CP and Discrete Flavour Symmetries

based on JHEP 1304 (2013) 122 [arXiv:1211.6953] with Manfred Lindner (HD) and Michael A. Schmidt (Melbourne)

Planck 2013, Bonn

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Heidelberg

Lepton mixing from discrete groups

complete flavour group

see talk by K.S. Lim tuesday

G_f

$e \to \rho(g_e)e$

$\nu \to \rho(g_\nu)\nu$

$L \to \rho(g)L$

LH leptons 3-dim rep.

residual symmetry of ($M_e M_e^+$)

$$\rho(g_e)^T M_e M_e^\dagger \rho(g_e)^* = M_e M_e^\dagger$$

residual symmetry of M_ν

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

$G_e=\langle T \rangle = Z_3$

$$\rho(T) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

(Z_3 smallest choice, but can also be continuous)

$G_\nu=\langle S,U \rangle = Z_2 x Z_2$

$$\rho(S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \rho(U) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

($Z_2 x Z_2$ most general choice if mixing angles do not depend on masses & Majorana ν s)

misaligned non-commuting symmetries lead to

$$\Omega_e^\dagger \rho(g_e)\Omega_e = \rho(g_e)_{diag}$$

$$U_{\text{PMNS}} = \Omega_e^\dagger \Omega_\nu$$

$$\Omega_\nu^\dagger \rho(g_\nu)\Omega_\nu = \rho(g_\nu)_{diag}$$

mixing matrix determined from symmetry up to interchanging of rows/columns and diagonal phase matrix

[He, Keum, Volkas '06; Lam' 07, '08; Altarelli,Feruglio' 05, Feruglio, Hagedorn,Toroop' 11]

$G_\nu=\langle S,U \rangle = Z_2 x Z_2$

$$U_{\text{HPS}} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{1}{3}, \quad \sin^2\theta_{23} = \frac{1}{2}, \quad \sin^2\theta_{13} = 0$$

tri-bimaximal mixing (TBM)

$G_\nu=\langle S \rangle = Z_2$

$$U_{TMM} = U_{TBM} \begin{pmatrix} \cos\theta & 0 & e^{i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

tri-maximal mixing (TM2)

[Lin' 10, Shimizu et al. '11,Luhn,King' 11,...]

Since it worked so nicely last time...

Θ_{13}

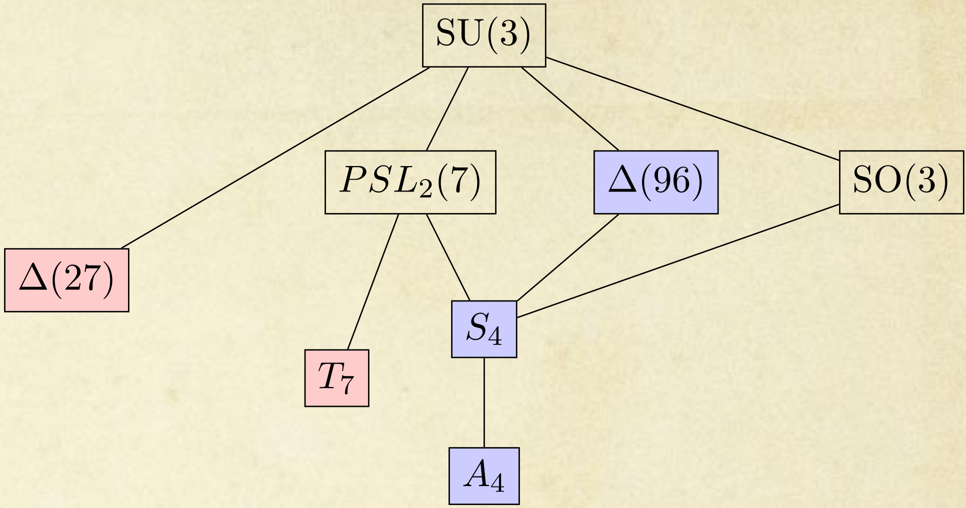
[Chen, Mahanpatra, Feruglio, Hagedorn, Ziegler, Ding, King, Luhn, Steward, Ivanov, Lavoura, Antusch, Spinrath, Meroni, Petcov, de Medeiros Varzielas, Leser, Ahn, Kang, Kim,...]

