Discrete Symmetries in Global MSSM-like D-Brane Models

Gabriele Honecker

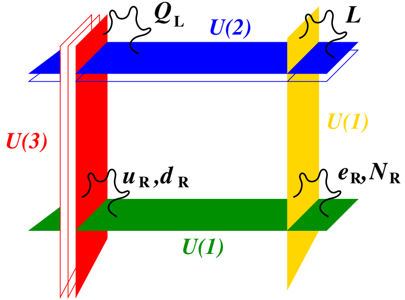
Cluster of Excellence PRISMA & Institut für Physik, JG|U Mainz

based on arXiv:1303.4415 [hep-th] with **Wieland Staessens**

Planck 2013, Bonn, 22 May 2013

Motivation: D-Branes & U(1) Symmetries

Spanish Quiver:



$U(3) \times U(2) \times U(1) \times U(1)$

$\xrightarrow{\text{Green-Schwarz mechanism}}$

$$SU(3) \times SU(2) \times Y \times \begin{cases} U(1)^3_{\text{massive}} \\ (B-L) \times U(1)^2_{\text{massive}} \end{cases}$$

- $U(1)_{\text{massive}}$ remains as *perturbative* **global** symmetry
- $U(1)_{\text{massive}}$ broken by *non-perturvative* effects
  e.g. D-brane instantons
- $\mathbb{Z}_n \subset U(1)_{\text{massive}}$ remains as global **discrete** symmetry
  $\rightsquigarrow$ constraints on couplings

This talk:

- Conditions on the existence of $\mathbb{Z}_n$ gauge symmetries?
- Which $\mathbb{Z}_n$ occur in global D-brane models?

Related Works on Abelian Discrete Symmetries

SUSY field theory:

- *Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model* L.E.Ibáñez, G.G.Ross: Nucl.Phys.B368(1992)3-37
- *What is the discrete gauge symmetry of the MSSM?* H.K.Dreiner, C.Luhn, M.Thormeier: Phys.Rev.D73(2006)075007

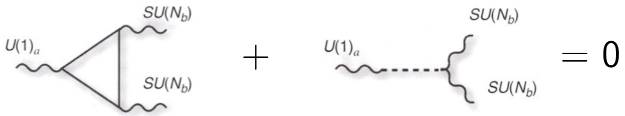
$\rightsquigarrow$ R-parity ($\mathbb{Z}_2$), baryon triality ($\mathbb{Z}_3$), proton hexality ($\mathbb{Z}_6$) for e.g. **proton stability**

D-brane models:

- *Discrete gauge symmetries in D-brane models* M.Berasaluce-Gonzalez, L.E.Ibáñez, P.Soler, A.M.Uranga: JHEP1112(2011)113
- *Discrete Gauge Symmetries in Discrete MSSM-like Orientifolds* L.E.Ibáñez, A.N.Schellekens, A.M.Uranga: Nucl.Phys.B865(2012)509-540
- *String Constraints on Discrete Symmetries in MSSM Type II Quivers* P.Anastasopoulos, M.Cvetič, R.Richter, P.K.S.Vaudrevange: JHEP1303(2013)011
- *Zp charged branes in flux compactifications* M.Berasaluce-Gonzalez, P.G.Camara, F.Marchesano, A.M.Uranga: JHEP1304(2013)138

Two ingredients: (1) Green-Schwarz Mechanism

- **Global model** $\rightsquigarrow$ Non-Abelian $SU(N_b)$ gauge anomalies=0: $\left[\sum_a N_a\,(\Pi_a + \Pi'_a) - 4\,\Pi_{O6}\right] \circ \Pi_b = 0$
- Mixed anomalies canceled by the **Green-Schwarz** mechanism:



$$\mathcal{S}_{CS} \supset \int_{\mathbb{R}^{1,3}} \sum_{i=0}^{h_{21}} \left(B_a^i\, \mathcal{B}_2^{(i)} \wedge \mathrm{tr} F_a + A_b^i\, \phi_{(i)}\, \mathrm{tr} F_b \wedge F_b \right)$$

with $\mathcal{B}_2^{(i)} \propto \int_{\Pi_i^{\mathrm{odd}}} C_5^{RR}$; $\phi_{(i)} \propto \int_{\Pi_i^{\mathrm{even}}} C_3^{RR}$

- $U(1)_a \subset U(N_a)$ remains massless if $B_a^{(i)} = 0\ \forall i$
- $U(1)_X = \sum_a q_a U(1)_a$ is massless if $\sum_a N_a q_a B_a^{(i)} = 0\ \forall i$
- $U(1)_{\text{massive}} = \sum_a k_a U(1)_a$:
  - $\text{mass}^{(i)} \propto \sum_a N_a k_a B_a^i$
  - if $\boxed{\sum_a N_a k_a \tilde{B}_a^i = 0 \bmod n\ \forall i}$ $\rightsquigarrow$ $\mathbb{Z}_n$ symmetry
  - How are $\tilde{B}_a^i$ (B_a^i) computed? Normalisations of k_a, $\tilde{B}_a^i$?

