# String theory and the AdS/CFT correspondence

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# What is string theory?

String theory is currently the most promising candidate for a quantum theory incorporating high energy physics as well as general relativity.

However, it is not a `complete' theory: there are many conceptual problems (in particular its off-shell formulation) that are not well understood.

# What is string theory?

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The strings can either be

or

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open

# String basics

They propagate (initially) in a given fixed background.

The consistency conditions (conformal beta equations) require that this background satisfies a modification of the

Einstein equations of general relativity.

Furthermore, the excitation spectrum of the string contains the `graviton‘ that describes fluctuations of the background --- thus the theory should be independent of the chosen initial background.

# String basics

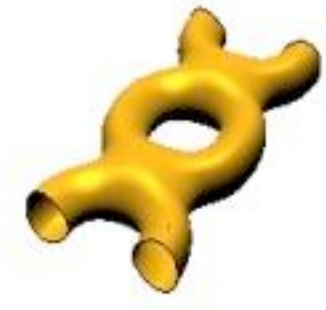
The other vibrational excitations of the string contain the quanta that correspond to the familiar elementary particles (electrons, quarks, gluons, etc.)

Thus string theory incorporates
(some extension of) the

standard model of particle physics.

# String basics

The interaction of these excitations is described geometrically by the joining and splitting of strings:



These diagrams replace the usual Feynman diagrams of quantum field theories:

instead of

# String basics

This smooth description suggests that string theory has in fact better UV properties than conventional quantum field theories or naive attempts to quantise gravity.

[On the other hand, string field theory is conceptually not that well developed: in particular one does not yet understand how to derive the `Feynman rules' of string field theory from first principles.]

# String scale

The string scale (size of strings) is usually believed to be of the order of the Planck scale, i.e.

$$L_{\text{Planck}} = \sqrt{\left(\frac{hG}{2\pi c^3}\right)} = 1,6 \cdot 10^{-35} m \ .$$

The Planck scale is the length (or energy) scale at which effects of quantum gravity should become important.

# Is string theory right?

Unfortunately, the Planck (and string) scale is very small indeed!

So while string theory makes very definite predictions at this scale, we cannot test them directly by experiments at present.

[Actually, this is true for any theory of quantum gravity.]

However, there are other (indirect) reasons why one may believe that string theory has something to do with the real world --- see below.

# String basics

So far I have described the standard string folklore....

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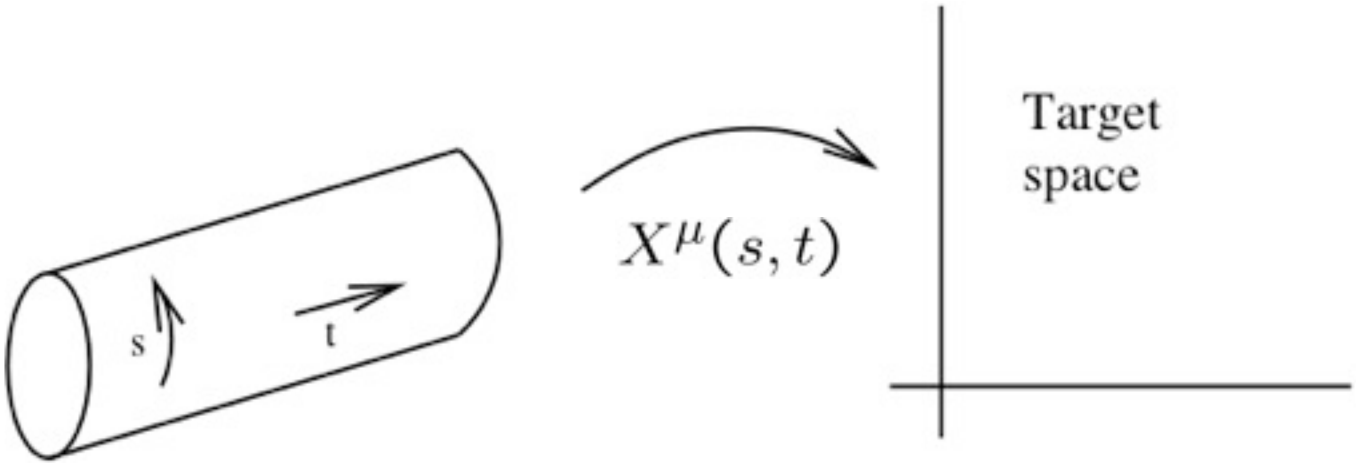
But how does one actually work with this theory quantitatively?