δ_{CP}

use discrete symmetry groups to predict leptonic CP phase(s)

CP Phase from Discrete Groups

- Take smallish group (A_4, S_4, T‘,….) and assume Θ_{13} is generated at NLO
  - TM1, TM2,… [e.g. King, Luhn 13]
- Impose CP on Lagrangian and break it spontaneously. [Lee 73, Branco 80]
- Can complex Clebsch-Gordon coefficients be the origin of CP violation?



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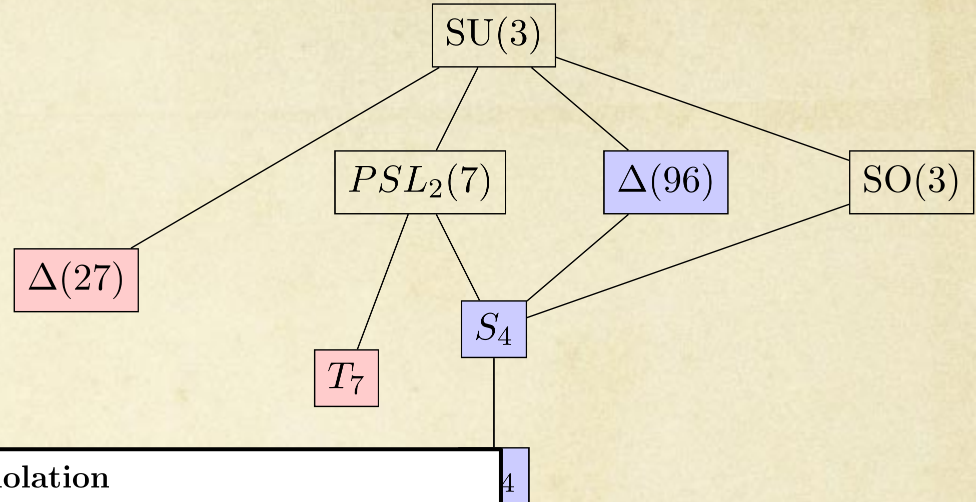

Group Theoretical Origin of CP Violation

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Abstract

We propose the complex group theoretical Clebsch-Gordon coefficients as a novel origin of CP violation. This is manifest in our model based on SU(5) combined with the T' group as the family symmetry. The complex CG coefficients in T' lead to explicit CP violation which is thus geometrical in origin. The predicted CP violation measures in the quark sector are consistent with the current experimental data. The corrections due to leptonic Dirac CP violating phase gives the experimental best fit value for the solar mixing angle, and we also gets the right amount of the baryonic asymmetry.

CP Phase from Discrete Groups

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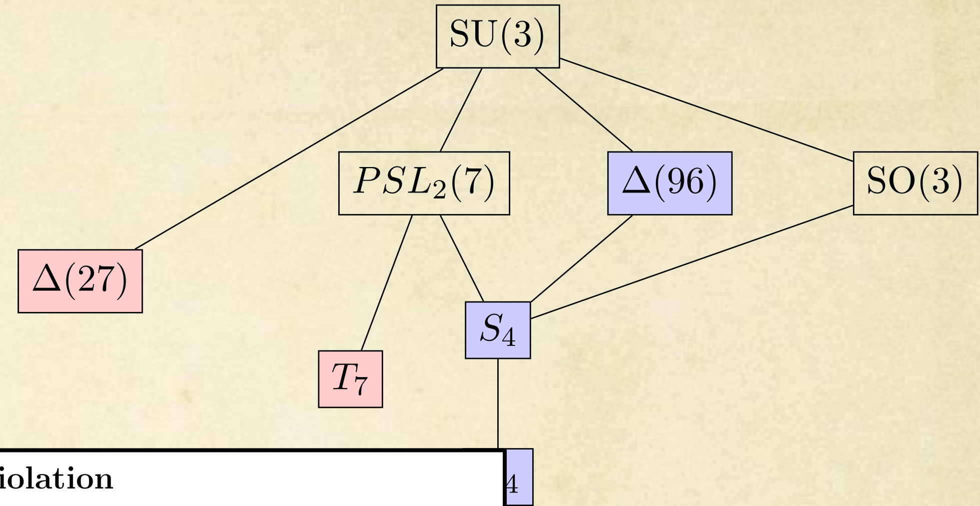

