

# **Particle production from expanding flux tubes**

**Naoto Tanji  
KEK/Saclay**

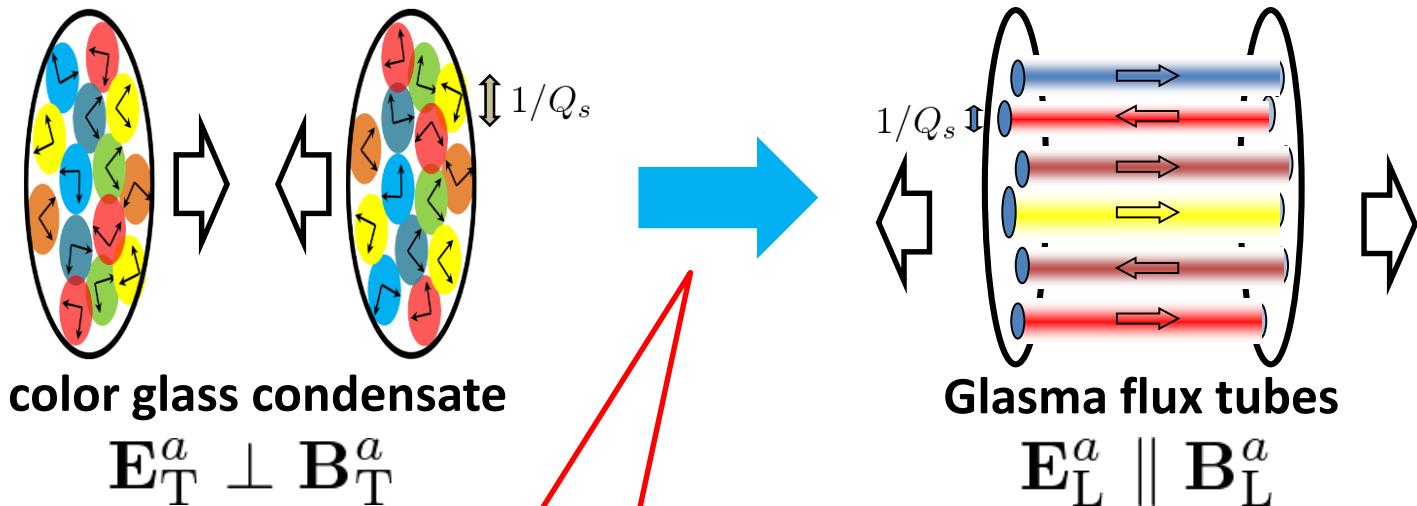
**collaboration with F. Gelis**

# Outline

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1. Strong fields in heavy-ion collisions
  - Non-uniform and time-dependent fields
2. Monte Carlo method to compute the Schwinger mechanism
3. Uniform field results
4. Expanding flux tubes

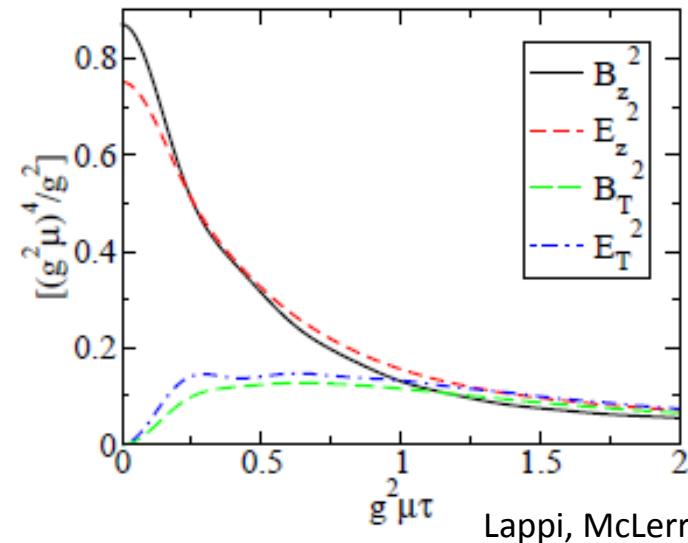
# Strong fields in heavy-ion collisions



non-Abelian Gauss's laws

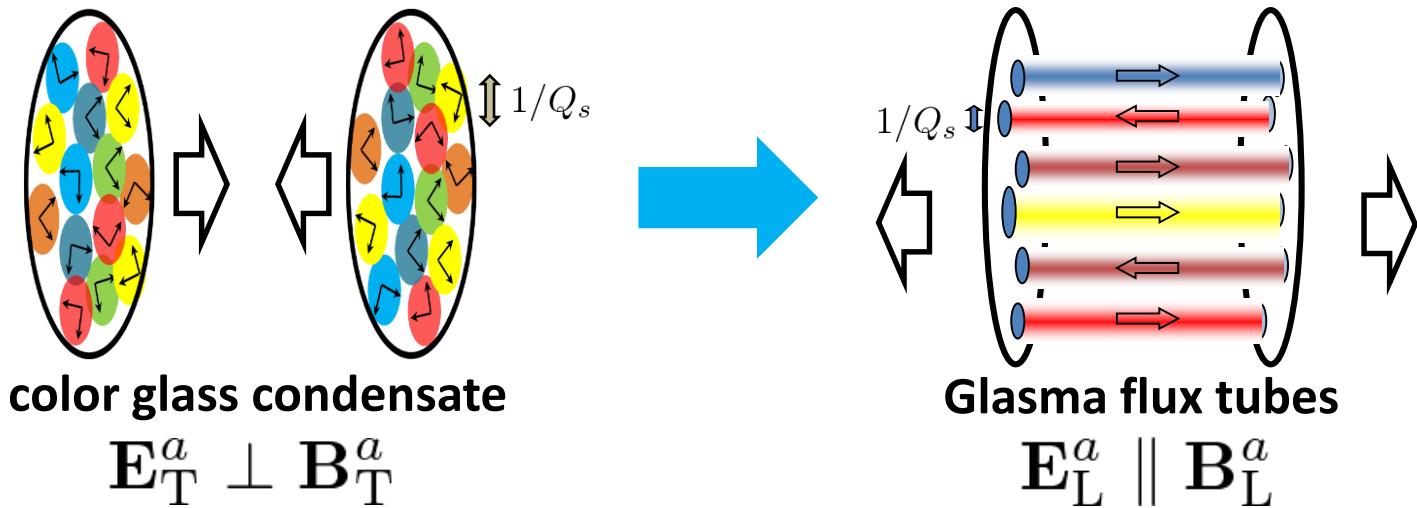
$$\nabla \cdot \mathbf{E}^a = -g f^{abc} \mathbf{A}^b \cdot \mathbf{E}^c$$

$$\nabla \cdot \mathbf{B}^a = -g f^{abc} \mathbf{A}^b \cdot \mathbf{B}^c$$



Lappi, McLerran (2006)

