Enhanced 1–Loop Corrections to WIMP Annihilation

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Introduction

Within standard cosmology: WIMP relic density can be computed from WIMP annihilation cross section!

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$\Longrightarrow$ Need to include "large" radiative corrections! ($> \alpha/\pi$)

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- 1.6% error true for standard 6–parameter ΛCDM;
  allowing more parameters (in particular, finite curvature) increases the uncertainty

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- 1.6% error true for standard 6–parameter ΛCDM; allowing more parameters (in particular, finite curvature) increases the uncertainty
- Also need to know values of relevant parameters to similar accuracy! Very challenging in SUSY scenarios.

Minimal Supersymmetric extension of the Standard Model

WIMP candidate: Lightest neutralino $\tilde{\chi}_1^0$.
$\tilde{\nu}$ excluded by direct searches.

Neutralinos are mixtures of $\tilde{B}$, $\widetilde{W}_3$, $\tilde{h}_1^0$, $\tilde{h}_2^0$.

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In order to compute loop corrections: need renormalization scheme!

Improved on–shell renormalization of $\tilde{\chi}$ sector

Chatterjee, MD, Kulkarni, Xu, arXiv:1107.5218 [hep-ph]

$\tilde{\chi}_i^0,\ i \in \{1,2,3,4\}$, have charged $SU(2)$ partners: charginos $\tilde{\chi}_a^{\pm}, a \in \{1,2\}$

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Mixing of current eigenstates described by mass matrices:

$$M^{\rm c} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}.$$

$$M^{\rm n} = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

M_1, M_2: gaugino masses; μ: higgsino mass.
Other parameters (M_W, M_Z, $\tan\beta$) are renormalized independently!

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Selection criterion: Perturbative expansion should converge quickly, i.e. corrections to the remaining masses should be small!

Reasons for perturbative instability

- $\exists$ parameter $\in \{M_1,\ M_2,\ \mu\}$ that does not affect input masses significantly. E.g. no input state has sizable $\tilde{B}$ component $\Longrightarrow$ finite part of δM_1 can be very large.

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- Finite parts can have poles in parameter space. E.g. if $\delta M_2,\ \delta\mu$ from $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_1^\pm}$: $\delta\mu \propto 1/(M_2^2 - \mu^2)$!

Solutions

- First condition $\Longrightarrow$ must have one $\tilde{B}-$like, one $\widetilde{W}-$like, one $\tilde{h}-$like input state. Choice of indices of input states depends on ordering of parameters! (Note: anyway need information on this ordering, since equations determining counterterms have multiple solutions.)

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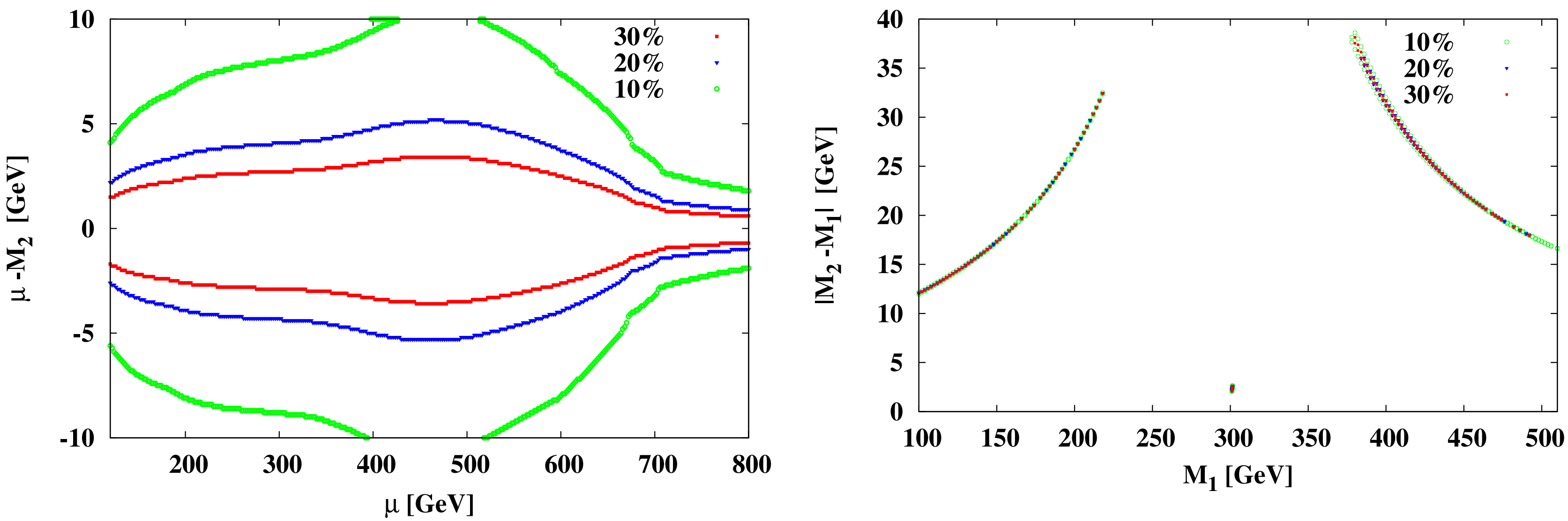
- Second condition $\Longrightarrow$ best choice for input states: Wino–like chargino; higgsino– and bino–like neutralino! All other choices have significant regions with perturbative instability.

Regions of instability



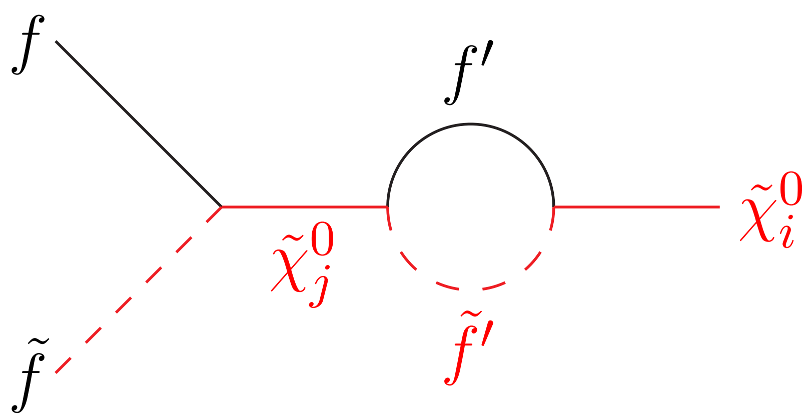
$\tilde{\chi}_1^{\pm}$, $\tilde{\chi}_2^{\pm}$, $\tilde{\chi}_{\tilde{B}}^0$ as input

$\tilde{\chi}_{\tilde{B}}^0$, $\tilde{\chi}_{\widetilde{W}}^0$, $\tilde{\chi}_{\tilde{h}}^0$ as input

1–loop corrections via effective $\tilde{\chi}$ couplings

Chatterjee, MD, Kulkarni, arXiv:1209.2328 [hep-ph]

Observation: matter (s)fermion correction to $\tilde{\chi}$ two–point fcts., plus appropriate counterterms, form finite, gauge invariant subset of corrections! Guasch, Hollik & Sola 2002



Is the *only* diagram (apart from CTs) involving $f' \neq f$!

