The MSSM Higgs Mass Revisited

Howard E. Haber

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The Higgs sector of the MSSM

The Higgs sector of the MSSM is a two Higgs doublet model, whose scalar potential and Yukawa couplings are constrained by supersymmetry (SUSY). The scalar potential of the MSSM is:

$$V = \left(m_d^2 + |\mu|^2\right) H_d^{i*} H_d^i + \left(m_u^2 + |\mu|^2\right) H_u^{i*} H_u^i - m_{ud}^2 \left(\epsilon^{ij} H_d^i H_u^j + \text{h.c.}\right)$$
$$+ \tfrac{1}{8}\left(g^2 + g'^{\,2}\right)\left[H_d^{i*} H_d^i - H_u^{j*} H_u^j\right]^2 + \tfrac{1}{2} g^2 |H_d^{i*} H_u^i|^2 \,,$$

where μ is a supersymmetric Higgsino mass parameter and m_d^2, m_u^2, m_{ud}^2 are soft-SUSY-breaking masses.

Minimizing the Higgs potential, the neutral components of the Higgs fields acquire vacuum expectation values (vevs), $\langle H_d^0 \rangle = v_d/\sqrt{2}$ and $\langle H_u^0 \rangle = v_u/\sqrt{2}$, where $v^2 \equiv v_d^2 + v_u^2 = 4m_W^2/g^2 = (246~\text{GeV})^2$. The ratio of the two vevs is an important parameter of the model:

$$\tan\beta \equiv \frac{v_u}{v_d}\,.$$

Tree-level neutral MSSM Higgs masses

The CP-even Higgs bosons h and H are eigenstates of the squared-mass matrix

$$\mathcal{M}_0^2 = \begin{pmatrix} m_A^2 \sin^2\beta + m_Z^2\cos^2\beta & -(m_A^2+m_Z^2)\sin\beta\cos\beta \\ -(m_A^2+m_Z^2)\sin\beta\cos\beta & m_A^2\cos^2\beta + m_Z^2\sin^2\beta \end{pmatrix}.$$

The eigenvalues of $\mathcal{M}_0^2$ are the squared-masses of the two CP-even Higgs scalars

$$m_{H,h}^2 = \tfrac{1}{2}\left(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2+m_Z^2)^2 - 4m_Z^2 m_A^2\cos^2 2\beta}\,\right),$$

and α is the angle that diagonalizes the CP-even Higgs squared-mass matrix. It follows that

$$m_h \leq m_Z|\cos 2\beta| \leq m_Z\,.$$

If this tree-level mass inequality were more generally satisfied, then the MSSM would be ruled out today!