Two ingredients: (1) cont'd

- Expand 3-cycles and $\Omega\mathcal{R}$-images in the Type IIA string as:

$$\Pi_a = \sum_{i=0}^{h_{21}} \left(A_a^i \, \Pi_i^{\text{even}} + B_a^i \, \Pi_i^{\text{odd}} \right), \quad \Pi_a' = \sum_{i=0}^{h_{21}} \left(A_a^i \, \Pi_i^{\text{even}} - B_a^i \, \Pi_i^{\text{odd}} \right)$$

- If $\boxed{\Pi_i^{\text{even}} \circ \Pi_j^{\text{odd}} = \delta_{ij}}$
  - $\{\Pi_i^{\text{even}}, \Pi_j^{\text{odd}}\}$ span full unimodular lattice of 3-cycles
  - wrapping numbers $A_a^i, B_a^i \in \mathbb{Z}$
  - **all** known global D-brane models **violate** this condition ↯
- If $\boxed{\Pi_i^{\text{even}} \circ \Pi_j^{\text{odd}} = m_i \, \delta_{ij}}$ with $m_i \in \mathbb{Z}$
  - $\{\Pi_i^{\text{even}}, \Pi_j^{\text{odd}}\}$ span only **sublattice** of finite index
  - **all** known global D-brane models of **this type**
  - $A_a^i, B_a^i \in \frac{1}{m_i} \mathbb{Z}$ - how exactly?
    $\rightsquigarrow$ compare with similar argument: K-theory constraint

Two Ingredients: (2) K-Theory Constraint

- D-branes are not fully described by $H_3(CY_3)$, but **K-theory**
- K-theory constraint $\Leftrightarrow$ absence of field theo. $SU(2)$ anomalies $[T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)]$
- **probe brane** argument: Uranga '02

$$\sum_a N_a \Pi_a \circ \Pi_{USp(2)_k} = -\sum_{i=0}^{h_{21}} \sum_a N_a \underbrace{m_i B_a^i}_{\equiv \tilde{B}_a^i \in \mathbb{Z}} A^i_{USp(2)_k} = 0 \text{ mod } 2$$

$$\stackrel{?}{\rightsquigarrow} \mathbb{Z}_2 \text{ symmetry for } k \in \{0 \ldots h_{21}\}$$

- N_a D-branes on Π_i^{even}: $U(N_a) \hookrightarrow \begin{cases} USp(2N_a) \\ SO(2N_a) \end{cases}$
- in general K-theory constraint $\not\Leftrightarrow$ $\mathbb{Z}_2$ symmetry
- $\Pi_a \circ \Pi_{USp(2)_k} \in \mathbb{Z}$ independent of basis $\{\Pi_i^{\text{even}}, \Pi_i^{\text{odd}}\}$ & normalisation $\{A_a^i, B_a^i\}$
  $\rightsquigarrow$ express also $\mathbb{Z}_n$ condition via **intersection numbers**

Conditions on the Existence of $\mathbb{Z}_n$ Symmetries

$U(1)_{\text{massless}}$	$\mathbb{Z}_n$
$\sum_a N_a q_a \Pi_a \circ \Pi_i^{\text{even}} = 0 \; \forall i$	$\sum_a N_a k_a \Pi_a \circ \Pi_i^{\text{even}} = 0 \text{ mod } n \; \forall i$
$\sum_a N_a q_a B_a^i = 0 \; \forall i$	$\sum_a N_a k_a \tilde{B}_a^i = 0 \text{ mod } n \; \forall i$ $\tilde{B}_a^i = m_i B_a^i \in \mathbb{Z}$
$q_a \in \mathbb{Q}$	$k_a \in \mathbb{Z}$, $0 \leqslant k_a < n$, $\gcd(k_a, n) = 1$

- cross-check: $(k_a, k_b, \ldots) = (1, 1, \ldots)$ identical to K-theory 'probe brane' argument if *all* $\Pi_i^{\text{even}} = \Pi_{USp(2)_i}^{\text{even}}$ ✓
- ambiguities of normalisation factors m_i in B_a^i and Π_i^{odd} cancel
- derivation of m_i, B_a^i for all orbifolds possible