Properties

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- Correction from heavy sfermion $\tilde{f}'$ is *enhanced* logarithmically: no decoupling!

Pure gaugino sector

Consider spectrum $m_{\tilde{q}} \gg m_{\tilde{\ell}} \simeq m_{\tilde{\chi}}$. Effective $\tilde{\chi}\ell\tilde{\ell}$ coupling after integrating out heavy squarks:

$\tilde{g}(m_{\tilde{\chi}}) = \tilde{g}(m_{\tilde{q}}) - \beta_{\ell,\tilde{\ell}} \log \frac{m_{\tilde{q}}}{m_{\tilde{\chi}}}$

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\end{aligned}$$

In effective theory approach: can also have non–decoupling corrections to off–diagonal entries of $\tilde{\chi}$ mass matrices, i.e. to gaugino–higgsino mixing!

Order of corrections

coupling	$\tilde{\chi} \simeq$ higgsino tree	$\tilde{\chi} \simeq$ higgsino one–loop	$\tilde{\chi} \simeq$ gaugino tree	$\tilde{\chi} \simeq$ gaugino one–loop
$\tilde{\chi}\ell_L\tilde{\ell}_L,\ \tilde{\chi}\ell_R\tilde{\ell}_R$	$\mathcal{O}(\epsilon g)$	$\mathcal{O}(\epsilon g^3,\ \epsilon g \lambda^2)$	$\mathcal{O}(g)$	$\mathcal{O}(g^3,\ \epsilon^2 g \lambda^2)$
$\tilde{\chi}\ell_L\tilde{\ell}_R,\ \tilde{\chi}\ell_R\tilde{\ell}_L$	$\mathcal{O}(\lambda_\ell)$	$\mathcal{O}(\lambda_\ell^3,\ \lambda_\ell\lambda_b^2,\ \epsilon^2\lambda_\ell g^2,\ \epsilon^2\lambda_\ell\lambda_t^2)$	$\mathcal{O}(\epsilon\lambda_\ell)$	$\mathcal{O}(\epsilon\lambda_\ell g^2,\ \epsilon\lambda_\ell\lambda^2)$

g: generic electroweak gauge coupling;

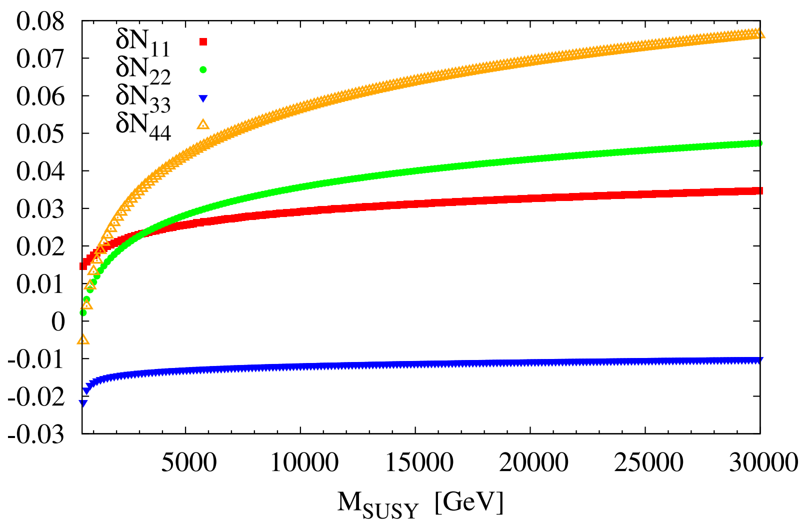
λ_ℓ: Yukawa coupling of lepton ℓ;

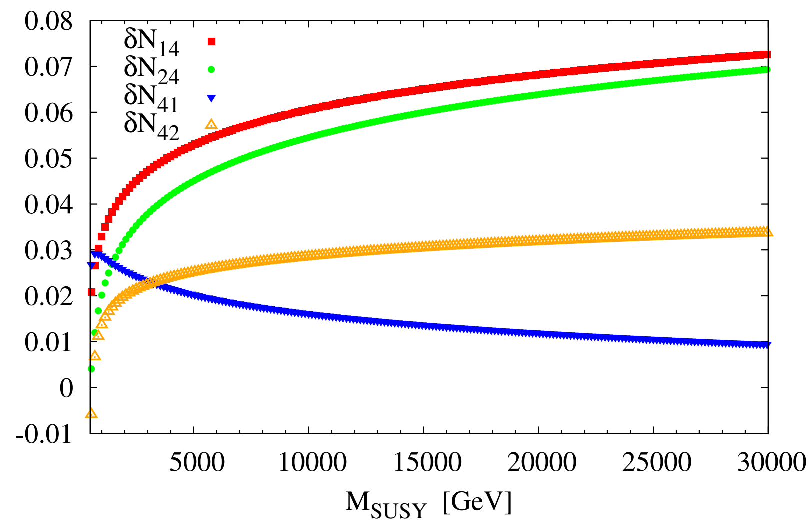
λ: generic superpotential coupling;

ϵ: one factor of (small) gaugino–higgsino mixing.

All corrections are non–decoupling!

Relative size of corrections to couplings

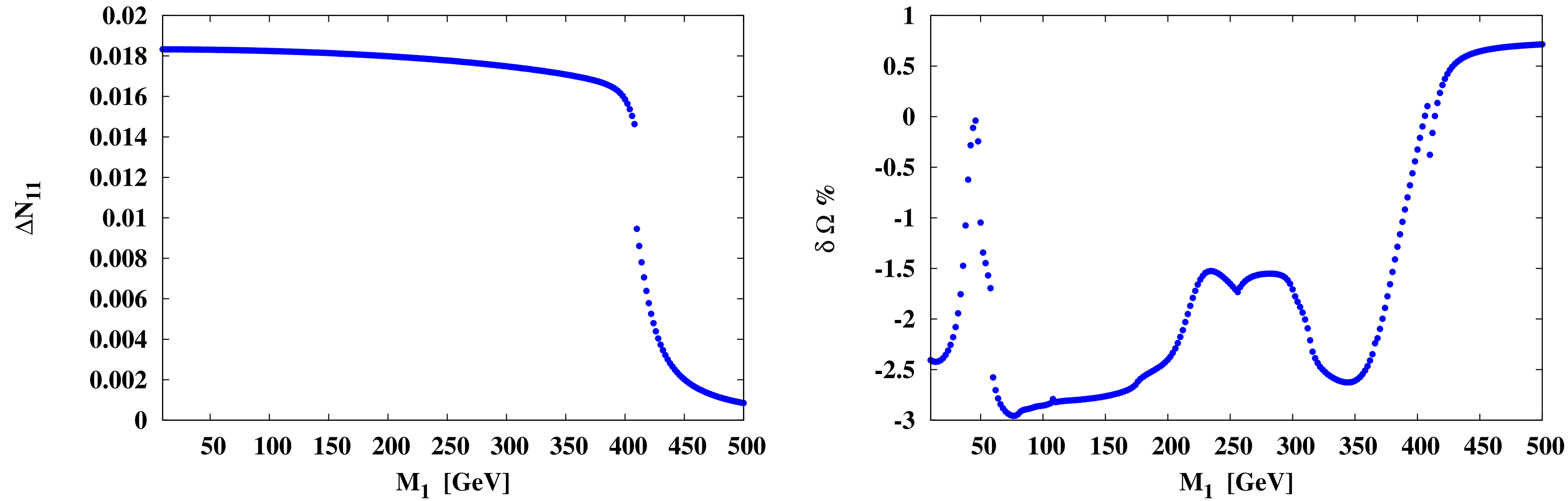




$M_2 = 3M_2 = 0.3$ TeV, $\mu = 0.6$ TeV, $\tan\beta = 10$, common $\tilde{f}$ masses

Note: Non–logarithmic non–decoupling corrections can be sizable, too!