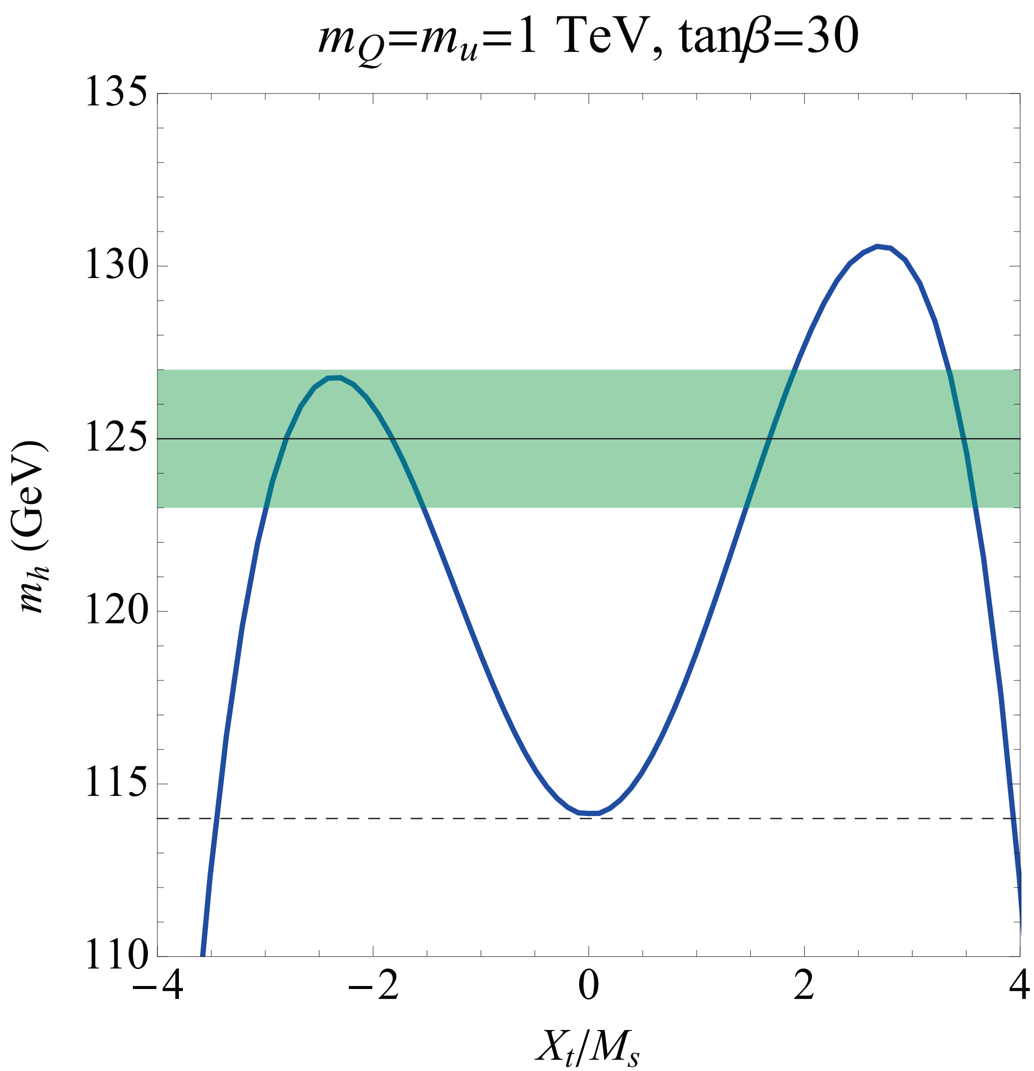
The radiatively-corrected mass of h^0

The tree-level inequality, $m_h \leq m_Z$, is significantly modified by quantum corrections. The Higgs mass can be shifted due to an incomplete cancellation from loops of particles and their superpartners [H.E. Haber and and R. Hempfling (1991); Y. Okada, M. Yamaguchi and T. Yanagida (1991); J.R. Ellis, G. Ridofi and F. Zwirner (1991)]:

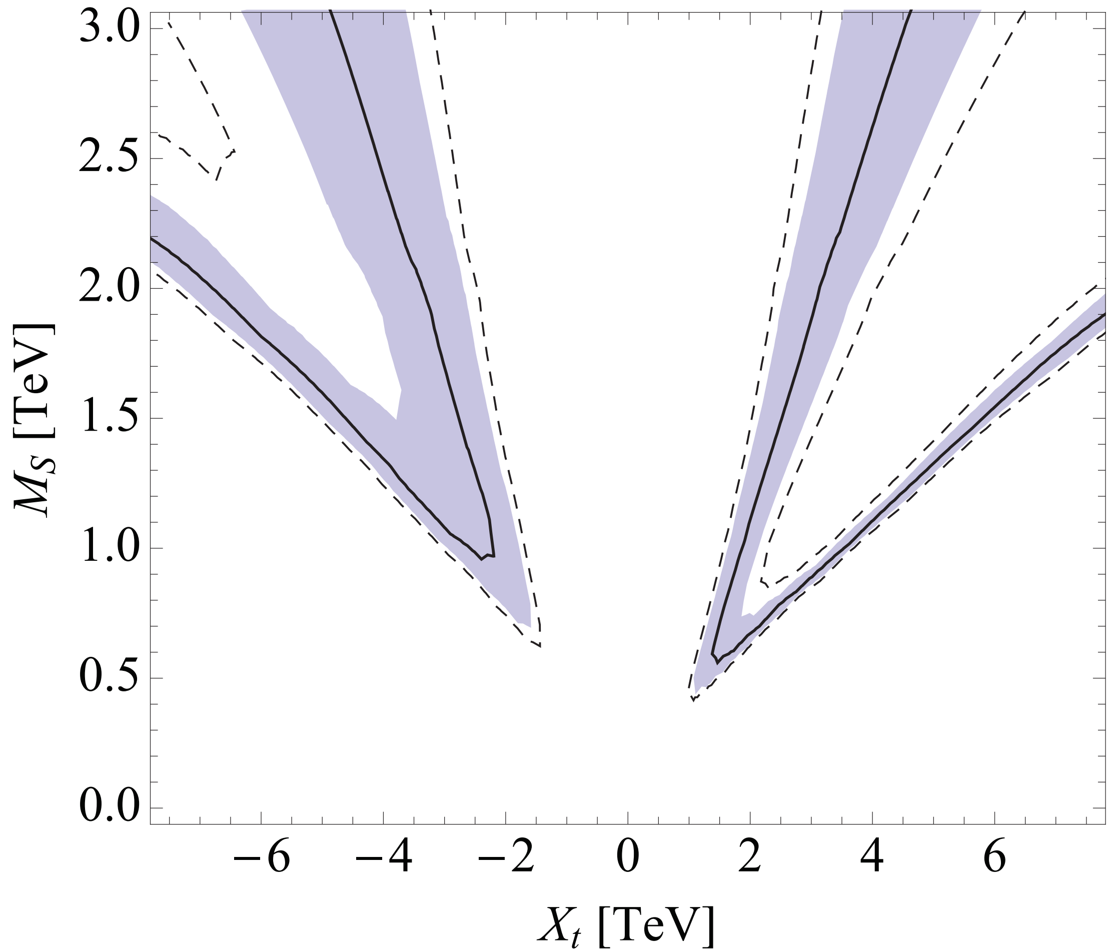
$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

where $X_t \equiv A_t - \mu\cot\beta$ governs stop mixing and M_S^2 is the average squared-mass of the top-squarks $\widetilde{t}_1$ and $\widetilde{t}_2$ (which are the mass-eigenstate combinations of the interaction eigenstates, $\widetilde{t}_L$ and $\widetilde{t}_R$). Here, only the leading one-loop log and leading squark mixing contributions are exhibited.