- first check: validity of $\mathbb{Z}_n$ symmetry in effective field theory

Conditions on the Existence of $\mathbb{Z}_n$ Symmetries

- RR scalars $\phi_{(i)} \simeq \phi_{(i)} + 1$ transform under $U(1)_{\text{massive}}$

$$d\mathcal{B}_2^{(i)} = m_i \ *_4 d\phi_{(i)}$$

$$A^\mu \to A^\mu + \partial^\mu \lambda \qquad \phi_{(i)} \to \phi_{(i)} + \left(\sum_a N_a k_a \tilde{B}_a^i \right) \lambda$$

$\rightsquigarrow$ $\mathbb{Z}_n$ symmetry preserved for $\boxed{\sum_a N_a k_a \tilde{B}_a^i = 0 \text{ mod } n \ \forall i}$

- **all** D2-brane instantons respect $\mathbb{Z}_n$ symmetry: $e^{-\mathcal{S}_{\text{D2}}}$ contains

$$\mathcal{S}_{\text{D2}} = -\frac{\text{Vol(D2)}}{g_s} + 2\pi i \phi \quad \text{with} \quad \phi = \int_{\Pi_{\text{D2}}} C_3^{RR} = \sum_{i=0}^{h_{21}} A^i_{\text{D2}} \, \phi_{(i)}$$

$$\mathcal{S}_{\text{D2}} \to \mathcal{S}_{\text{D2}} + 2\pi i \lambda \underbrace{\sum_{i=0}^{h_{21}} A^i_{\text{D2}} \left(\sum_a N_a k_a \tilde{B}_a^i \right)}_{= \Pi_{\text{D2}} \circ \Pi_{U(1)_{\text{massive}}} = 0 \text{ mod } n}$$

$\rightsquigarrow$ derive explicit conditions on orbifolds

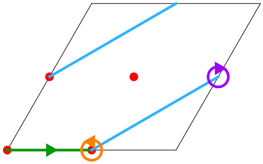
D6-Branes on 3-Cycles on Orbifolds

- $\boxed{T^6/\mathbb{Z}_{2N}}$

$$\Pi_a^{\text{frac}} = \frac{1}{2}\left(\Pi_a^{\text{bulk}} + \Pi_a^{\mathbb{Z}_2}\right)$$

- $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}}$ with discrete torsion:

$$\Pi_a^{\text{frac}} = \frac{1}{4}\left(\Pi_a^{\text{bulk}} + \sum_{i=1}^{3} \Pi_a^{\mathbb{Z}_2^{(i)}}\right)$$



8 discrete param. of **rigid D6-brane**:

- 3 displacements $\sigma \in \{0,1\}$
- 2 $\mathbb{Z}_2$ eigenvalues ± 1
- 3 Wilson lines $\tau \in \{0,1\}$

- very large number of 3-cycles *per given bulk cycle*
- but: only $(h_{21}+1)$ independent Π_i^{even} $\rightsquigarrow$ classify!

Classification of 'Probe' D-Branes: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$

$c \mid\mid$ to	$\Omega\mathcal{R}$ invariant for $(\eta_{(1)}, \eta_{(2)}, \eta_{(3)}) \stackrel{!}{=}$
$\Omega\mathcal{R}$	$\left(-(-1)^{\delta_2+\delta_3}, -(-1)^{\delta_1+\delta_3}, -(-1)^{\delta_1+\delta_2}\right)$
$\Omega\mathcal{R}\mathbb{Z}_2^{(1)}$	$\left(-(-1)^{\delta_2+\delta_3}, (-1)^{\delta_1+\delta_3}, (-1)^{\delta_1+\delta_2}\right)$
$\Omega\mathcal{R}\mathbb{Z}_2^{(2)}$	$\left((-1)^{\delta_2+\delta_3}, -(-1)^{\delta_1+\delta_3}, (-1)^{\delta_1+\delta_2}\right)$
$\Omega\mathcal{R}\mathbb{Z}_2^{(3)}$	$\left((-1)^{\delta_2+\delta_3}, (-1)^{\delta_1+\delta_3}, -(-1)^{\delta_1+\delta_2}\right)$

$\Omega\mathcal{R}$ inv. D6-branes c:

- $\delta_i \equiv 2 b_i \tau^i \sigma^i \in \{0, 1\}$ non-tivial for **tilted** tori
- indep. of $(-1)^{\tau^{\mathbb{Z}_2^{(i)}}}$

