Exponential hierarchy of scales and cosmology

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Outline:

- hierarchy of scales
- dynamically generated mass scales
- approximate scaling symmetries
- susy and non-susy models
- inflation
- vacuum selection in early universe
- summary

Standard Model

• hierarchy of scales:

$$M_{SM} \approx 100\,\mathrm{GeV}$$

vs

$$M_P \approx 10^{18}\ \mathrm{GeV}$$

$$\delta m_h^2 = O(\lambda, g^2, h^2; \text{ new physics})\, \Lambda^2$$

to keep the Higgs mass low
and to avoid unnatural cancellations

$$\Lambda = \Lambda_{NP} \sim 1\,\text{TeV}$$

CURRENT LHC BOUNDS ON NEW PHYSICS PROVIDING Λ_{NP}

Extra dimensions: ADD, RS, TC, Z', W'

$$\Lambda_{NP} > 1 - 3\,\mathrm{TeV}$$

Supersymmetry:

$$\Lambda_{NP} \sim 1\ \mathrm{TeV}$$

mass splitting in the smultiplets

with $M_{\tilde{g}} > 1.2\,\mathrm{TeV}$

Dimensional transmutation as the origin of mass scales

$$\mathcal{L}(h,\psi) = \frac{1}{M}\bar{\psi}\psi h^2 - \frac{\lambda}{4}h^4$$

$$M > 5\ \mathrm{TeV}$$

with scale invariance

$$V(h) = -\Lambda_\chi^2 \left(\frac{\Lambda_\chi}{M}\right)^{1+\gamma} e^{-(1+\gamma)\phi/F_D} h^2 + \frac{\lambda}{4}h^4$$

Coupling to the gauge bosons

$$\mathcal{L} = \frac{1}{4} D_\mu \Sigma D^\mu \Sigma v^2 \left((1-\gamma)\bar{U}/F_D + \frac{1}{2}(\gamma - 1)\gamma \bar{U}^2/F_D^2 + \ldots \right)$$

$$\text{where } v^2 = \frac{2\Lambda_\chi^{3+\gamma}}{\lambda M^{1+\gamma}}$$

$$\bar{U} = U - F_D$$

$$U = F_D e^{\phi/F_D}$$

different from $v^2(2\bar{U}/F_D + \bar{U}^2/F_D^2)$

coupling to fermion arises through

$$\delta L_U = \bar{U}/F_D T_\mu^\mu$$

Models with a modulus

$$\mathcal{L}_{kin} = \frac{1}{4}(\Delta_i + \frac{s}{8\pi^2})F_i^2$$

$$< s >= \frac{8\pi^2}{g^2(\Lambda_{UV})}$$

$\Lambda_{UV} = M$ - heavy particles

$$\frac{1}{4}(\Delta_i + \frac{s}{8\pi^2})F_i^2 = \frac{1}{4}(\Delta_i + \frac{b_i}{8\pi^2}\log\left(\frac{M}{\Lambda_i}\right))F_i^2$$

Stability wrt s means stability wrt UV cut-off scale M

Supersymmetric version

$$W = -M^3 e^{-\frac{3s}{b_1 M}} + BM^3 e^{-\frac{3s}{b_2 M}} + Me^{-\frac{3s}{b_1 M}} H_1 H_2$$

$$< s > \sim \frac{M b_1 b_2}{b_1 - b_2} \log\left(\frac{b_1 B}{b_2}\right)$$

$$L_1 = e^{-\frac{s}{b_1 M}} \qquad \mu = M L_1^3$$

$$m_s^2 = 162 M^2 \frac{(b_1 - b_2)^2}{b_1^4 b_2^2} L_1^6 = 162 \mu^2 \frac{(b_1 - b_2)^2}{b_1^4 b_2^2}$$

$b_1 \to b_2$ means $s \to s + \delta$

quartic $\delta V_4 = 9 L_1^6 h_1^2 h_2^2$

ZZs $\delta v^2 = ... + \frac{96}{g_1^2 + g_2^2} \frac{1}{b_1} \frac{\mu^2}{M} \delta s$