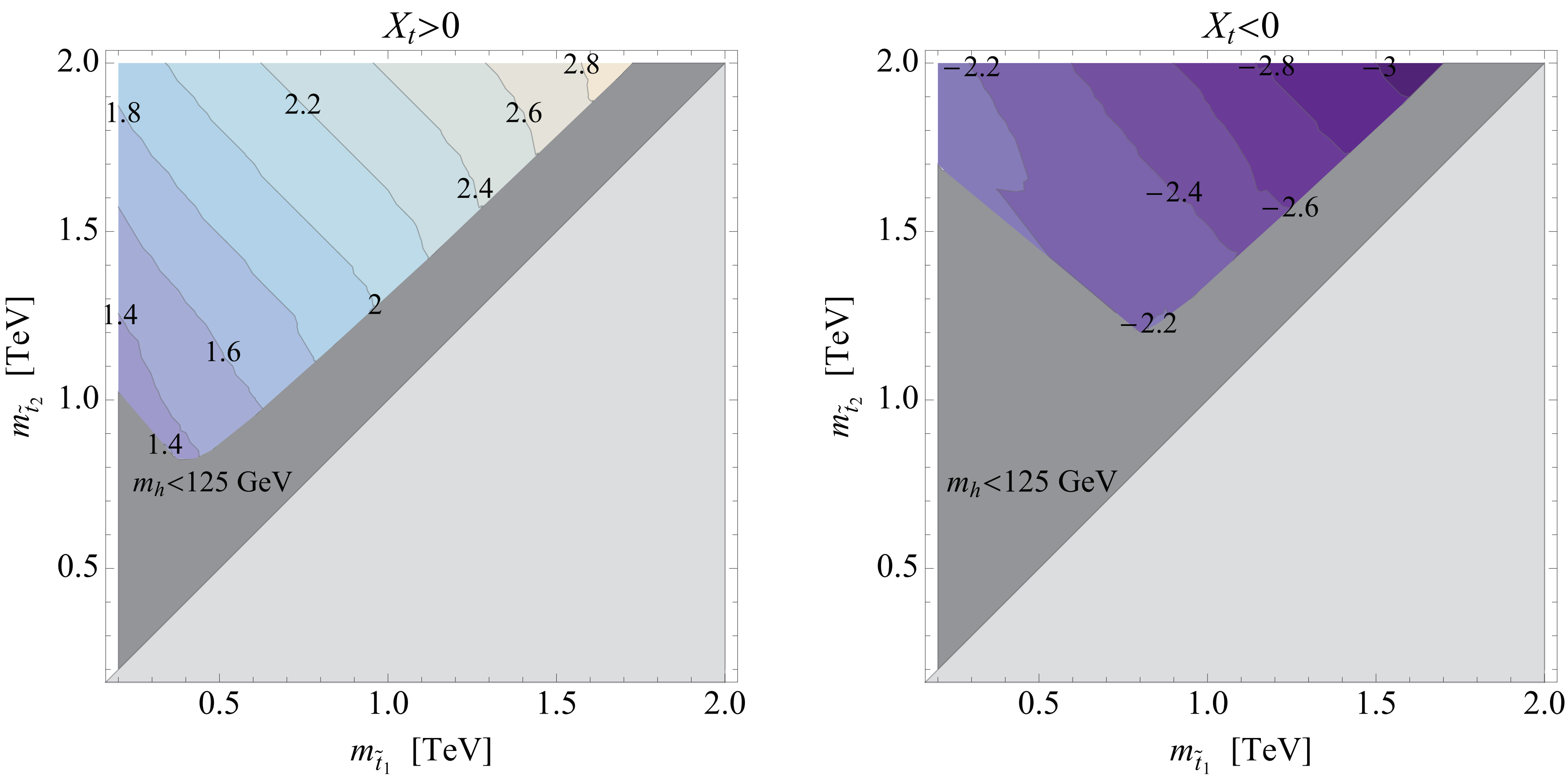
The state-of-the-art computation includes the full 1-loop result, all the significant 2-loop contributions, some of the leading 3-loop terms, and renormalization-group improvements.