# Quantitative strings

In order to describe strings quantitatively think of them in terms of a 2-dimensional field theory that is defined on the world-sheet of the string:



# Sigma model

The propagation of the string is then described by specifying to which point in the target space (space-time) every world-sheet point is mapped to.

This map is controlled by the sigma-model action

$$S = \frac{T}{2} \int ds dt \sqrt{h}\, h^{mn}\, \partial_m X^\mu\, \partial_n X^\nu\, G_{\mu\nu}$$

where G is the space-time metric (that may depend on the position X). [h is the world-sheet metric.]

# Sigma model

This sigma model is classically invariant under reparametrisations and Weyl rescalings of the world-sheet.

We can use this symmetry to go to `conformal gauge' in which we take the world-sheet metric to be proportional to the usual 2d Minkowski metric.

Then the residual symmetry is the conformal (Weyl) symmetry. The resulting 2d field theory is therefore a conformal field theory.

# Conformal transformations

The conformal transformations preserve the metric up to rescalings. They therefore preserve angles, but not necessarily lengths.

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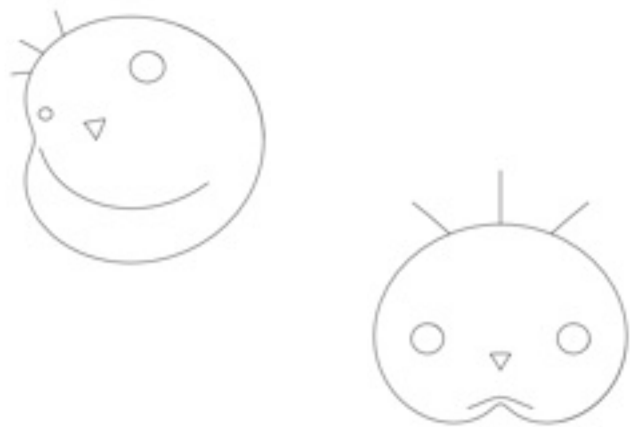


# Virasoro algebra

The conformal symmetry in 2 dimensions is therefore very powerful!

In fact, the algebra of infinitesimal local conformal transformations is infinite dimensional:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n}$$

[Virasoro algebra]

c: central charge

# Conformal symmetry

Because of this large symmetry, 2d conformal field theories can be solved essentially based on symmetry considerations alone.

[In fact, 2d conformal field theories have a very rich mathematical structure: they define vertex operator algebras, and have had a significant impact on many areas in modern mathematics, such as group theory, number theory, geometry, etc.]

# No ghost theorem

From the point of view of string theory, the conformal symmetry is a gauge symmetry. Just as in electrodynamics this gauge symmetry can then remove the negative norm states (ghosts) that the covariant theory initially has.

In the present context this requires that

$$c \leq 26$$

[Goddard, Thorn]

# Critical dimension

If the target space is flat space, then c=dimension.

Thus D=26 is the critical dimension of (bosonic) string theory!

In order to describe 4d physics, the idea is that

$$26 = c_{4d} + c_{int}\,, \qquad c_{int} = 22\,,$$

i.e. that 22 dimensions are compactified on some \`internal manifold': compactification.

# Fermions

The analysis for the (world-sheet) fermionic string is similar: in this case the residual gauge symmetry is the N=1 superconformal symmetry, and the no-ghost theorem requires that

$$c \leq 15$$

Since each flat direction contributes c=1+1/2=3/2 to the central charge, this corresponds to a critical dimension

D=10.

# D-branes

So far we have (implicitly) only discussed closed strings. For open strings we need in addition to specify the boundary conditions at the end-points of the open string.

In flat space, the relevant boundary conditions are for every direction either

Neumann (endpoint can freely move)

or

Dirichlet (endpoint has fixed position).