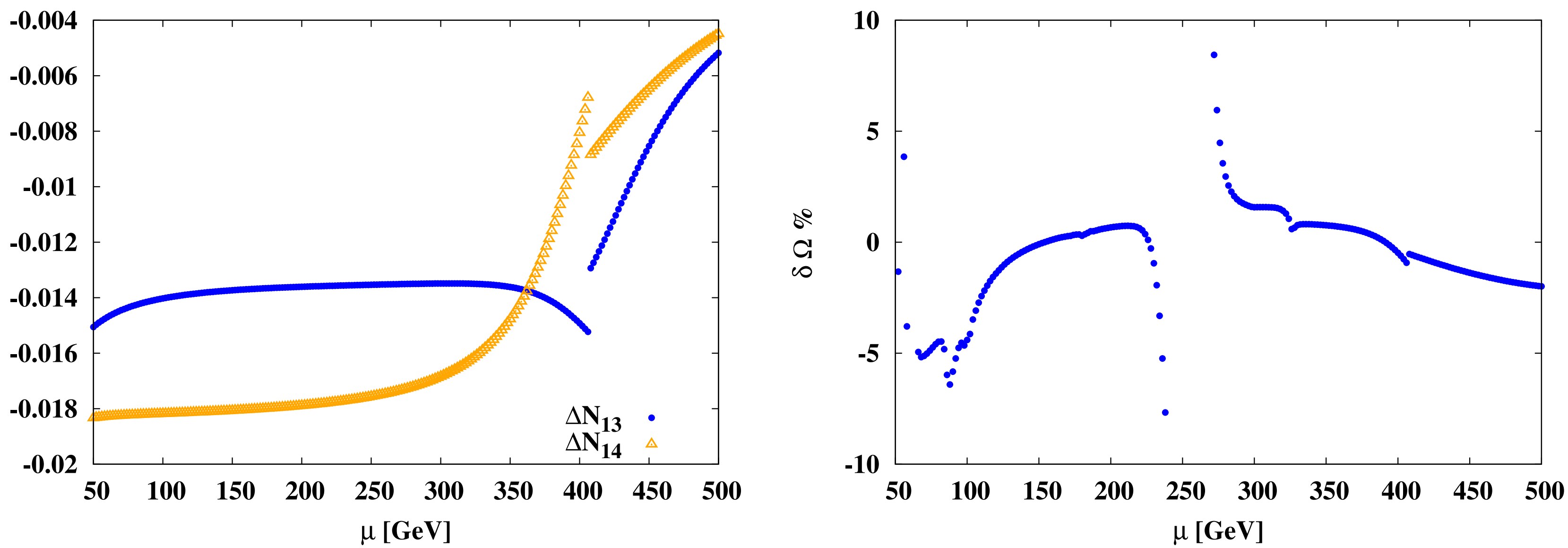
Correction for (mostly) $\tilde{B}-$like LSP



$m_{\tilde{q}} = 1.5, M_3 = 1.2, M_2 = 0.4, m_{\tilde{l}_L} = 0.55, m_{\tilde{l}_R} = 0.5,$
$\mu = 0.6, m_A = 0.5,\ A_3 = 1.0,\ \tan(\beta)(M_Z) = 10$
(All dim.–ful parameters in TeV)

Obtained with modified `micrOMEGAs`.

Correction for $\tilde{B}-$ or $\tilde{h}-$like LSP



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Bug in `micrOMEGAs`

Co–annihilation $\tilde{\tau}_1 + \tilde{\chi}_1^0 \to A + \tau$ can diverge if $m_A \gtrsim 2 m_{\tilde{\chi}_1^0}$!

Reason: Can be due to *on–shell* $\tilde{\tau}_1 \to \tau + \tilde{\chi}_1^0$ decay, followed by $\tilde{\chi}_1^0 + \tilde{\chi}_1^0 \to A$!

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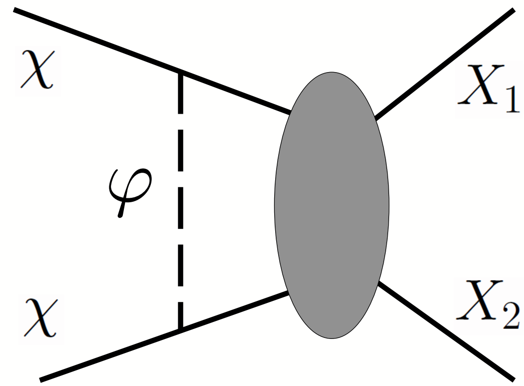
However, $\tilde{\tau}_1 \leftrightarrow \tau + \tilde{\chi}_1^0$ is part of the (fast) processes maintaining relative equilibrium between $\tilde{\chi}_1^0$ and $\tilde{\tau}_1 \Longrightarrow$ this kinematic configuration should *not* be included in the explicit co–annihilation cross section!

Sommerfeld enhanced 1–loop corrections

MD, J.M. Kim, Nagao, arXiv:0911.3795 [hep-ph]

Are due to exchange of a relatively light boson in the initial state:



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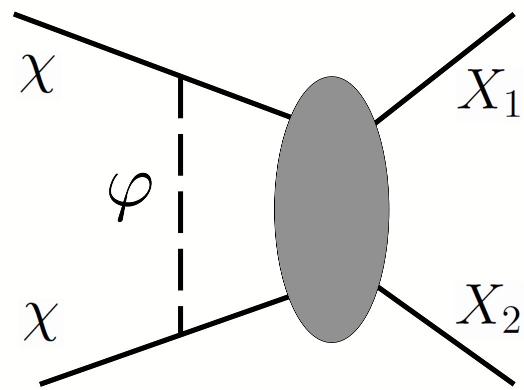

Are interested in case where 1–loop correction is sizable, but still a correction: $\alpha m_\chi > \mu$. (α : relevant coupling; μ: mass of exchanged boson.)

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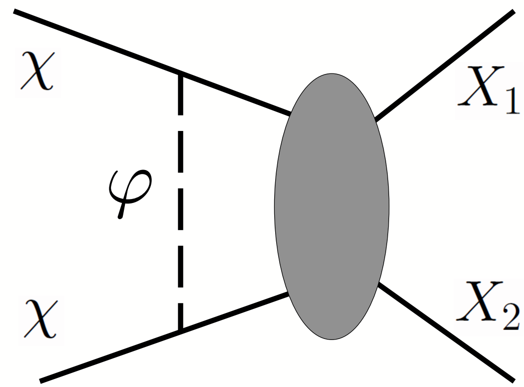

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Note: $\mu < \alpha m_\chi$ may be technically unnatural: expect loop corrections $\delta\mu^2 = \mathcal{O}(\alpha m_\chi^2/\pi)$!