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Mu-Chun Chen[1,∗] and K.T. Mahanthappa[2,†]

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Geometrical CP violation from non-renormalisable scalar potentials

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(Dated: April 18, 2012)

We consider in detail the non-renormalisable scalar potential of three Higgs doublets transforming as an irreducible triplet of $\Delta(27)$ or $\Delta(54)$. We start from a renormalisable potential that spontaneously leads to a vacuum with CP-violating phases independent of arbitrary parameters – geometrical CP violation. Then we analyse to arbitrarily high order non-renormalisable terms that are consistent with the symmetry and we demonstrate that inclusion of non-renormalisable terms in the potential can preserve the geometrical CP-violating vacuum.

PACS numbers: 11.30.Hv, 12.60.Fr
Keywords: CP violation; Flavour symmetries; Extensions of Higgs sector

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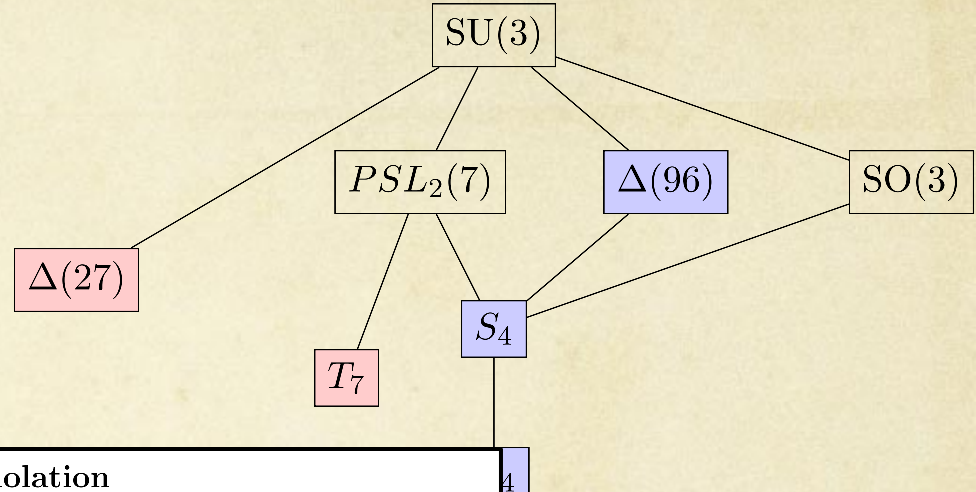

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PHYSICAL REVIEW D **86,** 113003 (2012)

A supersymmetric $SU(5) \times T'$ unified model of flavor with large θ_{13}

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(Received 12 September 2012; published 3 December 2012)

We present a SUSY $SU(5) \times T'$ unified flavor model with type I seesaw mechanism of neutrino mass generation, which predicts the reactor neutrino angle to be $\theta_{13} \approx 0.14$ close to the recent results from the Daya Bay and RENO experiments. The model predicts also values of the solar and atmospheric neutrino mixing angles, which are compatible with the existing data. The T' breaking leads to tribimaximal mixing in the neutrino sector, which is perturbed by sizeable corrections from the charged lepton sector. The model exhibits geometrical *CP* violation, where all complex phases have their origin from the complex Clebsch-Gordan coefficients of T'. The values of the Dirac and Majorana *CP* violating phases are predicted. For the Dirac phase in the standard parametrization of the neutrino mixing matrix we get a value close to 90°: $\delta \cong \pi/2 - 0.45\theta^c \cong 84.3°$, θ^c being the Cabibbo angle. The neutrino mass spectrum can be with normal ordering (2 cases) or inverted ordering. In each case the values of the three light neutrino masses are predicted with relatively small uncertainties, which allows one to get also unambiguous predictions for the neutrinoless double beta decay effective Majorana mass.