Solutions in non-supersymmetric models

$$V(h) = -\left(\frac{\Lambda_1^3}{M}\right) h^2 + \frac{\lambda}{4} h^4 - \frac{\Lambda_1^6}{M^2} + B\frac{\Lambda_2^6}{M^2}$$

$$\Lambda = Me^{-s/(Mb)}$$

approximate symmetry broken by Λ_2^6

$$h(x) \to \sigma h(\sigma x) \text{ and } s(x) \to s(\sigma x) - 2/3\, Mb \log(\sigma)$$

$$< h^2 >= \frac{2M^2 L_1^3}{\lambda},\ < s >= M\frac{b_1 b_2}{6(b_1 - b_2)} \log\left(\frac{Bb_1}{b_2}\frac{1}{1+1/\lambda}\right)$$

$b_1 > b_2$ and $B > \lambda/(1+\lambda)$ and $L_{1,2} = e^{-s/(Mb_{1,2})}$

$$m_2^2 = L_1^3 m_1^2,\ m_1^2 = 4M^2 L_1^3$$

$$< h^2 >= 2M^2 L^3/\lambda = (246\,\mathrm{GeV})^2$$

$\lambda = 0.13$ and $L_1 = 10^{-11}$

$m_2 = 1.25 \times 10^{-5.5}\,\mathrm{eV}$

Higher dimension operators

$$V(s,h) = -M^2 h^2 L_1^{3+\gamma} + \frac{\lambda}{4} h^4 - M^4(\Lambda_1^{p_1+\gamma_1} - B\Lambda_2^{p_2+\gamma_2})$$

$$< h^2 >= \frac{2M^2 L_1^{3+\gamma}}{\lambda}, \quad \frac{s}{M} = \frac{b_1 b_2}{(p_2+\gamma_2)b_1 - (6+2\gamma)b_2} \log\left(\frac{B(p_2+\gamma_2)b_1\lambda}{(6+2\gamma)b_2}\right)$$

$$p_2 + \gamma_2 \approx (6+2\gamma)\frac{b_2}{b_1}$$

$$m_2^2 \sim m_1^2 L_1^{3+\gamma}((p_2+\gamma_2)b_1 - (6+2\gamma)b_2)/(b_1 b_2)$$

$$\mathcal{L} = \frac{1}{4} D_\mu \Sigma D^\mu \Sigma v^2 \left(1 + 2a\frac{s}{v} + b(\frac{s}{v})^2 + \ldots\right) - \sum_q m_q \bar{q} q \left(1 + c_q \frac{s}{v}\right)$$

one finds $b = 2a^2$, $c = a$ and

$$a = -\frac{(3+\gamma)v}{b_1 M}$$

Inflation from a modulus/UV cut-off

$$V(h) = -\left(\frac{\Lambda_1^3}{M}\right)h^2 + \frac{\lambda}{4}h^4 - \frac{\Lambda_1^6}{M^2} + B\frac{\Lambda_2^6}{M^2}$$

$$\Lambda = Me^{-s/(Mb)}$$

This simple structure insufficient to generate inflation along s

Ways out:

Pseudogoldstone (warm) inflation:

$$M^4\, f(s,T)\, \left(1 - \cos(\tfrac{G}{f(T)})\right)$$

New moduli via threshold corrections/phase transitions

If at the scale m some states become heavy/light and $b \to b_0$

$$\Lambda = M e^{-\frac{s}{bM}} \left(\frac{M}{m}\right)^{(b'-b)/b}$$

$m = \alpha + \beta z$ (in SUSY $\delta W = (\alpha + \beta z)\Psi\Psi$)

and

$$V(s,z) = M^4 e^{-6s/b_1} \left(\frac{M}{\alpha + \beta\, z}\right)^{\gamma_1} - B M^4 e^{-6s/b_2} \left(\frac{M}{\alpha + \beta\, z}\right)^{\gamma_2} + h.c.$$

$$z = x e^{i\theta}$$

After simplifying

$$V = 2M^4 \left(e^{-6s/b_1} \frac{\cos(\psi\gamma_1)}{r_1^{\gamma}} - B e^{-6s/b_2} \frac{\cos(\psi\gamma_2)}{r_2^{\gamma}} \right),$$

where $\tan(\psi) = \frac{\beta x \sin(\theta)}{\alpha + \beta x \cos(\theta)}$ and $r^2 = (\alpha^2 + \beta^2 x^2 + 2\alpha\beta x \cos(\theta))/M^2$