Calculation of 1–loop Correction

Corrections are calculated in non–relativistic kinematics, with several approximations; these are necessary to give a finite result!

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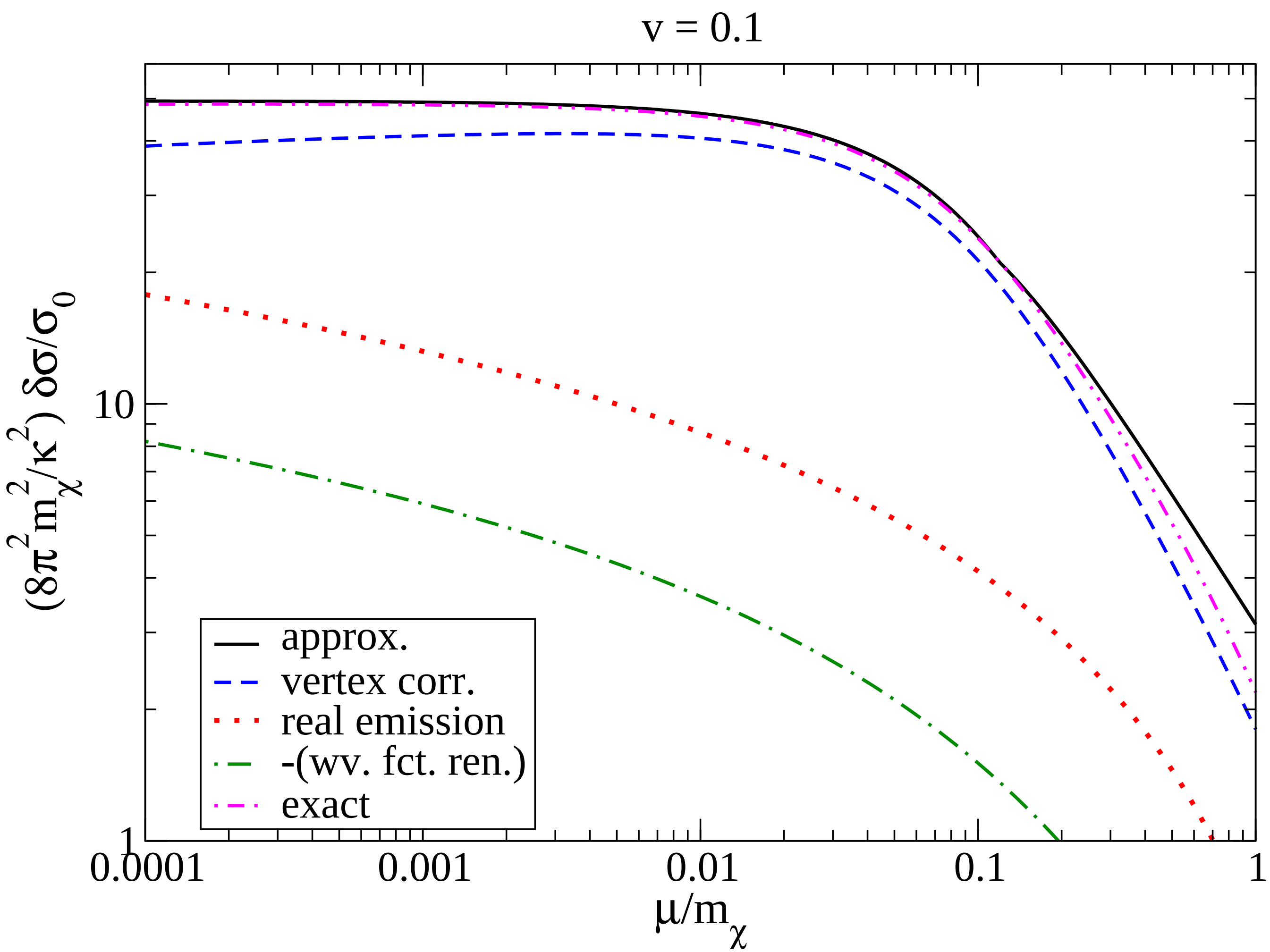
- Ignore energy dependence of φ propagator

Check against exact calculation



For scalar WIMP χ coupling to (light) boson φ with (dimensionful) coupling κ

Result

- Correction factorizes!

$$\sigma_\ell^{1-\mathrm{loop}}(\chi\chi \to \mathrm{any}) = \sigma_\ell^{\mathrm{tree}}(\chi\chi \to \mathrm{any}) \cdot \left(1 + \frac{\mathrm{g}^2}{2\pi^2 \mathrm{v}_\chi} \mathrm{I}_\ell(\mu/|\tilde{\mathrm{p}}_\chi|)\right)$$

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- Loop function:

$$I_S = I_P = \frac{\pi^2}{2} \quad \text{for } \mu \ll |\vec{p}_\chi|$$

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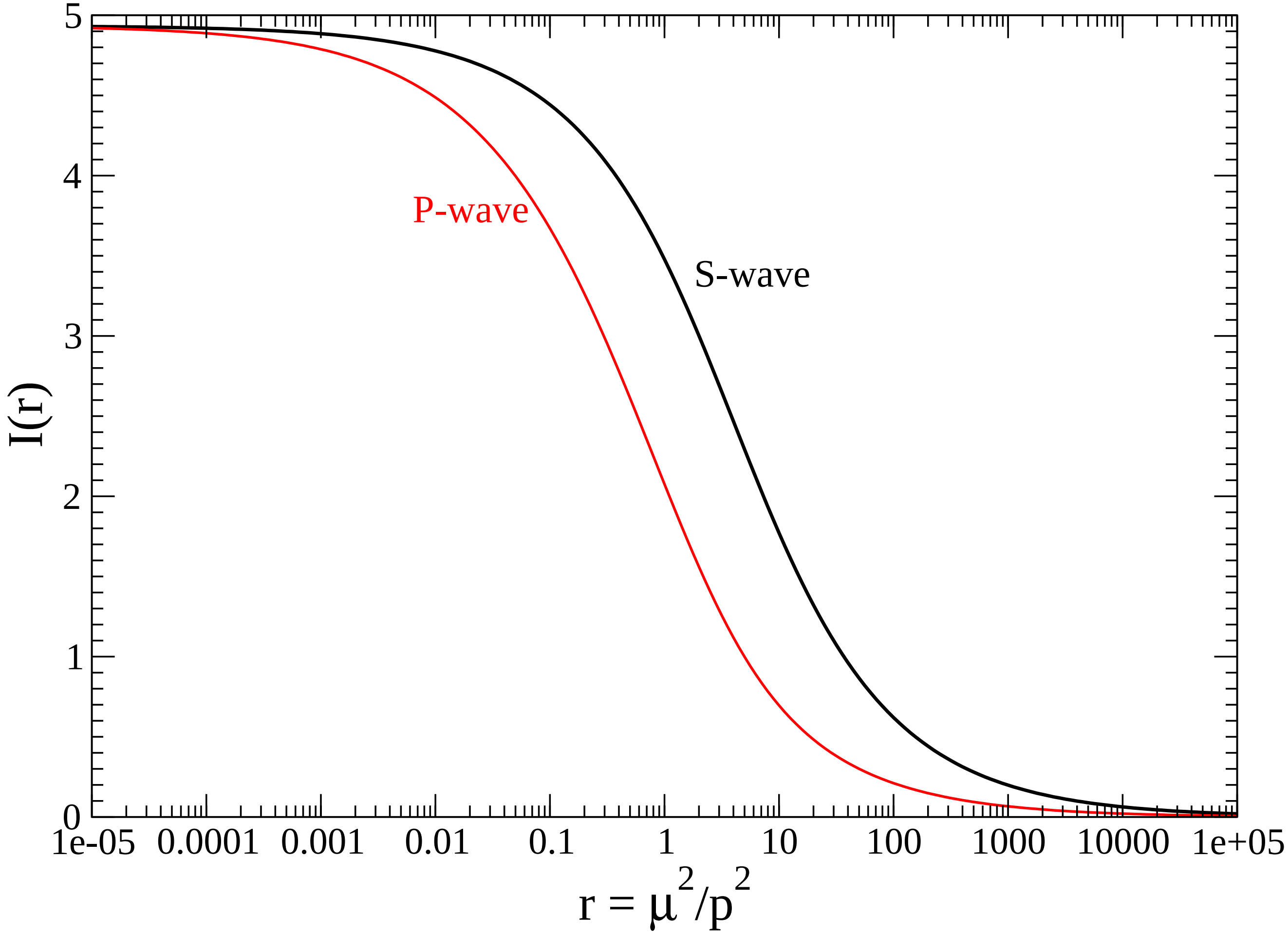
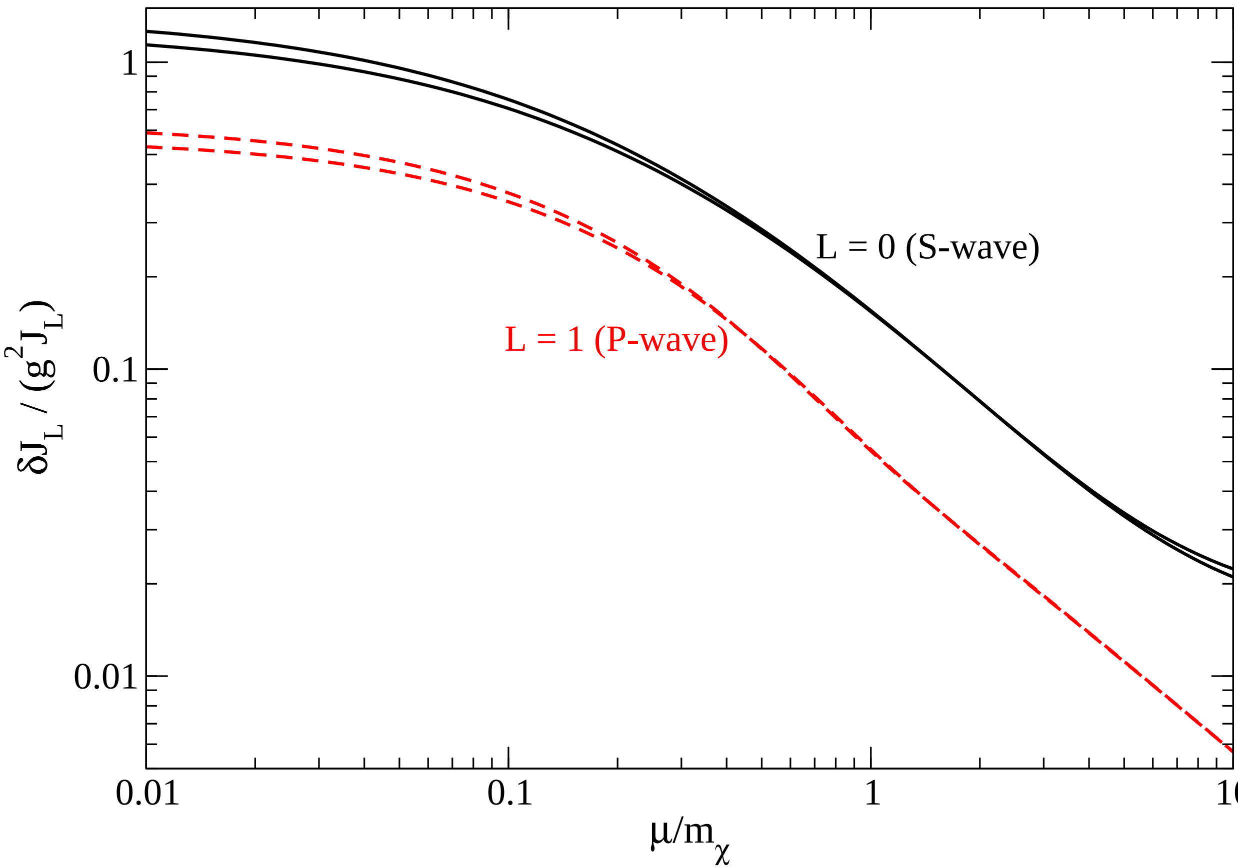
- Correction depends only on partial wave, not on final state!

