels: $|\text{magn. flux}| = |qB| \cdot \text{area}$

Magnetic catalysis

- in QCD the chiral symmetry is broken by the chiral condensate

$$\langle \bar{\psi}\psi \rangle \sim \rho(\lambda = 0)$$

Banks, Casher '80

like mass term

hadrons not paired, pions as pseudo-Goldstone bosons ...

- strong magnetic fields generate many small eigenvalues

magnetic catalysis: stronger breaking of chiral symmetry

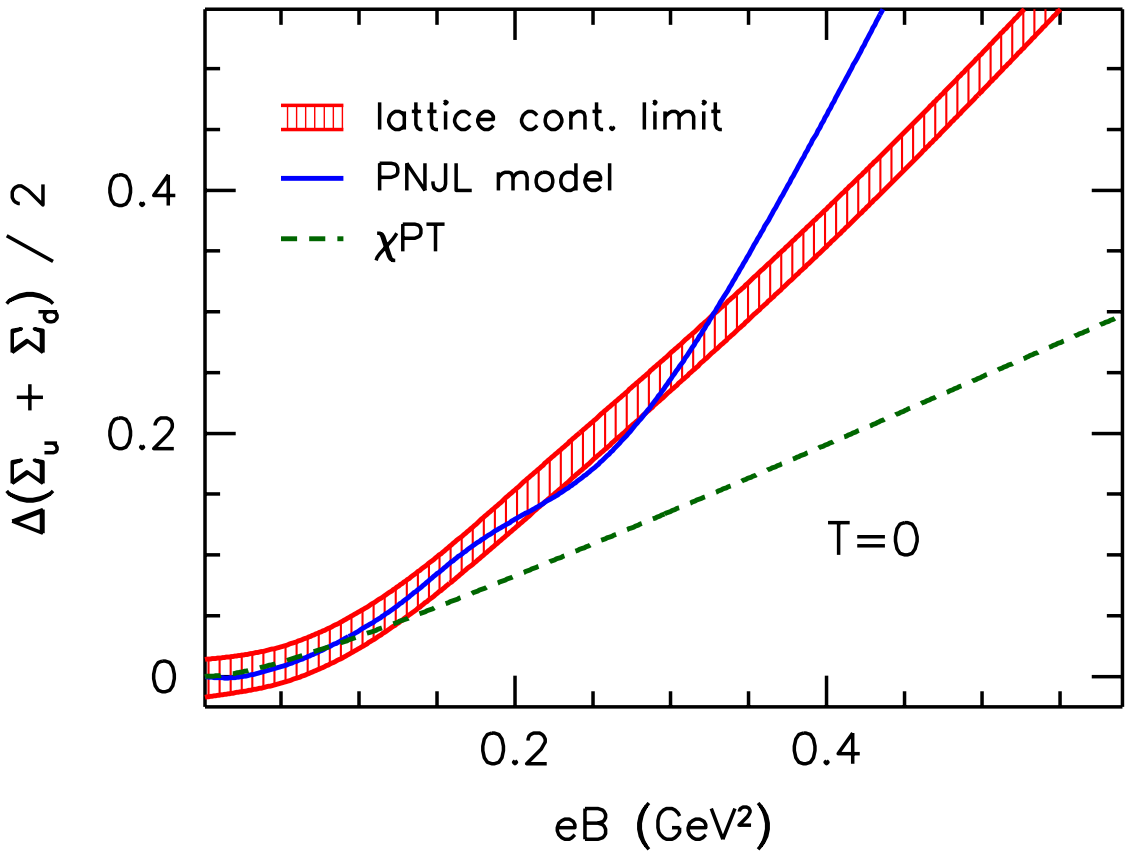
robust in all models

Müller, Schramm^2 '92

Gusynin, Miransky, Shovkovy '96

Magnetic catalysis: lattice and other approaches

- change of light quark condensate with B: FB et al. '12



chiral perturbation theory Cohen, McGady, Werbos '07, Andersen '12
& NJL model Gatto, Ruggieri '10

$\Rightarrow$ well approximated unless $eB > \begin{matrix} 0.1 \\ 0.3 \end{matrix}$ GeV^2 (approaches valid there?)

