Spontaneous CP violation in A_4 x SU(5) with Constrained Sequential Dominance 2

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Based on collaborations with:
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arXiv:0910.5127, arXiv:1103.5930, arXiv:1108.4278, arXiv:1301.6764

SISSA
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Outline

- **Motivation & Strategy**
  - CSD2
  - GUT corrections
  - Spontaneous CP violation
- The Model
- Summary & Conclusions

The Flavour Puzzle

- Why do we have three generations?
- Why are the SM fermion masses so vastly different?
- What is the origin of CP violation?
- Why are the quark mixing angles rather small and the leptonic mixing angles rather large?

Great Recent Progress in the Lepton Sector

	March 2011, NH	Jun 2012, NH
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	$0.307^{+0.018}_{-0.016}$
$\sin^2\theta_{23}$	0.51 ± 0.06	$0.386^{+0.024}_{-0.021}$
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$	0.0241 ± 0.0025
δ	?	$(1.08^{+0.28}_{-0.31})\pi$
	[Schwetz, Tortola, Valle 2011]	**[Fogli, Lisi *et al.* 2012]**

So what?

Many opinions...

A. Adulpravitchai, Y. H. Ahn, C. H. Albright, G. Altarelli, S. Antusch, A. Aranda, T. Araki, F. Bazzocchi, W. Buchmüller, P. S. Bhupal Dev, G. C. Branco, Q.-H. Cao, H.-Y. Cheng, I. K. Cooper, S. Dev, G. Blankenburg, C. Bonilla, F. Gonzalez Canales, W. Chao, J.-M. Chen, M.-C. Chen, X. Chu, A. Datta, K. N. Deepthi, M. Dhen, D. A. Dicus, G.-J. Ding, P. V. Dong, V. Domcke, L. Dorame, B. Dutta, D. A. Eby, L. Everett, R. P. Feger, F. Feruglio, P. Ferreira, P. H. Frampton, M. Fukugita, R. R. Gautam, S.-F. Ge, D. K. Ghosh, R. Gonzalez Felipe, S. Gollu, S. Gupta, W. Grimus,C. Gross, N. Haba, C. Hagedorn, T. Hambye, J. E. Kim, Y. Koide, K. Hashimoto, K. Harigaya, H.-J. He, X.-G. He, J. Heek, D. Hernandez, M. Holthausen, R. S. Hundi, M. Ibe, H. Ishimori, F. R. Joaquim, A. S. Joshipura, S. K. Kang, T. W. Kephart, S. Khalil, S. F. King, T. Kobayashi, S. Kumar, L. Lavoura, X.-Q. Li, H. N. Long, P. O. Ludl, C. Luhn, B. Q. Ma, E. Ma, S. K. Majee, K.T. Mahanthappa, D. Marzocca, V. Maurer, D. Meloni, A. Merle, A. Meroni, R. Mohanta, R. N. Mohapatra, E. Molinaro, A. Mondragon, M. Mondragon, S. Morisi, C. H. Nam, H. Nishiura, S. Oh, H. Okada, K. M. Patel, K. M. Parattu, E. Peinado, S. T. Petcov, N. Qin, A. Rashed, W. W. Repko, A. D. Rojas, W. Rodejohann, A. Romanino, G. G. Ross, S. Rigolin, M. A. Schmidt, K. Schmitz, M. Severson, M.-S. Seo, H. Serodio, Y. Shimizu, J. I. Silva-Marcos, L. Singh, K. Siyeon, C. Sluka, A. Yu. Smirnov, M. Spinrath, E. Stamou, A. J. Stuart, R. Takahashi, M. Tanimoto, R. d. A. Toorop, J. W. F. Valle, I. d. M. Varzielas, L. Velasco-Sevilla, V. V. Vien, B. Wang, Q. Wang, A. Watanabe, D. Wegman, A. Wingerter, Yue-Liang Wu, Z. -Z. Xing, T. T. Yanagida, W.-M. Yang, B. Zaldívar, F.-R. Yin, A. Zee, H. Zhang, Y.-j. Zheng, J.-J. Zhong, S. Zhou, R. Zwicky, …

Please let me know if I missed someone...

So what?

- Very popular before θ_{13} measurement: (Tri-)Bimaximal Mixing

$$\sin^2\theta_{12} = \left(\frac{1}{3}\right)\frac{1}{2}\,, \quad \sin^2\theta_{23} = \frac{1}{2}\,, \quad \sin^2\theta_{13} = 0$$

- Modifications? Alternatives? Plenty...
  - Here: A GUT version of Trimaximal Mixing

[Antusch, King, Luhn, MS '11; Antusch, King, MS '13]

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Sequential Dominance

[King '98-'02; Antusch, Boudjemaa, King '10]

- Type I seesaw with the basis (A, B, C vectors)

$$Y_\nu = (A, B, C) \quad \text{and} \quad M_R = \text{diag}(M_A, M_B, M_C)$$

- The light neutrino mass matrix reads then

$$M_\nu = \frac{v^2 A A^T}{M_A} + \frac{v^2 B B^T}{M_B} + \frac{v^2 C C^T}{M_C}$$

- For a strong hierarchy $A^2/M_A \gg B^2/M_B \gg C^2/M_C$

$$A \to \theta_{23}\ , \quad B \to \theta_{12}\ , \quad A, B \to \theta_{13}$$

Simple Patterns

- Minimalistic pattern for TBM (CSD):

[King '05]

$$Y_\nu = \begin{pmatrix} 0 & b \\ a & b \\ -a & b \end{pmatrix}, \qquad M_R = \begin{pmatrix} M_A & 0 \\ 0 & M_B \end{pmatrix}$$

- Pattern for trimaximal mixing (CSD2):