Implications of the observed Higgs state with $m_h \simeq \mathbf{125}$ GeV

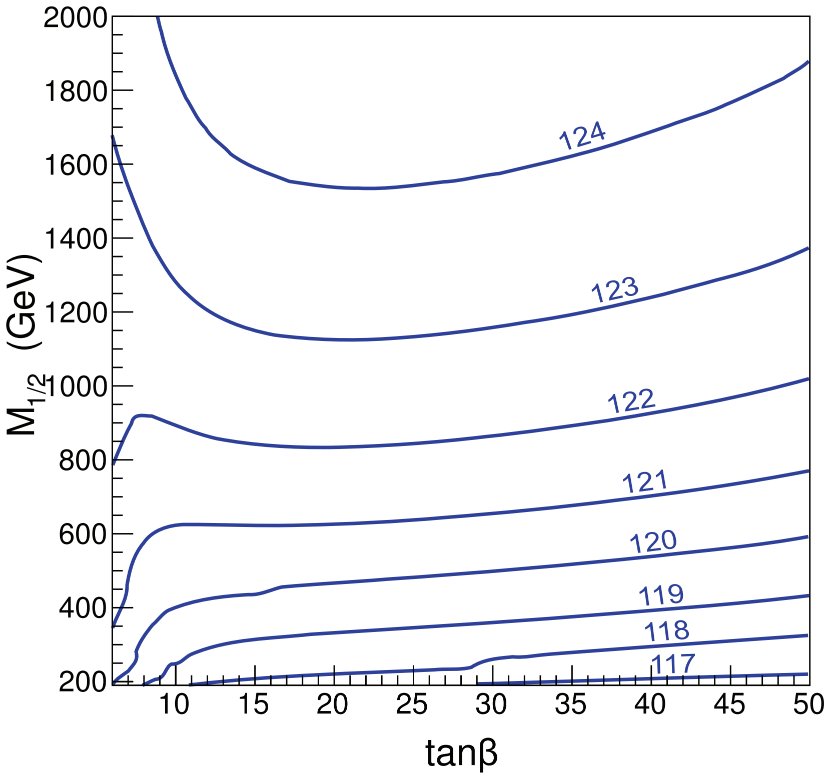


Contour plot of m_h in the M_S vs. X_t plane, with $\tan\beta = 30$ and $M_Q = M_U = M_S$. The solid curve is $m_h = 125$ GeV with $m_t = 173.2$ GeV. The band around the solid curve corresponds to $m_h = 125 \pm 2$ GeV. The dashed lines correspond to varying m_t from 172–174 GeV. Taken from P. Draper, P. Meade, M. Reece and D. Shih, Phys. Rev. D **85**, 095007 (2012).



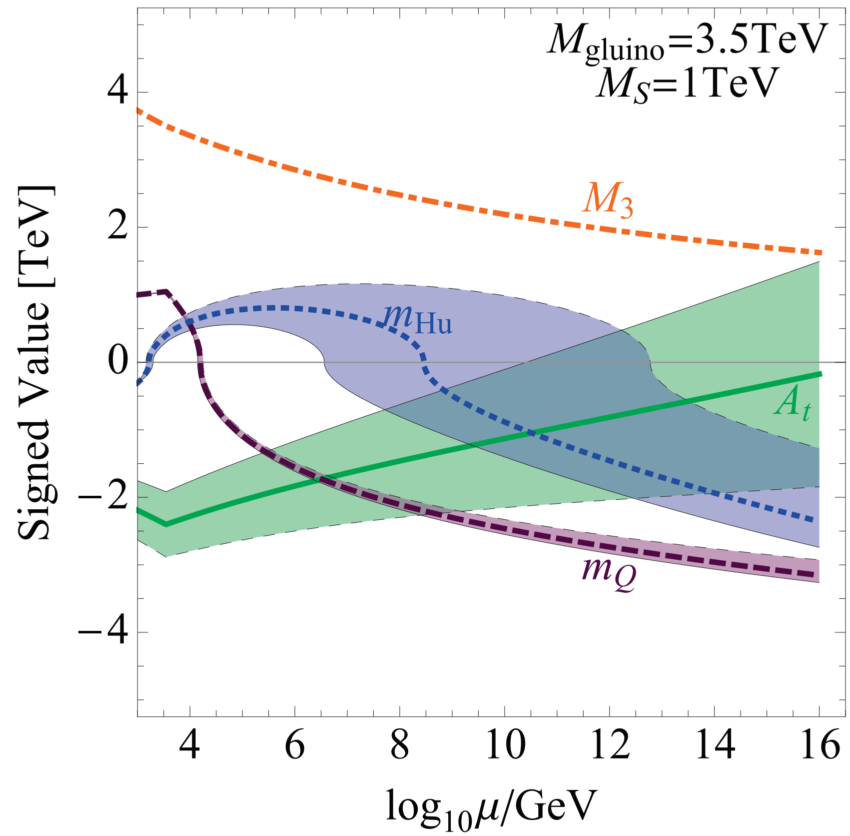
Contour plot of X_t in the plane of physical stop masses ($m_{\tilde{t}_1}$; $m_{\tilde{t}_2}$). Here X_t is fixed to be the minimum positive (left) or negative (right) solution to $m_h = 125$ GeV. Taken from P. Draper, P. Meade, M. Reece and D. Shih (2012).