Catalysis at high temperatures

- at high temperatures, the quark condensate vanishes approximately and chiral symmetry gets restored: quark-gluon “plasma”
  lattice: crossover at $T_c \simeq 150$ MeV $= 2.3 \cdot 10^{12}$ K Aoki et al. ’06, hotQCD

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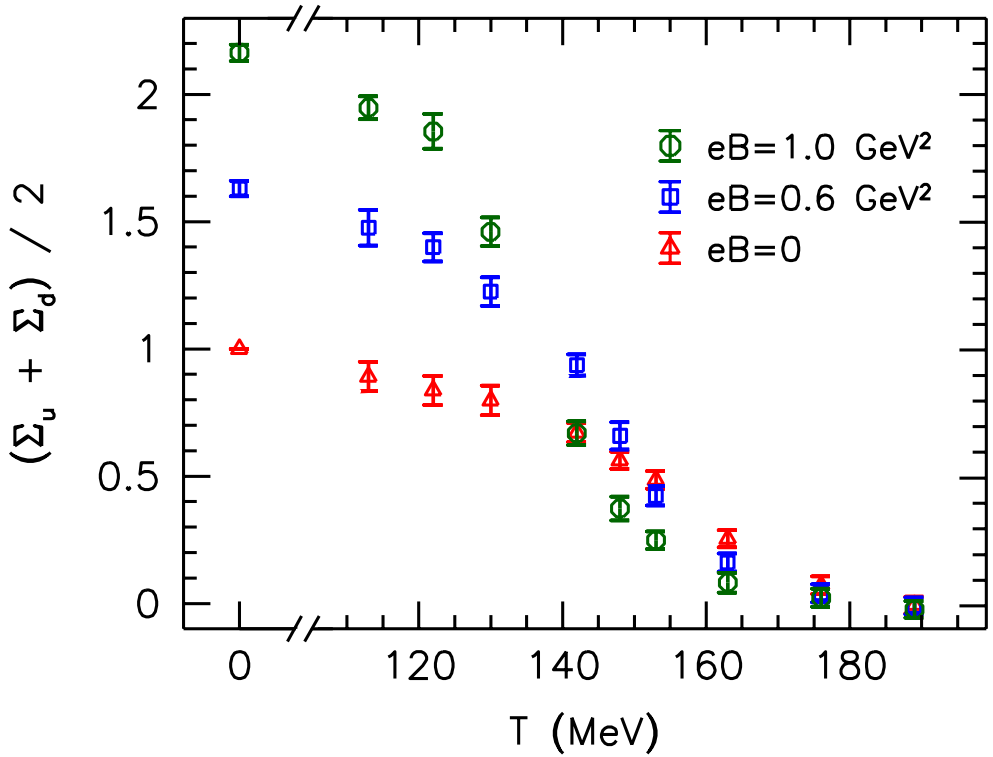
- with B

non-monotonic!



