The heterotic string on magnetized orbifolds

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based on

JHEP 1303 (2013) 142 (arXiv:1212.4033)

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DESY, Hamburg

Planck'13, Bonn, 22 May 2013

Geometrical heterotic string backgrounds

Calabi–Yau compactification with gauge bundles:

- lowest order in α'
- only the string zero modes
- generic point in moduli space

Orbifold compactification with shifts and Wilson lines:

- exact CFT constructions
- special point in moduli space
- full string spectrum accessible

Both approaches have lead to many MSSM–like models

Orbifolds with vanishing Euler number

Orbifolds that have been used for string model building so far have non–vanishing Euler number, e.g.: Erler,Klemm'92

- $T^6/\mathbb{Z}_3$: $(h_{11}, h_{21}) = (36, 0)$; $\chi = 2(h_{11} - h_{21}) = 72$
- $T^6/\mathbb{Z}_{6\text{-II}}$: $(h_{11}, h_{21}) = (35, 11)$; $\chi = 2(h_{11} - h_{21}) = 48$
- $T^6/\mathbb{Z}_{12\text{-I}}$: $(h_{11}, h_{21}) = (29, 5)$; $\chi = 2(h_{11} - h_{21}) = 48$

Orbifolds with vanishing Euler number are not considered for phenomenology since they always give rise to a non–chiral spectrum.

Recent classification reveals that 23 of the 138 orbifolds with Abelian point group have vanishing Euler number;
they all $\mathbb{Z}_2 \times \mathbb{Z}_2$ variants. Fischer,Ratz,Torrado,Vaudrevange'12

Orbifold resolution description of the Schoen manifold

A specific Calabi–Yau, the so–called Schoen manifold has been used to construct MSSM–like models with bundles.

Bouchard,Donagi'05 Braun,He,Ovrut,Pantev'05

A specific singular limit of the Schoen manifold leads to a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with vanishing Euler number. Donagi,Wendland'08

Paradox: The smooth compactification admits chirality but its orbifold version does not.

Using the procedure of orbifold resolutions, giving an alternative description of the Schoen manifold, we hope to resolve this paradox.

Strategy of this talk

In this talk we will see that one should not write off all these orbifolds with vanishing Euler number just yet.

To this end, we:

- consider the Schoen orbifold as a concrete orbifold with vanishing Euler number,
- show that by putting magnetic flux on its tori, it is possible to obtain 4D chirality,
- and construct an MSSM-like model in this way.

Overview of this talk

1. Introduction
2. $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds
3. Orbifold resolution
4. Example: A semi–realistic MSSM model
5. Conclusion

Some $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds are defined as

$$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$$

where the orbifold elements, g_θ, g_ω, involve $\mathbb{Z}_2 \times \mathbb{Z}_2$ rotations

$$\theta \ : \ (z_1, z_2, z_3) \longrightarrow (z_1, -z_2, -z_3) \ , \quad \omega \ : \ (z_1, z_2, z_3) \longrightarrow (-z_1, z_2, -z_3)$$

possibly combined with some torus translations.

We will consider two types: Donagi,Wendland'08

- **DW(0–1)**: The standard $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold
- **DW(0–2)**: A roto–translational $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

The standard $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

The orbifold action of the DW(0–1) orbifold is given by:

$$g_\theta\big(z_1, z_2, z_3\big) = \big(z_1, -z_2, -z_3\big) ,$$

$$g_\omega\big(z_1, z_2, z_3\big) = \big(- z_1, z_2, -z_3\big) ,$$

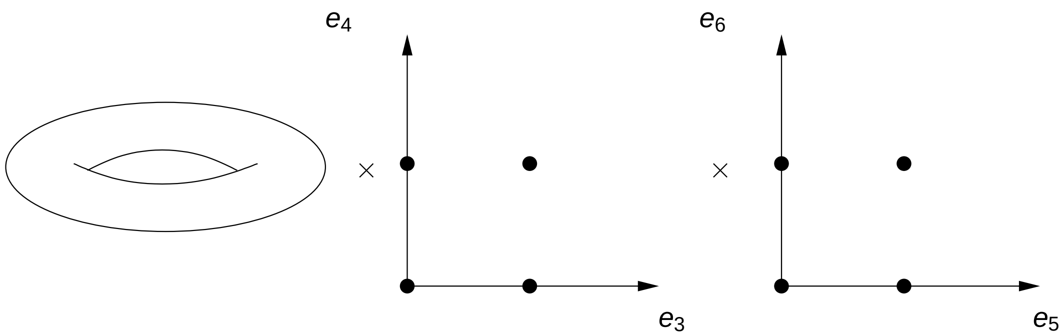
$$g_\theta g_\omega\big(z_1, z_2, z_3\big) = \big(- z_1, -z_2, z_3\big) .$$

The Hodge / Euler numbers of the DW(0–1) orbifold read:

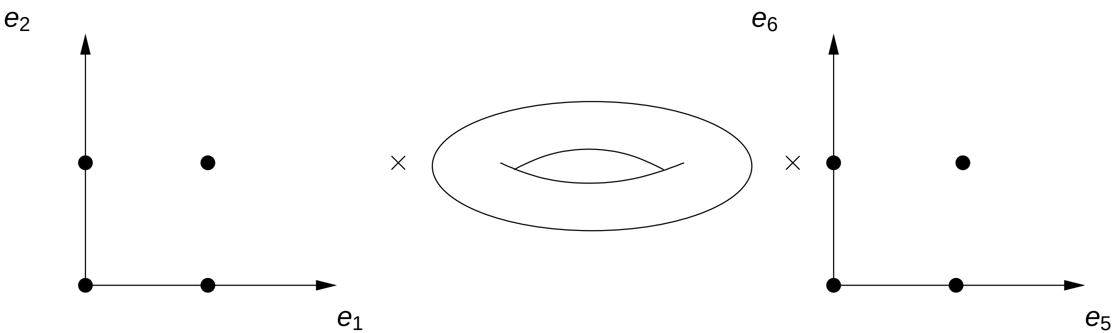
$$(h_{11}, h_{21}) = (51, 3) , \qquad \chi = 2(h_{11} - h_{21}) = 72$$

The standard $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

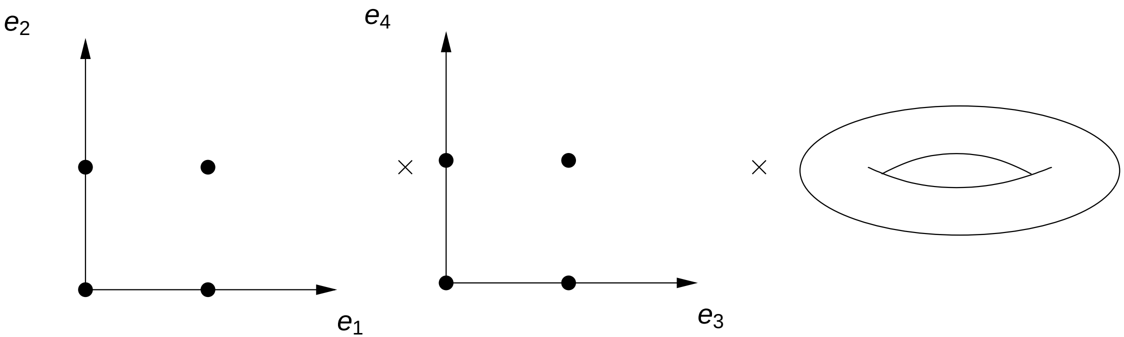
θ–sector:



ω–sector:



$\theta\omega$–sector:



A roto–translational $\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ orbifold

The orbifold actions of the DW(0–2) orbifold is given by:

Donagi,Wendland'08

$$g_\theta\big(z_1, z_2, z_3\big) = \big(z_1, -z_2, -z_3\big)\ ,$$

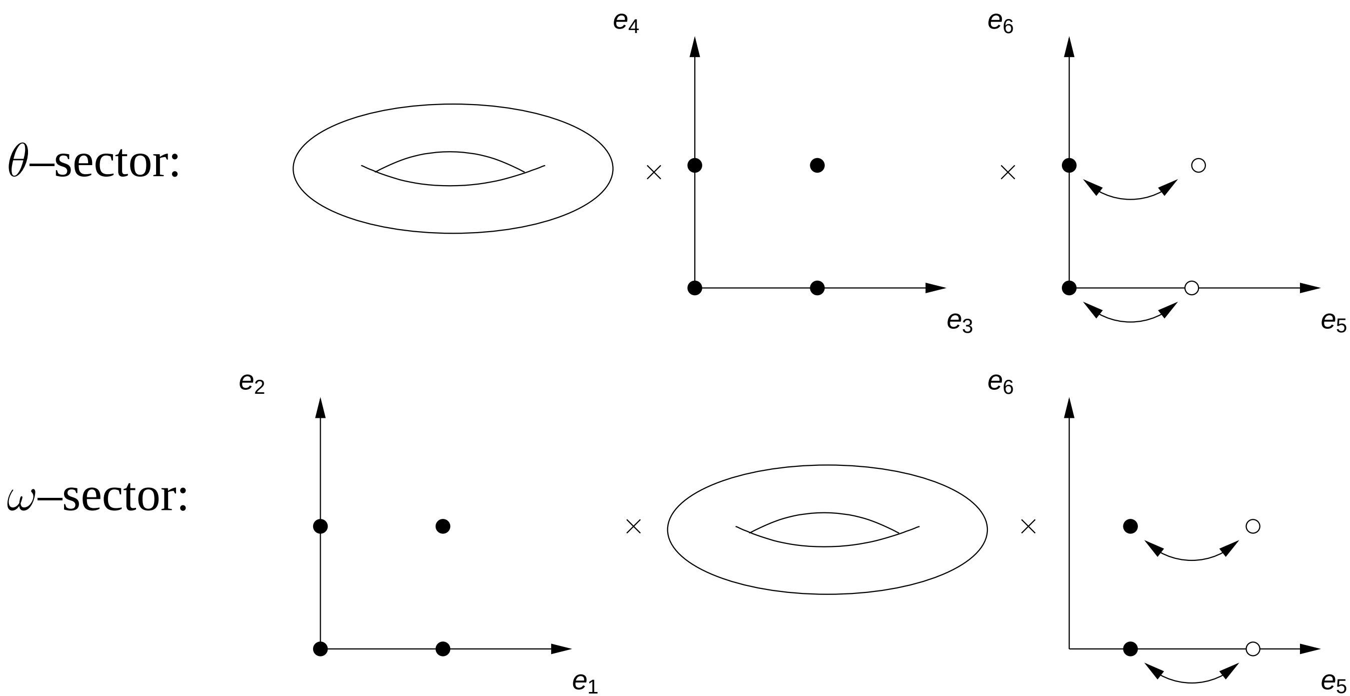
$$g_\omega\big(z_1, z_2, z_3\big) = \big(-z_1, z_2, -z_3 + \tfrac{1}{2}\big)\ ,$$

$$g_\theta g_\omega\big(z_1, z_2, z_3\big) = \big(-z_1, -z_2, z_3 - \tfrac{1}{2}\big)\ .$$

The Hodge / Euler numbers of the DW(0–2) orbifold read:

$$(h_{11}, h_{21}) = (19, 19)\ , \qquad \chi = 2(h_{11} - h_{21}) = 0$$

A roto–translational $\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ orbifold



Magnetic fluxes on the orbifold

To obtain chirality we propose to put magnetic fluxes on the tori of this orbifold.

But unfortunately, as far as we are aware, there is no exact CFT description for magnetized orbifolds...

Therefore we have to take an indirect route:

- construct the orbifold resolution
- put an Abelian gauge flux background
- compute the spectrum

Constructing the orbifold resolution

To construct an orbifold resolution we have to identify:

Denef,Douglas,Florea'04, Luest,Reffert,Scheidegger,Stieberger'06, SGN,Held,Ruehle,Trapletti,Vaudrevange'09

- a complete set of divisors of the resolution,
- their intersection numbers.

For an orbifold resolution there are two types of divisors:

- inherited divisors (four–tori within the orbifold),
- exceptional divisors (blow–up cycles).