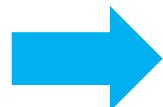
# Strong fields in heavy-ion collisions



Strong fields

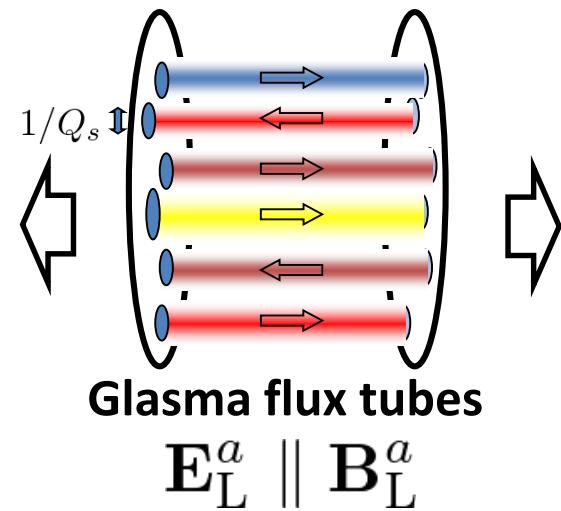
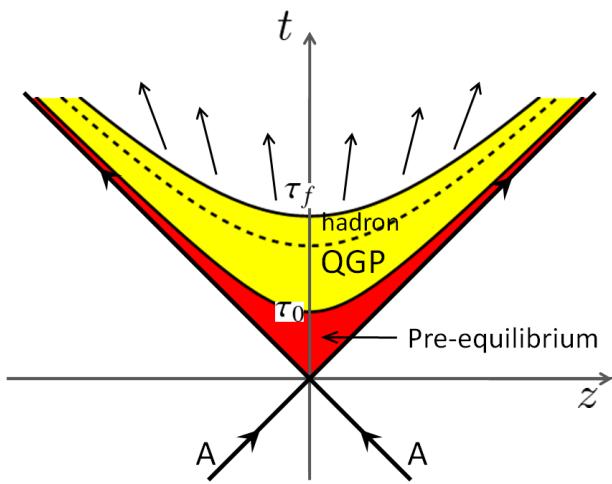
$$gE \simeq gB \simeq Q_s^2 \sim 1 \text{ GeV}^2$$

$$\frac{m^2}{gE} \ll 1$$



Matter formation via the Schwinger mechanism

# Strong fields in heavy-ion collisions



## Inhomogeneous and time-dependent fields

- boost-invariant expansion in the longitudinal beam direction
- flux-tube like structure in the transverse plane

Need a real-time QFT method for the particle production in such fields

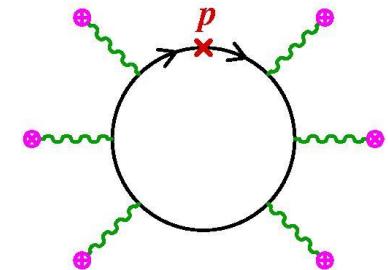
# Mode expansion

Need to calculate....

the behavior of a charged *quantum* field interacting with *classical* gauge fields

linear eq. of motion

$$(\mathcal{D}_\mu \mathcal{D}^\mu + m^2)\phi(x) = 0$$



expansion by mode functions

$$\phi(x) = \int \frac{d^3 q}{(2\pi)^3 2E_q} \left[ \varphi_{\mathbf{q}}(x) a_{\text{in}}(\mathbf{q}) + \varphi_{\mathbf{q}}^*(x) b_{\text{in}}^\dagger(\mathbf{q}) \right]$$

$$(\mathcal{D}_\mu \mathcal{D}^\mu + m^2)\varphi_{\mathbf{q}}(x) = 0 \quad \lim_{x^0 \rightarrow -\infty} \varphi_{\mathbf{q}}(x) = e^{-iq \cdot x}$$



Field quantities can be calculated.

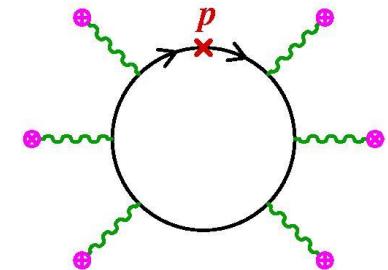
$$\langle 0_{\text{in}} | \mathcal{O}[\phi, \dot{\phi}] | 0_{\text{in}} \rangle$$

# Mode expansion

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Field quantities can be calc

$$\langle 0_{\text{in}} | \mathcal{O} |$$

Numerical cost

$$N_t \times N_{\text{latt}}^2$$

expensive in 3+1dim.

# Monte Carlo method

---

1. Assume that mode functions at  $t = t_0$  are known or easily computed.

2. Construct initial conditions at  $t = t_0$

$$\varphi_0(\mathbf{x}) = \int \frac{d^3q}{(2\pi)^3 2E_q} [c_{\mathbf{q}} \varphi_{\mathbf{q}}(t_0, \mathbf{x}) + d_{\mathbf{q}} \varphi_{\mathbf{q}}^*(t_0, \mathbf{x})]$$

$$\pi_0(\mathbf{x}) = \int \frac{d^3q}{(2\pi)^3 2E_q} [c_{\mathbf{q}} \dot{\varphi}_{\mathbf{q}}(t_0, \mathbf{x}) + d_{\mathbf{q}} \dot{\varphi}_{\mathbf{q}}^*(t_0, \mathbf{x})]$$

with random c-numbers

$$\langle c_{\mathbf{q}} c_{\mathbf{q}'}^* \rangle = \langle d_{\mathbf{q}} d_{\mathbf{q}'}^* \rangle = (2\pi)^3 E_q \delta(\mathbf{q} - \mathbf{q}') , \quad \text{others} = 0$$