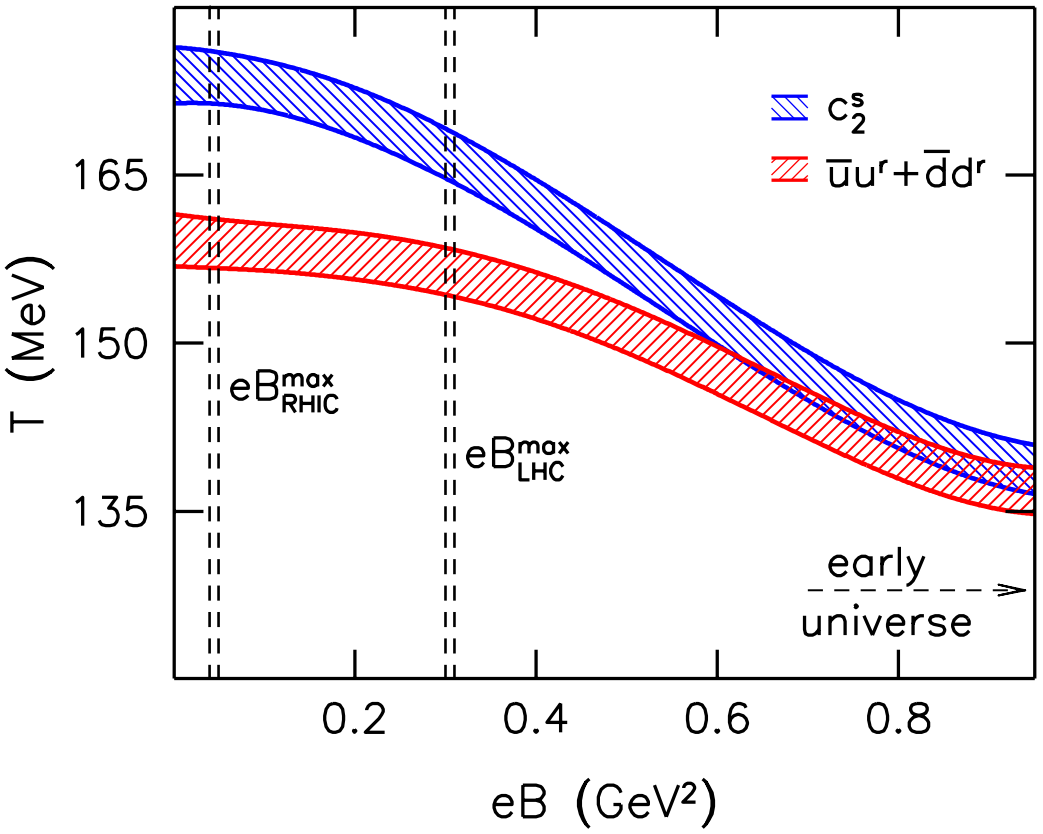
FB et al. '11

magn. catalysis turns into inverse magnetic catalysis around T_c

- order parameter: T_c from inflection points $\Rightarrow$ decreases with B...

QCD phase diagram with B

- pseudo-critical temperatures $T_c(B)$: FB et al. '11



light quark condensate & strange number suscept.

▸ conventional phase diagram

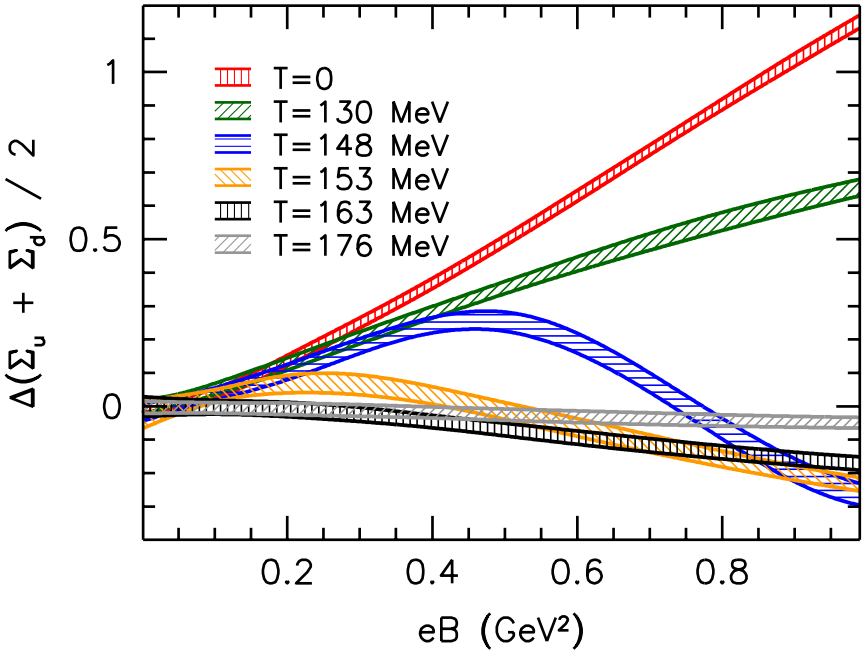
T_c decreases by O(10) MeV, relevant for LHC?? (setting!)