Loop functions

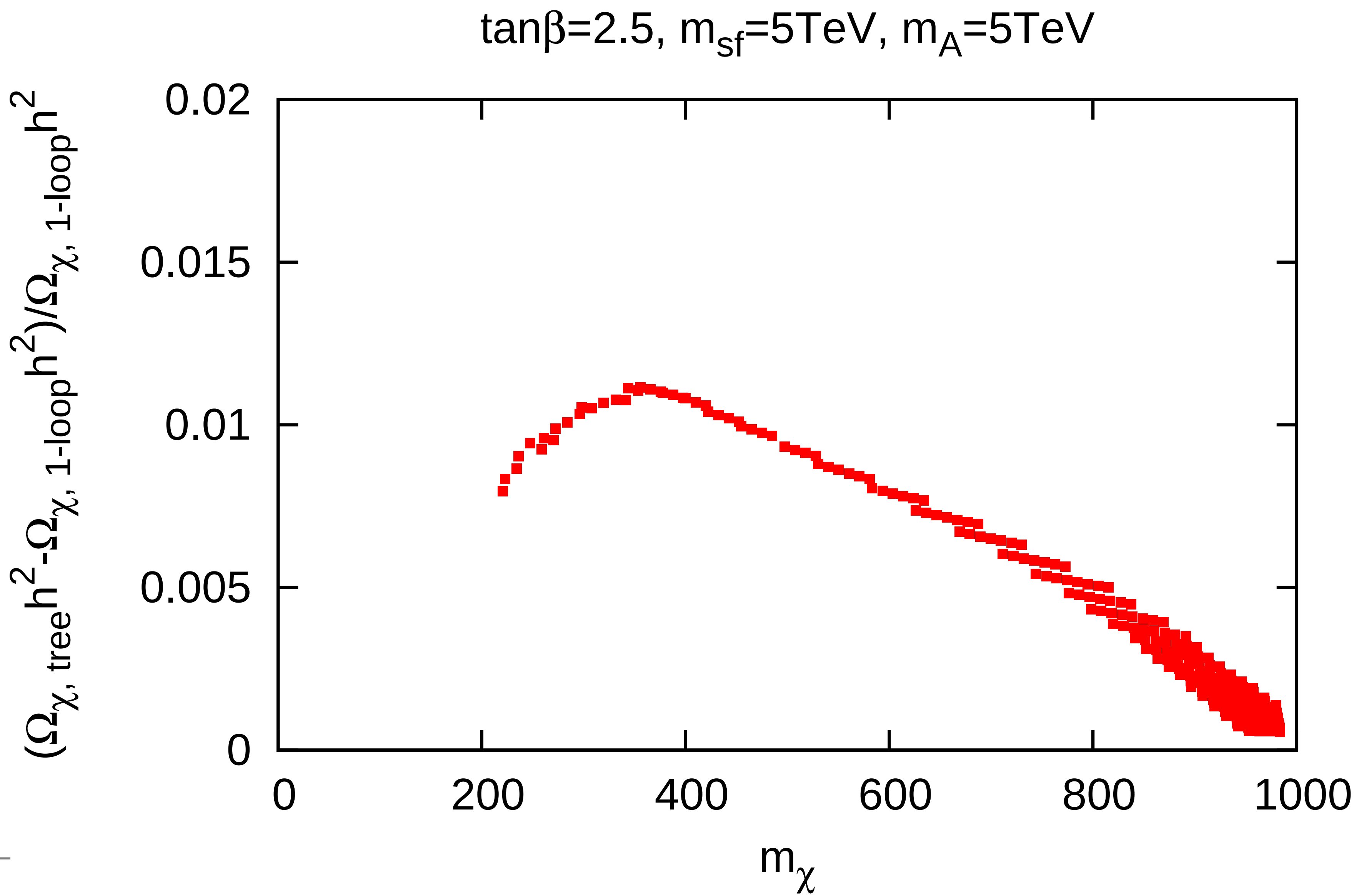


Correction to annihilation integral



$$J(x_F) = \int_{x_F}^{\infty} \frac{\langle \sigma(\chi\chi \to \text{any}) v_\chi \rangle}{x^2} dx$$

Corrections for “well–tempered neutralino” in MSSM



Incorporating Co–Annihilation

MD, J. Gu, 2013

Complications:

- Masses in intermediate state differ (slightly) from those in the final state; can be bigger or smaller. Described by $\kappa = \frac{m_3 m_4}{m_1 m_2} \frac{m_1+m_2}{m_3+m_4} - \frac{2 m_3 m_4}{m_3+m_4} \frac{1}{p^2} (m_3 + m_4 - m_1 - m_2)$

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- In general, have to sum over several intermediate states $\chi_3\chi_4$!

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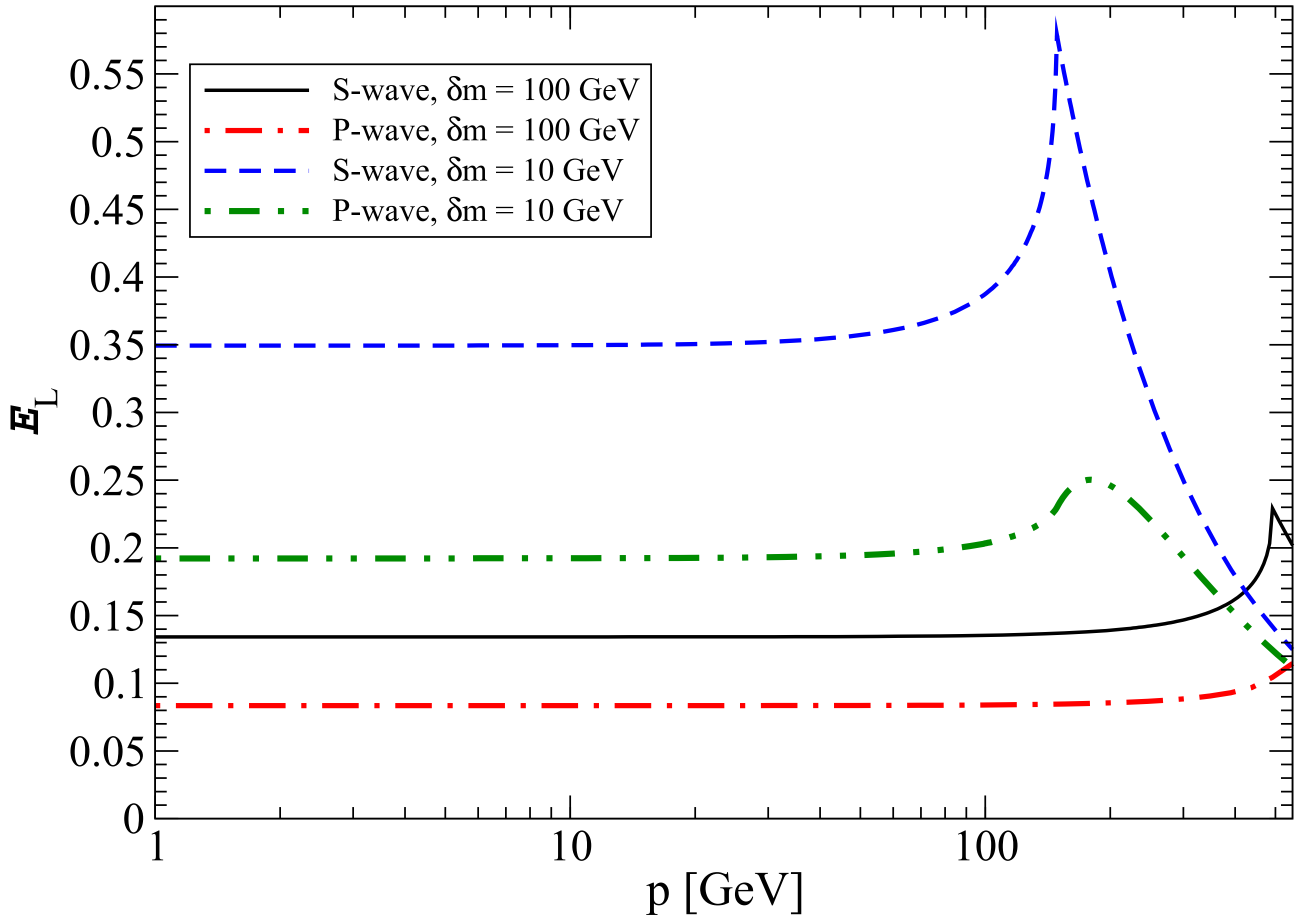
MD, J. Gu, 2013

Complications:

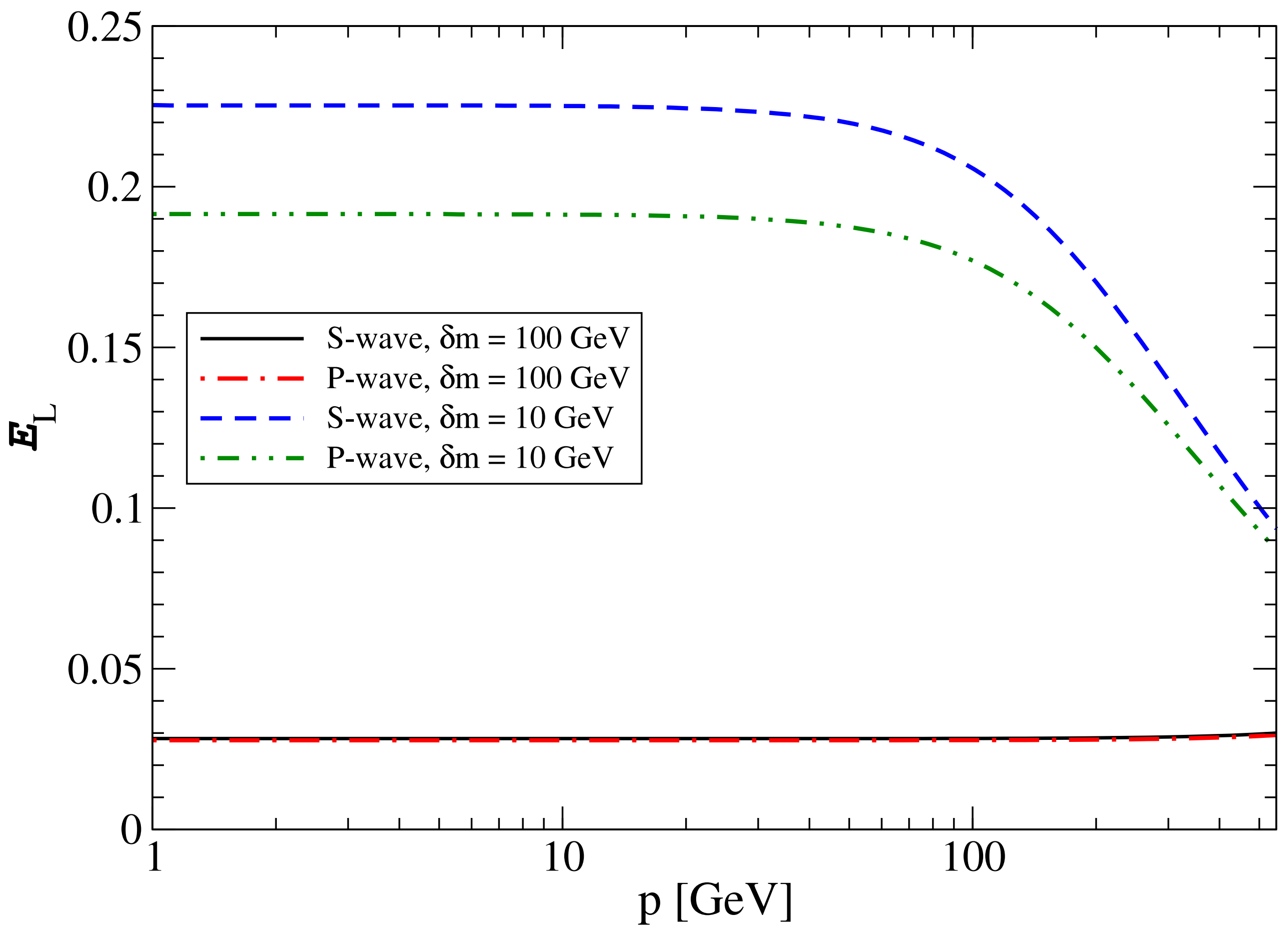
- Masses in intermediate state differ (slightly) from those in the final state; can be bigger or smaller. Described by $\kappa = \frac{m_3 m_4}{m_1 m_2} \frac{m_1+m_2}{m_3+m_4} - \frac{2 m_3 m_4}{m_3+m_4} \frac{1}{p^2} (m_3 + m_4 - m_1 - m_2)$
- Correction now only factorizes on amplitude level: $\delta A_\ell^{\chi_1 \chi_2} |_{\text{1-loop}} = \frac{g_{\phi \chi_1 \chi_3} g_{\phi \chi_2 \chi_4}}{8 \pi^2} \frac{c_N^\ell}{c_D |\vec{p}|} \sqrt{\frac{m_1 m_2}{m_3 m_4}} I_\ell(r, \kappa) A_{0,\ell}^{\chi_3 \chi_4}$,
- In general, have to sum over several intermediate states $\chi_3 \chi_4$!
- Need to worry about signs (or phases)!

Loop functions: heavier intermediate state



Correction suppressed, except near threshold for on-shell production of intermediate state

Loop functions: lighter intermediate state



Correction always suppressed.

Application to MSSM

Need (co–)annihilation *amplitudes* $\Longrightarrow$ use `DarkSUSY`

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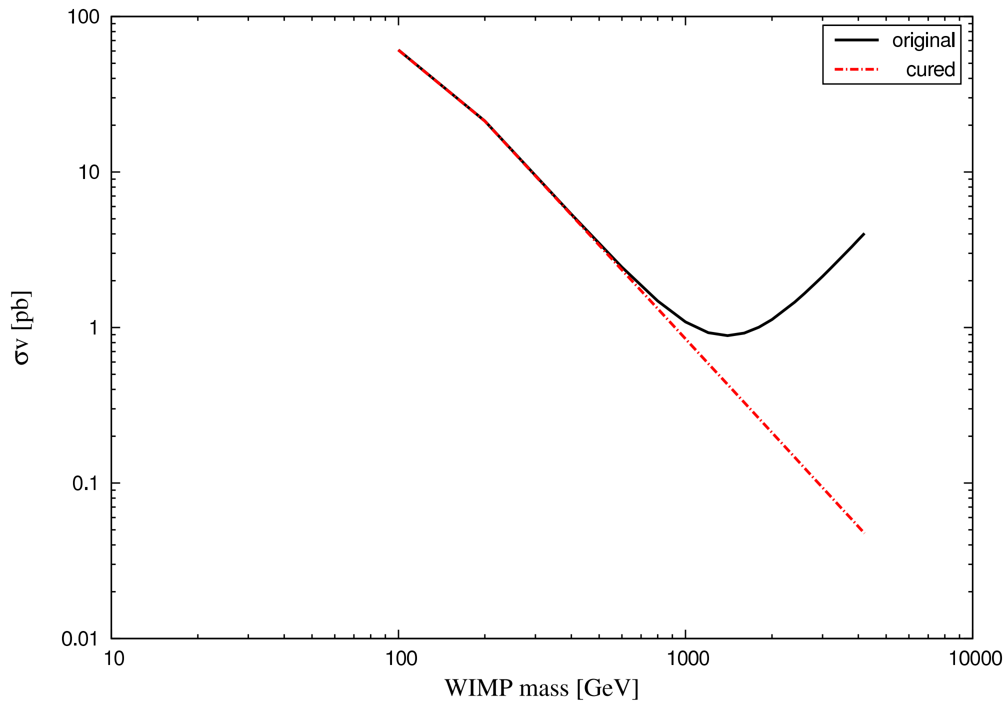
Unfortunately, original (summer 2012) version of `DarkSUSY` violated unitarity badly:

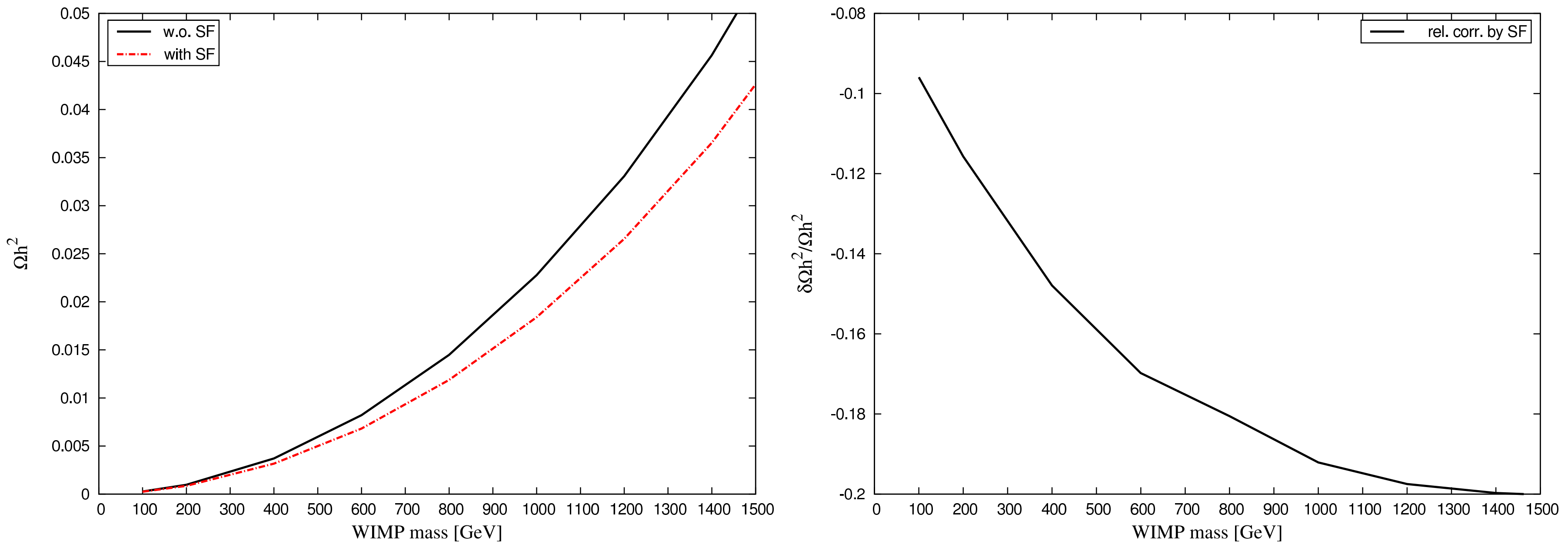


$v_\chi = 0.001$

Wino–like LSP

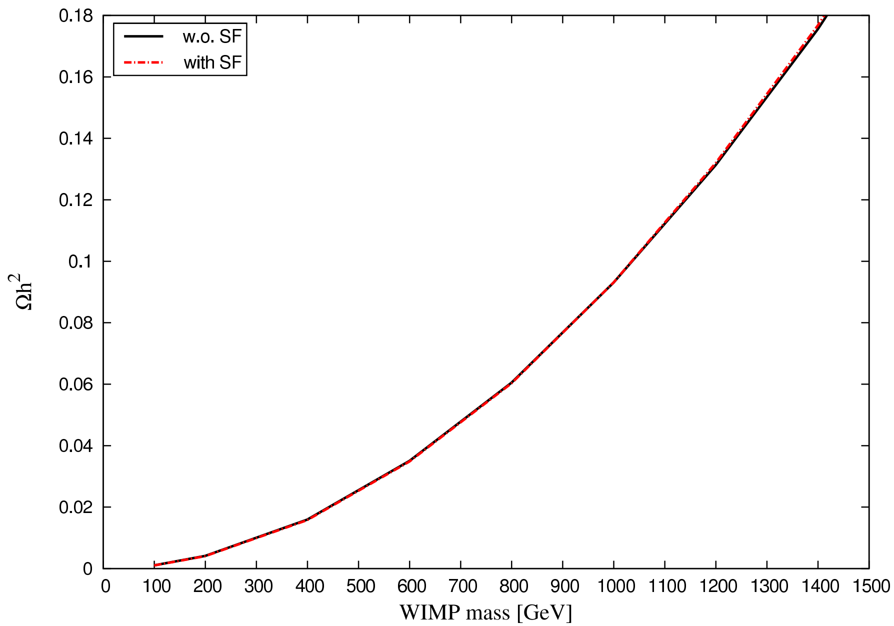
Problem due to use of finite widths for $t-, u-$channel propagators.

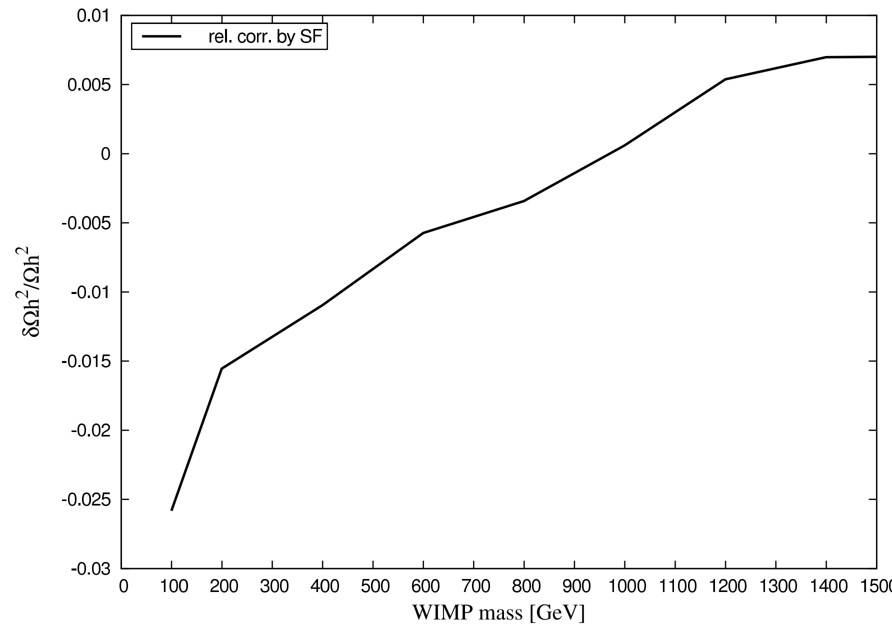
Results for wino–like LSP



Size of effect increases with LSP mass

Results for higgsino–like LSP





Strong cancellations, e.g. between two neutral higgsinos in loop!

Difference between higgsinos and winos

Wino–like states form (approximate) $SU(2)$ triplet of Majorana fermions: no cancellations.

Higgsino–like states form (approximate) $SU(2)$ doublet of Dirac fermions: strong cancellations.

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- Applied formalism of effective couplings to describe non–decoupling corrections to neutralino annihilation; typically few percent effect for TeV–ish squarks
- Developed model–independent method to compute one–loop Sommerfeld enhanced corrections; small for $\tilde{\chi}_1^0$ annihilation in MSSM
- Extended this method to include co–annihilation: large effects for wino–like neutralino, small for higgsino–like neutralino.