3. Solve for each ensemble

$$(\mathcal{D}_\mu \mathcal{D}^\mu + m^2) \varphi = 0 \quad \varphi(t_0, \mathbf{x}) = \varphi_0(\mathbf{x}), \quad \dot{\varphi}(t_0, \mathbf{x}) = \pi_0(\mathbf{x})$$

4. Take the ensemble average

$$\langle \mathcal{O} [\varphi(t, \mathbf{x}), \dot{\varphi}(t, \mathbf{x})] \rangle$$

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3. Solve

The result of the mode expansion method is reproduced

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# Monte Carlo method

---

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2. Construct initial conditions at  $t = t_0$

**Numerical cost**  
 $N_{\text{ens}} \times N_{\text{latt}}^2$

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4. Take

$$N_{\text{ens}} \times N_{\text{latt}} \times (N_{\text{latt}} + N_t) \ll N_t \times N_{\text{latt}}^2$$

if  $N_{\text{ens}} \ll N_{\text{latt}}$  and  $N_{\text{ens}} \ll N_t$

# Monte Carlo method

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- Applicable to a non-uniform system with a reasonable numerical cost
- Applicable to fermion fields      → The talk by F. Hebenstreit
- Can be viewed as a special case of the classical statistical approach

# Monte Carlo method

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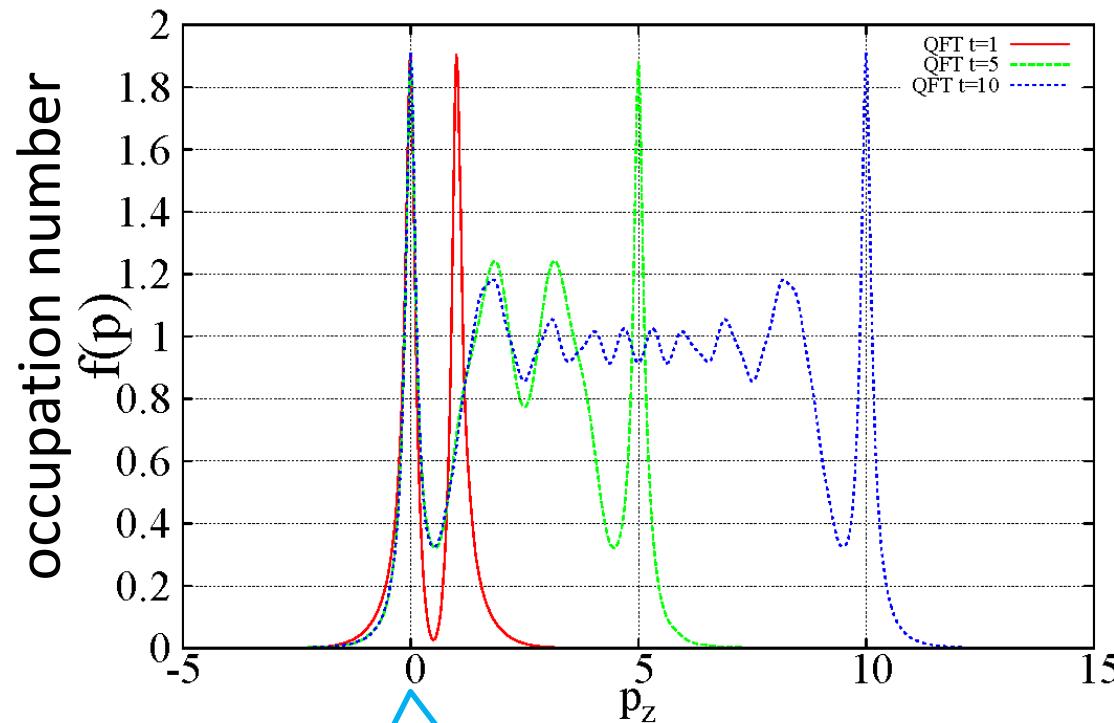
- Applicable to a non-uniform system with a reasonable numerical cost
  - Applicable to fermion fields      → The talk by F. Hebenstreit
  - Can be viewed as a special case of the **classical statistical approach**
- 
- A non-perturbative method to compute the real-time evolution of interacting quantum fields
  - If typical occupancy of a field is large ( $f \gg 1$ ), its dynamics shows classical behavior.
  - Quantum corrections are partly taken into account by taking ensemble average.

# Constant electric field – a benchmark of the MC method

Time-evolution of the longitudinal momentum distribution  
a result by the mode expansion method

N.T. (2009)