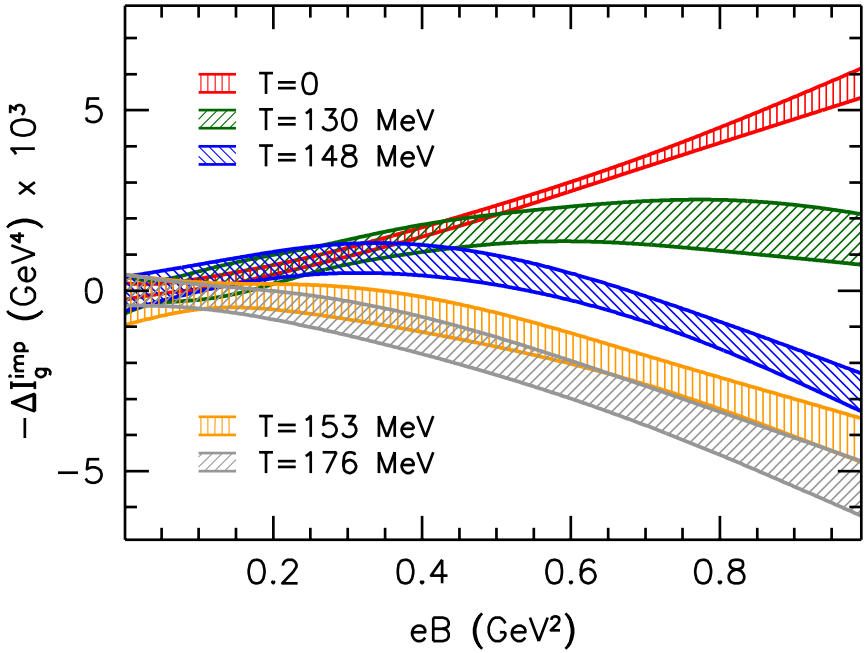
contradiction to many models and lattice simulations with heavier quarks D'Elia et al. '10, Ilgenfritz et al. '12

- transition remains a crossover (from volume scaling)

Catalysis in the gluon sector

- change of condensate and gluonic action FB et al. '13





non-monotonic, very similar shape

gluons inherit magnetic catalysis and inverse magnetic catalysis from quarks

- strongly coupled, related via trace anomaly

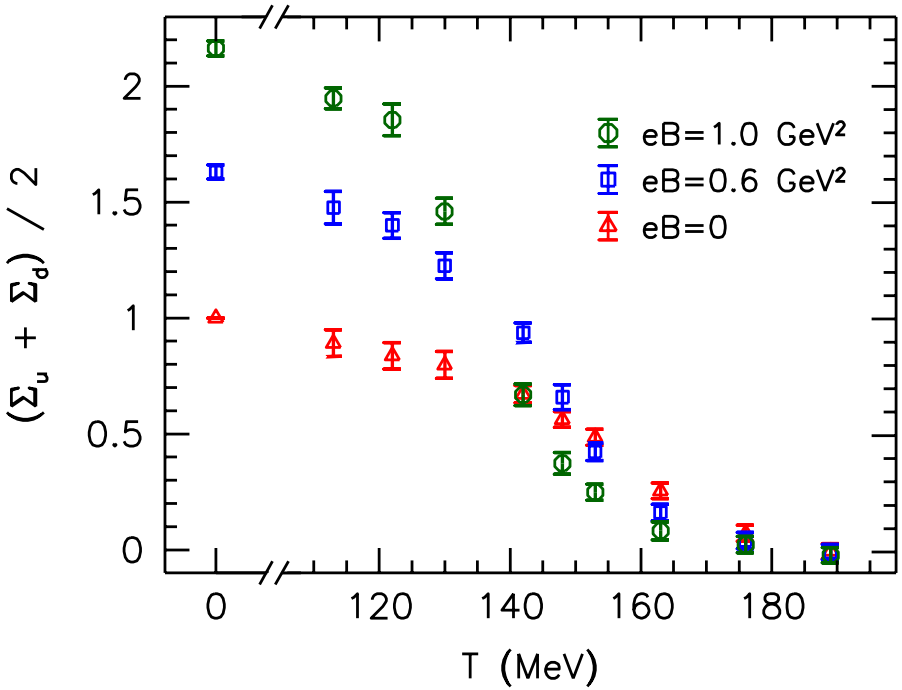
details

Deconfinement order parameter

- T-dependence:

FB, Endrődi, Kovács '13

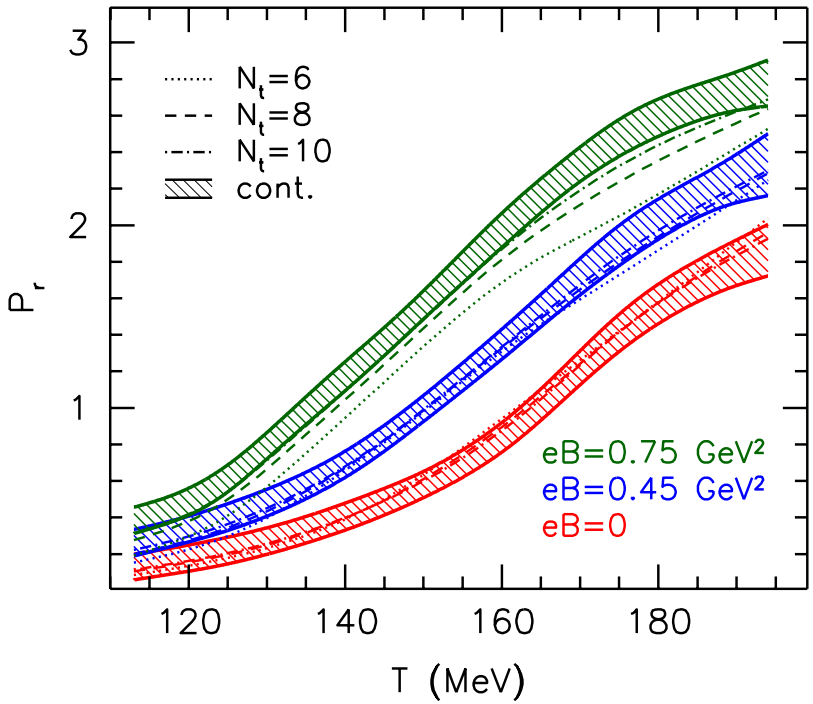
condensate (again)



Polyakov loop

meaning

$P \sim e^{-\beta F_{\text{heavy quark}}}$



deconfinement temperature also decreases (harder to determine)

- no splitting wrt. the chiral restoration temperature

Matsubara picture

- Polyakov loop P effectively changes quark boundary conditions in Euclidean time away from antiperiodic

  less twist $\Rightarrow$ lower eigenvalues of $\lambda(\not{D})$... 'Matsubara frequencies'

- one-loop effective action from fermion determinant with B:

$$S^{\text{eff}} = -\log\det(\not{D}[B, P; T] + m) \qquad \text{no gluons, just } P$$

  from Schwinger's proper time representation:

  prefers deconfining P the smaller the quark mass (lattice ✓)

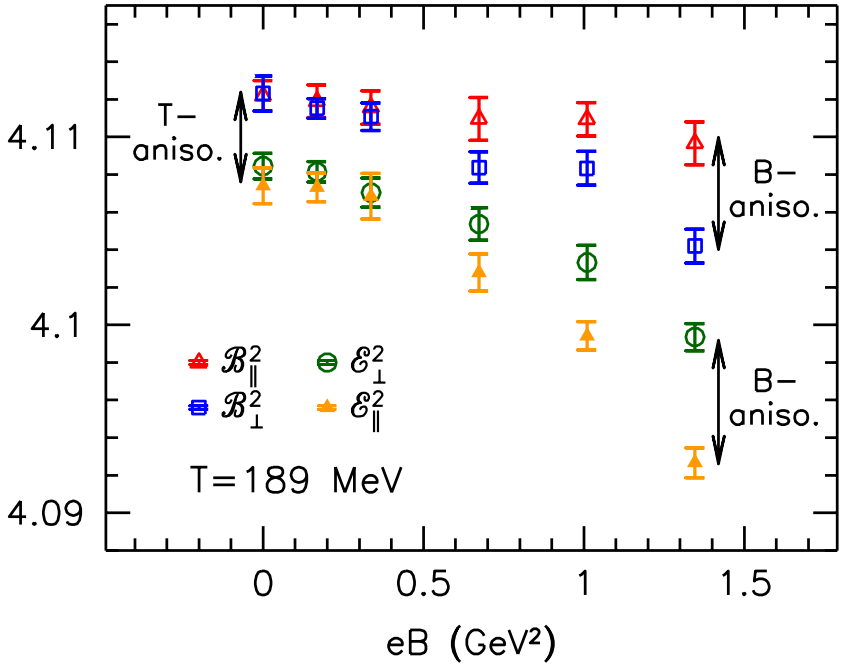
  prefers deconfining P the larger the magnetic field

