A predictive scheme for triplet leptogenesis

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Based on work done in collaboration with Stéphane Lavignac

Introduction

Baryon asymmetry of the universe (BAU)

$$\frac{n_B}{n_\gamma} = \begin{cases} (5.1 - 6.5) \times 10^{-10} \text{ (BBN)} \\ 6.04 \pm 0.8 \times 10^{-10} \text{ (CMB)} \end{cases}$$

Sakharov's conditions

- B violation
- CP violation
- Processes that violate B and CP out of equilibrium

Baryogenesis through leptogenesis

[Fukugita, Yanagida]

- Creation of a lepton asymmetry in the decay of heavy particles
- Conversion to a baryon asymmetry by electroweak sphalerons

$$C = \frac{Y_B}{Y_{B-L}} = \frac{28}{79}$$

1. Presentation of the model
2. *CP* asymmetry
3. Boltzmann equations
4. Results

Model based on a Grand Unified Theory with gauge group $SO(10)$.
[M. Frigerio, P. Hosteins, S. Lavignac, A. Romanino (2009)]

Particle content

- 1 complex scalar triplet $\Delta = (\Delta^{++}, \Delta^{+}, \Delta^{0})$
- 3 pairs of vector-like heavy lepton doublets

$$\underbrace{\mathcal{L}_\alpha = \begin{pmatrix} \mathcal{N}_\alpha \\ \mathcal{E}_\alpha \end{pmatrix}}_{L=1}, \quad \underbrace{\bar{\mathcal{L}}_\alpha = \begin{pmatrix} \bar{\mathcal{N}}_\alpha \\ \bar{\mathcal{E}}_\alpha \end{pmatrix}}_{L=-1}$$

- 1 real scalar triplet $T = (T^+, T^0, T^-)$ & 1 real scalar singlet S

New couplings

- $f_{\alpha\beta}\Delta\ell_\alpha\ell_\beta$ $(\Delta L = 2)$
- $f_{\alpha\beta}\Delta^\dagger\bar{\mathcal{L}}_\alpha\bar{\mathcal{L}}_\beta$ $(\Delta L = 2)$
- $\mu\Delta^\dagger HH$
- $c_R f_{\alpha\beta} R\bar{\mathcal{L}}_\alpha\ell_\beta$ $(R = S$ or $T)$

The Yukawa couplings are related by $SO(10)$ symmetry

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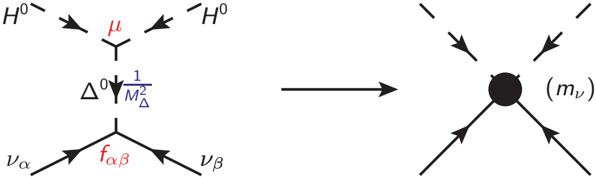
[Schechter & al. - Lazarides & al. - Mohapatra & al. - Wetterich]



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Coupling matrix

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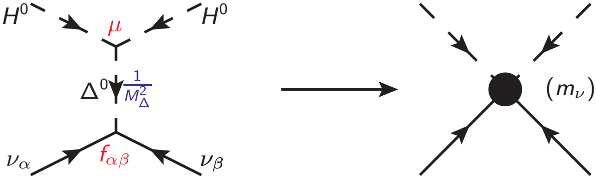

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We consider the *CP* asymmetries in the decays of the three scalars.

CP asymmetries

$$\epsilon_\Delta = 2\frac{\Gamma(\Delta^\dagger \to \ell\ell) - \Gamma(\Delta \to \ell^c\ell^c)}{\Gamma_\Delta + \Gamma_{\Delta^\dagger}}$$

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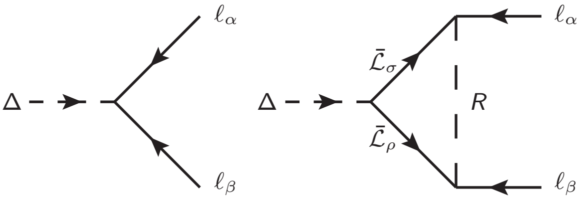

The asymmetry vanishes if $M_{\bar{\mathcal{L}}_1}, M_{\bar{\mathcal{L}}_2}, M_{\bar{\mathcal{L}}_3} > M_\Delta$ or if $M_{\bar{\mathcal{L}}_1}, M_{\bar{\mathcal{L}}_2}, M_{\bar{\mathcal{L}}_3} \ll M_\Delta$.