DOI: 10.1103/PhysRevD.86.113003 PACS numbers: 14.60.Pq, 12.10.Dm, 12.15.Ff, 12.60.Jv

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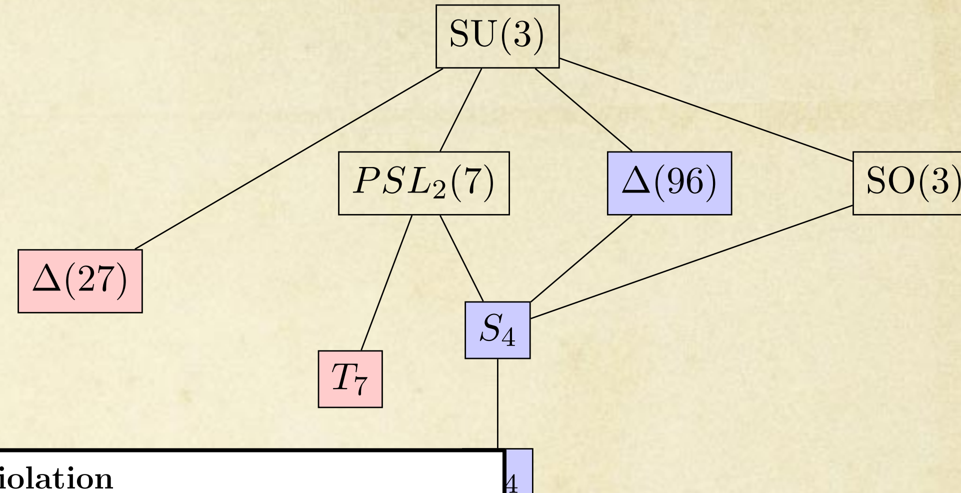

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Important Question for Model Building:

How can CP be defined consistently in a theory with a discrete flavour symmetry?

Problem

consider the group A4:

$$A_4 = \langle S, T | S^2 = T^3 = (ST)^3 = E \rangle$$

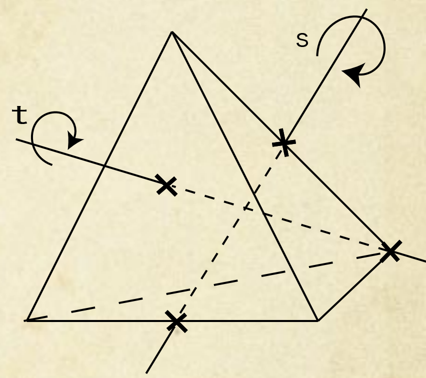
	E	T	T^2	S
$\underline{\mathbf{1}}_\mathbf{1}$	1	1	1	1
$\underline{\mathbf{1}}_\mathbf{2}$	1	ω	ω^2	1
$\underline{\mathbf{1}}_\mathbf{3}$	1	ω^2	ω	1
$\underline{\mathbf{3}}$	3	0	0	-1

$u : (S, T) \to (S, T^2)$.

consider a triplet $\chi \sim \underline{\mathbf{3}}$ transforming as

$$\rho_{\underline{\mathbf{3}}_\mathbf{1}}(S) = S_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \rho_{\underline{\mathbf{3}}_\mathbf{1}}(T) = T_3 \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

and a non-trivial singlet $\xi \sim \underline{\mathbf{1}}_\mathbf{3}$ ($\rho_{\underline{\mathbf{1}}_\mathbf{3}}(S) = 1 \quad \rho_{\underline{\mathbf{1}}_\mathbf{3}}(T) = \omega^2$)



under the CP transformation $\chi \to \chi^* \quad \xi \to \xi^*$

the A4 invariant

$$I = \xi \left(\chi_1\chi_1 + \omega^2\chi_2\chi_2 + \omega\chi_3\chi_3\right) \sim \underline{\mathbf{1}}_\mathbf{1}$$

$$\omega = e^{\mathrm{i}\frac{2\pi}{3}}$$

is mapped to sth. not invariant:

$$CP[I] = \xi^* \left(\chi_1^*\chi_1^* + \omega^2\chi_2^*\chi_2^* + \omega\chi_3^*\chi_3^*\right) \sim \underline{\mathbf{1}}_\mathbf{2}$$

- CP extends the group A4 and forbids this invariant??
- Is is possible to impose CP without enlarging the group?