  washed out for heavy quarks

Anisotropies

- anisotropy in gluonic field strength: FB et al. '13

$$\langle \mathrm{tr}\mathcal{B}_{\parallel}^2\rangle > \langle \mathrm{tr}\mathcal{B}_{\perp}^2\rangle \overset{T>0}{>} \langle \mathrm{tr}\mathcal{E}_{\perp}^2\rangle > \langle \mathrm{tr}\mathcal{E}_{\parallel}^2\rangle$$



same hierarchy in Euler-Heisenberg effective action

$$-\log\det(\not{D}[B]+m) \sim \frac{(qB)^2}{m^4}\left[\frac{5}{2}\mathrm{tr}\,\mathcal{E}_{\parallel}^2 - \mathrm{tr}\,\mathcal{B}_{\perp}^2 - \mathrm{tr}\,\mathcal{E}_{\perp}^2 - 3\mathrm{tr}\,\mathcal{B}_{\parallel}^2\right]$$

Anisotropies

- fermionic anisotropy:

$$\langle \bar{\psi} \not{D}_{\parallel} \psi \rangle > \langle \bar{\psi} \not{D}_{\perp} \psi \rangle$$

enters magnetization via anisotropic preasures

QCD vacuum is paramagnetic

- magnetic susceptibility:

$$\langle \bar{\psi} \sigma_{12} \psi \rangle = \tau B \qquad (B = F_{12})$$

lattice simulations: $\tau = -40$ MeV FB et al. '12

- topological correlator:

$$\langle q(0) q(r_{\parallel}) \rangle \approx \langle q(0) q(r_{\perp}) \rangle$$

no significant anisotropy(!)

Summary

strong external (constant) magnetic fields and equilibrium QCD:

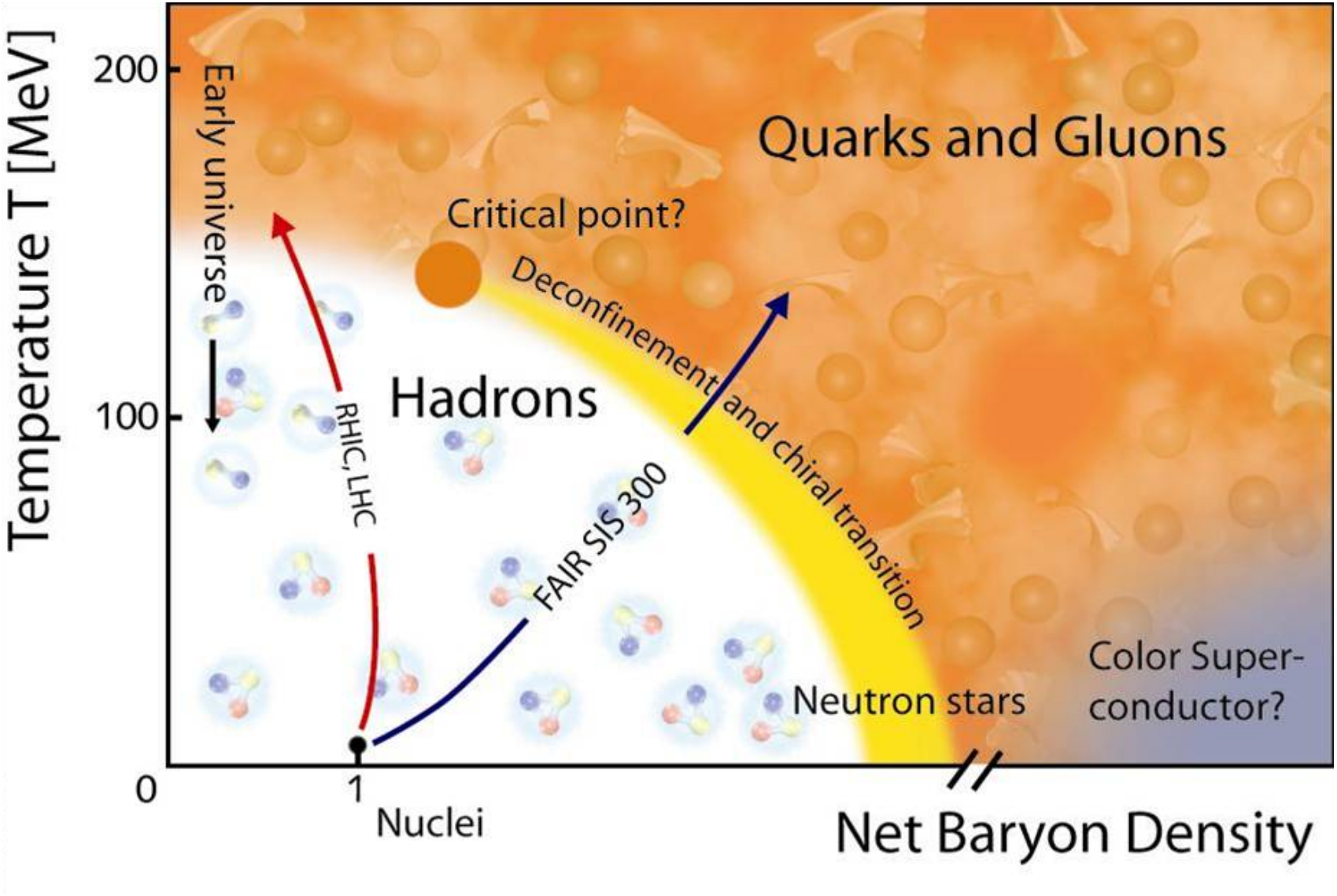
- magnetic catalysis: $\langle\bar{\psi}\psi\rangle(B) \nearrow$ at $T = 0$
  - from degeneracy of Landau levels, robust in all models
  - also for gluons
- inverse magnetic catalysis: $\langle\bar{\psi}\psi\rangle(B) \searrow$ at $T \simeq T_c$
  - quark back reaction, only for light (phys.) quark masses
  - Polyakov loop: $\langle P\rangle(B) \nearrow$ (Matsubara picture)
- QCD crossover: $T_c(B)$ decreases slightly
  - phenomenologically relevant?
- anisotropies in field strength, fermion action:
  - QCD vacuum paramagnetic

under the continuum . . .

. . . the lattice

[thanks to Hendrik]

QCD phase diagram



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Backup: Simulation details

as for transition studies at $B = 0$ Budapest-Wuppertal

- tree-level improved gauge action
- stout smeared staggered fermions (rooting trick)
- 2 light quarks + strange quark, charges $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$
- lattice spacing set at $T = 0, B = 0$

  physical pion masses

  set by f_K, f_K/m_π and f_K/m_K
- $T = 0$: $24^3 \times 32$, $32^3 \times 48$ and $40^3 \times 48$ lattices
- $T > 0$: $N_t = 6, 8, 10$ meaning $a = 0.2, 0.15, 0.12$ fm

  $N_s = 16, 24, 32$ for finite volumes
- condensates from stochastic estimator method with 40 vectors
- magn. flux quanta: $N_B \leq 70 < \frac{N_x N_y}{4} = 144$