Constructing the orbifold resolution

For the resolution of the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ we have:

SGN,Vaudrevange'12

- Three inherited divisors:

$$R_1 := \{z_1 = c_1\} \cup \{z_1 = -c_1\} \ , \ R_2 := \{z_2 = c_2\} \cup \{z_2 = -c_2\}$$

$$R_3 := \{z_3 = c_3\} \cup \{z_3 = -c_3\} \cup \{z_3 = \tfrac{1}{2} + c_3\} \cup \{z_1 = \tfrac{1}{2} - c_1\}$$

- 16 exceptional divisors:

$$\theta\text{–sector: } E_r = E_{n_3 n_4 n_6} \ , \qquad \omega\text{–sector: } E'_{r'} = E'_{n_1 n_2 n'_6}$$

Constructing the orbifold resolution

The non–vanishing self–intersections between these divisors are:

$$R_1 R_2 R_3 = 4 \,, \qquad R_2 (E'_{n_1 n_2 n'_6})^2 = R_1 (E_{n_3 n_4 n_6})^2 = -4 \,.$$

The total Chern class $\mathrm{c}(TX)$ is computed from the splitting principle:

$$\mathrm{c}(TX) = \prod (1+D) \prod (1+E) \prod (1-R)^2 \,,$$

This provides an alternative description of the Schoen manifold.

Abelian gauge flux background

The gauge flux is expanded as:

$$\frac{\mathcal{F}}{2\pi} = \sum_a R_a\, H_{B_a} + \sum_r E_r\, H_{V_r} + \sum_{r'} E'_{r'}\, H_{V'_{r'}}$$

where $H_A = A_I H_I$, with H_I are the Cartan generators of $\mathrm{E}_8 \times \mathrm{E}_8{}'$.

This embedding is characterized by 16-dimensional vectors:

- line bundle vectors $V_r\,,\, V'_{r'}$
- and the magnetic fluxes B_a.

They are subject to sets of flux quantization and Bianchi identities.

Determining the massless spectrum

The spectrum in four dimensions is determined by the multiplicity operator SGN,Trapletti,Walter'07

$$N_{4D} = \int_X \left\{ \frac{1}{6} \left(\frac{\mathcal{F}}{2\pi} \right)^3 + \frac{1}{12} c_2(TX) \frac{\mathcal{F}}{2\pi} \right\} .$$

This operator counts the number of chiral states arise for each of the $248 + 248$ gaugino components.

For the orbifold resolution of in interest in this talk, it is readily computed:

$$N_{4D} = 2\Big(1 - \sum_r H_{V_r}^2\Big) H_{B_1} + 2\Big(1 - \sum_{r'} H_{V'_{r'}}^2\Big) H_{B_2} + 4\, H_{B_1} H_{B_2} H_{B_3} .$$

This result shows that without magnetized tori, i.e. $B_a = 0$, no chiral states in four dimensions.

Schoen line bundle MSSM

- Input data: gauge fluxes
- double GUT spectra
- Wilson line GUT $\longrightarrow$ MSSM breaking

Choice gauge fluxes

We define a line bundle model on the Schoen manifold with the flux vectors

$$B_1 = \big(3, -3, 0^6\big)\big(3, 3, 0^6\big) \quad \text{and} \quad B_2 = B_3 = 0\ ,$$

on the ordinary divisors R_a,

$$V_{(0,0,0)} = V_{(0,1,0)} = -V_{(0,0,1)} = -V_{(0,1,1)} = \big(\tfrac{1}{4}^8\big)\big(0, 0, 0, \tfrac{1}{2}, 0, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}\big)\ ,$$

$$V_{(1,0,0)} = V_{(1,1,0)} = -V_{(1,0,1)} = -V_{(1,1,1)} = \big(0, \tfrac{1}{2}, \tfrac{1}{2}, 0^5\big)\big(0, \tfrac{1}{2}, 0, 0, 0, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}\big)\ ,$$