$$eE = 1$$
$$m = 0.1$$



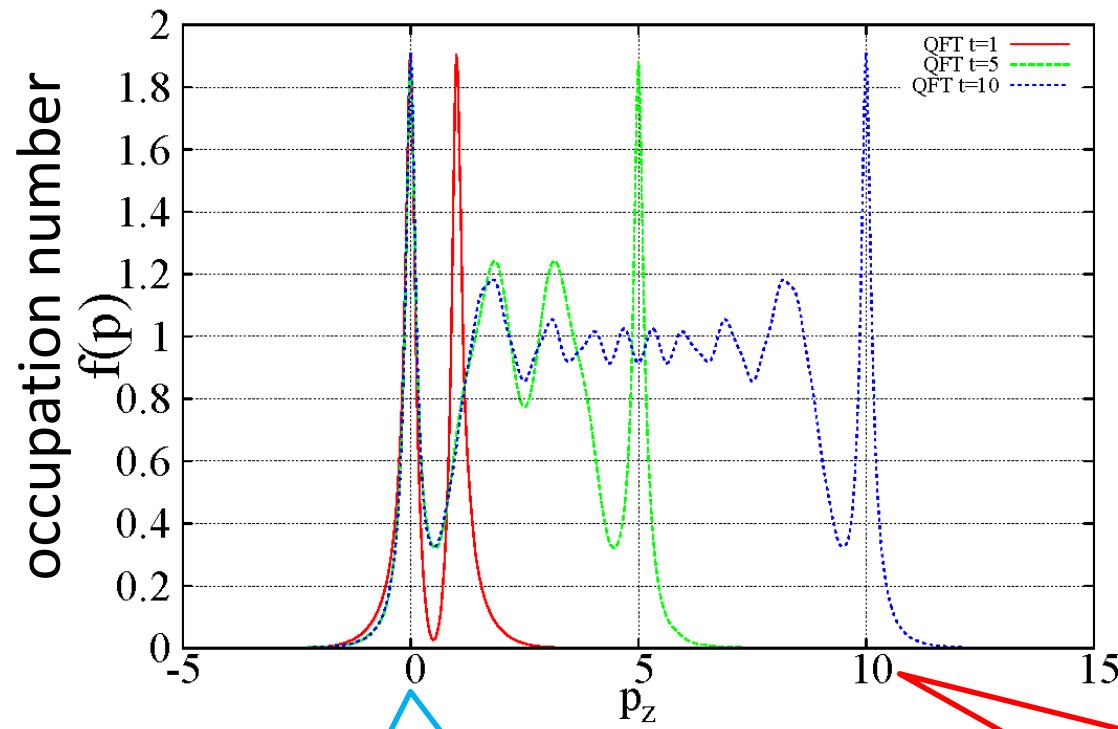
created with approximately  
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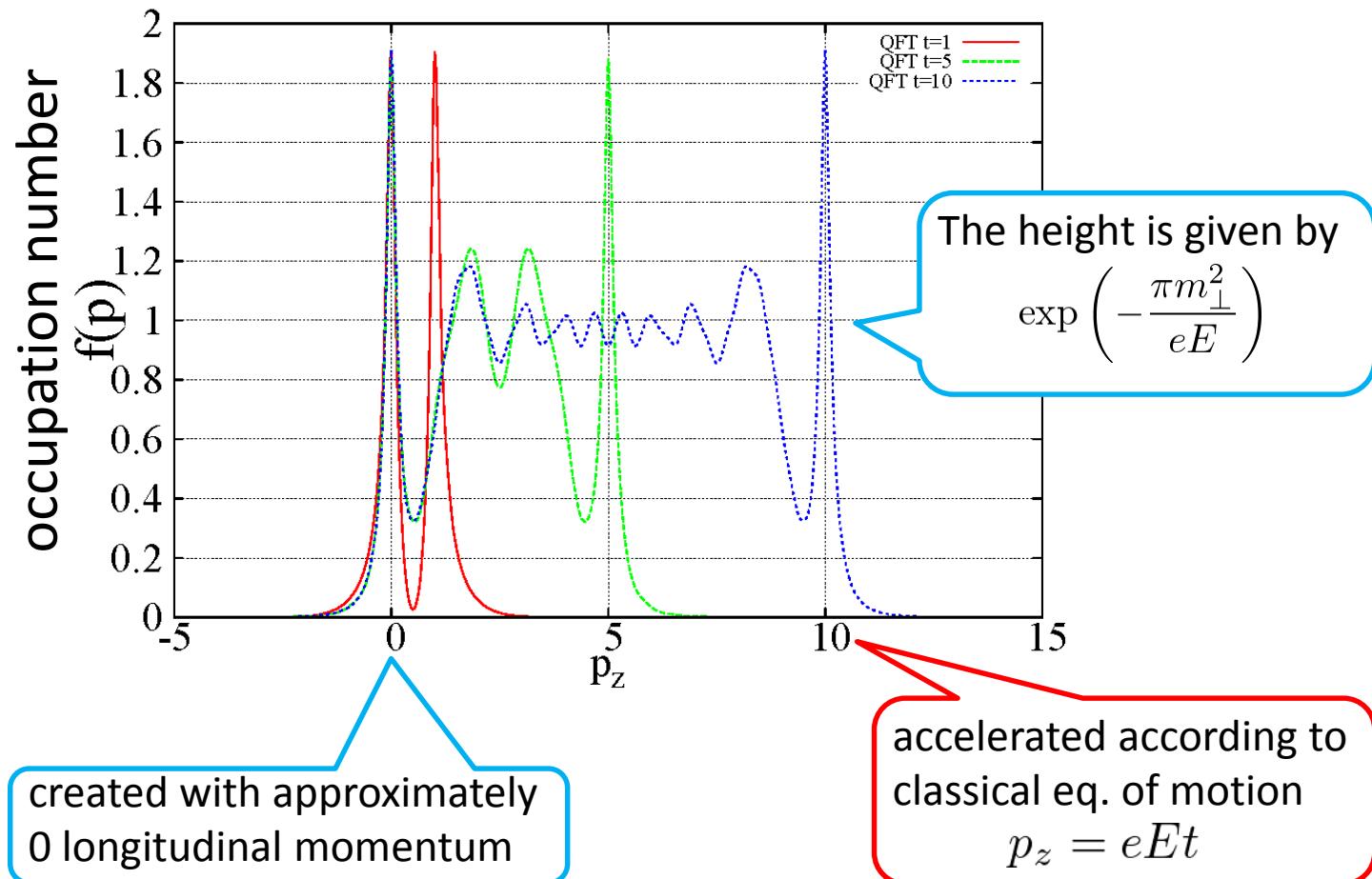
accelerated according to  
classical eq. of motion  
 $p_z = eEt$