- **untilted tori** ($b_i \equiv 0 \forall i$): $\Omega\mathcal{R}$ inv. only for $c \mid\mid$ exotic O6 & any $(\vec{\sigma}, \vec{\tau}) \rightsquigarrow USp(2N)$ — $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$: Blumenhagen, Cvetič, Marchesano, Shiu '05
- **tilted tori** ($b_i \equiv \frac{1}{2} \forall i$): $\Omega\mathcal{R}$ invariance for — G.H., Ripka, Staessens '12
  - $c \mid\mid$ exotic O6 & $\tau^i \sigma^i \equiv 0 \;\forall i \rightsquigarrow USp(2N)$
  - $c \mid\mid$ exotic O6 & $\tau^i \sigma^i \equiv 1 \;\forall i \rightsquigarrow SO(2N)$
  - $c \perp$ exotic O6 & $\tau^i \sigma^i \neq \tau^j \sigma^j = \tau^k \sigma^k = 1 \rightsquigarrow SO(2N)$
  - $c \perp$ exotic O6 & $\tau^i \sigma^i \neq \tau^j \sigma^j = \tau^k \sigma^k = 0 \rightsquigarrow USp(2N)$

  256 cycles, but only $(h_{21} + 1)$ linearly independent

  $\rightsquigarrow$ K-theory constraint $\stackrel{?}{\not\Rightarrow}$ $\mathbb{Z}_2$ symmetry??
- can be **truncated** for $\boxed{T^6/\mathbb{Z}_{2N}}$: $\eta_{(k)} \stackrel{!}{=} 1$ for $\mathbb{Z}_2^{(k)} \subset \mathbb{Z}_{2N}$

General Characteristics of Global D6-Brane Models

Field theory / MSSM:

- 3 generators for $\mathbb{Z}_n$ in MSSM: $\boxed{g_n = e^{i2\pi \mathcal{R}\frac{m}{n}} \cdot e^{i2\pi \mathcal{A}\frac{k}{n}} \cdot e^{i2\pi \mathfrak{L}\frac{p}{n}}}$
  - R-parity: $\mathcal{R}_2$
  - baryon triality: $\mathcal{L}_3\mathcal{R}_3$
  - proton hexality: $\mathcal{L}_6^2\mathcal{R}_6^5$
- Q_L charge can be rotated away by $U(1)_Y$

Charges of generation-independent $\mathbb{Z}_n$ symmetries in the MSSM

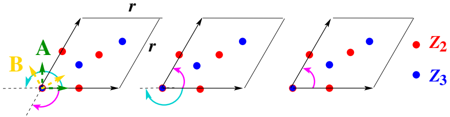
Generator	Q_L	$\overline{U}_R$	$\overline{D}_R$	L	$\overline{E}_R$	$\overline{N}_R$	H_u	H_d
$\mathcal{R}$	0	$n-1$	1	0	1	$n-1$	1	$n-1$
$\mathfrak{L}$	0	0	0	$n-1$	1	1	0	0
$\mathcal{A}$	0	0	$n-1$	$n-1$	0	1	0	1

Global D-brane models: G.H., Staessens '13

- $U(1)_{B-L}$ in L-R sym. models makes $\mathbb{Z}_2$'s (R-parity) trivial
- L-R models: $\mathbb{Z}_3 \subset U(1)_a \subset U(3)_a$ trivial
- Pati-Salam models: no $\mathbb{Z}_3 \not\subset U(1)_a \subset U(4)_a$

Example: A Pati-Salam Model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

- $\mathbb{Z}_2 \times \mathbb{Z}_6'$ shifts: $\vec{v} = (\frac{1}{2}, \frac{-1}{2}, 0)$, $\vec{w}' = (\frac{-1}{3}, \frac{1}{6}, \frac{1}{6})$ on $SU(3)^3$



- $\Pi_a^{\text{frac}} = \frac{1}{4}\left(X_a\,\rho_1 + Y_a\,\rho_2 + \sum_{k=1}^{3}\sum_{\alpha=1}^{5}\left[x_{a,\alpha}^{(k)}\,\varepsilon_\alpha^{(k)} + y_{a,\alpha}^{(k)}\,\tilde{\varepsilon}_\alpha^{(k)}\right]\right)$
  with $\rho_1 \circ \rho_2 = -\varepsilon_\alpha^{(k)} \circ \tilde{\varepsilon}_\alpha^{(k)} = 4$