[Antusch, King, Luhn, MS '11; Antusch, King, MS '13]

$$Y_\nu = \begin{pmatrix} 0 & b \\ a & 0 \\ -a & 2b \end{pmatrix}, \qquad M_R = \begin{pmatrix} M_A & 0 \\ 0 & M_B \end{pmatrix}$$

[Trimaximal variant we use based on C. S. Lam '06; C. H. Albright, W. Rodejohann '09; C. H. Albright, A. Dueck, W. Rodejohann '10]

Phenomenology

[Antusch, King, Luhn, MS '11]

- The light neutrino mass matrix:

$$M_\nu = m_a \begin{pmatrix} \eta & 0 & 2\eta \\ 0 & 1 & -1 \\ 2\eta & -1 & 1+4\eta \end{pmatrix} \ , \quad \eta = \epsilon \ \mathrm{e}^{\mathrm{i}\alpha} \ , \quad \epsilon \ll 1$$

- Approximate formulas:

$$\sin\theta^\nu_{23} \approx \frac{1}{\sqrt{2}} - \frac{\epsilon}{\sqrt{2}} \cos\alpha \ , \qquad \delta^\nu_{13} \approx \pi - \alpha + \epsilon \frac{5}{2} \sin\alpha \ ,$$

$$\sin\theta^\nu_{12} \approx \frac{1}{\sqrt{3}} \ , \qquad \alpha_2 \approx -\alpha + 2\epsilon \sin\alpha \ ,$$

$$\sin\theta^\nu_{13} \approx \frac{\epsilon}{\sqrt{2}} \longleftarrow \theta_{13} \sim 5°\text{-}6°$$ (ϵ determined by neutrino masses)

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Charged Lepton Mixing Contributions in GUTs I

- Typical structure for the charged lepton Yukawa matrix:

$$Y_e \approx \begin{pmatrix} 0 & c\,\mathrm{e}^{\mathrm{i}\,\beta} & 0 \\ * & d & 0 \\ 0 & * & * \end{pmatrix}$$

- We use the conventions:

$$U_e^\dagger = U_{23}^e U_{13}^e U_{12}^e \text{ with } U_{12}^e = \begin{pmatrix} c_{12}^e & s_{12}^e \mathrm{e}^{-\,\mathrm{i}\,\delta_{12}^e} & 0 \\ -s_{12}^e \mathrm{e}^{\mathrm{i}\,\delta_{12}^e} & c_{12}^e & 0 \\ 0 & 0 & 1 \end{pmatrix} (U_e M_e M_e^\dagger U_e^\dagger = \text{diag.})$$

- Then we find (in our model later c/d < 0):

$$\theta_{12}^e \approx \left|\frac{c}{d}\right| \text{ and } \delta_{12}^e = \begin{cases} -\beta & \text{for } c/d > 0 \\ -\beta + \pi & \text{for } c/d < 0 \end{cases}$$

Charged Lepton Mixing Contributions in GUTs II

- With the previous structures:

$$\theta_{23} \approx 45^\circ - \epsilon \cos\alpha$$

$$\theta_{13} \approx \frac{\epsilon}{\sqrt{2}} - \cos(\beta - \alpha)\,\frac{\theta^e_{12}}{\sqrt{2}}$$

$$\theta_{12} \approx 35.3^\circ + \cos\beta\,\frac{\theta^e_{12}}{\sqrt{2}}$$

- θ_{23} about 40° for $\alpha = \pi/3$ (→ Spontaneous CP)
- Right θ_{13} for $\beta = \alpha + \pi$ and $\theta^e_{12} = 3°$ (→ Spontaneous CP + GUTs)

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Discrete Vacuum Alignment

[Antusch, King, Luhn, MS '11]

- Use family symmetry to align flavon vevs (Yukawa entries), e.g.

$$\langle\phi\rangle \propto (0,0,x)^T \quad \text{or} \quad \langle\phi\rangle \propto (x,x,x)^T$$

- Add term to W compatible with Z_n symmetry

$$\mathcal{W} \supset P\left(\kappa\frac{\phi^n}{\Lambda^{n-2}} \mp \lambda M^2\right)$$

- Solve F-term conditions + CP symmetry ($|F_P| = 0$; κ, λ real)

$$\arg(\langle\phi\rangle) = \arg(x) = \begin{cases} \frac{2\pi}{n}q\ , \quad q = 1,\ldots,n & \text{for "}-\text{"} \\ \frac{2\pi}{n}q + \frac{\pi}{n}\ , \quad q = 1,\ldots,n & \text{for "}+\text{"} \end{cases}$$

Applications to Quarks: The Phase Sum Rule

[Antusch, King, Malinsky, MS '10]

- For mass matrices with vanishing 1-3 element:

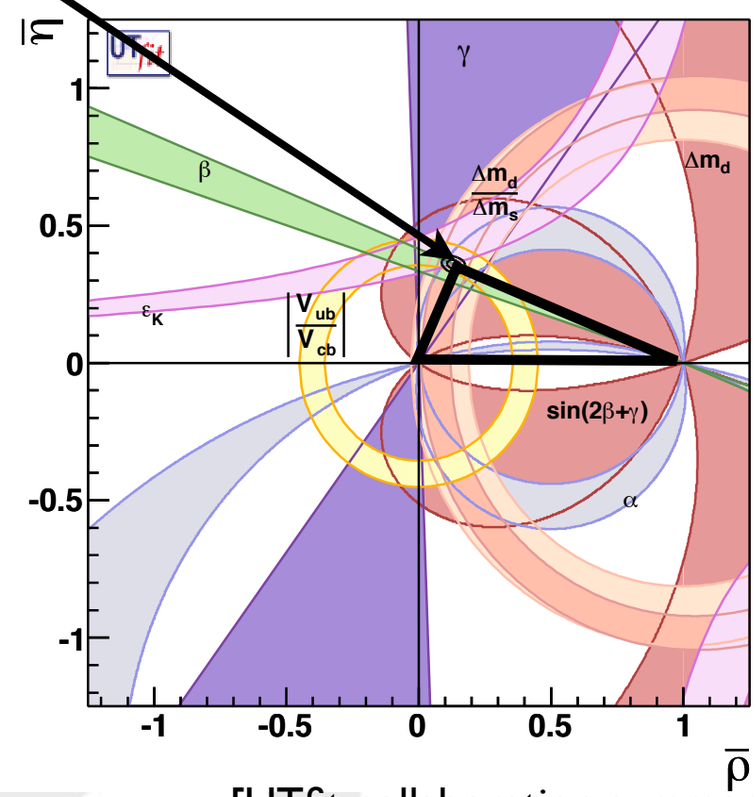
$$\alpha \approx \delta_{12}^d - \delta_{12}^u \approx 90^\circ$$