Even without assumptions about the SUSY-breaking mechanism, the observed Higgs mass tends to pushes some MSSM parameters into the multi-TeV regime. This provides significant tension with naturalness constraints. The tension is exacerbated in specific SUSY breaking models.



CMSSM in the focus point region with dark matter constraints

from J.L. Feng, K.T. Matchev and D. Sanford (2012)



Gauge-Mediated SUSY breaking with $A_t = 0$ at the high scale

from P. Draper, P. Meade, M. Reece and D. Shih (2012)

But, is the MSSM Higgs mass prediction reliable? Could it potentially be modified by new physics that lies significantly above the TeV scale?

The Higgs mass prediction relies on decoupling—very heavy states that do not receive their masses from electroweak symmetry breaking should have a negligible impact on the Higgs mass prediction.

In 2011, S. Heinemeyer, M.J. Herrero, S. Penaranda and A.M. Rodriguez-Sanchez [JHEP **1105**, 063 (2011)] analyzed corrections to the MSSM Higgs mass in the seesaw-extended MSSM. Some have interpreted their results as suggesting that contributions from the right-handed neutrino sector could alter the Higgs mass prediction by a few GeV. If true, one might accommodate the observed Higgs mass more comfortably within some SUSY-breaking scenarios.

Patrick Draper and I argue in arXiv:1304.6103 [hep-ph] that this interpretation is not correct. Decoupling of heavy-scale physics does hold as expected, and the impact of the right-handed neutrino sector of the seesaw-extended MSSM is utterly negligible and thus can be safely ignored in the Higgs mass prediction.

The MSSM Higgs mass at one-loop

Although rarely displayed, the complete 1-loop expressions for the pole masses of the CP-even neutral MSSM Higgs bosons are given by:

$$m_h^2 = (m_h^2)^{\rm tree} - \frac{\sqrt{2}A_h}{v} s_{\beta-\alpha} - \Sigma_{ZZ}(m_Z^2) s^2_{\beta+\alpha} + \Sigma_{hh}(m_h^2)$$
$$- \Sigma_{AA}(m_A^2) c^2_{\beta-\alpha} + \Sigma_{GG}(0) s^2_{\beta-\alpha} - 4m_Z^2 c_\beta^2\, s_{\beta+\alpha} c_{\beta+\alpha} \delta \tan\beta \ ,$$

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$$- \Sigma_{AA}(m_A^2) s^2_{\beta-\alpha} + \Sigma_{GG}(0) c^2_{\beta-\alpha} + 4m_Z^2 c_\beta^2\, s_{\beta+\alpha} c_{\beta+\alpha} \delta \tan\beta \ ,$$

where $s_{\beta-\alpha} \equiv \sin(\beta-\alpha)$, $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$, etc., and the Higgs mixing angle α is determined implicitly via its tree-level relation,

$$\frac{m_A^2}{m_Z^2} = -\frac{c_{\beta+\alpha} s_{\beta+\alpha}}{c_{\beta-\alpha} s_{\beta-\alpha}} \,.$$

For consistency of the one-loop approximation, the arguments of the self-energies are evaluated by their *tree-level* values.

The Σ functions for the scalars (vectors) are the real parts of one loop self-energies (proportional to $g^{\mu\nu}$). Here, the on-shell scheme is used in defining the renormalized physical boson masses.

In terms of the one-loop tadpoles, A_u and A_d, of the hypercharge ± 1 neutral Higgs fields (which are determined by the requirement that they cancel the corresponding tree-level tadpoles), we have defined,

$$A_h \equiv A_u \cos\alpha - A_d \sin\alpha\,, \qquad\qquad A_H \equiv A_u \sin\alpha + A_d \cos\alpha\,.$$

These are related to the Goldstone self-energy,

$$\sqrt{2} v \Sigma_{GG}(0) = A_H c_{\beta-\alpha} + A_h s_{\beta-\alpha}\ ,$$

which follows from the requirement that the one-loop Goldstone boson mass vanishes.

In order to make use of the above formulae, we must decide on a method for fixing the $\tan\beta$ counterterm, denoted above by $\delta\tan\beta$.

A consistent low energy definition of $\tan\beta$

Consider the seesaw-extended MSSM. How does the heavy right-handed neutrino sector affect the predicted values of m_h^2 and m_H^2? The answer depends on the definition of $\tan\beta$. If you define $\tan\beta$ based on a physical quantity that can be measured in the low-energy theory, then the effects of the heavy right-handed neutrino sector are completely negligible.

Here is a simple example of a consistent low energy definition of $\tan\beta$. We call this scheme the Higgs mass (HM) scheme. In this scheme, we use m_H as an input parameter in place of $\tan\beta$. In this case,

$$m_h^2 = m_A^2 + m_Z^2 - m_H^2 + A_h(m_h^2) + A_H(m_H^2) - \Sigma_{ZZ}(m_Z^2) - \Sigma_{AA}(m_A^2) - \Sigma_{GG}(0)\,,$$

a result originally obtained by M. Berger in 1990. [Theoretical issues associated with the definition of $\tan\beta$ have also been considered by A. Freitas and D. Stockinger (2002).]