How to define CP consistently

- Consider the vector made up out of all real(R), pseudo-real (P) and complex (C) representations of a given model

$$\phi = \left(\varphi_R, \quad \varphi_P, \quad \varphi_P^*, \quad \varphi_C, \quad \varphi_C^* \right)^T$$

- under the group G it transforms as $\phi \xrightarrow{G} \rho(g)\phi, \qquad g \in G.$
- the (reducible) representation $\rho : G \to U(N)$ is assumed to be faithful and complex
  - if not faithful then real symmetry group of theory is $G/\ker\rho$
  - ρ is homomorphism: ρ (a*b)= ρ (a) ρ (b)
- definition implies the existence of matrix W $\phi^* = W\phi$ or $\rho(g)^* = W\rho(g)W^{-1}$

$$P : \varphi(t, \vec{x}) \to \varphi(t, -\vec{x})$$

$$C : \varphi(t, \vec{x}) \to \varphi^*(t, \vec{x})$$

$$CP : \varphi(t, \vec{x}) \to \varphi^*(t, -\vec{x})$$

- here only Lorentz-scalars, generalization straightforward

How to define CP consistently

- A generalized CP acts upon the vector

[Bernabeu, Branco, Gronau 86,]

$$\boxed{\phi \xrightarrow{CP} U\phi^*}$$

where U is unitary, to leave the kinetic term invariant.

for gauge groups this has been investigated by [Grimus, Rebelo 95]

CONSISTENCY CONDITION:

- If G is the complete symmetry group, CP has to close in G:

$$\begin{tikzcd}
 & U\phi^* \arrow[dr, bend left, "g"] & \\
\phi \arrow[ur, bend left, "CP"] \arrow[dr, bend right, "g'"'] & \boxed{U\rho(g)^*U^{-1} \in \mathrm{Im}\rho} & U\rho(g)^*\phi^* \arrow[dl, bend left, "CP^{-1}"] \\
 & \rho(g')\phi = \quad U\rho(g)^*U^{-1}\phi &
\end{tikzcd}$$

CP and the automorphism group

- The consistency condition $U\rho(g)^*U^{-1} \in \mathrm{Im}\rho$ defines an automorphism

$$\begin{tikzcd}
 & \rho(g)^* \arrow[r] & U\rho(g)^*U^{-1} = \rho(g') \arrow[dr, bend left, "\rho^{-1}"] & \\
g \in G \arrow[ur, bend left, "\rho"] \arrow[rrr, "u : G \to G"] & & & u(g) = g' \in G
\end{tikzcd}$$

$$\boxed{U\rho(g)^*U^{-1} = \rho(u(g))}$$

- the matrcies {U} furnish a representation of the automorphism group

$$\begin{aligned}
\rho\left((a \circ b)(g)\right) = \rho(a(b(g))) &= U(a)\rho(b(g))^*U(a)^{-1} \\
&= U(a)W\rho(b(g))W^{-1}U(a)^{-1} \\
&= U(a)WU(b)\rho(g)^*U(b)^{-1}W^{-1}U^{-1}(a)
\end{aligned}$$

$$\boxed{U(a \circ b) = U(a)WU(b)}$$

remember $\rho(g)^* = W\rho(g)W^{-1}$

neutral: $U(id) = W$

inverse: $U(u^{-1}) = WU^{-1}(u)W^{-1}$

CP and the automorphism group

Inverse Direction: : Each automorphism u of G may be represented by such a matrix U.

$$U\rho(g)^*U^{-1} = \rho(u(g))$$

Proof:

- Construct group extended by automorphism u (u^n=id)

$$G' = G \rtimes_\theta Z_n \quad \theta : \{0, ..., n-1\} \to Aut(G) \quad \theta(1) = u$$
$$(g_1, z_1) \star (g_2, z_2) = (g_1\theta_{z_1}(g_2), z_1 + z_2)$$

- u acts as conjugation within this group

$$(E,1) \star (g,0) \star (E,1)^{-1} = (u(g),0)$$

- Consider representation $\rho' : G' \to U(M)$ induced via $\rho'(g,0) = \rho(g)$

- automorphism u is represented by matrix

$$U(u) = \rho'((E,1))W$$

$$\begin{aligned}\rho(u(g)) &= \rho'(u(g),0)\\ &= \rho'((E,1) \star (g,0) \star (E,1)^{-1})\\ &= \rho'((E,1))\rho'((g,0))\rho'((E,1))^{-1}\\ &= \rho'((E,1))W\rho(g)^*W^{-1}\rho'((E,1))^{-1}\end{aligned}$$