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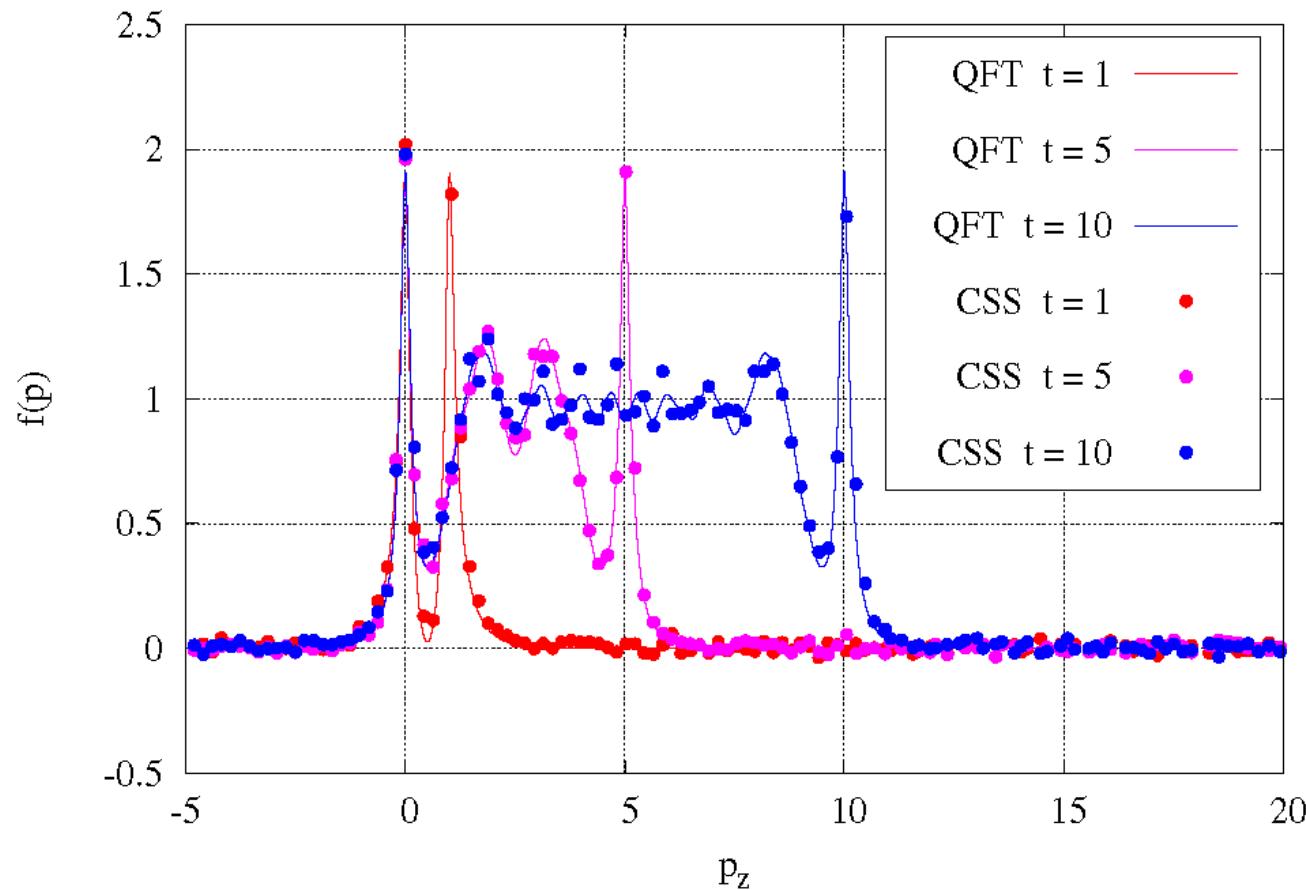
$$eE = 1 \\ m = 0.1$$



# Constant electric field – a benchmark of the MC method

the mode expansion method vs. the Classical Statistical Simulations

$$\begin{aligned} eE &= 1 \\ m &= 0.1 \\ N_x = N_y &= 32 \\ N_z &= 256 \\ N_{\text{ens}} &= 1024 \end{aligned}$$

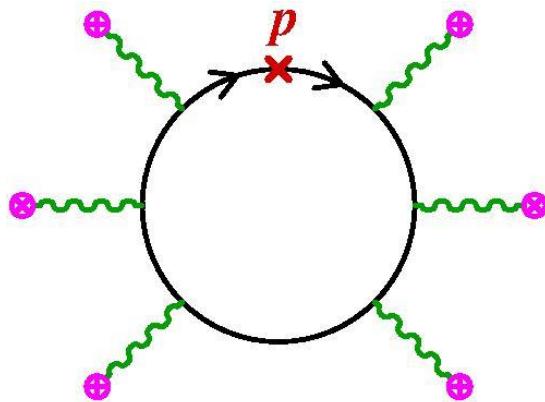


# Back reaction

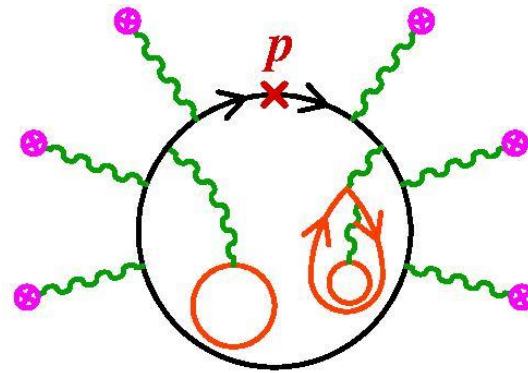
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Solve the Maxwell eq.  $\partial_\mu F^{\mu\nu} = \langle J^\nu \rangle$  with  $(D^2 + m^2) \phi = 0$