- $\Omega\mathcal{R}$-even & odd 3-cycles:

$$\Pi_0^{\text{even},\mathbb{1}} = \rho_1, \qquad \Pi_0^{\text{odd},\mathbb{1}} = -\rho_1 + 2\,\rho_2,$$
$$\Pi_{\alpha\in\{1,2,3\}}^{\text{even},\mathbb{Z}_2^{(k)}} = \varepsilon_\alpha^{(k)}, \qquad \Pi_{\alpha\in\{1,2,3\}}^{\text{odd},\mathbb{Z}_2^{(k)}} = -\varepsilon_\alpha^{(k)} + 2\,\tilde{\varepsilon}_\alpha^{(k)},$$
$$\Pi_4^{\text{even},\mathbb{Z}_2^{(k)}} = \varepsilon_4^{(k)} + \varepsilon_5^{(k)}, \qquad \Pi_4^{\text{odd},\mathbb{Z}_2^{(k)}} = 2\,(\tilde{\varepsilon}_4^{(k)} + \tilde{\varepsilon}_5^{(k)}) - (\varepsilon_4^{(k)} + \varepsilon_5^{(k)}),$$
$$\Pi_5^{\text{even},\mathbb{Z}_2^{(k)}} = 2\,(\tilde{\varepsilon}_4^{(k)} - \tilde{\varepsilon}_5^{(k)}) - (\varepsilon_4^{(k)} - \varepsilon_5^{(k)}), \qquad \Pi_5^{\text{odd},\mathbb{Z}_2^{(k)}} = \varepsilon_4^{(k)} - \varepsilon_5^{(k)},$$

- Intersection numbers

$$\Pi_{\tilde{\alpha}}^{\text{even},\mathbb{Z}_2^{(k)}} \circ \Pi_{\tilde{\beta}}^{\text{odd},\mathbb{Z}_2^{(l)}} = \delta^{kl}\delta_{\tilde{\alpha}\tilde{\beta}} \times \begin{cases} 8 & \tilde{\alpha} = 0 \\ -8 & 1\ldots 3 \\ -16 & 4 \\ 16 & 5 \end{cases} \quad \text{with} \quad \mathbb{Z}_2^{(0)} \equiv \mathbb{1}$$

- wrapping numbers *a priori* $A_a^i, B_a^i \in \frac{1}{8}\,\mathbb{Z}$

A Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: $\mathbb{Z}_n$ conditions

$$
\sum_a k_a N_a \left(\begin{array}{c}
Y_a \\ \hline
-y^{(1)}_{a,1} \\
-y^{(1)}_{a,2} \\
-y^{(1)}_{a,3} \\
-(y^{(1)}_{a,4} + y^{(1)}_{a,5}) \\
2\,(x^{(1)}_{a,4} - x^{(1)}_{a,5}) + (y^{(1)}_{a,4} - y^{(1)}_{a,5}) \\ \hline
-y^{(2)}_{a,1} \\
-y^{(2)}_{a,2} \\
-y^{(2)}_{a,3} \\
-(y^{(2)}_{a,4} + y^{(2)}_{a,5}) \\
2\,(x^{(2)}_{a,4} - x^{(2)}_{a,5}) + (y^{(2)}_{a,4} - y^{(2)}_{a,5}) \\ \hline
-y^{(3)}_{a,1} \\
-y^{(3)}_{a,2} \\
-y^{(3)}_{a,3} \\
-(y^{(3)}_{a,4} + y^{(3)}_{a,5}) \\
2\,(x^{(3)}_{a,4} - x^{(3)}_{a,5}) + (y^{(3)}_{a,4} - y^{(3)}_{a,5})
\end{array} \right)
\stackrel{!}{=} 0 \bmod n \stackrel{!}{=} \sum_a k_a N_a \left(\begin{array}{c}
\frac{Y_a - \sum_{i=1}^3 [y^{(i)}_{a,1} + y^{(i)}_{a,2} + y^{(i)}_{a,3}]}{4} \\
\frac{Y_a - [y^{(1)}_{a,1} + y^{(1)}_{a,2} + y^{(1)}_{a,3}]}{2} \\
\frac{Y_a - [y^{(2)}_{a,1} + y^{(2)}_{a,2} + y^{(2)}_{a,3}]}{2} \\
-\frac{y^{(2)}_{a,1} + y^{(2)}_{a,3} + y^{(3)}_{a,1} + y^{(3)}_{a,3}}{2} \\
-\frac{y^{(1)}_{a,1} + y^{(1)}_{a,3} + y^{(3)}_{a,2} + y^{(3)}_{a,3}}{2} \\
-\frac{y^{(1)}_{a,2} + y^{(1)}_{a,3} + y^{(2)}_{a,2} + y^{(2)}_{a,3}}{2} \\ \hline
\frac{Y_a + [y^{(1)}_{a,3} + x^{(1)}_{a,4} + y^{(1)}_{a,4} - x^{(1)}_{a,5}] + \sum_{j=2}^3 [y^{(j)}_{a,2} - (y^{(j)}_{a,4} + y^{(j)}_{a,5})]}{4} \\
\frac{Y_a + \sum_{j=1,2} [y^{(j)}_{a,1} - x^{(j)}_{a,4} + x^{(j)}_{a,5} + y^{(j)}_{a,5}] + [y^{(3)}_{a,3} + x^{(3)}_{a,4} + y^{(3)}_{a,4} - x^{(3)}_{a,5}]}{4} \\
\frac{Y_a + [y^{(2)}_{a,2} - (y^{(2)}_{a,4} + y^{(2)}_{a,5})]}{2} \\
\frac{Y_a + [y^{(1)}_{a,1} - x^{(1)}_{a,4} + x^{(1)}_{a,5} + y^{(1)}_{a,5}]}{2} \\
-\frac{y^{(2)}_{a,4} + y^{(2)}_{a,5} + y^{(3)}_{a,4} + y^{(3)}_{a,5}}{2} \\
-\frac{x^{(1)}_{a,4} - x^{(1)}_{a,5} + y^{(1)}_{a,5} + x^{(2)}_{a,4} - x^{(2)}_{a,5} + y^{(2)}_{a,5}}{2} \\ \hline
\frac{Y_a + \sum_{i=1}^3 [y^{(i)}_{a,3} + x^{(i)}_{a,4} + y^{(i)}_{a,4} - x^{(i)}_{a,5}]}{4} \\
\frac{Y_a + y^{(1)}_{a,3} + x^{(1)}_{a,4} + y^{(1)}_{a,4} - x^{(1)}_{a,5}}{2} \\
\frac{Y_a + y^{(2)}_{a,3} + x^{(2)}_{a,4} + y^{(2)}_{a,4} - x^{(2)}_{a,5}}{2} \\
\frac{Y_a + \sum_{i=1}^3 y^{(i)}_{a,3}}{2}
\end{array} \right)
$$

A Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: spectrum

$$SU(4)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d \times SU(2)_e \times U(1)^5_{\text{massive}}$$

- Standard Model particles plus **one Higgs**

$$(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + 2\,(\mathbf{4}, \mathbf{2}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1}) + 2\,(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}; \mathbf{1}, \mathbf{1})$$

$\rightsquigarrow$ **one massive generation** at leading order by charge selection rules

- chiral w.r.t. anomalous $U(1)^5_{\text{massive}}$

$$(\mathbf{1}, \mathbf{2}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + 3(\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \overline{\mathbf{2}}) + (\mathbf{1}, \mathbf{1}, \overline{\mathbf{2}}; \mathbf{2}, \mathbf{1}) + 3(\mathbf{1}, \mathbf{1}, \mathbf{2}; \mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{2})$$

but non-chiral w.r.t. $SU(4)_a \times SU(2)_b \times SU(2)_c$

- non-chiral w.r.t. to full $U(4)_a \times U(2)^4$ with **GUT Higgses**

$$2\left[(\mathbf{4}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + h.c.\right] + [(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{2}) + h.c.] + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{4}_{\mathbf{Adj}}, \mathbf{1})$$
$$+\, 2\left[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{3}_{\mathbf{S}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}_{\mathbf{A}}, \mathbf{1}) + h.c.\right] + [(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{3}_{\mathbf{S}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}_{\mathbf{A}}) + h.c.]$$

Pati-Salam model cont'd: $\mathbb{Z}_n$ Symmetries in $U(1)^5_{\text{massive}}$

- 5 independent conditions on $\mathbb{Z}_n$ symmetries ($h_{21} = 15$)
- family-independent:
  - $\mathbb{Z}_4 \subset U(1)_a \subset U(4)_a$
  - $\mathbb{Z}_2 \subset U(1)_x \subset U(2)_{x, x \in \{b,c,d,e\}}$
- **family-dependent**:
  - $\mathbb{Z}_4 \subset \sum_{x \in \{b,c,d,e\}} U(1)_x$

Discrete charges for the five-stack Pati-Salam model on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6' \times \Omega\mathcal{R})$

Discrete symmetries		Charge assignment for the 'chiral' states										
$\mathbb{Z}_n$	$U(1) = \sum_x k_x\, U(1)_x$	(Q_L, L) ab	(Q_L, L) ab'	(Q_R, R) ac	(Q_R, R) ac'	(H_d, H_u)	X_{bd}	$X_{bd'}$	$X_{be'}$	X_{cd}	$X_{cd'}$	$X_{ce'}$
$\mathbb{Z}_2$	$U(1)_e$	0	0	0	0	0	0	0	1	0	0	1
	$U(1)_d$	0	0	0	0	0	1	1	0	1	1	0
	$U(1)_c$	0	0	1	1	1	0	0	0	1	1	1
	$U(1)_b$	1	1	0	0	1	1	1	1	0	0	0
$\mathbb{Z}_4$	$U(1)_a$	1	1	3	3	0	0	0	0	0	0	0
	$U(1)_b + U(1)_c + {}+U(1)_d + U(1)_e$	3	1	1	3	0	0	2	2	0	2	2