CP asymmetries

With the assumption $M_{\bar{\mathcal{L}}_1} \ll M_{\Delta,S,T} \ll M_{\bar{\mathcal{L}}_{2,3}}$ (so that $\bar{\mathcal{L}}_2$ and $\bar{\mathcal{L}}_3$ decouple from the dynamics) one gets

$$\epsilon_\Delta = \frac{1}{4\pi} \frac{\mathcal{I}m[f_{11}(f^\dagger f f^\dagger)_{11}]}{\mathrm{Tr}(f f^\dagger)} \sum_{R=S,T} c_R^2 g\left(\frac{M_R^2}{M_\Delta^2}\right)$$

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The CP asymmetry depends only on the scalar masses, the coupling μ, and neutrino parameters

In particular

$$\mathcal{I}m[f_{11}(f^\dagger f f^\dagger)_{11}] = \left(\frac{2M_\Delta^2}{\mu v^2}\right)^4 \left(-m_1 m_2 \Delta m_{21}^2 c_{12}^2 c_{13}^4 s_{12}^2 \sin 2\rho \right.$$
$$\left. + m_1 m_3 \Delta m_{31}^2 c_{12}^2 c_{13}^2 s_{13}^2 \sin 2(\sigma - \rho) + m_2 m_3 \Delta m_{32}^2 c_{13}^2 s_{12}^2 s_{13}^2 \sin 2\sigma \right) ,$$

- m_i: eigenvalues of m_ν (physical neutrino masses) $\bar{m} = \sqrt{m_1^2 + m_2^2 + m_3^2}$
- $\Delta m_{ij}^2 = m_i^2 - m_j^2$
- $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$
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Boltzmann equations

- 3 equations for the scalar densities with the general form

$$sHz\frac{dY_a}{dz} = -(D_a + S_a), \quad z = \frac{M_\Delta}{T}$$

$D_a \propto \Gamma_a$: decays and inverse decays of particle a
S_a: scatterings consuming a (typically electroweak annihilations)

- We also need the asymmetries in Standard Model leptons Δ_ℓ, in heavy leptons $\Delta_{\bar{\mathcal{L}}_1}$, in Higgs doublets Δ_H and in triplets Δ_Δ

$$sHz\frac{d\Delta_a}{dz} = \epsilon_a^b D_b - W_a$$

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[R. Barbieri, P. Creminelli, A. Strumia, N. Tetradis, '99]

- If charged lepton Yukawa interactions are in equilibrium ($T < 10^{12}$ GeV for τ, $T < 10^{9}$ GeV for μ) lepton flavours are distinguishable
  $\Rightarrow$ For $T < 10^{9}$ GeV, write 3 Boltzmann equations for Δ_{ℓ_e}, Δ_{ℓ_μ} and Δ_{ℓ_τ}.
- In the opposite case, lepton flavors are undistinguishable
  $\Rightarrow$ For $T > 10^{12}$ GeV, there are quantum correlations between the various flavours to take into account.

Density matrix

$$\Delta n_{\ell_\alpha} = n_{\ell_\alpha} - n_{\ell^c_\alpha} = \langle : \ell^\dagger_\alpha \ell_\alpha : \rangle \to \Delta n_{\alpha\beta} = \langle : \ell^\dagger_\alpha \ell_\beta : \rangle$$

We need to derive the Boltzmann equation for $(\Delta_\ell)_{\alpha\beta} = \dfrac{\Delta n_{\alpha\beta}}{s}$

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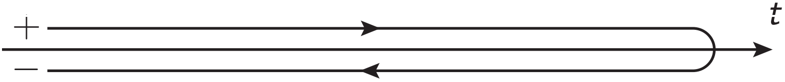
Closed time-path formalism