In the HM scheme, the $\tan\beta$ counterterm is obtained by setting $m_H^2 = (m_H^2)^{\rm tree}$ in the one-loop expression for m_H^2, which *defines* $\tan\beta$ in terms of the physical parameters m_Z, m_H and m_A,

$$(\delta\tan\beta)_{\rm HM} = \frac{1}{4m_Z^2 c_\beta^2 s_{\beta+\alpha} c_{\beta+\alpha}}\left(\frac{\sqrt{2}A_H}{v}c_{\beta-\alpha} + \Sigma_{ZZ}(m_Z)c^2_{\beta+\alpha} - \Sigma_{HH}(m_H)\right.$$
$$\left. + \Sigma_{AA}(m_A)s^2_{\beta-\alpha} - \Sigma_{GG}(0)c^2_{\beta-\alpha}\right)\,.$$

A second possible scheme is to define $\tan\beta$ via the decay $A^0 \to \tau\tau$. The $\tan\beta$ counterterm would then depend on the measured partial width. Both these schemes are complete on-shell schemes.

An alternative strategy: define $\tan\beta$ via the $\overline{\rm DR}$ scheme. Not surprisingly, the effects of the high-scale physics do not decouple. In this case, $\tan\beta_{\overline{\rm DR}}$ is not directly a physical parameter. One would then measure some low-energy process that depends on $\tan\beta_{\overline{\rm DR}}$. Eliminating $\tan\beta_{\overline{\rm DR}}$ in terms of the low energy observable, all effects of high-scale physics must then decouple.

Can $\overline{\mathrm{DR}}$ be modified to respect decoupling? The m$\overline{\mathrm{DR}}$ scheme of Heinemeyer et al. attempts to do this by removing by hand terms that are logarithmically sensitive to the high energy scale. But, this procedure fails to remove constant terms induced by the high-scale physics that can be of order a few GeV. Such terms are absent in the physical schemes described previously.

We have developed an alternative scheme that automatically removes both the large logarithms and the constant terms induced by the high-scale physics. After renormalizing the vevs,

$$v_u \to \mathcal{Z}_{H_u}^{-1/2} v_u = v_u(1 + \tfrac{1}{2}\delta\mathcal{Z}_{H_u}) \ , \qquad v_d \to \mathcal{Z}_{H_d}^{-1/2} v_d = v_d(1 + \tfrac{1}{2}\delta\mathcal{Z}_{H_d}) \ ,$$

the $\tan\beta$ counterterm, defined by $\tan\beta \to \tan\beta - \delta\tan\beta$ is given by:

$$\delta\tan\beta \equiv \tfrac{1}{2}(\delta\mathcal{Z}_{H_d} - \delta\mathcal{Z}_{H_u})\tan\beta \ .$$

We now introduce the decoupling scheme (DEC) to fix the wave function renormalization,

$$\left(\delta\mathcal{Z}_{H_d}\right)_{\mathrm{DEC}} = -\frac{d\Sigma_{HH}(p^2)}{dp^2}\bigg|_{\alpha=0} \ , \qquad (\delta\mathcal{Z}_{H_u})_{\mathrm{DEC}} = -\frac{d\Sigma_{hh}(p^2)}{dp^2}\bigg|_{\alpha=0} \ .$$

The large logarithms and the constant terms induced by the high-scale physics are manifestly removed. Although $\tan\beta_{\mathrm{DEC}}$ is not directly a physical parameter, its definition is completely insensitive to the high-scale physics.

The seesaw extended MSSM

Introduce a right-handed neutrino superfield N and a superpotential

$$W = \mu H_d H_u + y_\nu L H_u N - y_l L H_d R + \frac{1}{2} m_M N N \, .$$

Add soft SUSY-breaking terms,

$$V_{\rm soft} = m^2_{\tilde{R}} \tilde{N}^* \tilde{N} + (y_\nu A_\nu H^0_U \tilde{\nu}_L \tilde{N}^* + m_M B_\nu \tilde{N} \tilde{N} + {\rm h.c.}) \, .$$

As a result, one obtains the seesaw neutrino mass matrix,

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \, ,$$

where $m_D \equiv y_\nu v_u$. The CP-even/odd $(+/-)$ sneutrino mass matrices are given by:

$$\mathcal{M}^2_{\tilde{\nu}\pm} = \begin{pmatrix} m^2_{\tilde{L}} + m^2_D + \frac{1}{2} m^2_Z \cos 2\beta & m_D(A_\nu - \mu \cot\beta \pm m_M) \\ m_D(A_\nu - \mu \cot\beta \pm m_M) & m^2_{\tilde{R}} + m^2_D + m^2_M \pm 2 B_\nu m_M \end{pmatrix} \, ,$$

where $m^2_{\tilde{L}}$ is the usual soft-breaking mass for the left-handed sneutrinos present in the MSSM.