[Antusch, King, Malinsky, MS '09] see also [Fritzsch and Xing; Masina and Savoy '06; Harrison, Dallison, Roythorne, Scott '09]

- Simple solution, e.g.

$$M_d = \begin{pmatrix} 0 & \mathrm{i}\,\epsilon_2 & 0 \\ * & \epsilon_{23} & * \\ 0 & 0 & * \end{pmatrix}, \quad M_u \text{ real}$$

$$\Rightarrow \delta_{12}^d = 90^\circ\,, \quad \delta_{12}^u = 0$$



[UTfit collaboration summer 2010 results (pre-ICHEP)]

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The Model

[Antusch, King, MS '13]

- $SU(5) \times A_4 \times Z_4^{\,2} \times Z_3^{\,4}$
  - $F \sim (d^c, L) \sim 3$, $T_i \sim (Q, u^c, e^c)_i \sim 1$, $N_i \sim 1$
  - Higgs fields in 5, 24, 45 dimensional representations
- The A_4 triplet flavon vevs are real
- Singlet flavons acquire complex vevs, e.g.

$$\mathcal{W} \supset \frac{P}{\Lambda}(\xi_2^3 + M^3) \;\Rightarrow\; \langle \xi_2 \rangle \sim \mathrm{e}^{-\mathrm{i}\,\frac{\pi}{3}} \;\Rightarrow\; \alpha = \frac{\pi}{3}$$

How to get the (1,0,2) alignment?

[Antusch, King, Luhn, MS '11]

- Two sets of flavons:

1st column of U_{PMNS} (good θ_{12})

$$\langle\phi_1^\nu\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \langle\phi_2^\nu\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \langle\phi_3^\nu\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

$$\langle\phi_1^e\rangle \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\phi_2^e\rangle \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \langle\phi_3^e\rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- Use orthogonality ($F_O = 0$):

$$\mathcal{W} = O_1(\phi_2^e \cdot \phi_{102}) + O_2(\phi_3^\nu \cdot \phi_{102})$$

The Yukawa and Mass Matrices

[Antusch, King, MS '13]

- Neutrino mass matrix as before ($\alpha = \pi/3$)
- Down-type quark and charged lepton Yukawa (unusual SU(5) relations [Antusch, Spinrath '09], $\beta = 4\pi/3$)

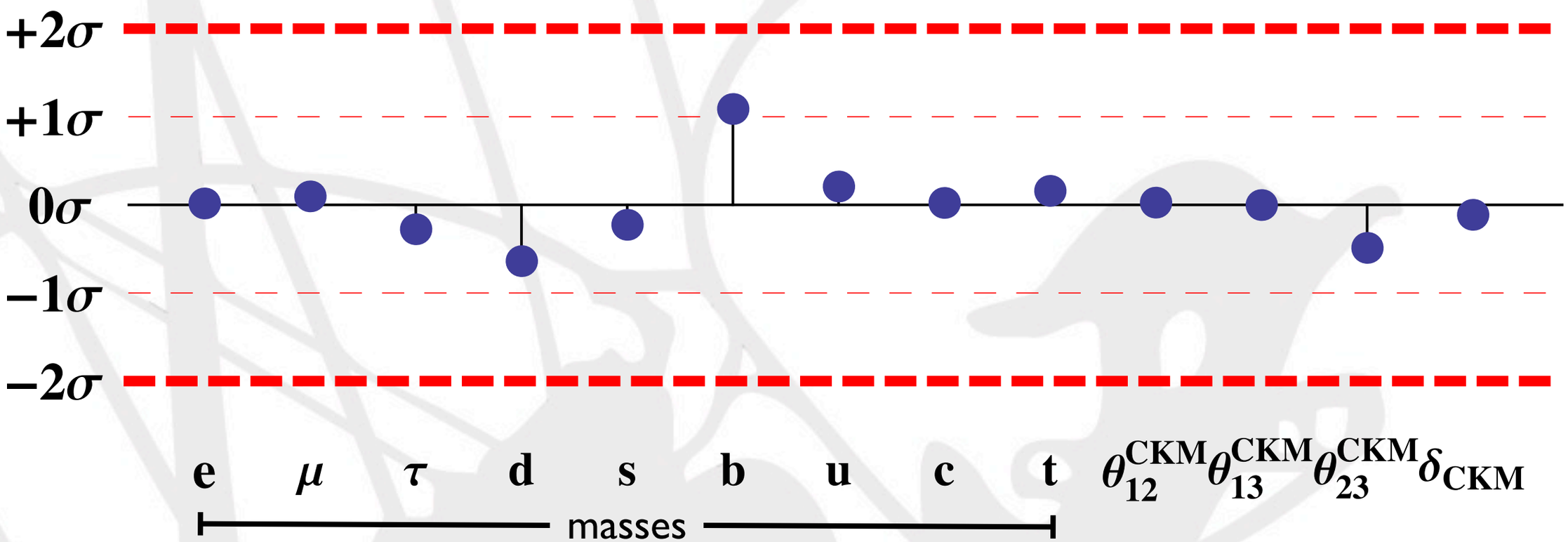
$$Y_e = \begin{pmatrix} 0 & -3/2\,\mathrm{e}^{4\pi\mathrm{i}/3}\epsilon_{102} & 0 \\ -3/2\,\mathrm{i}\,\epsilon_2 & 9/2\,\epsilon_{23} & 0 \\ 0 & \left(-3\,\mathrm{e}^{4\pi\mathrm{i}/3}\epsilon_{102} - 9/2\,\epsilon_{23}\right) & \epsilon_3 \end{pmatrix} \text{ and } Y_d \sim Y_e^T$$