Formalism used to describe quantum out of equilibrium phenomena, applied to leptogenesis [W. Buchmüller & al., De Simone & al., Garbrecht & al.]
$\mathcal{C}$ = time-path that goes from 0 to ∞ and back

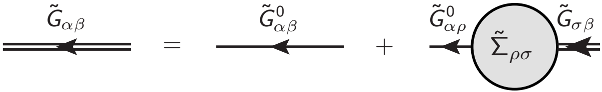


$\tilde{G}_{\alpha\beta} = \langle \mathcal{T}_{\mathcal{C}} \ell_\alpha \bar{\ell}_\beta \rangle$ Green's function, time-ordered **following the contour**.

$$\tilde{G} = \begin{pmatrix} G^{++} & -G^{+-} \\ G^{-+} & -G^{--} \end{pmatrix}$$

For instance $G^{-+}_{\alpha\beta}(x,y) = -i\langle \ell_\alpha(x)\bar{\ell}_\beta(y)\rangle$
Idea: Deduce the evolution equation of $\Delta n_{\alpha\beta} = \langle : \ell^\dagger_\alpha \ell_\beta : \rangle$ From the equation of motion of $\tilde{G}_{\beta\alpha}$

- **Schwinger-Dyson equation** expresses $\tilde{G}$ as a function of the free Green's function $\tilde{G}^0$ and the 1PI self-energy $\tilde{\Sigma}$



- One obtains the evolution equation for the density matrix by noticing that $\frac{d\Delta n_{\alpha\beta}}{dt} = -\mathrm{Tr}((i\overrightarrow{\partial\!\!\!/}_x + i\overleftarrow{\partial\!\!\!/}_y)G^{-+}_{\beta\alpha}(x,y))_{|y=x}$ and by replacing $\frac{d\Delta n_{\alpha\beta}}{dt} \rightarrow \frac{d\Delta n_{\alpha\beta}}{dt} + 3H\Delta n_{\alpha\beta} = sHz\frac{d(\Delta_\ell)_{\alpha\beta}}{dz}$
- In the end, taking the classical limit, one obtains a Boltzmann equation for the density matrix of lepton asymmetry $(\Delta_\ell)_{\alpha\beta}$

$$sHz\frac{d(\Delta_\ell)_{\alpha\beta}}{dz} = \epsilon^{\Delta}_{\alpha\beta}D_{\Delta} + \epsilon^{S}_{\alpha\beta}D_S + \epsilon^{T}_{\alpha\beta}D_T - \mathcal{W}_{\alpha\beta}$$

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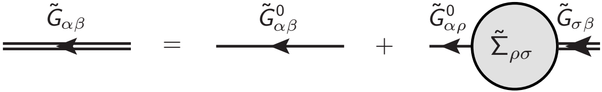

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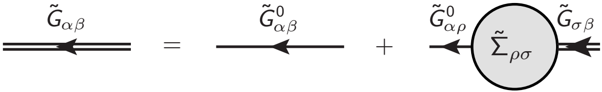

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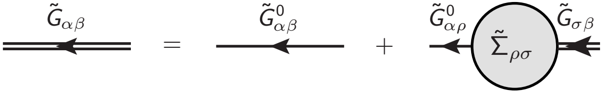

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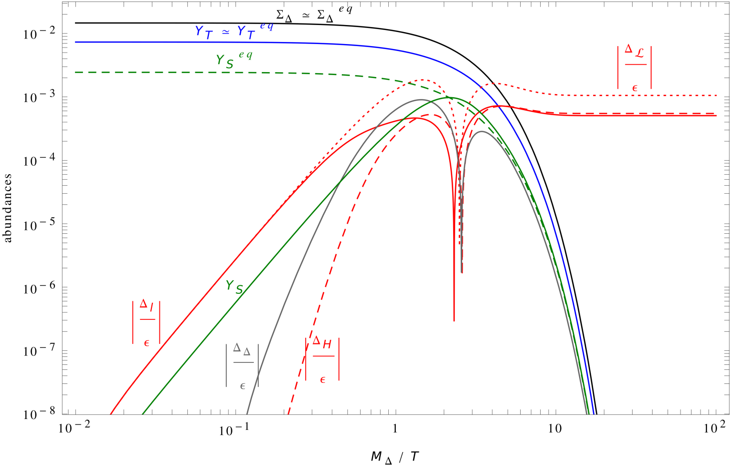