# D-branes

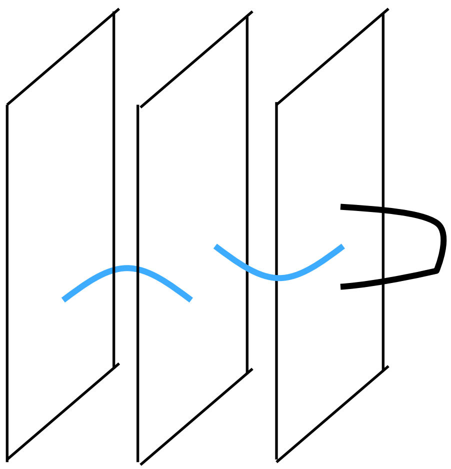
Thus can describe the boundary conditions geometrically.

The brane is the hypersurface on which open strings can end.


[Polchinski]

# Non-abelian gauge theories

Consider stack of such D-branes



N=3 D-branes

In the limit in which the branes come to lie on top of each other, the blue strings become massless: U(N) gauge theory on world-volume of D-brane.

# Gauge gravity duality

On the other hand, D-branes have also got an effect on the (closed string) background in which they are placed.

In the large N limit (of 4-dimensional D-branes), then have two equivalent (or dual) descriptions:

supergravity on $\mathrm{AdS}_5 \times S^5$ = SU(N) super Yang-Mills theory in 4 dimensions in planar limit

AdS/CFT correspondence [Maldacena]

# AdS/CFT correspondence

Yang-Mills theory = theory of standard model of particle physics.

So can use string theory ideas and techniques to say something about quantum field theories that are known to be relevant for the description of the real world!

# AdS/CFT correspondence

More concerely, the relation between the parameters of the two descriptions is

$$\left(\frac{R}{l_{\rm Pl}}\right)^4 = N \qquad g_{\rm string} = g^2_{\rm YM} \qquad \left(\frac{R}{l_{\rm s}}\right)^4 = g^2_{\rm YM} N = \lambda$$

AdS radius in Planck units

AdS radius in string units

't Hooft parameter

# Strong weak duality

For example, in the large N limit of gauge theory at large ‘t Hooft coupling

$$\left(\frac{R}{l_{\rm Pl}}\right)^4 = N \qquad g_{\rm string} = g_{\rm YM}^2 \qquad \left(\frac{R}{l_{\rm s}}\right)^4 = g_{\rm YM}^2 N = \lambda$$

large small large

Supergravity (point particle) approximation is good for AdS description.

# AdS/CFT correspondence

Dramatic progress in recent years: using this point of view can determine, for example, the anomalous dimension of certain gauge theory operators to arbitrary order in perturbation theory!

[Beisert, Eden, Staudacher], ...

Very detailed and quantitative predictions --- to the extent that perturbation theory calculations can be performed, they have been verified.

[Bern et al, Lipatov et al]
[Bartels,...]

# AdS/CFT correspondence

Recently the AdS/CFT correspondence has also been applied more generally to study at least qualitative aspects of strongly coupled field theories (that are inaccessible otherwise), e.g. in the context of

- Quark gluon plasma (RHIC)
- Cold atoms
- Condensed matter systems at their critical points

[see talk by Sachdev]

# Conceptual understanding

However, at present, we are far from a conceptual understanding of why the duality works, and what ingredients are crucial for it, e.g. whether it requires

supersymmetry
integrability
...

This is obviously an important question since in many applications these features are absent.

# Strings on AdS

One main problem is that string theory (rather than supergravity) on AdS spaces is only poorly understood since sigma model isn't solvable.

Some recent progress using supergroup WZW models and their deformations (that also play a role in condensed matter problems).

[Berkovits, Vafa, Witten]
[Schomerus, Saleur, et al]
[Zirnbauer]

# Weakly coupled gauge theory

Another idea is to start with a different corner of AdS/CFT: consider case where gauge theory is weakly coupled

$$\left(\frac{R}{l_{\rm Pl}}\right)^4 = N \qquad g_{\rm string} = g_{\rm YM}^2 \qquad \left(\frac{R}{l_{\rm s}}\right)^4 = g_{\rm YM}^2 N = \lambda$$