- Up-type quark Yukawa matrix real, symmetric with vanishing 1-3 element

Fit Results

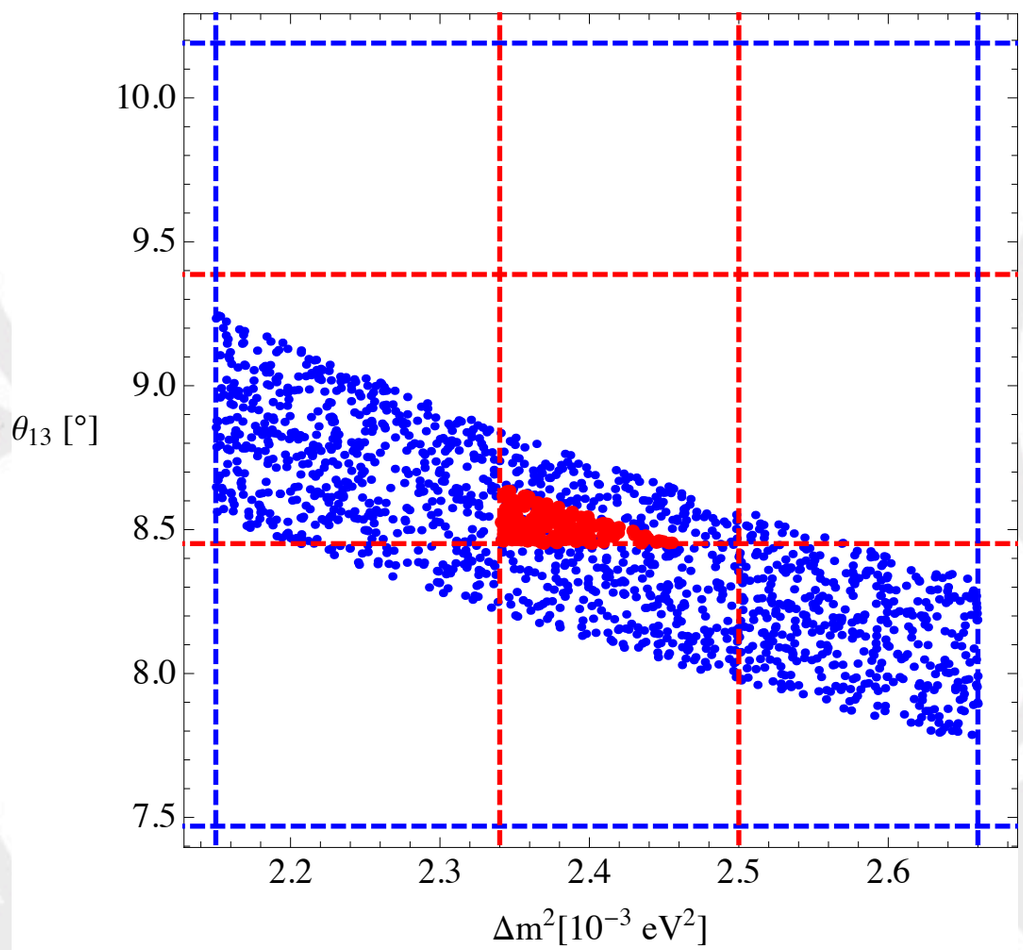
[Antusch, King, MS '13]

