# Radiation reaction effects on the interaction of an electron with an intense laser pulse

Adam Noble, Yevgen Kravets, Sam Yoffe and Dino Jaroszynski

SILIS, Department of Physics, University of Strathclyde, UK

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# Introduction

- Charged particles subject to electromagnetic fields accelerate
- Accelerating charges radiate
- Upcoming laser facilities require description of both processes
- ‘Correct’ description of radiation reaction remains obscure

# Lorentz-Abraham-Dirac equation

Equation of motion for point charge developed by Lorentz, Abraham and Dirac:

$$\ddot{x}^a = -\frac{q}{m} F^a{}_b \dot{x}^b + \tau \Delta^a{}_b \dddot{x}^b. \qquad (1)$$

$\Delta = \dot{x}$-orthogonal projection; $\tau = q^2/6\pi m \simeq 10^{-23}$s.

**Problems:**

- exponential acceleration ('runaway solutions')
- causality violation ('preacceleration')

# Landau-Lifshitz equation

Landau and Lifshitz introduced an alternative equation, valid to order $\tau$, by replacing $\dddot{x}$ in (1) with derivative of applied force:

$$\ddot{x}^a = -\frac{q}{m} F^a{}_b \dot{x}^b - \tau \frac{q}{m} \dot{x}^c \partial_c F^a{}_b \dot{x}^b + \tau \frac{q^2}{m^2} \Delta^a{}_b F^b{}_c F^c{}_d \dot{x}^d. \qquad (2)$$

This equation is free of runaways and preacceleration.

**Problems:**

- What happens when radiation reaction is not a small perturbation?
- Why should we trust an approximation to an equation we do not trust?

# Ford-O'Connell equation

For a charged particle of a given structure, Ford and O'Connell derived an 'exact' equation of motion:

$$\ddot{x}^a = f^a + \tau \Delta^a{}_b \dot{f}^b. \tag{3}$$

If the applied force $f^a$ is the Lorentz force, this becomes

$$M^a{}_b \ddot{x}^b = -\frac{q}{m}(F^a{}_b + \tau \dot{x}^c \partial_c F^a{}_b)\dot{x}^b, \tag{4}$$

where $M^a{}_b = \Delta^a{}_b + \tau G^a{}_b$ and $G^a{}_b = \frac{q}{m}\Delta^a{}_c F^c{}_d \Delta^d{}_b$.

# Agreement with LL

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The FO equation agrees with LL to order $\tau$, i.e. if $M$ is 'close to' the unit matrix $\Delta$. Given its determinant,

$$\det M = 1 + \frac{\tau^2}{2} G^{ab} G_{ab}, \tag{5}$$

this gives rise to the condition

$$\tau \sqrt{G^{ab} G_{ab}/2} \ll 1. \tag{6}$$

In terms of the fields 'seen' by the particle, this is

$$\tau \frac{q}{m} B \ll 1. \tag{7}$$

Note: this involves the *magnetic* field, not the *electric* field.

# Example: Electron in a plane wave

Radiation reaction will be important when high energy electrons collide head-on with intense laser pulses.
We model the latter as a plane wave,

$$\frac{q}{m}F_{ab} = \mathcal{E}(\phi)(\epsilon_a n_b - \epsilon_b n_a), \tag{8}$$

where $\epsilon$ is the polarization, $n$ the (null) propagation direction, and $\phi = n_a x^a$.

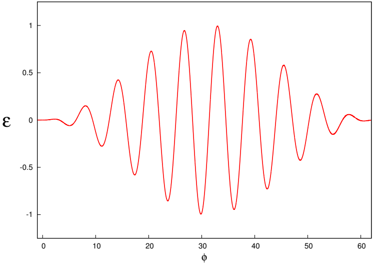


Figure 1 : 10 cycle pulse $\mathcal{E}$ with $\sin^2\phi$ envelope

# Moderately high energies, intensities

In this field, the condition for validity of LL is

$$\mathcal{T} := \tau\mathcal{E}\dot{\phi} \ll 1. \tag{9}$$

This is satisfied for electrons of energy $\gamma = 100$ in a pulse of intensity $a_0 = 100$.

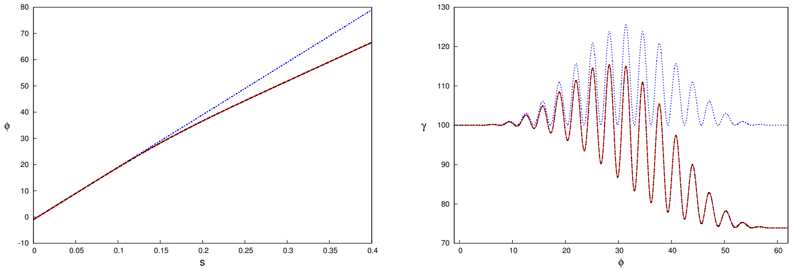


Figure 2 : $\phi$ vs. $s$ and $\gamma$ vs. $\phi$ for $\gamma_{\text{in}} = 100$ and $a_0 = 100$. No radiation reaction; LL radiation reaction; FO radiation reaction.

# Ultra-high energies, intensities

For the highest energy electrons produced, $\gamma = 10^5$; for the most powerful lasers under development, $a_0 = 1000$.
With these parameters

$$\mathcal{T} := \tau \mathcal{E} \dot{\phi} > 1. \tag{10}$$

We expect predictions of LL and FO to diverge.

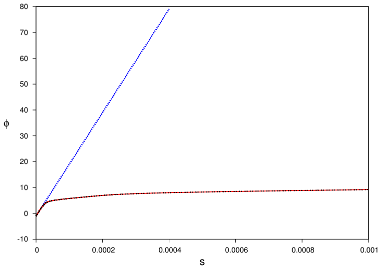


Figure 3 : $\phi$ vs. $s$ and $\gamma$ vs. $\phi$ for $\gamma_{\text{in}} = 10^5$ and $a_0 = 1000$. No radiation reaction; LL radiation reaction; FO radiation reaction.

# Dynamical paramters

$a_0$ and $\gamma_{\text{in}}$ are constants characterising the interaction. But $\mathcal{T}$ depends on the instantaneous values of $\dot{\phi}$ and $\mathcal{E}$, which vary along the particle's trajectory.

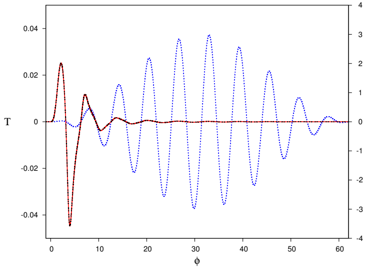


Figure 4 : $\mathcal{T}$ vs. $\phi$. Right axis, no radiation reaction; left axis, LL radiation reaction; left axis FO radiation reaction.

For large enough $\gamma_{\text{in}}$, the particle loses sufficient energy before reaching the high-intensity region of the pulse that the LL approximation never breaks down.

# Other concerns

Current analysis has limitations:

- Restricted to plane wave.
- Ignored quantum effects.

Transverse structure should not affect conclusions: ponderomotive force should drive particle to region of lower intensity, reinforcing LL.

Quantum effects *appear* important only as the electron first enters the pulse.

# Conclusions

- Issues with the LAD and LL equations for radiation reaction must be addressed.
- FO is an alternative equation of motion that is free of these problems, which may or may not describe radiating electrons.
- FO can be used to test validity of LL.
- FO reproduces predictions of LL in the linear regime.
- Rapid radiative damping prevents the particle entering the nonlinear regime.