CP vs. A_4

outer automorphism group Z_2, there is one outer automorphism:

$$u : (S, T) \to (S, T^2).$$

	E	T	T^2	S
$\underline{\mathbf{1}}_{\mathbf{1}}$	1	1	1	1
$\underline{\mathbf{1}}_{\mathbf{2}}$	1	ω	ω^2	1
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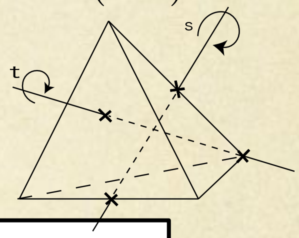

on 3-dim representation

$$\rho_{\underline{\mathbf{3}}_{\mathbf{1}}}(S) = S_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \rho_{\underline{\mathbf{3}}_{\mathbf{1}}}(T) = T_3 \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

as long as the theory only contains 3-dimensional representations, there are 2 possibilities for $U\rho(g)^* U^{-1} \in \mathrm{Im}\rho$

$$U = U_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$U_3 \rho(T)^* U_3^{-1} = \rho(T^2)$$

$$u : (S, T) \to (S, T^2).$$

$$\underline{\mathbf{3}} \to U_3 \underline{\mathbf{3}}^*$$

outer automorphism

$$U = \mathbb{1}_3$$

$$\rho(T)^* = \rho(T)$$

trivial map

$$\underline{\mathbf{3}} \to \underline{\mathbf{3}}^*$$

inner automorphism

CP vs. A_4

- the ‚CP transformation‘ that is trivial with regard to A_4 runs into trouble if one considers a non-trivial singlet $\xi \sim \underline{\mathbf{1}}_{\mathbf{3}}$ in addition to the triplet $\chi \sim \underline{\mathbf{3}}$

- if one would use $\chi \to \chi^*$ and $\xi \to \xi^*$ one finds that the invariant is mapped to sth. non-invariant

$$\underline{\mathbf{1}}_{\mathbf{1}} \sim (\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}}\, \xi \to (\chi^*\chi^*)_{\underline{\mathbf{1}}_{\mathbf{2}}}\, \xi^* \sim \underline{\mathbf{1}}_{\mathbf{2}}$$

with $(\phi\phi)_{\underline{\mathbf{1}}_{\mathbf{2}}} = \frac{1}{\sqrt{3}}\left(\phi_1\phi_1 + \omega^2\phi_2\phi_2 + \omega\phi_3\phi_3\right)$

- this can be readily understood if one looks at how this ‚CP transformation‘ $\phi \to U \phi^*$ acts upon $\phi = (\xi, \xi^*, \chi)^T$
  - naive CP corresponds to $U=1_5$
  - A_4 does not close under this CP:

$$U\rho(T)^* U^{-1} = \rho(T)^* \notin \rho(G)$$

$$\rho(T) = \mathrm{diag}(\omega, \omega^2, T_3)$$
$$\rho(S) = \mathrm{diag}(1, 1, S_3)$$

- the real flavour group is larger, this has to be considered when constructing Lagrangian

often overlooked in literature [Toorop et. al. 2011, Ferreira, Lavoura 2011,....]

CP vs. A_4

if one does not want to extend the group one therefore has the options

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & U_3 \end{pmatrix} .$$

$$u : (S, T) \to (S, T^2).$$

$$\chi \to U_3 \chi^*$$

$$\xi \to \xi^*$$

$$U = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \mathbb{1}_3 \end{pmatrix} .$$

trivial map

$$\chi \to \chi^*$$

$$\xi \to \xi$$

to fulfil the consistency condition

$$U \rho(g)^* U^{-1} = \rho(u(g))$$

Note that complex VEVs of the type (1,z,z*) conserve this CP

$$U_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

CP vs. A_4 - Application

- consider a triplet of Higgs doublets $\chi = (\chi_1, \chi_2, \chi_3)^T \sim \underline{\mathbf{3}}$
- there is one phase-dependent term in the potential

$$\lambda_5 \; (\chi^\dagger \chi)_{\underline{\mathbf{3}}_{\mathbf{1}}} \left(\chi^\dagger \chi\right)_{\underline{\mathbf{3}}_{\mathbf{1}}} + \text{h.c.} = \lambda_5 \; \left[\left(\chi_1^\dagger \chi_2\right)^2 + \left(\chi_2^\dagger \chi_3\right)^2 + \left(\chi_3^\dagger \chi_1\right)^2\right] + \text{h.c.}$$