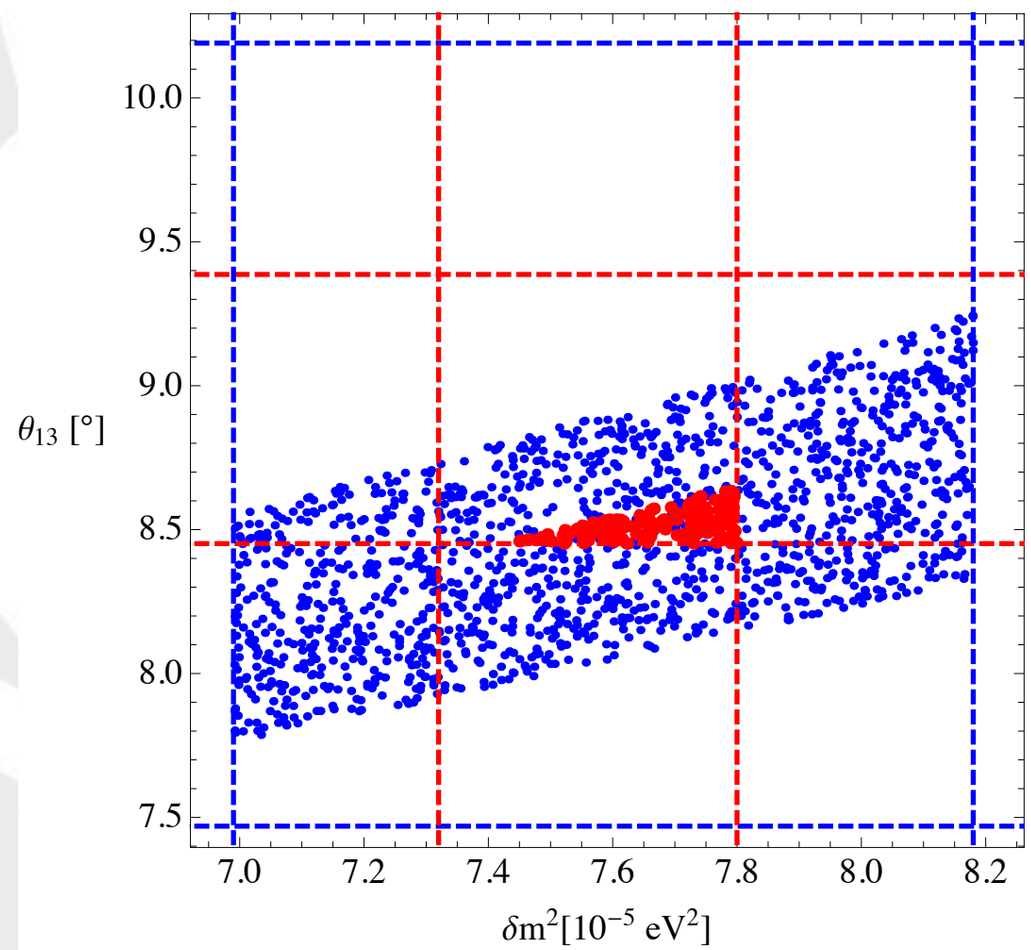
$\chi^2/\mathrm{dof} = 2.05/3$



The error for the lepton masses is estimated to 1%, the quark masses are taken from [Xing, Zhang, Zhou '07], the CKM parameters from [PDG '10].