Details of a supersymmetric model

$$W_{\rm npert} = \chi^p A N_1 M^3 {\rm e}^{-S/N_1} - \chi^{p'} B N_2 M^3 {\rm e}^{-S/N_2} \left(\frac{M^2}{(\alpha+\beta\chi)^2}\right)^{3(N_2'-N_2)/(2N_2)}$$

with $p = p'$

$$\begin{aligned} V(s,\phi,x,\theta) \;=\; & \frac{e^{x^2}}{2s}\kappa x^{2p}\Big(A^2(2s+N_1)^2{\rm e}^{-2s/N_1} + B^2(2s+N_2)^2{\rm e}^{-2s/N_2} r^{-4\gamma}(x,\theta) \\ & -2AB(2s+N_1)(2s+N_2){\rm e}^{-s(\frac{1}{N_1}+\frac{1}{N_2})} r^{-2\gamma}(x,\theta)\cos[\epsilon\phi + 2\gamma\delta(x,\theta)]\;\Big) \\ & +\frac{e^{x^2}}{2s}\kappa x^{2p}(1+\frac{p}{x^2})^2\Big(x^2A^2N_1^2{\rm e}^{-2s/N_1} + B^2N_2^2{\rm e}^{-2s/N_2} r^{-4\gamma}(x,\theta) r'^2(x,\theta) \\ & -2xABN_1N_2{\rm e}^{-s(\frac{1}{N_1}+\frac{1}{N_2})} r^{-2\gamma}(x,\theta) r'(x,\theta)\cos(\epsilon\phi + 2\gamma\delta(x,\theta) - \delta'(x,\theta))\;\Big) \end{aligned}$$

where

$$r^2(x,\theta) = [\alpha + \beta x\cos(\theta)]^2 + \beta^2 x^2 \sin^2(\theta),$$

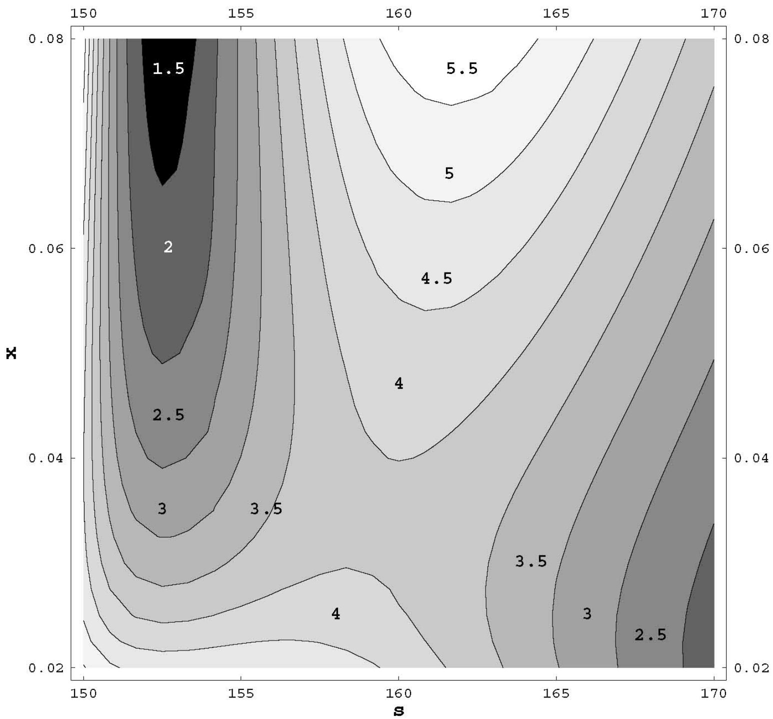
$$r'^2(x,\theta) = \left(x - 2\tilde\gamma(x)\beta\frac{\beta x + \alpha\cos(\theta)}{\beta^2x^2+\alpha^2+2\alpha\beta x\cos(\theta)}\right)^2 + \frac{4\tilde\gamma^2(x)\beta^2\alpha^2\sin^2(\theta)}{(\beta^2x^2+\alpha^2+2\alpha\beta x\cos(\theta))^2}\ ,$$

$$\tan[\delta(x,\theta)] = \frac{\beta x\sin(\theta)}{\alpha+\beta x\cos(\theta)}\ ,$$