Decoupling of the right-handed neutrino sector in the one-loop expression for m_h

For simplicity, set $\mu = A_\nu = B_\nu = 0$ and fix $m_{\tilde{L}} = m_{\tilde{R}} \equiv m_S$. Then expand to first order in m^2/m_M^2, where $m \in \{m_Z, m_S, m_D\}$, and to leading order in powers of m_Z. At leading-log order, the lightest Higgs mass squared is shifted relative to its tree level value in the HM and DEC schemes by an amount:

$$\left(\Delta m_h^2\right)_{\rm HM} \simeq \frac{g^2}{48\pi^2 c_W^2} m_Z^2 \log\frac{m_S}{m_Z} - \frac{g^2 m_D^4 m_S^2}{4\pi^2 c_W^2 m_M^2 m_Z^2 \sin^2\beta} \log\frac{m_M}{m_S}\,,$$

$$\left(\Delta m_h^2\right)_{\rm DEC} \simeq \frac{g^2}{48\pi^2 c_W^2} m_Z^2 \cos^2 2\beta \log\frac{m_S}{m_Z} - \frac{g^2 m_D^4 m_S^2}{4\pi^2 c_W^2 m_M^2 m_Z^2} \log\frac{m_M}{m_S}\,,$$

where $c_W \equiv \cos\theta_W = m_W/m_Z$. The second term in both expressions above, which is generated by the right-handed neutrino sector, yields a correction $\Delta m_h \sim m_\nu^2/m_h$ (where $m_\nu \sim m_D^2/m_M$) and is thus utterly negligible.

The difference in the two results can be accounted for by the different definitions of $\tan\beta$. Indeed,

$$\tan\beta_{\rm HM} = \tan\beta_{\rm DEC} + \delta\tan\beta_{\rm HM} - \delta\tan\beta_{\rm DEC}\,,$$

implies that

$$\left(\Delta m_h^2\right)_{\rm DEC} - \left(\Delta m_h^2\right)_{\rm HM} \simeq -2m_Z^2\cos^2\beta\sin 4\beta\Big[\delta\tan\beta_{\rm HM} - \delta\tan\beta_{\rm DEC}\Big]\,.$$

If we evaluate $\delta\tan\beta$ in the two schemes employing the same approximations used to obtain Δm_h^2 above, we obtain:

$$\frac{\delta\tan\beta_{\rm HM} - \delta\tan\beta_{\rm DEC}}{\tan\beta} \simeq c_{2\beta}\left(\frac{g^2}{96\pi^2 c_W^2}\log\frac{m_S}{m_Z} - \frac{g^2 m_D^4 m_S^2}{32\pi^2 c_W^2 m_M^2 m_Z^4 s_\beta^4}\log\frac{m_M}{m_S}\right)\,.$$

Substituting this expression above then reproduces the difference in the two expressions for Δm_h^2 previously obtained.

We can compare the mass shift in the DEC scheme to that of the $\overline{\mathrm{DR}}$ scheme for defining the $\tan\beta$ counterterm,

$$\left(\Delta m_h^2\right)_{\overline{\mathrm{DR}}} \simeq \left(\Delta m_h^2\right)_{\mathrm{DEC}} + \frac{g^2 m_D^2}{8\pi^2 c_W^2}\left\{\cos^2\beta\cos 2\beta\log\frac{m_M^2}{Q^2} + 1\right\} ,$$

where Q is the arbitrary mass scale of $\overline{\mathrm{DR}}$ scheme. Even if one sets $Q = m_M$, which removes the large logs by hand, one is left with a term of $\mathcal{O}(m_D^2)$ which can be as large as a few GeV. This is a remnant of the right-handed neutrino scale and must also be removed to restore the decoupling behavior.

Numerical instability

Even if the definition of $\tan\beta$ respects decoupling, intermediate results will exhibit sensitivity to the right-handed neutrino sector. This sensitivity is removed only when all the relevant terms are combined to obtain the physical Higgs masses. This typically requires a cancellation of terms with an accuracy of 20 significant figures or more. Achieving this cancellation numerically is challenging, in contrast to the semi-analytical analysis exhibited previously.

Effective field theory estimates of the Higgs mass shift

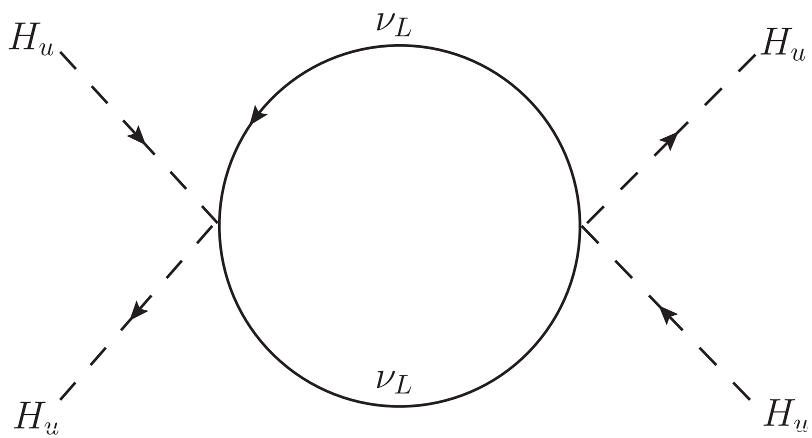
Suppose that we integrate out the right-handed neutrino and sneutrino at the right-handed neutrino mass threshold. Above this scale, the running of the Higgs quartic coupling is supersymmetric, but the TeV-scale soft mass splits the scalar and fermion states, leading to a logarithmic correction to the quartic coupling from the right-handed sneutrino bubble diagram:

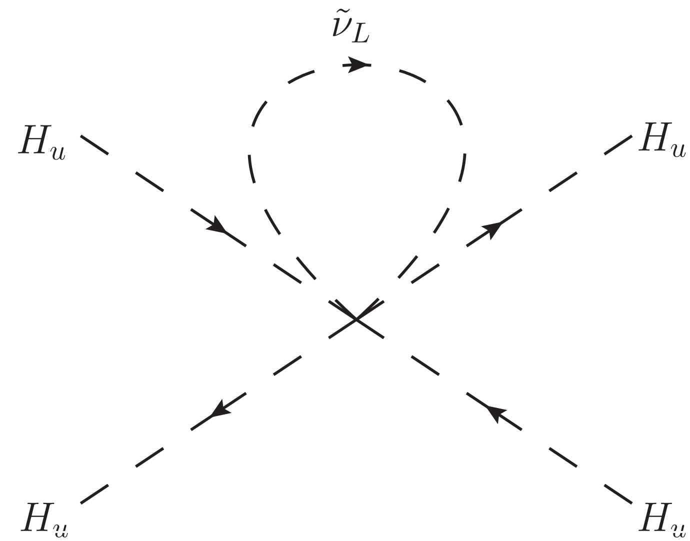
$$\Delta m_h^2 \sim \frac{m_D^4}{v^2} \log \frac{m_{\tilde{N}}^2}{m_N^2} \sim \frac{m_D^4 m_S^2}{v^2 m_M^2} \ .$$

This term is m_M-suppressed and has no log enhancement. In addition to direct contributions to the Higgs quartic coupling, we also generate an approximately supersymmetric higher-dimensional coupling,

$$\Delta W \sim \frac{y_\nu^2}{m_M} LHLH \ .$$

This coupling affects the running of the quartic coupling when supersymmetry is broken via the diagrams exhibited below:





From the vertices, we obtain a factor $m_D^4/v^4 m_M^2$, and at one-loop leading logarithmic order, we obtain a factor $m_S^2 \log(m_M^2/m_S^2)$ from the sum of the loop integrals (running from m_M to m_S.) Thus the Higgs mass shift is

$$\Delta m_h^2 \sim \frac{m_D^4 m_S^2}{v^2 m_M^2} \log \frac{m_M^2}{m_S^2} \ ,$$

reproducing the leading term generated by the right-handed neutrino sector.

Large SUSY-breaking in the right-handed neutrino sector?

One might be tempted to consider the possibility of choosing large values for the SUSY-breaking parameters $m^2_{\tilde{R}}$ and B_ν. An effective field theory estimate shows that

$$\Delta m_h^2 \sim \frac{m_D^4}{v^2} \log\left(\frac{m_M^2 + m^2_{\tilde{R}}}{m_M^2}\right) ,$$

and

$$\Delta m_h^2 \sim \frac{m_D^4}{v^2} \log\left(\frac{m_M^2 - B_\nu^2}{m_M^2}\right) ,$$

respectively, which would yield a Higgs mass shift of order $\Delta m_h \sim m_D^4/(v^2 m_h)$.

However, naturalness constraints suggest that $m_{\tilde{R}}$ should not be larger than other soft-SUSY-breaking parameters, and B_ν cannot be larger than about $10^3 m_{\tilde{\nu}_L}$ in order to avoid generating too large a one-loop mass for neutrinos via $\tilde{\nu}_L$–$\tilde{\nu}_R$ mixing [Y. Grossman and H.E. Haber (1997)].

Conclusions

- It is possible to accommodate $m_h \sim 125$ GeV within the MSSM. However, this value strongly suggests that the relevant SUSY-breaking parameters must be at least of $\mathcal{O}(1\ \text{TeV})$ or higher, which provides tension with expectations of naturalness. This tension is often exacerbated in specific SUSY-breaking models.

- The MSSM predictions for the masses of the neutral CP-even Higgs bosons are robust. Potential contributions to these masses due to additional physics at a very high mass scale are strongly suppressed (decoupling!).

- In practice, one must employ a sensible definition of $\tan\beta$ that respects decoupling. As an example, the contribution in the seesaw-extended MSSM from the heavy right-handed neutrino sector to the one-loop MSSM Higgs mass (when expressed in terms of m_A, $\tan\beta$ and other parameters of the low-energy MSSM spectrum) is utterly negligible.