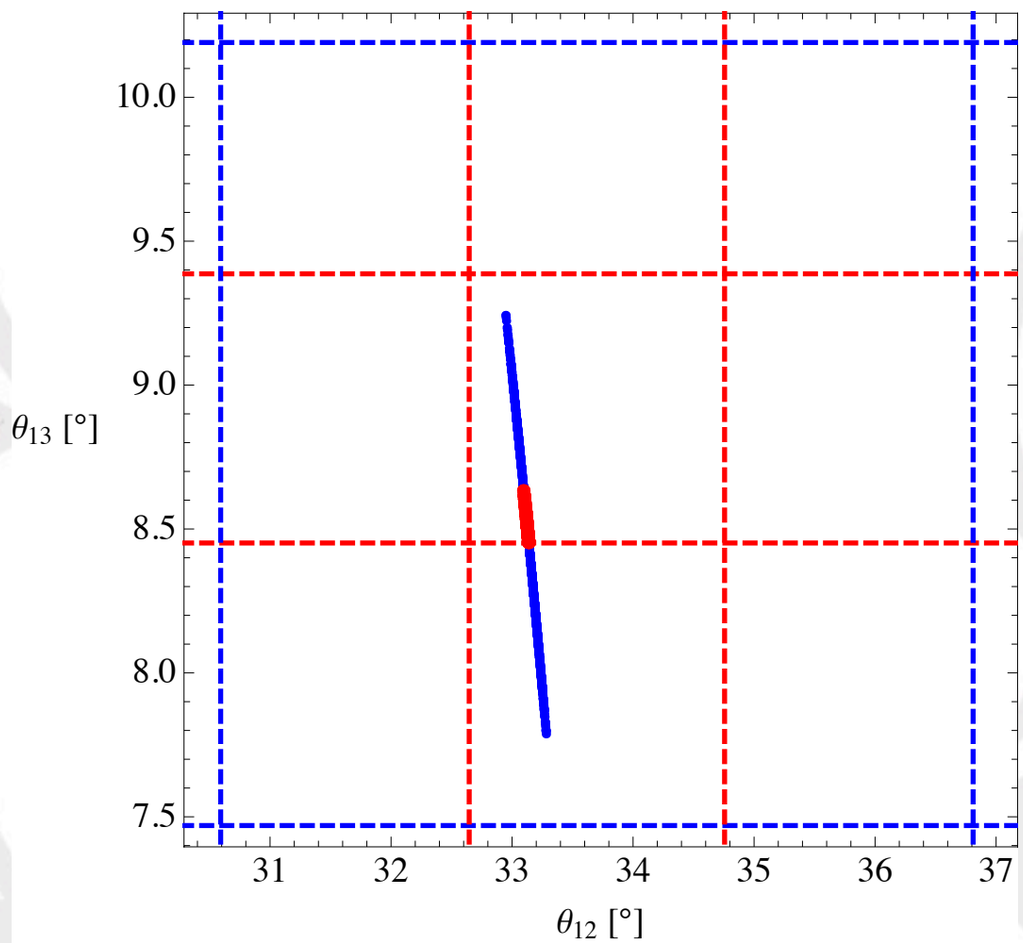
Results for the neutrino sector I

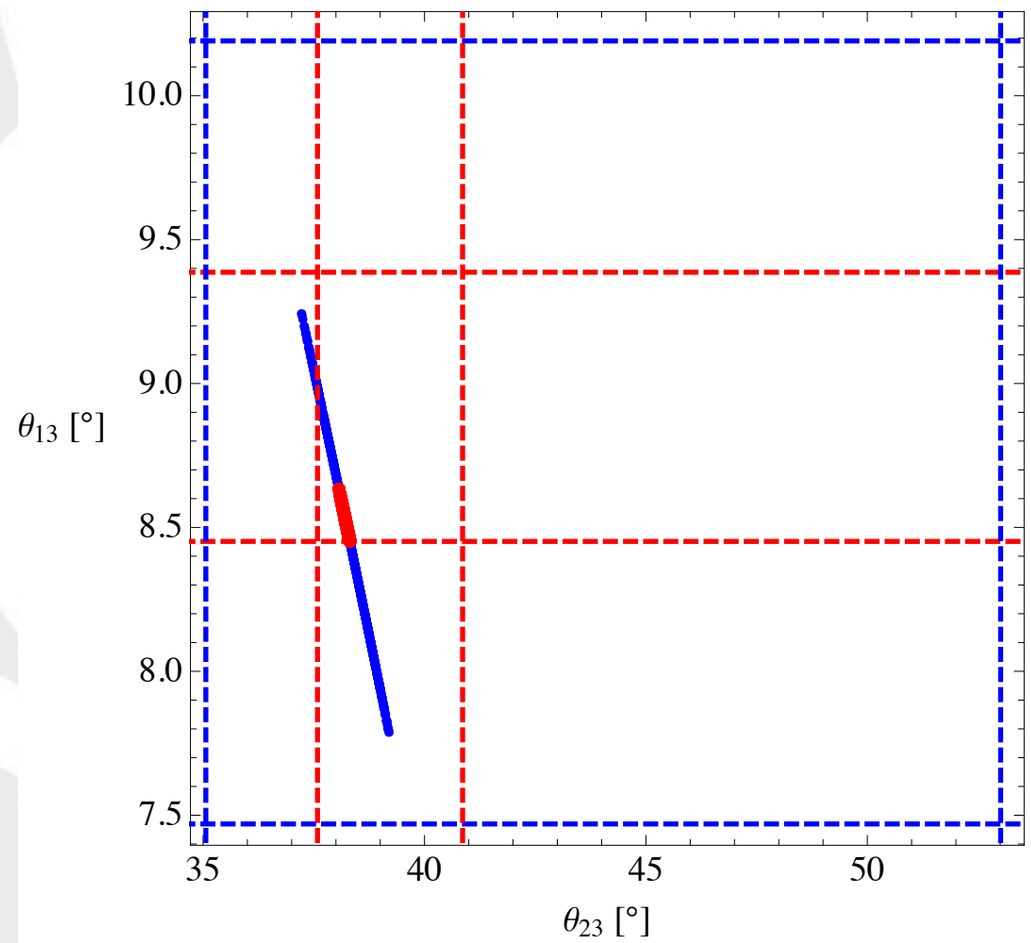




Red (blue) dashed lines are the 1(3)σ ranges of [Lisi et al. '12]
The dots are from a random scan in our model [Antusch, King, MS '13]