Backup: Trace anomaly

$$I = \epsilon - 3p \qquad \ldots \text{interaction measure, since free gas: } \epsilon = 3p$$

$$\overset{\text{lattice}}{=} -\frac{T}{V}\frac{d\log Z}{d\log a} \qquad \ldots \text{scale anomaly}$$

$$= -\frac{T}{V}\left(\frac{\partial \log Z}{\partial \beta}\frac{\partial \beta}{\partial \log a} + \frac{\partial \log Z}{\partial \log am}\frac{\partial \log am}{\partial \log a}\right) \qquad \beta = \frac{6}{g^2}$$

$$= -\left(\langle s_g\rangle \frac{-\partial\beta}{\partial \log a} + m\,\langle\bar\psi\psi\rangle \frac{\partial \log am}{\partial \log a}\right)$$

$$\Delta I = -\left(\Delta\langle s_g\rangle \frac{-\partial\beta}{\partial \log a} + m\,\Delta\langle\bar\psi\psi\rangle \frac{\partial \log am}{\partial \log a}\right)$$

- change of gluonic action density and condensate add up

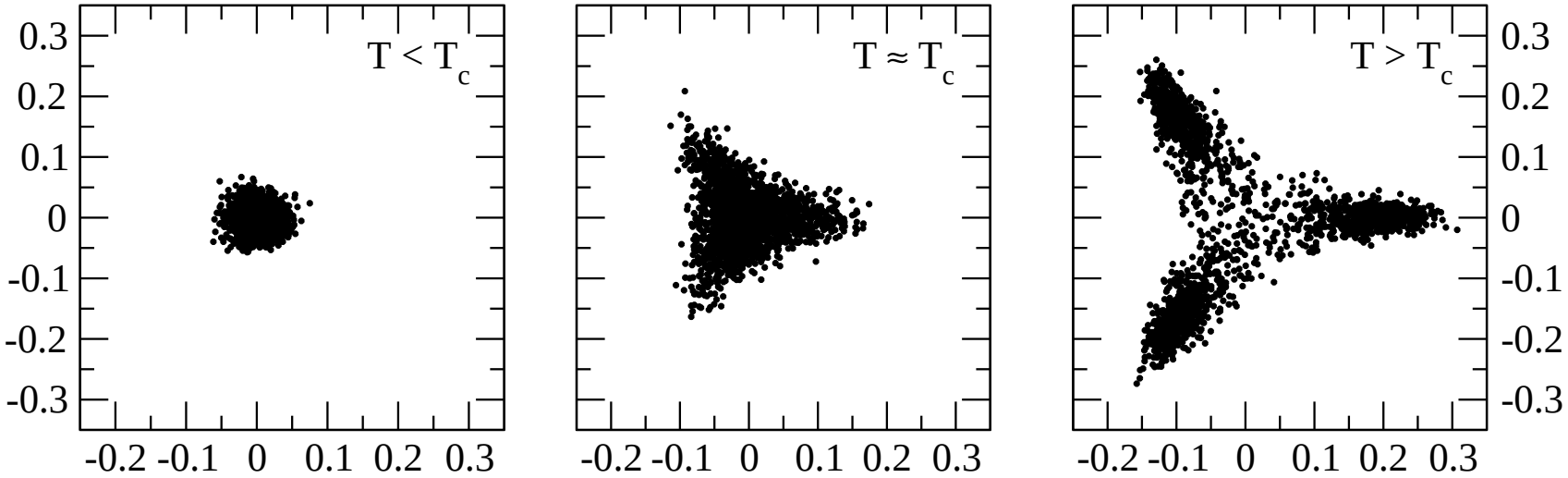
$\overset{!?}{\Rightarrow}$ similarity in B-dependence

lattice beta and gamma function enter

Deconfinement

- free energy of a single (infinitely heavy) quark $\langle \overline{\mathrm{tr}\, P} \rangle$ Polyakov '78

  quenched ensembles around T_c from the lattice:



- order parameter like magnetization, but inverse behavior:

$$\langle \overline{\mathrm{tr}\, P} \rangle \sim e^{-\beta F_{\mathrm{quark}}} = \begin{cases} e^{-\infty} = 0 & T < T_c \\ e^{-\#} \neq 0 & T > T_c \end{cases}$$

$\Rightarrow$ respects/breaks center symmetry $\simeq \frac{2\pi}{3}$ rotations of tr P

- finite quark mass breaks it explicitly

$\Rightarrow$ approximate order parameter in realistic QCD