Diagrammatically,



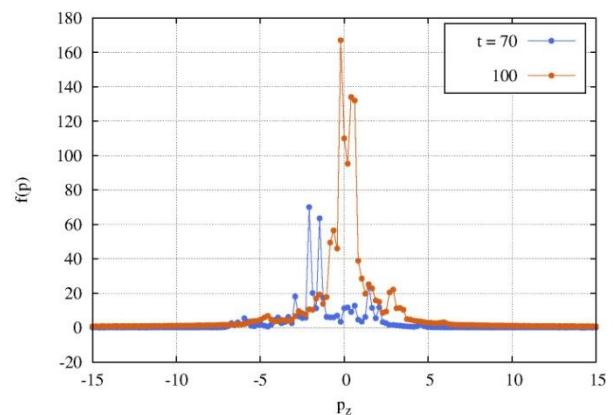
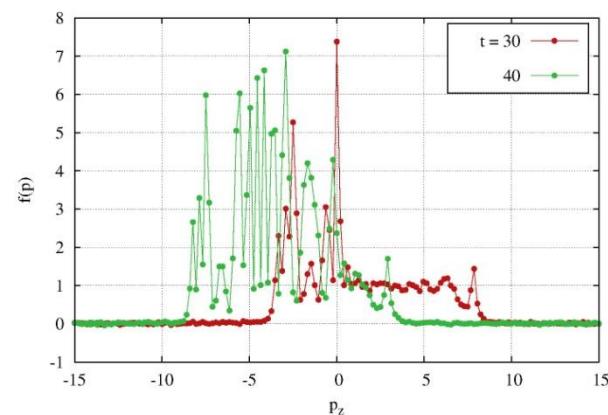
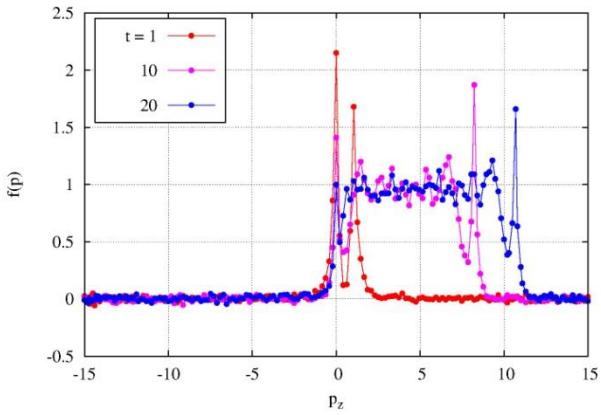
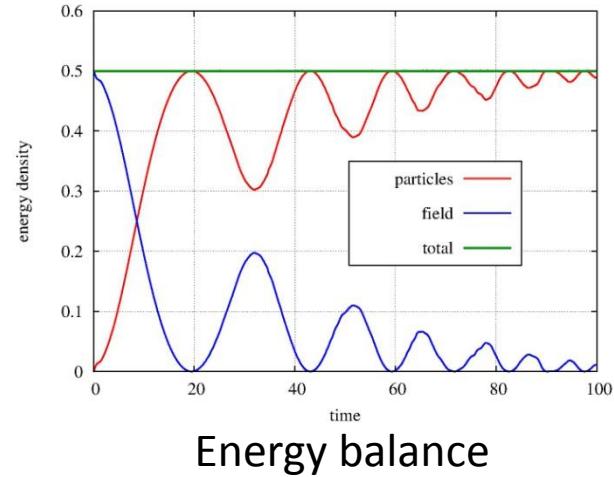
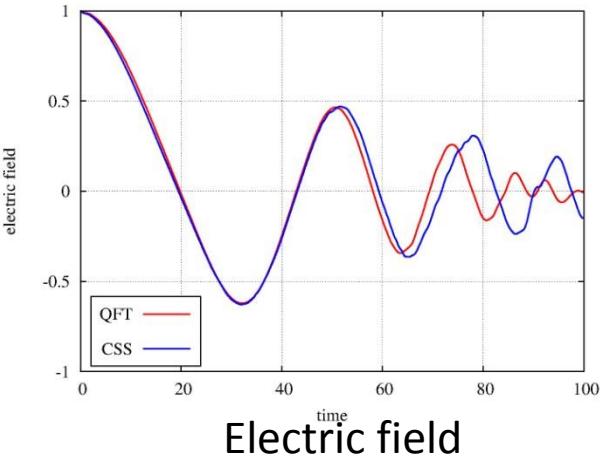
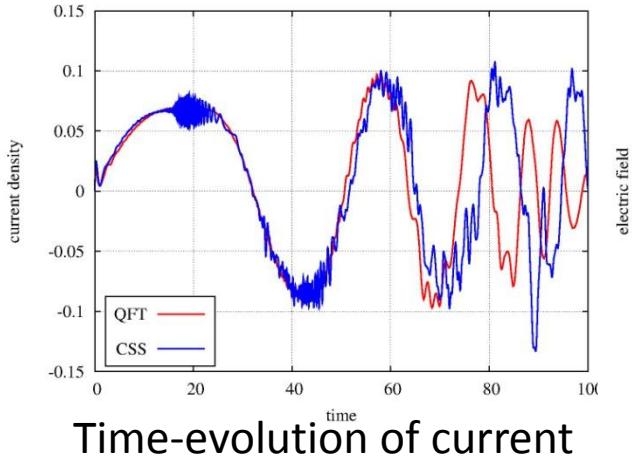
without back reaction