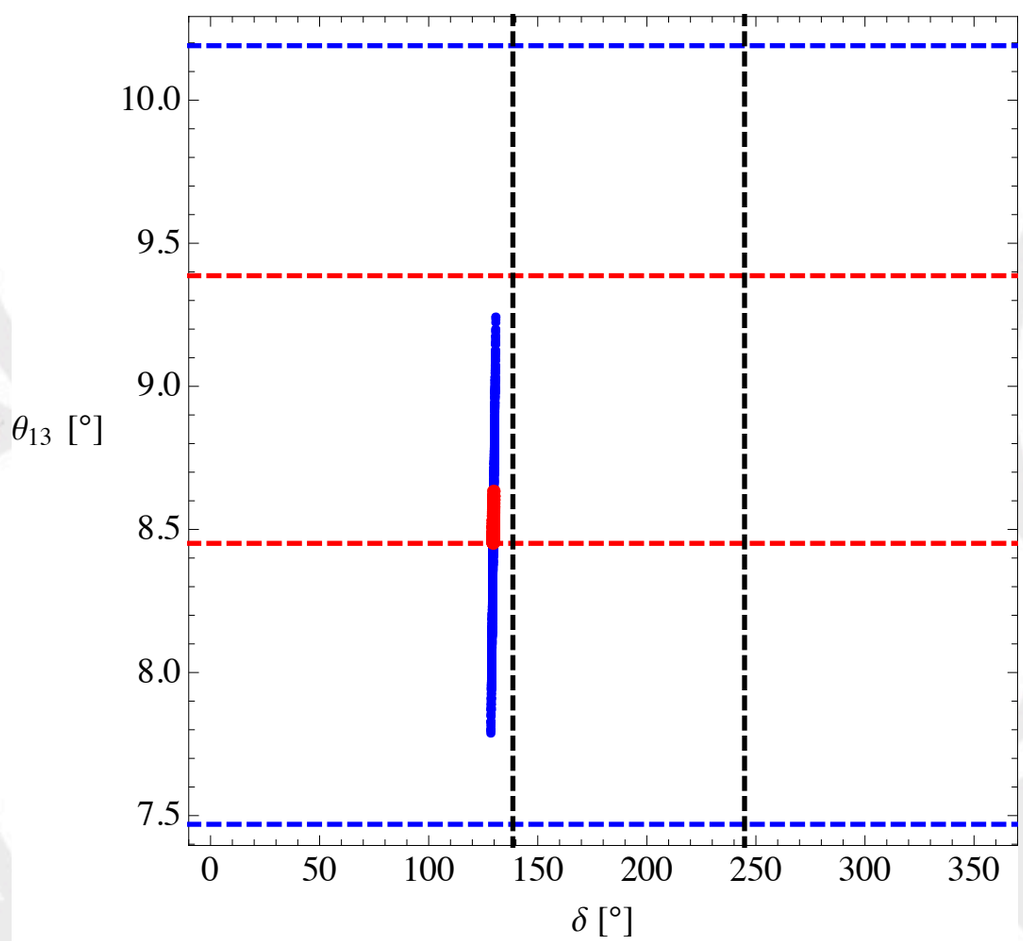
Results for the neutrino sector II

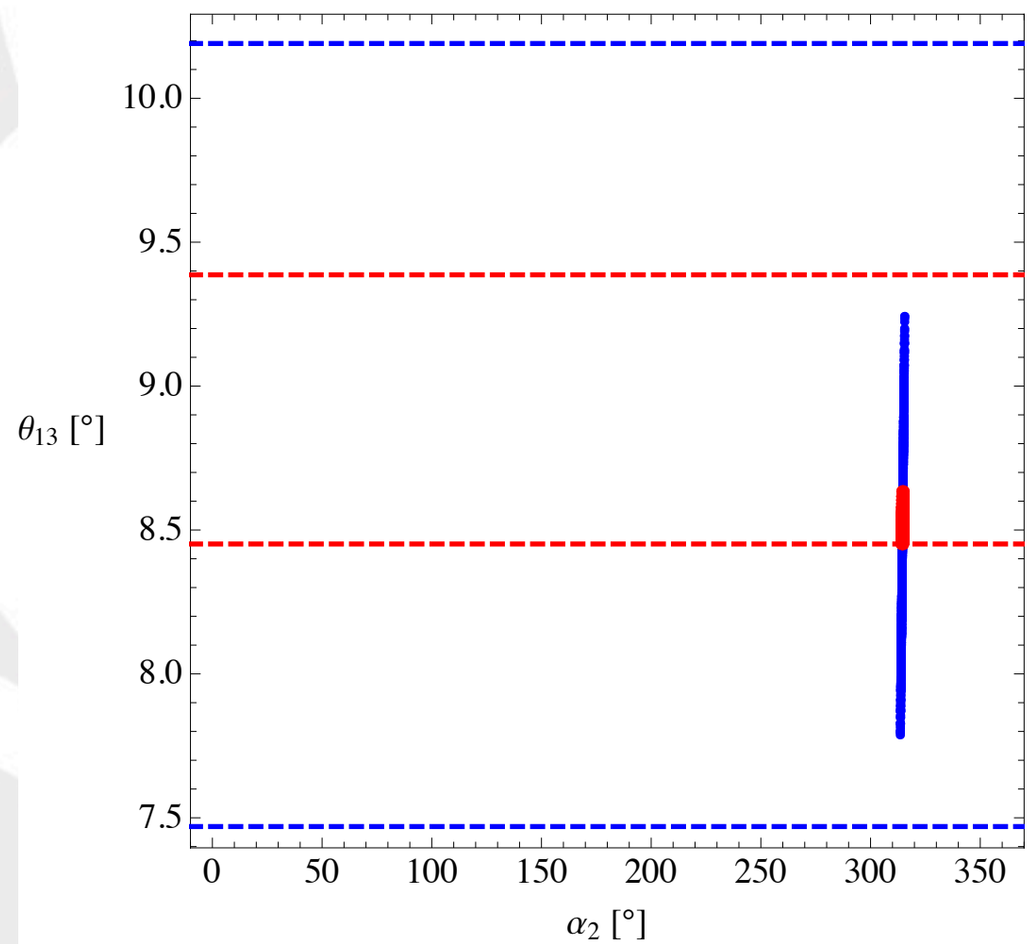




Red (blue) dashed lines are the 1(3)σ ranges of [Lisi et al. '12]
The dots are from a random scan in our model [Antusch, King, MS '13]

Results for the neutrino sector III





Red (blue) dashed lines are the 1(3)σ ranges of [Lisi et al. '12]
Black dashed line is the 1 σ hint for the Dirac CP phase of [Lisi et al. '12]
The dots are from a random scan in our model [Antusch, King, MS '13]

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Summary and Conclusions

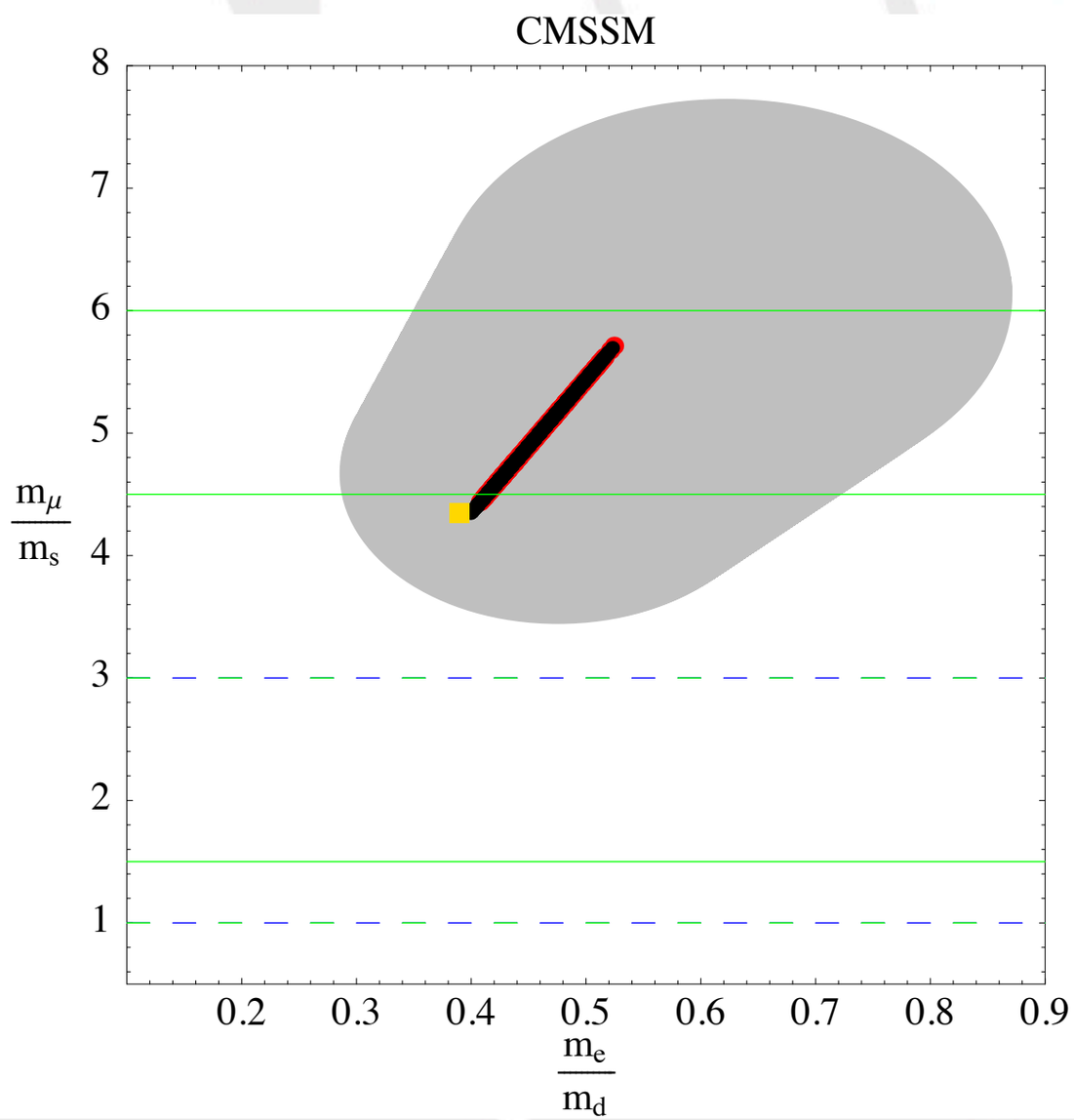
- Huge experimental progress in the leptonic flavour sector
- Challenges for the Model Builders!
  - Trimaximal Mixing (+ Spontaneous CP violation)