Figure: Evolution of the abundances for $M_\Delta = M_S = M_T = 10^{13}$ GeV, $m_1 = 10^{-3}$ eV and $\mu/M_\Delta = 0.2$

Final baryon asymmetry

- Before the action of sphalerons

$$Y_{B-L} = \Delta_{\bar{\mathcal{L}}_1} - \mathrm{Tr}(\Delta_\ell)$$

- In the end, we obtain the BAU

$$\frac{n_B}{n_\gamma} = 7.04 \times C \times Y_{B-L}$$

- To be viable, the model must allow

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1. Presentation of the model
2. CP asymmetry
3. Boltzmann equations
4. Results

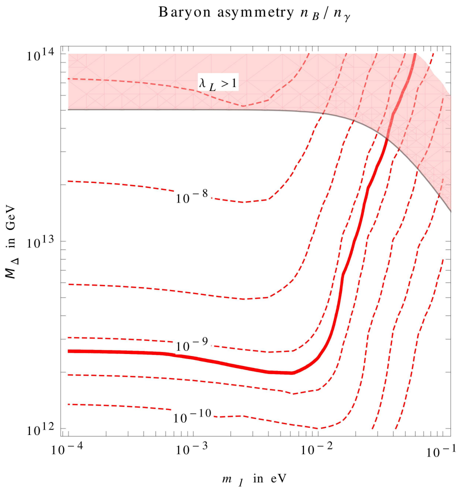

Figure: Final baryon asymmetry as a function of m_1 and $M_\Delta = M_S = M_T$ for $\mu / M_\Delta = 0.2$. The red line indicates the observed BAU $\sim 6 \times 10^{-10}$.

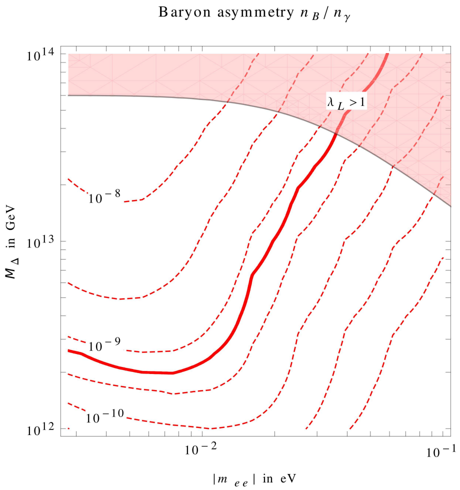

Figure: Final baryon asymmetry as a function of $|(m_\nu)_{ee}|$ and $M_\Delta = M_S = M_T$ for $\mu/M_\Delta = 0.2$.

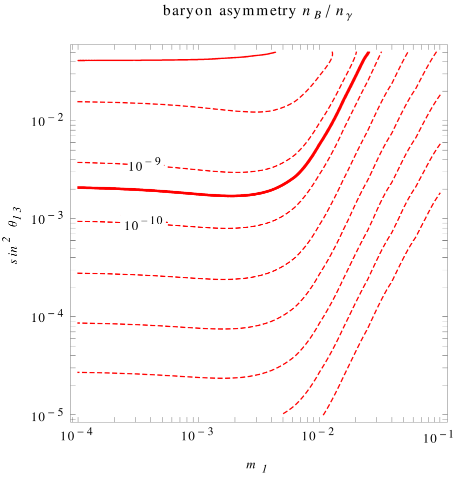

Figure: Final baryon asymmetry as a function of m_1 and $\sin^2 \theta_{13}$ for $M_\Delta = M_S = M_T = 10^{13}$ GeV, $\mu / M\Delta = 0.2$.