with back reaction

# Back reaction

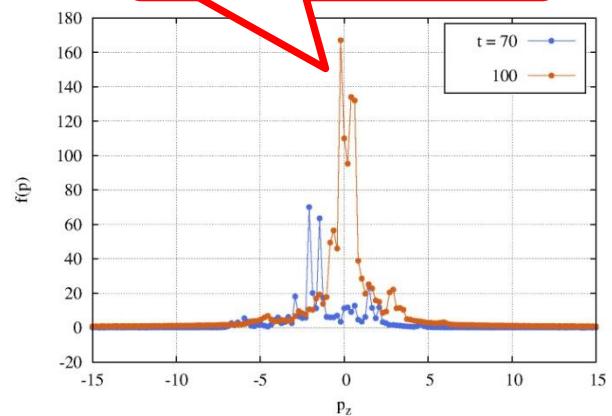
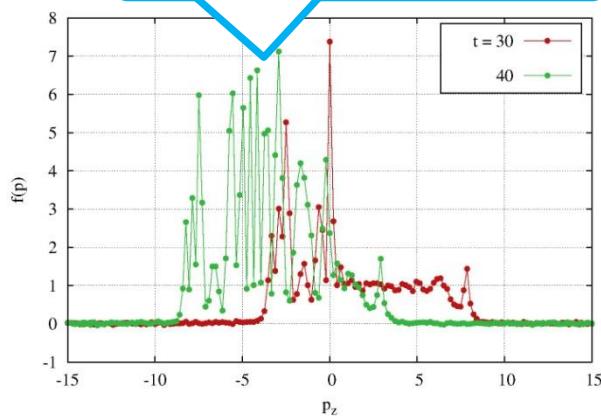
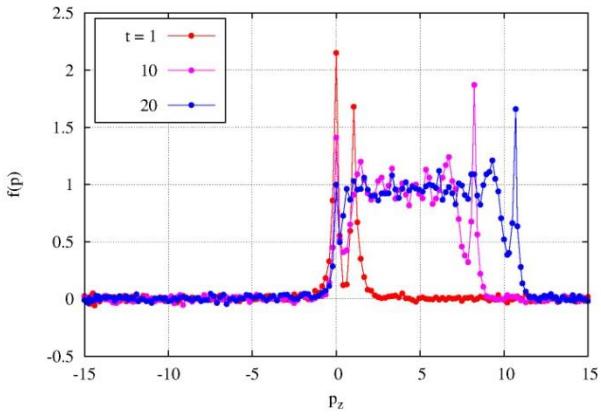
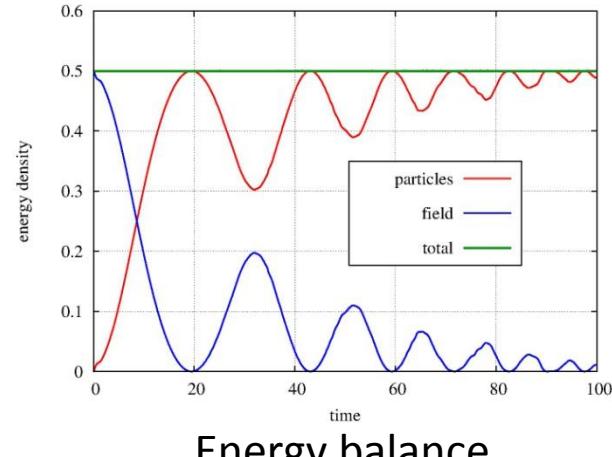
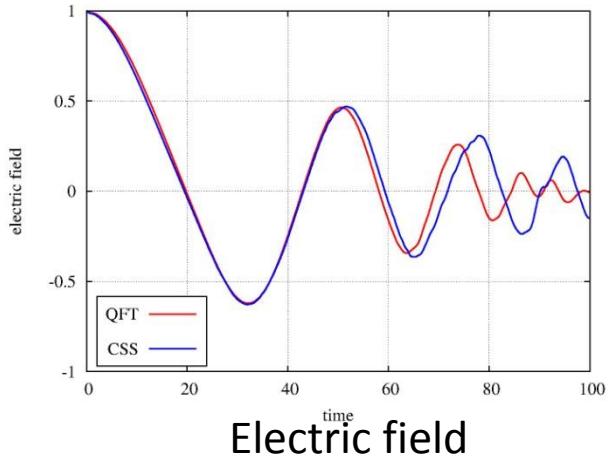
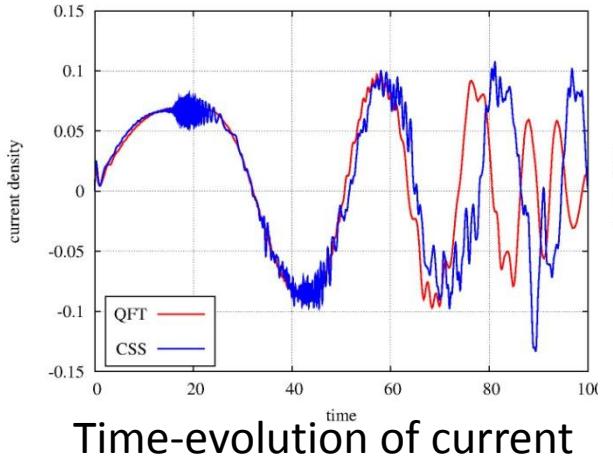
$$eE_0 = 1, e = 1, m = 0.1$$



Time-evolution of the distribution function

# Back reaction

$$eE_0 = 1, e = 1, m = 0.1$$



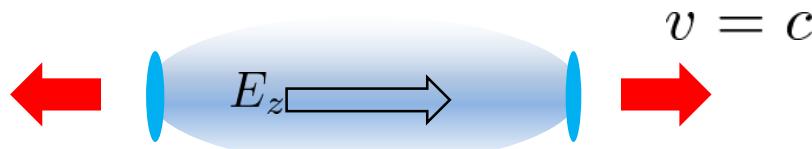
Quantum interference

Bose enhancement

# Expanding flux tube

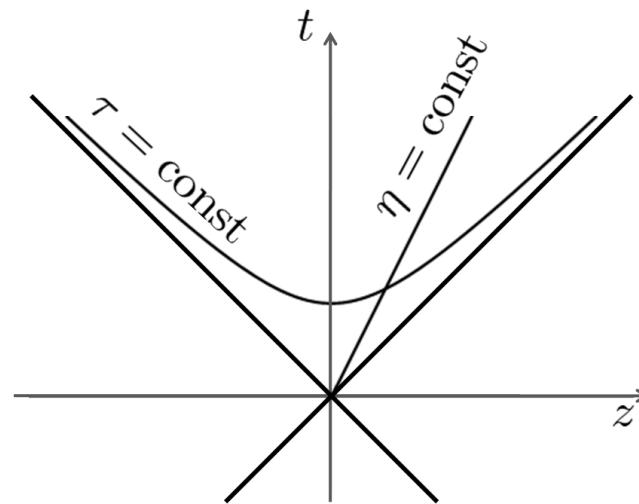
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The particle production from a single flux tube in scalar QED