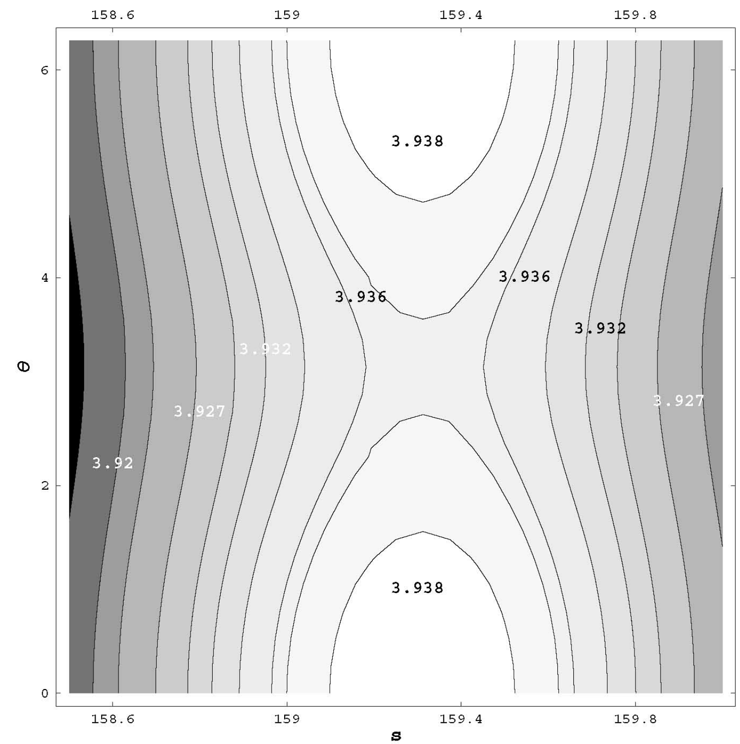
$$\tan[\delta'(x,\theta)] = \frac{2\tilde\gamma(x)\beta\alpha\sin(\theta)}{\beta^2x^2+\alpha^2+2\alpha\beta x\cos(\theta)}\left(x - 2\tilde\gamma(x)\beta\frac{\beta x+\alpha\cos(\theta)}{\beta^2x^2+\alpha^2+2\alpha\beta x\cos(\theta)}\right)^{-1}\ ,$$