Conclusion

- This scenario can account for a successful baryogenesis
- The result is closely related to neutrino parameters thanks to the relation

$$(m_\nu)_{\alpha\beta} = \frac{\mu v^2}{M_\Delta^2} f_{\alpha\beta}$$

- This scenario happens at a huge energy scale since $M_\Delta > 10^{12}$ GeV $\rightarrow$ it cannot be tested directly
- But this scenario could be ruled out
  -for some values of $|(m_\nu)_{ee}|$
  -if neutrinos are quasi-degenerate with $m_1 \gtrsim 0.1$ eV
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The underlying $SO(10)$ model

$$\overbrace{16_i}^{SO(10)} = \overbrace{\underbrace{10_i^{16}}_{(Q_i, u_i^c, e_i^c)} \oplus \underbrace{\bar{5}_i^{16}}_{(\mathcal{L}_i, \bar{\mathcal{D}}_i)} \oplus \underbrace{1_i^{16}}_{\nu_i^c}}^{SU(5)}$$

$$10_i = \underbrace{5_i^{10}}_{(\bar{\mathcal{L}}_i, \mathcal{D}_i)} \oplus \underbrace{\bar{5}_i^{10}}_{(\ell_i, d_i^c)}$$

$$54 = \underbrace{15 \oplus \overline{15}}_{(\Delta, \Delta^\dagger)} \oplus \underbrace{24}_{(S, T)}$$

Lagrangian

$$\mathcal{L} = \frac{1}{2} f_{ij} 10_i\, 10_j\, 54 + \frac{1}{2} \mu 10_H\, 10_H\, 54 + \frac{1}{2} M_{54}^2 54^2$$

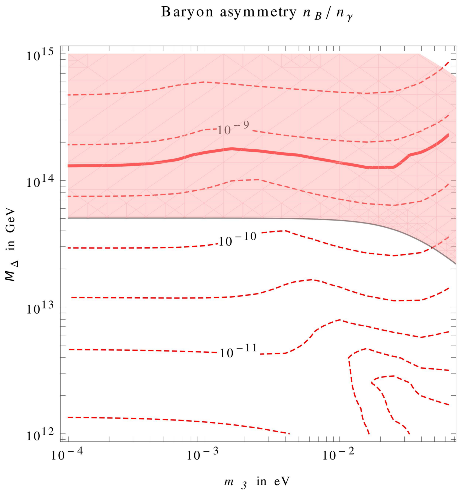


Figure: Final baryon asymmetry as a function of m_3 (inverted hierarchy) and $M_\Delta = M_S = M_T$ for $\mu / M_\Delta = 0.2$.

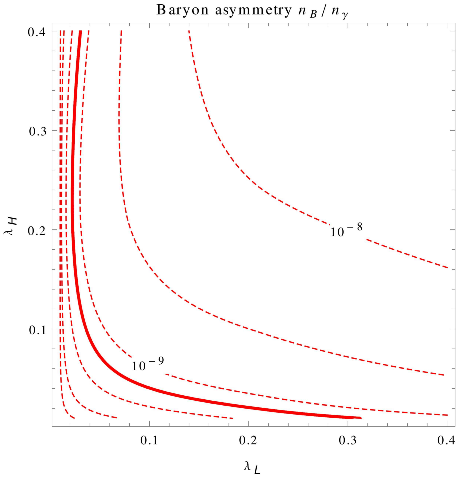
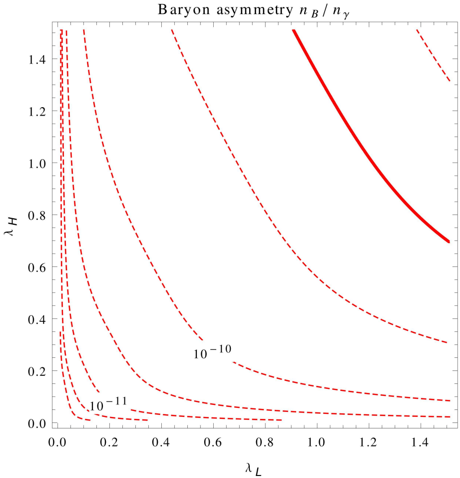


Figure: Final baryon asymmetry as a function of $\lambda_\ell = \sqrt{\mathrm{Tr}(ff^\dagger)}$ and $\lambda_H = \mu/M_\Delta$ for $m_1 = 10^{-3}$ eV (normal hierarchy).



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