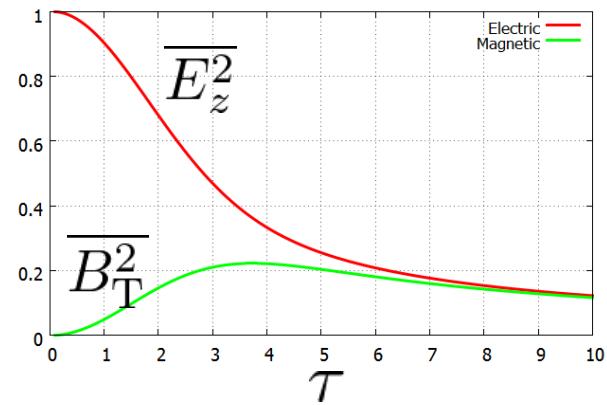
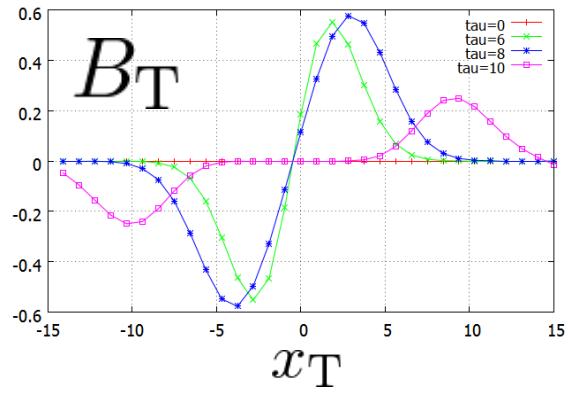
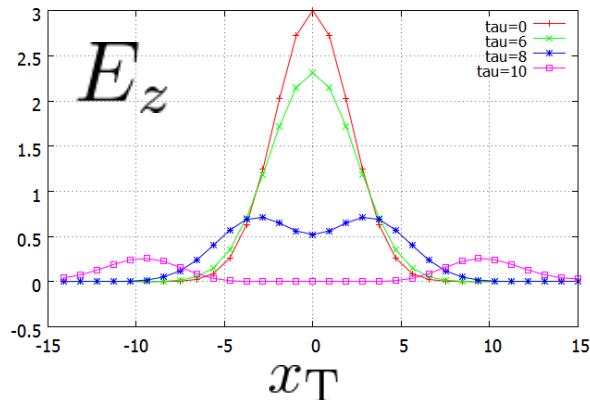
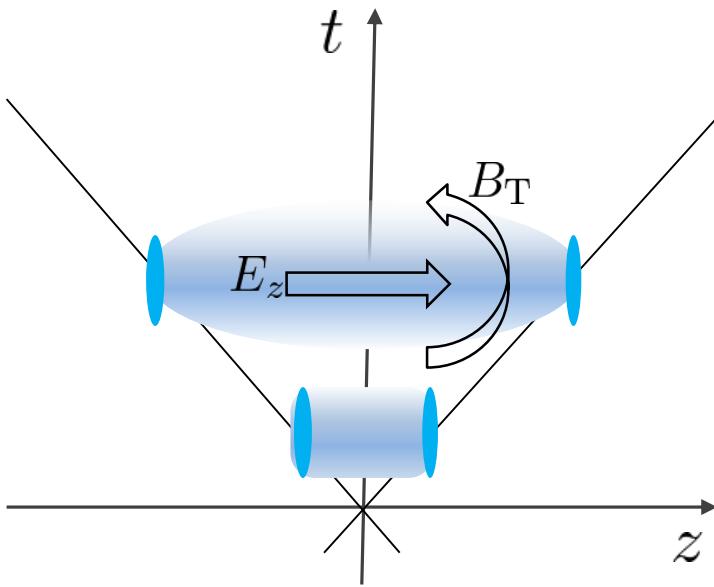


The  $\tau$ - $\eta$  coordinate

$$\begin{cases} \tau = \sqrt{t^2 - z^2} \\ \eta = \frac{1}{2} \ln \frac{t+z}{t-z} \end{cases}$$



# Expanding flux tube



See Fujii, Itakura (2008) for analytical computations of expanding color flux tubes.

# Particle production from the expanding flux tube

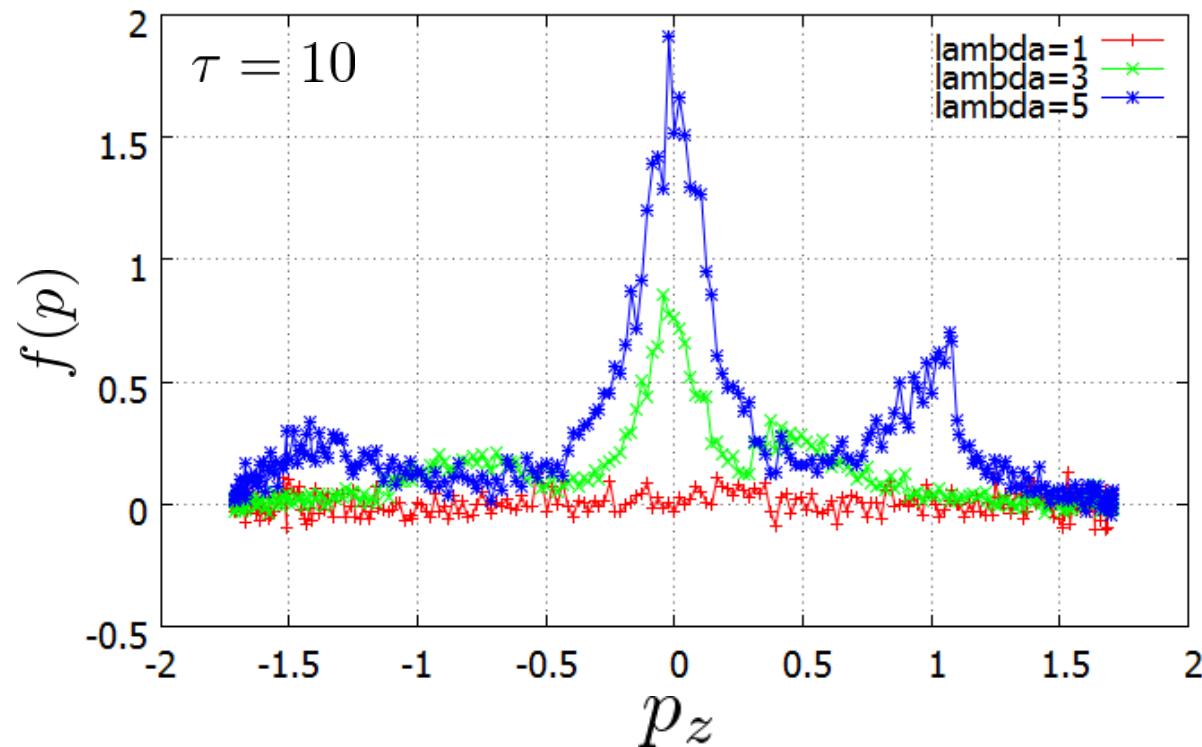
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$$N_x = N_y = 32$$

$$N_z = 256$$

$$N_{\text{ens}} = 512$$

$$E_{\text{ini}}(\mathbf{x}_T) = 3E_0 e^{-(x_T/\lambda)^2}$$



# Summary and outlook

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- The Schwinger mechanism in non-uniform and time-dependent fields can be computed by the Monte Carlo method with reasonable numerical costs.
- The particle production from expanding flux tubes has been computed.
  - ✓ Quark production in glasma
  - ✓ Dynamical photon/gluon cascading?