- the CP trafo $\chi \rightarrow \chi^*$ would restrict the phase to be zero
- even for non-vanishing phase, the VEV configuration $\langle \chi \rangle = V(1,1,1)$, $V \in \mathbb{R}$ can be obtained. [Toorop et. al. 2011]
- Spontaneous CP restoration??
- This can be understood if one considers the CP transformation $\chi \rightarrow U_3 \chi^*$
  - this is a symmetry of the potential for any phase of λ_5
  - also the VEVs preserve the CP transformation
  - therefore this CP is conserved in this case
- accidental CP transformations seem to be origin of ‚calculable phases‘

$$U_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Geometric CP violation in $\Delta\,(27)$

$$\Delta(27) = \langle A, B | A^3 = B^3 = (AB)^3 = E \rangle$$

automorphism group generated by

$u_2 : (A, B) \to (ABAB, B^2)$ $\quad$ $u_1 : (A, B) \to (ABA^2, B^2AB)$

red $\quad$ blue

	E	$BABA$	ABA	A	BAB	AB	A^2	B^2	B	BA^2BAB	AB^2ABA
$\underline{\mathbf{1}}_{\mathbf{1}}$	1	1	1	1	1	1	1	1	1	1	1
$\underline{\mathbf{1}}_{\mathbf{2}}$	1	ω	ω^2	1	ω	ω^2	1	ω	ω^2	1	1
$\underline{\mathbf{1}}_{\mathbf{3}}$	1	ω^2	ω	1	ω^2	ω	1	ω^2	ω	1	1
$\underline{\mathbf{1}}_{\mathbf{4}}$	1	ω	ω	ω^2	ω^2	ω^2	ω	1	1	1	1
$\underline{\mathbf{1}}_{\mathbf{5}}$	1	ω^2	1	ω^2	1	ω	ω	ω	ω^2	1	1
$\underline{\mathbf{1}}_{\mathbf{6}}$	1	1	ω^2	ω^2	ω	1	ω	ω^2	ω	1	1
$\underline{\mathbf{1}}_{\mathbf{7}}$	1	ω^2	ω^2	ω	ω	ω	ω^2	1	1	1	1
$\underline{\mathbf{1}}_{\mathbf{8}}$	1	1	ω	ω	ω^2	1	ω^2	ω	ω^2	1	1
$\underline{\mathbf{1}}_{\mathbf{9}}$	1	ω	1	ω	1	ω^2	ω^2	ω^2	ω	1	1
$\underline{\mathbf{3}}$	3	.	.	.	.	.	.	.	.	3ω	$3\omega^2$
$\underline{\mathbf{3}}^*$	3	.	.	.	.	.	.	.	.	$3\omega^2$	3ω

What are calculable phases?

- consider again a triplet of Higgs doublets $H = (H_1, H_2, H_3) \sim \underline{\mathbf{3}}$ which transforms as

$$\rho(A) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad \rho(B) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

- the potential only contains one phase dependent term

$$I \equiv (H_1^\dagger H_2)(H_1^\dagger H_3) + (H_2^\dagger H_3)(H_2^\dagger H_1) + (H_3^\dagger H_1)(H_3^\dagger H_2)$$

  - if coupling λ_4 multiplying I is positive, the global minimum is at $\langle H \rangle = \frac{v}{\sqrt{3}}(\omega^2, 1, 1)$
    (or a configuration that can be obtained by acting on this vacuum with a group element)
  - if coupling λ_4 is negative, the global minimum is at $\langle H \rangle = \frac{v}{\sqrt{3}}(1, \omega, \omega^2)$
    (or a configuration that can be obtained by acting on this vacuum with a group element)
- These phases do not depend on potential parameters!
  - can this be used to predict (leptonic) CP phases?
  - can they be understood in terms of generalized CP?