on the exceptional divisors E_r, and finally,

$$\begin{aligned}
V'_{(0,0,0)} = -V'_{(0,1,1)} &= \big(0, -\tfrac{1}{2}, -\tfrac{1}{2}, 0^5\big)\big(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, 0, -\tfrac{1}{2}, 0, 0, 0\big)\ ,\\
V'_{(0,1,0)} = -V'_{(0,0,1)} &= \big(0, -\tfrac{1}{2}, -\tfrac{1}{2}, 0^5\big)\big(\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}, 0, \tfrac{1}{2}, 0, 0, 0\big)\ ,\\
V'_{(1,0,0)} = V'_{(1,1,0)} &= \big(0, 1, 0, 0^5\big)\big(-\tfrac{1}{2}, -\tfrac{1}{2}, 0, 0, 0, 0, 0, 0\big)\ ,\\
V'_{(1,1,1)} = V'_{(1,0,1)} &= \big(-1, 0^7\big)\big(-\tfrac{1}{2}, -\tfrac{1}{2}, 0^6\big)\ ,
\end{aligned}$$

on the exceptional divisors $E'_{r'}$.

Double six generation GUT

Superfield multiplicity	Representation $\mathrm{SU}(5) \times \mathrm{SU}(5)'$	U(1) charges q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7
6	$(\overline{\mathbf{10}}, \mathbf{1})$	0	0	0	0	1	0	-3	0
6	$(\mathbf{5}, \mathbf{1})$	0	0	0	0	0	0	-6	0
6	$(\overline{\mathbf{5}}, \mathbf{1})$	1	0	1	0	-1	0	1	0
6	$(\mathbf{5}, \mathbf{1})$	1	0	1	0	0	0	4	0
24	$(\mathbf{1}, \mathbf{1})$	2	0	0	0	0	0	0	0
6	$(\mathbf{1}, \mathbf{1})$	-1	0	-1	0	-1	0	5	0
6	$(\mathbf{1}, \mathbf{1})$	1	0	-3	0	0	0	0	0
6	$(\mathbf{1}, \mathbf{1})$	0	0	0	0	2	0	0	0
6	$(\mathbf{1}, \overline{\mathbf{10}})$	0	0	0	2	0	0	0	-6
24	$(\mathbf{1}, \mathbf{5})$	0	1	0	3	0	0	0	-2
6	$(\mathbf{1}, \overline{\mathbf{5}})$	0	0	0	-2	0	0	0	-8
6	$(\mathbf{1}, \overline{\mathbf{5}})$	0	0	0	0	0	1	0	7
6	$(\mathbf{1}, \overline{\mathbf{5}})$	0	0	0	0	0	-1	0	7
42	$(\mathbf{1}, \mathbf{1})$	0	0	0	4	0	1	0	-5
42	$(\mathbf{1}, \mathbf{1})$	0	0	0	4	0	-1	0	-5
24	$(\mathbf{1}, \mathbf{1})$	0	1	0	-3	0	1	0	-5
24	$(\mathbf{1}, \mathbf{1})$	0	1	0	-3	0	-1	0	-5
6	$(\mathbf{1}, \mathbf{1})$	0	2	0	0	0	0	0	0

Wilson line GUT $\rightarrow$ MSSM breaking

Both the orbifold and the resolution admit a freely acting involution:

$$\big(z_1, z_2, z_3\big) \longrightarrow \big(z_1 + \tfrac{i}{2}, z_2 + \tfrac{i}{2}, z_3 + \tfrac{i}{2}\big) \ .$$

The freely acting involution can be embedded as a Wilson line

$$W_{\text{free}} = \big(0^3, 1, 1, 1, -\tfrac{3}{2}, -\tfrac{3}{2}\big)\big(0^8\big) \ ,$$

that breaks $\mathrm{SU}(5)$ to $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$.

This choice leads to an MSSM–like model with three generations.

Summary

We have studied a specific class of heterotic orbifolds with vanishing Euler number:

- these do not give chirality in 4D,
- unless the tori become magnetized.

To determine the spectrum two methods are available:

- gauge fluxes on the full resolution,
- intermediate 6D models on magnetized tori.

A concrete example of this all was provided by the orbifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$. We construct a MSSM–like model based on it.