$$\tilde\gamma(x) = \gamma(1+\frac{p}{x^2})^{-1}$$

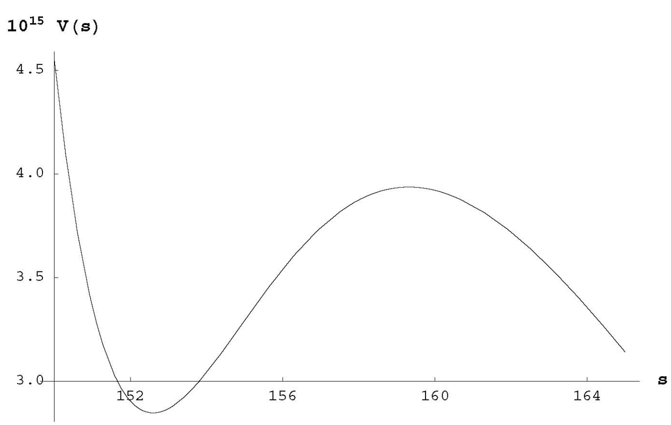
Solution



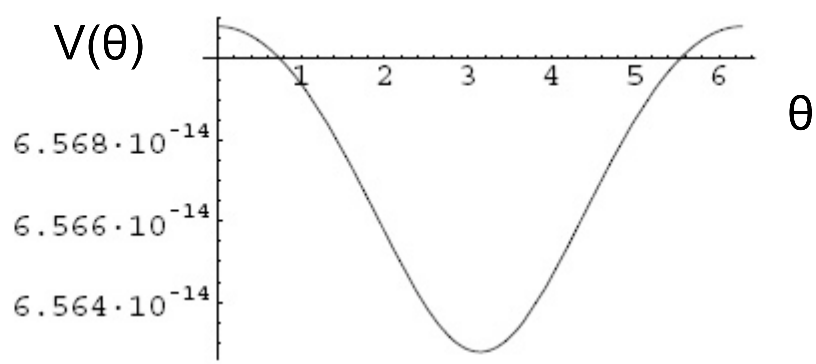
150 155 160 165 170
0.08 0.06 0.04 0.02
x
s
1.5
5.5
5
2
4.5
4
2.5
3
3.5
3.5
4
3
2.5



158.6 159 159.4 159.8
0 2 4 6
θ
s
3.938
3.936
3.936
3.932
3.932
3.927
3.927
3.92
3.938

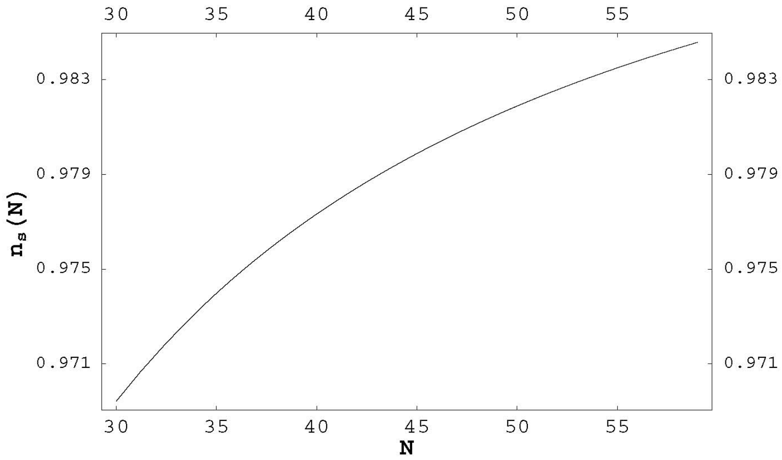


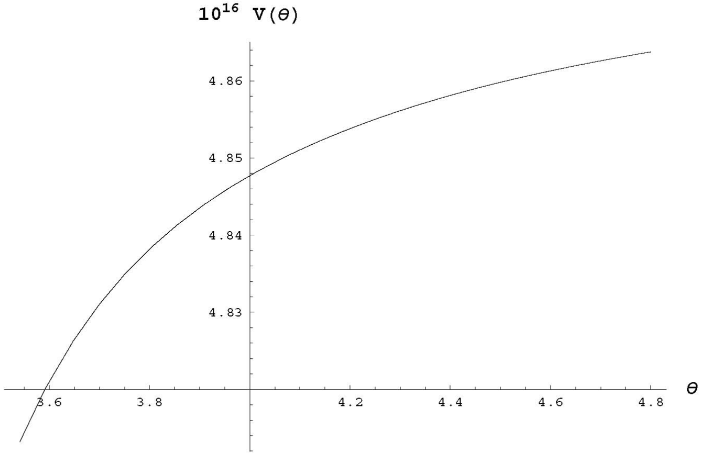
10^15 V(s)
4.5
4.0
3.5
3.0
152 156 160 164
s



V(θ)
θ
1 2 3 4 5 6
6.568·10^-14
6.566·10^-14
6.564·10^-14

$$A = 1.5,\ B = 8.2,\ N_1 = 10,\ N_2 = 9,$$
$$p = 0.5,\ \alpha = 1,\ \beta = 2.3,\ \gamma = 10^{-4}$$





Minimum:

$s = 152.6,\ \phi = 0,\ x = 0.42,\ \theta = 3.16$

Inflation:

$\theta_\star = 4.71,\ \theta_e = 3.54,\ \eta_\star = -0.0089,\ n_\star = 0.98$

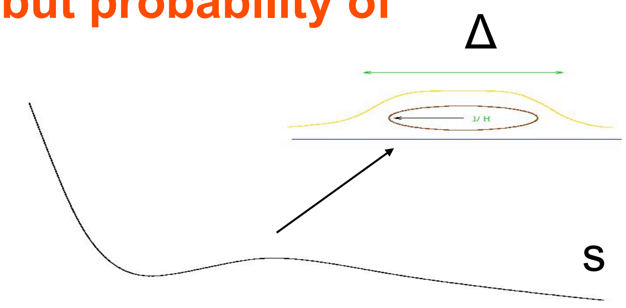
$N_e \approx 8000$

Comments on susy case:

$$V_{infl}^{1/4} > T_{eq} \rightarrow m_{3/2} > 1\,TeV$$

- Reducing $m_{3/2}$ reduces the ridge between the finite and the noninteracting vacua

Topological trapping may still work, but probability of populating finite vacuum reduced



- $$W = XW_R \rightarrow V = |W_R|^2 + |X|^2|W_R'|^2$$

Produce gravitino mass somewhere else

Nearly scale invariant SM

$$V = a + \mu\Phi^\dagger\Phi + \lambda/2(\Phi^\dagger\Phi)^2 + \frac{1}{(8\pi)^2}\Big(3/2(\mu+\lambda\Phi^\dagger\Phi)^2(\log\frac{(\mu+\lambda\Phi^\dagger\Phi)^2}{M^4} - 1)$$
$$+1/2(\mu+3\lambda\Phi^\dagger\Phi)^2(\log\frac{(\mu+3\lambda\Phi^\dagger\Phi)^2}{M^4} - 1)$$
$$+ (3/2g^4 + 3/4(g^2+g'^2)^2 - 12h_t^4)(\Phi^\dagger\Phi)^2(\log\frac{\Phi^\dagger\Phi}{M^2} - 1/2)\Big),$$

scale invariance broken by the dilaton kinetic term

$$a \to ae^{4\sigma/f},\ \mu \to \mu e^{2\sigma/f}$$

$$a(f) \sim f^4 \text{ and } \mu(f) \sim f^2.$$

But:

$$m_\sigma^2 = -\frac{4}{f^2}M\frac{\partial V}{\partial M} = \frac{1}{8\pi^2 f^2}\,Str\,\mathcal{M}^4$$

In SM

$$Str\,\mathcal{M}^4 < 0$$

Improvement

additional breaking of scale invariance

$$ae^{4\sigma/f} \to ae^{4\sigma/f} - be^{p\sigma/f}$$

this results in an acceptable solution

$$m^2_{\sigma\, tr} = \frac{4(4-p)}{f^2} ae^{4\sigma_0/f},$$

$$\frac{\sigma_0}{f} = \frac{1}{4-p} \log(\frac{bp}{4a}).$$

tuning

Temperature corrections

$$V = a(\sigma/f)e^{4\sigma/f} + \frac{\mu}{2}e^{2\sigma/f}\phi^2 + \mu e^{2\sigma/f}\frac{T^2}{24} + \frac{\alpha T^2}{24}\phi^2$$

$$\alpha = \frac{1}{v^2}\left(\frac{3}{2}m_h^2 + 6m_W^2 + 3m_Z^2 + 6m_t^2\right)$$

$$T_c^2 = 6\frac{m_h^2}{\alpha}$$

for $T > T_c$

$$V(\sigma) = a(\sigma/f)e^{4\sigma/f} + \mu e^{2\sigma/f}\frac{T^2}{24}$$

take $a = f^4$ and $\mu = -f^2$

$$e^{2<\sigma>/f} = \frac{T^2}{48f^2},\ m_\sigma^2 = \frac{T^4}{288f^2}$$

Dilaton trapped and driven to the proper low-energy vacuum

Summary

- One needs to understand generation of mass scales hierarchically smaller than the Planck scale - even in supersymmetry one needs to generate a relatively small scale at rather high energies

- Dimensional transmutaion in strongly coupled gauge theories is a reliable source of hierarchically small scales, with or without supersymmetry

- Dynamical scales may depend on moduli, which mix with „standard" degrees of freedom

- Some eigenstates of the complete mass matrix may be light or very light and/or may have phenomenological or cosmological consequences

- Supersymmetric or nonsupersymmetric moduli associated with dynamical mass scales may provide a natural source of cosmological inflation

- Thermal trapping of the modulus at high temperature