Potential Dependence of Phases

- in general you expect two different kinds of vacua of a CP conserving potential
  - either VEV is real, conserves CP and phase does not depend on potential parameters
  - or VEV is complex, breaks CP and phase depends on potential parameters

Example:

all parameters real

$$\begin{aligned} V &= m_1^2\varphi^*\varphi + m_2^2(\varphi^2 + \varphi^{*2}) + \lambda_1(\varphi^*\varphi)^2 + \lambda_2(\varphi^4 + \varphi^{*4}) \\ &= m_1^2 A^2 + m_2^2 A^2 \cos 2\alpha + \lambda_1 A^4 + \lambda_2 A^4 \cos 4\alpha \end{aligned}$$

invariant under $\varphi \to \varphi^*$

$$\varphi = A e^{i\alpha}$$

$$\alpha = 0$$

$$A = -\frac{\sqrt{-\mathrm{m1}^2 - 2\mathrm{m2}^2}}{\sqrt{2}\sqrt{\lambda 1 + 2\lambda 2}}$$

$$\cos^2\alpha = \frac{2\lambda_2 m_1^2 + \lambda_1 m_2^2 - 2\lambda_2 m_2^2}{4\lambda_2 m_1^2}$$

$$A = \frac{m_1}{\sqrt{2}\sqrt{2\lambda_2 - \lambda 1}}$$

What are calculable phases?

- The vacuum of the form $\langle H \rangle = \frac{v}{\sqrt{3}}(1, \omega, \omega^2)$ leaves invariant the CP transformation

$$H \to \rho(B^2)H^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} H^*$$

- which is a symmetry of I+I*
  - no surprise there, CP symmetric potential has CP symmetric ground state
- for the other solution $\langle H \rangle = \frac{v}{\sqrt{3}}(\omega^2, 1, 1)$ there is no group element that leaves H invariant $\langle H \rangle = \rho(g)\langle H \rangle^*$
  - this was interpreted as geometrical CP violation

GEOMETRICAL *T*-VIOLATION

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[Branco, Gerard and Grimus 1984; de Medeiros Varzielas, Emmanuel-Costa 2011; Battacharyya, de Medeiros Varzielas, Leser 2012]

Calculable Phases as a Result of an accidental generalized CP transformation

- every automorphism corresponds to a generalized CP transformation
  - automorphism group of $\Delta(27)$ is of order 432, generated by $u_1 : (A, B) \to (ABA^2, B^2AB)$ $\quad u_2 : (A, B) \to (ABAB, B^2)$
- this allows one to search for CP transformation that leaves $\langle H \rangle = \frac{v}{\sqrt{3}}(\omega^2, 1, 1)$ invariant and gives a real λ_4
- indeed there is such a CP transformation:

$$H \to \tilde{U} H$$

$$\tilde{U} = \begin{pmatrix} 0 & 0 & \omega^2 \\ 0 & 1 & 0 \\ \omega & 0 & 0 \end{pmatrix}$$

$$CP_u[\langle H \rangle] = \langle H \rangle$$

$$CP_u[I] = I$$

$$u \,:\, (A, B) \to (AB^2AB, AB^2A^2)$$

$$\underline{\mathbf{1}}_{\mathbf{2}} \leftrightarrow \underline{\mathbf{1}}_{\mathbf{3}}, \qquad \underline{\mathbf{1}}_{\mathbf{5}} \leftrightarrow \underline{\mathbf{1}}_{\mathbf{9}}, \qquad \underline{\mathbf{1}}_{\mathbf{6}} \leftrightarrow \underline{\mathbf{1}}_{\mathbf{8}},$$

Calculable Phases as a Result of an accidental generalized CP transformation

- it seems that geometric CP violation can always be explained as the result of an accidental generalized CP symmetry of the potential
- a symmetric potential can have a symmetric ground state
  - phases are dictated by accidental CP symmetry
  - explains the independence from potential parameters
- this setup might still be interesting for phenomenlogy:
  - if accidental symmetry only of potential, not of Yukawas, it can be used to predict phases etc.
- need groups with large outer automorphism group

Conclusions

- Consistency Conditions should be kept in mind when constructing models that contain CP and Flavour Symmetries
- generalized CP transformations may be interpreted as furnishing a representation of the automorphism group
- geometrical CP violation seems to be a consequence of (accidental) generalized CP symmetries of the potential
- maybe automorphisms may be used in model building more generally