- $\mathbb{Z}_2 \subset U(1)c \subset U(2)_c$: R-parity $\mathcal{R}_2$
- $\mathbb{Z}_2 \in U(1)_{d,e}$: only non-trivial on exotic matter

Conclusions & Outlook

Conclusions:

- Conditions on $\mathbb{Z}_n$ sym. expressed via **intersection numbers**:
  - independent of choice of basis & parameterisation: correct **normalisations**
  - many 'probe branes', but only $(h_{21}+1)$ conditions per orbifold
- $\mathbb{Z}_N \subset U(N)$ automatic
- in presence of $U(1)_{\text{massless}}$, e.g. $Y, (B-L)$: many $\mathbb{Z}_n$ **trivial**
- Example: Pati-Salam model
  - R-parity $\subset U(2)_R$
  - two $\mathbb{Z}_2$'s non-trivial only on exotic states
  - **family-dependent** $\mathbb{Z}_4$ ...more examples & details in G.H., Staessens '13

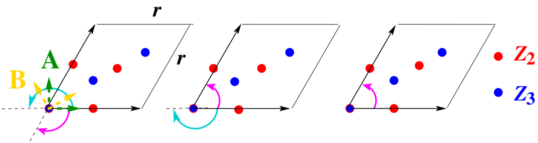
Outlook:

- compute D-instanton contributions to effective action
- which $\mathbb{Z}_n$ survive field theoretic breakings, e.g. $SU(4) \times SU(2)_L \times SU(2)_R \to SU(3) \times SU(2)_L \times Y$ via GUT Higgs *vev*?

Technical Details

IIA/$\Omega\mathcal{R}$ on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ with discrete torsion: geometry

$\mathbb{Z}_2 \times \mathbb{Z}_6'$ shifts: $\vec{v} = (\frac{1}{2}, \frac{-1}{2}, 0)$, $\vec{w}' = (\frac{-1}{3}, \frac{1}{6}, \frac{1}{6})$ on $SU(3)^3$



- $\boxed{\Pi_a^{\text{rigid}} = \frac{1}{4}\left(\Pi_a^{\text{bulk}} + \sum_{i=1}^3 \Pi_a^{\mathbb{Z}_2^{(i)}}\right)}$
- $\boxed{\Pi_a^{\text{bulk}} = X_a\,\rho_1 + Y_a\,\rho_2}$ with

Förste, G.H. JHEP 1101 (2011) 091

$$X_a \equiv n_a^1 n_a^2 n_a^3 - m_a^1 m_a^2 m_a^3 - \sum_{i \neq j \neq k \neq i} n_a^i m_a^j m_a^k \in \mathbb{Z}, \quad Y_a \equiv \sum_{i \neq j \neq k \neq i} \left(n_a^i n_a^j m_a^k + n_a^i m_a^j m_a^k\right) \in \mathbb{Z}$$

$$\rho_1 \equiv 2 \sum_{k=0}^{5} \omega^k(\pi_{135}), \qquad \rho_1 \equiv 2 \sum_{k=0}^{5} \omega^k(\pi_{136}) \qquad \text{with} \quad \rho_1 \circ \rho_2 = 4$$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ geometry cont'd

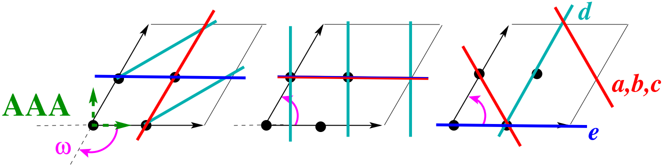
- $\boxed{\Pi_a^{\text{rigid}} = \frac{1}{4}\left(\Pi_a^{\text{bulk}} + \sum_{i=1}^{3} \Pi_a^{\mathbb{Z}_2^{(i)}}\right)}$
- $\boxed{\Pi^{\mathbb{Z}_2^{(i)}} = \sum_{\alpha=1}^{5} \left(x_{\alpha,a}^i\, \varepsilon_\alpha^{(i)} + y_{\alpha,a}^i\, \tilde{\varepsilon}_\alpha^{(i)}\right)}$ with
  - 3 equivalent $\mathbb{Z}_2^{(i)}$ twisted sectors:
    $\varepsilon_{\alpha=1}^{(i)} = 2\sum_{k=0}^{2} \omega^k (e_{41}^{(i)} \otimes \pi_{2i-1})$,
    $\tilde{\varepsilon}_{\alpha=1}^{(i)} = 2\sum_{k=0}^{2} \omega^k (e_{41}^{(i)} \otimes \pi_{2i})$
    with $\varepsilon_\alpha^{(i)} \circ \tilde{\varepsilon}_{\beta=1}^{(j)} = -4\,\delta^{ij}\,\delta_{\alpha\beta}$
  - exceptional wrappings $(x_{\alpha,a}^i, y_{\alpha,a}^i) \sim (n_a^i, m_a^i)$
  - sign factors from
    - $\mathbb{Z}_2$ eigenvalues ± 1
    - Wilson lines $\tau \in \{0, 1\}$
- example for a short $\Omega\mathcal{R}$-even cycle:

$$\Pi_{(\sigma^i)=(1,1,1)}^{\text{frac},\Omega\mathcal{R}} \overset{\tau^i \equiv \tau}{=} \frac{\Pi_0^{\text{even}}}{4} + \sum_{i=1}^{3} \frac{(-1)^{\tau^{\mathbb{Z}_2^{(i)}}}}{4} \left(-\Pi_3^{\text{even},\mathbb{Z}_2^{(i)}} + (-1)^\tau \frac{-\Pi_4^{\text{even},\mathbb{Z}_2^{(i)}} + \Pi_5^{\text{even},\mathbb{Z}_2^{(i)}}}{2}\right)$$

A typical global Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

G.H., Ripka, Staessens '12

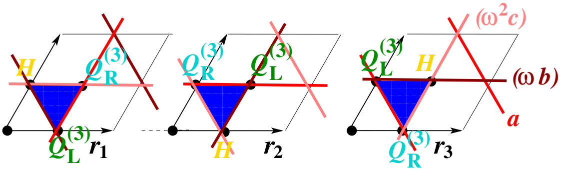
brane	$(n^i, m^i)_{i=1,2,3}$	$\mathbb{Z}_2$	$(\vec{\tau})$	$(\vec{\sigma})$	group	(X,Y)
a	(0,1;1,0,1,-1)	$(+++)$	$(0,0,1)$	$(\vec{1})$	$U(4)_a$	(1,0)
b		$(--+)$	$(0,1,1)$		$U(2)_b$	
c		$(-+-)$	$(1,0,1)$		$U(2)_c$	
d	(1,1;1,-2;0,1)	$(+++)$	$(0,0,1)$	$(\vec{1})$	$U(2)_d$	(3,0)
e	(1,0;1,0;1,0)	$(+--)$	$(1,1,1)$	$(1,1,0)$	$U(2)_e$	(1,0)



- a, b, c at $(\frac{\pi}{3}, 0, -\frac{\pi}{3})$, d at $(\frac{\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{3})$ e at $(0,0,0)$
- all $U(1)^5$ anomalous & massive at $M_{\text{string}} \leftrightarrow h_{21} = 15(\mathbb{Z}_2)$
- $SU(4)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d \times SU(2)_e$ with
  - 3 generations of quarks + leptons
  - **one Higgs** (H_d, H_u)
  - ~~**Adj**~~ on a, b, c, $e \longleftrightarrow 1 \times \textbf{Adj}_d$

Yukawa interactions for the typical Pati-Salam model

- charge selection rules not sufficient on $T^6/\mathbb{Z}_{2N}$, $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ due to various sectors $a(\omega^k b)_{k\in\{0,1,2\}}$ G.H., Vanhoof '12



- Pati-Salam model: one heavy generation by G.H., Ripka, Staessens '12
  $W_{Q_L^{(3)} Q_R^{(3)} H} \sim e^{-\sum_{i=1}^3 v_i/8}$ with Kähler moduli $v_i \equiv \frac{\sqrt{3}}{2}\frac{r_i^2}{\alpha'}$
- non-chiral $\left[(\mathbf{4},\mathbf{1},\mathbf{1};\overline{\mathbf{2}},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1_A},\mathbf{1}) + h.c.\right]$ massive via couplings to $(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{4_{Adj}},\mathbf{1})$
- several types of $(\mathbf{1},\mathbf{2}_x,\mathbf{2}_y,\mathbf{1},\mathbf{1})_{x,y\in\{b,c,d,e\}}$ massive through 3-point couplings among each other and with SM Higgs
- other masses through higher order or non-perturbative (instanton) couplings $\rightsquigarrow$ need to be computed!