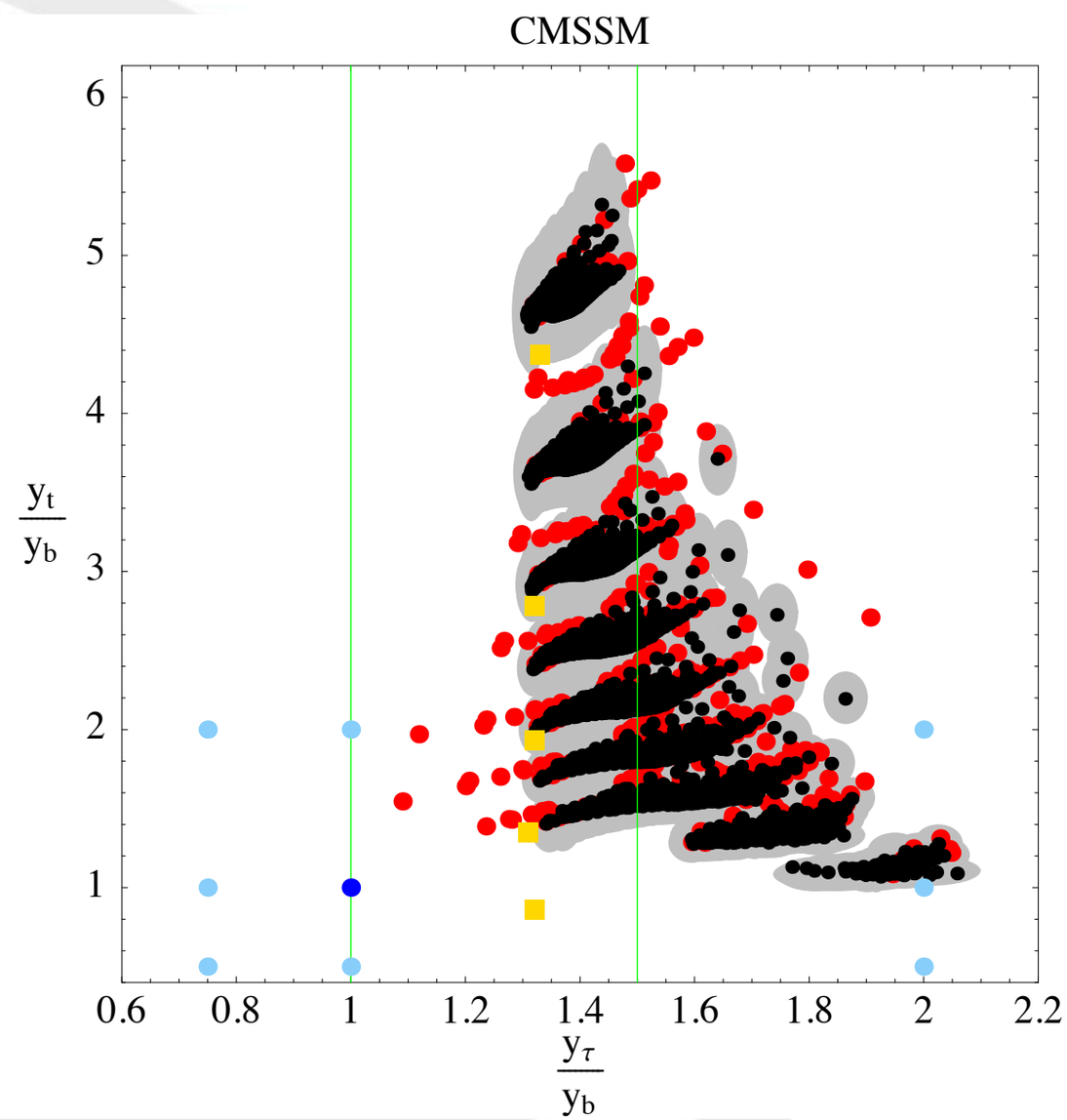
    [Antusch, King, Luhn, MS '11; Antusch, King, MS '13]
- In the future:
  - θ_{23}, mass hierarchy, CP violation?
  - LHC will test TeV scale supersymmetry, rare decays, ...

Thank you for your attention!

Backup: SUSY and GUT Relations

no SUSY thresholds	excluded
SU(5)	allowed
Pati-Salam	mass error





[Antusch, MS '09]