large

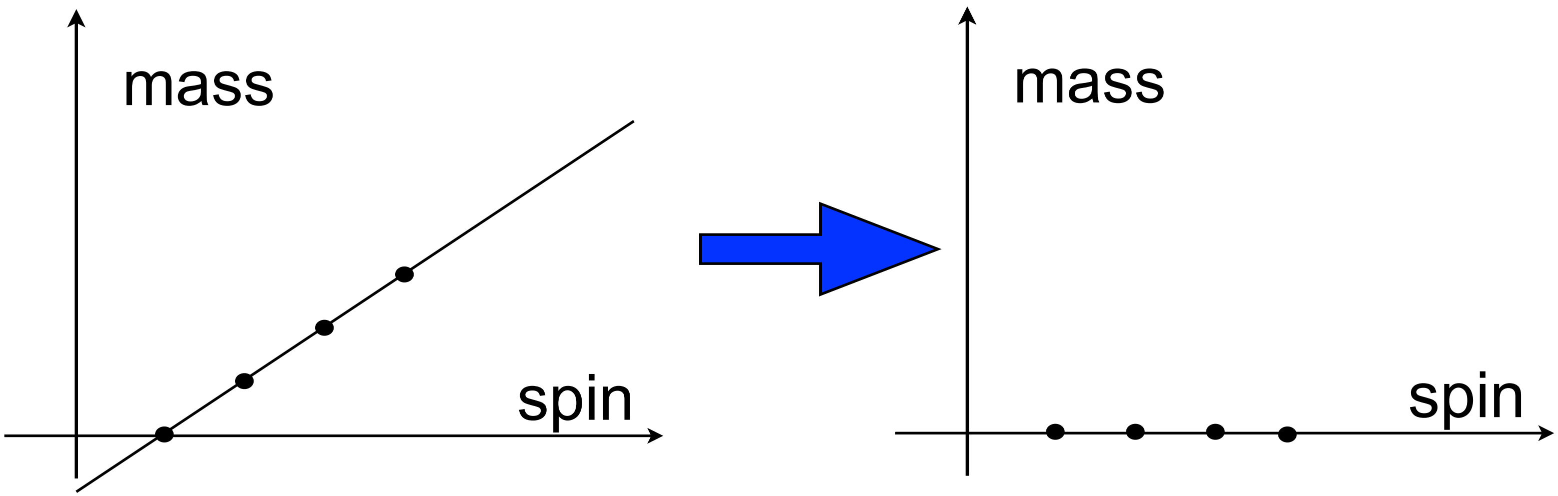
small

[Sundborg], [Witten]
[Sezgin,Sundell]

$l_{\rm s} \rightarrow \infty$ `tensionless strings’

# Tensionless limit

In tensionless limit all string excitations become massless:




leading
Regge trajectory

# Higher spin theory

Resulting theory has an infinite number of massless higher spin fields, which generate a very large gauge symmetry:

[Vasiliev]

maximally unbroken phase of string theory

Idea: try to understand AdS/CFT correspondence starting from this highly symmetric theory!

# Higher spin duality

Concrete proposal for HS - CFT duality

[Klebanov-Polyakov]
[Sezgin-Sundell]

| higher spin theory on AdS4 | $\longleftrightarrow$ | 3d O(N) vector model in large N limit |
|---|---|---|

During the last few years impressive checks of the duality have been performed, in particular through the work of Giombi & Yin.

# 3d proposal

More recently, a lower dimensional version of this sort of duality has been proposed:

[MRG,Gopakumar]

**AdS3:**
higher spin theory
with a complex
scalar of mass M

$\longleftrightarrow$

**2d CFT:**
$\mathcal{W}_{N,k}$ minimal models
in large N 't Hooft limit
with coupling $\lambda$

where $\lambda = \dfrac{N}{N+k}$ and $M^2 = -(1-\lambda^2)$

# Lower dimensional model

Lower dimensional version interesting

- 2d CFTs well understood (Virasoro algebra)
- Higher spin theories simpler in 3d

Allows for very detailed precision tests, and maybe even a proof of the AdS/CFT correspondence (at least for this simplified example).

# Conclusions

String theory is a quantum theory that combines in a consistent manner general relativity with (supersymmetric extensions of) the standard model of particle physics.

The AdS/CFT correspondence gives rise to a non-trivial duality relating supergravity/superstring theories and gauge theories (similar to those that appear in the standard model).

# Conclusion

The AdS/CFT correspondence has led to remarkable predictions for gauge theories that could be verified!

It has also led to new insights into other strongly coupled quantum systems.

# Interdisciplinarity

Good example of `interdisciplinary' research, bringing together insights from different areas of physics and mathematics,

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i.e. of the sort of activity that the

Wolfgang-Pauli Centre

should foster....

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